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The 10 semiotic dual systems in 4 contextures and 3 number structures

1. This article has more the character of a handout and gives a complete oversight over the 10 Peircean sign classes, their 10 reality thematics in 1, 2, 3 and 4 semiotic contextures, whereby each time proto-, deutero- and trito-number structuress are differentiated. As I have already pointed out (Toth 2009), the term "structure" avoids the term field, since in non-identity based qualitative mathematics, there are no such things as rings or fields.

2. The 10 semiotic dual systems in 1 contexture

(3.1 2.1 1.1) × (1.1 1.2 1.3)
(3.1 2.1 1.2) × (2.1 1.2 1.3)
(3.1 2.1 1.3) × (3.1 1.2 1.3)
(3.1 2.2 1.2) × (2.1 2.2 1.3)
(3.1 2.2 1.3) × (3.1 2.2 1.3)
(3.1 2.3 1.3) × (3.1 3.2 1.3)
(3.2 2.2 1.2) × (2.1 2.2 2.3)
(3.2 2.2 1.3) × (3.1 2.2 2.3)
(3.2 2.3 1.3) × (3.1 3.2 2.3)
(3.3 2.3 1.3) × (3.1 3.2 3.3)

There is no differentiation into proto-, deutero- and trito-structures.

3. The 10 semiotic dual systems in 2 contextures

2-contextural dual-systems correspond to the complex dual systems introduced in Toth (2007, pp. 52 ss.). Instead of number 0 and 1 we used here the already introduced notation with + and -:

(±3.±1 ±2.±1 ±1.±1) × (±1.±1 ±1.±2 ±1 ±3)
(±3.±1 ±2.±1 ±1.±2) × (±2.±1 ±1.±2 ±1.±3)
(±3.±1 ±2.±1 ±1±.3) × (±3.±1 ±1.±2 ±1.±3)
(±3.±1 ±2.±2 ±1.±2) × (±2.±1 ±2.±2 ±1.±3)

(±3.±1 ±2.±2 ±1.±3) × (±3.±1 ±2.±2 ±1.±3)
(±3.±1 ±2.±3 ±1.±3) × (±3.±1 ±3.±2 ±1.±3)
(±3.±2 ±2.±2 ±1.±2) × (±2.±1 ±2.±2 ±2.±3)
(±3.±2 ±2.±2 ±1.±3) × (±3.±1 ±2.±2 ±2.±3)
(±3.±2 ±2.±3 ±1.±3) × (±3.±1 ±3.±2 ±2.±3)
(±3.±3 ±2.±3 ±1.±3) × (±3.±1 ±3.±2 ±3.±3)

There is no differentiation into proto-, deutero- and trito-structures.

4. The 10 semiotic dual systems in 3 contextures

4.1. The 10 3-contextural dual systems on proto- and deutero- structure

$(3.1_{[[000],[001],[012]]}\ 2.1_{[0]}\ 1.1_{<[0],[[[000],[001],[012]]>}) \times$
$(1.1_{<[[000],[001],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.1_{[0]}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 1.2_{[0]}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.1_{[0]}\ 1.3_{[[000],[001],[012]]}) \times$
$(1.1_{<[[000],[001],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]}\ 3.2_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]})\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]})\ 3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]})$

$(3.3_{<[[00],[01], [[000],[001],[012]]>}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]}\ 3.2_{[[00],[01]]}\ 3.3_{<[[000],[001],[012]], [[00],[01]>})$

4.2. The 10 3-contextural dual systems on trito-structure

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.1_{[0]}\ 1.1_{<[0],[[000],[001],[010],[011][012]]>}) \times$
$(1.1_{<[[000],[001],[010],[011],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011][012]]}\ 2.1_{[0]}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 1.2_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.1_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(1.1_{<[[000],[001],[010],[011],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]}\ 3.2_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]})\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]})\ 3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]})$

$(3.3_{<[[00],[01], [[000],[001],[010],[011],[012]]>}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]}\ 3.2_{[[00],[01]]}\ 3.3_{<[[000],[001],]010],[011],[012]], [[00],[01]>})$

