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Addition of contextures of sign relations

1. According to Kronthaler (1986), "qualitative mathematics" (orig.: "Mathematik der Qualitäten") does not only allow to add objects that lie in different contextures, or to continue counting over contexture borders, but also to count contextures themselves. In this little contribution, I want to show this at the hand of the 4 systems of 3-contextures for the 9 Peircean sub-signs introduced in Toth (2009).

2. The 4 systems of 3-contextures – arbitrarily chosen and displayed in Toth (2009) – are:

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)	
K2	—	—	—	—	(2.2)	(2.3)	—	(3.2)	—	S1
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—	

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)	
K2	—	(1.2)	(1.3)	—	(2.2)	(2.3)	—	—	—	S2
K1	—	—	—	(2.1)	(2.2)	—	(1.3)	(3.2)	—	

K3	—	—	—	(2.1)	—	(2.3)	(3.1)	—	(3.3)	
K2	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.3)	S3
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—	

K3	—	—	—	(2.1)	(2.2)	—	(3.1)	(3.2)	—	
K2	(1.1)	—	(1.3)	(2.1)	—	(2.3)	—	—	—	S4
K1	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.3)	

3. If we know look which sub-sign appears in which contextures of the 4 systems, we get

(1.1): K1 + K2 (S1) + K2 (S4)
(1.2): K1 (S1) + K2 (S2) – no representation for (1.2) in K3
(1.3): K2 + K3 (S2) + K1 (S4)
(2.1): K1 (S1) + K3 + K4 (S4)
(2.2): K1 + K2 (S1) + K4 (S4
(2.3): K2 (S2) + K3 (S3) – no representation for (2.3) in K1
(3.1): K3 (S1) – no representation for (3.1) in K1 and K2
(3.2): K2 (S1) + K1 (S2) + K3 (S4)
(3.3): K3 (S1) + K2 (S3) + K1 (S4)

In order to "fill up" the "gaps" of representation, all we would need to do is construct additional matrices, or to map the sub-signs to different contextures, respectively. Therefore, there is no need to do that but to state that unlike the mapping of natural numbers to contextures (cf. Toth 2003, pp. 56 ss.), the mapping of sub-signs to contextures is surjective. (Amongst natural numbers, e.g., the number 2 cannot be mapped to any trito-number according to Kronthaler 1986).

From the surjectivity of the mappings of sub-signs to semiotic contextures, it follows that the ideal or "balanced" representation for a dyadic sub-sign of a 3-adic sign relation is to be represented in 3 contextures. Thus, we can add the contextural systems S1 – S4 and obtain the following structure S^{1+4}:

K3	(1.1)	—	(1.3)	(2.1)	(2.2)	(2.3)	(3.1)	(3.2)	(3.3)	
K2	(1.1)	(1.2)	(1.3)	(2.1)	(2.2)	(2.3)	—	(3.2)	(3.3)	S^{1+4}
K1	(1.2)	(1.2)	(1.3)	(2.1)	(2.3)	—	—	(3.2)	(3.3)	

When the gaps are filled, each of the ten Peircean sign classes can be represented in 3 different ways, provided that we work with 3 semiotic contextures. In the homogeneous case, we thus have

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k}) \rightarrow$

1. $(3.a_i\ 2.b_i\ 1.c_i)$
2. $(3.a_j\ 2.b_j\ 1.c_j)$
3. $(3.a_k\ 2.b_k\ 1.c_k)$,

and in the inhomogeneous cases

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k}) \rightarrow$

1. $(3.a_i\ 2.b_i\ 1.c_j)$
2. $(3.a_i\ 2.b_j\ 1.c_i)$
3. $(3.a_j\ 2.b_i\ 1.c_i)$
4. $(3.a_i\ 2.b_i\ 1.c_k)$
5. $(3.a_i\ 2.b_k\ 1.c_i)$
6. $(3.a_k\ 2.b_i\ 1.c_i)$
7. $(3.a_i\ 2.b_j\ 1.c_j)$
8. $(3.a_j\ 2.b_j\ 1.c_i)$
9. $(3.a_j\ 2.b_i\ 1.c_j)$
10. $(3.a_i\ 2.b_k\ 1.c_k)$
11. $(3.a_k\ 2.b_k\ 1.c_i)$
12. $(3.a_k\ 2.b_i\ 1.c_k)$
13. $(3.a_i\ 2.b_j\ 1.c_k)$
14. $(3.a_i\ 2.b_k\ 1.c_j)$
15. $(3.a_j\ 2.b_i\ 1.c_k)$
16. $(3.a_j\ 2.b_k\ 1.c_i)$
17. $(3.a_k\ 2.b_i\ 1.c_j)$
18. $(3.a_k\ 2.b_j\ 1.c_i)$

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