5. The 10 semiotic dual systems in 4 contextures

The proto-structure of C = 4 has 4, the deutero-structure has 5, and the trito-structure of C = 4 has 15 qualitative numbers. Therefore, the dual system, (3.1 2.2 1.3) × (3.1 2.2 1.3) in 4 contexures looks

in the proto-structure like

$$(3.1_{[[0000],[0001],[0012],[0123]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[0000],[0001],[0012],[0123]]}) \times$$
$$(3.1_{[[0000],[0001],[0012],[0123]]}\ 2.2_{<[[00],[01]],\ [0]>}\ 1.3_{[[0000],[0001],[0012],[0123]]}),$$

in the deutero-structure like

$$(3.1_{[[0000],[0001],(0011)l[0012],[0123]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[0000],[0001],(0011),\ [0012],[0123]]}) \times$$
$$(3.1_{[[0000],[0001],(0011),[0012],[0123]]}\ 2.2_{<[[00],[01]],\ [0]>}\ 1.3_{[[0000],[0001],(00110012],[0123]]}),$$

and in the trito-structure like

$$(3.1_{[[0000],[0001],[0010],(0011)l[0012],[0100],[0101],[0102],[0110],[0111[,[0112],[0120[,[121]\ ,[122],[0123]]}\ 2.2_{<[0],[[00],[01]]>}$$
$$1.3_{[[0000],[0001],(0011),\ [0012],[0123]]}) \times$$
$$(3.1_{[[0000],[0001],[0010],(0011)l[0012],[0100],[0101],[0102],[0110],[0111[,[0112],[0120[,[121],[122],[0123]]}$$
$$1.3_{[[0000],[0001],(00110012],[0123]]}),$$

6. Abbreviations

Especially in notations of sign relation for C ≥ 4, the index notation is awkward, although complete. Thus, according to

$$C = 2 \quad (\pm 3.\pm 1\ \pm 2.\pm 1\ \pm 1.\pm 3) \equiv ({}^{0}_{1}3.{}^{0}_{1}1\ \ {}^{0}_{1}2.{}^{0}_{1}1\ \ {}^{0}_{1}1.{}^{0}_{1}3),$$

we suggest the following notations:

$C = 3$ $({}^{0}_{3}3.{}^{0}_{3}1\ \ {}^{0}_{3}2.{}^{0}_{3}1\ \ {}^{0}_{3}1.{}^{0}_{3}3)$ for proto- and deutero-structure

$C = 3$ $({}^{0}_{5}3.{}^{0}_{5}1\ \ {}^{0}_{5}2.{}^{0}_{5}1\ \ {}^{0}_{5}1.{}^{0}_{5}3)$ for trito-structure

$C = 4$ $({}^{0}_{4}3.{}^{0}_{4}1\ \ {}^{0}_{4}2.{}^{0}_{4}1\ \ {}^{0}_{4}1.{}^{0}_{4}3)$ for proto-structure

$C = 4$ $({}^{0}_{5}3.{}^{0}_{5}1\ \ {}^{0}_{5}2.{}^{0}_{5}1\ \ {}^{0}_{5}1.{}^{0}_{5}3)$ for deutero-structure

$C = 4$ $({}^{0}_{15}3.{}^{0}_{15}1\ \ {}^{0}_{15}2.{}^{0}_{15}1\ \ {}^{0}_{15}1.{}^{0}_{15}3)$ for trito-structure, and so on for the higher contextures (e.g., C = 5: proto = 5; deutero = 7; trito = 203, according to the Sterling Numbers of the 2nd kind). However, the drawback of this short notation is that the contexture cannot be seen from the notation of the sign relation; cf.

C = 3 $({}^{0}_{5}3.{}^{0}_{5}1 \;\; {}^{0}_{5}2.{}^{0}_{5}1 \;\; {}^{0}_{5}1.{}^{0}_{5}3)$ for trito-structure

C = 4 $({}^{0}_{5}3.{}^{0}_{5}1 \;\; {}^{0}_{5}2.{}^{0}_{5}1 \;\; {}^{0}_{5}1.{}^{0}_{5}3)$ for deutero-structure,

but this problem can be solved by an index o.s.

Bibliography

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