Prof. Dr. Alfred Toth

Antichains from prime-signs

1. An antichain is a subset of a partially ordered set such that any two elements in the subset are incomparable. If S is a partially ordered set, then two elements a, b of S are comparable if $x \leq y$ or $y \leq x$, and incomparable if neither $x \leq y$ nor $y \leq x$. Since we have already dealt with semiotic chains in Toth (1996), we will show semiotic antichains very briefly, referring for the theoretic background to Weisstein (1999).

2. Since the semiotic zero-sign ∅ has already been introduced in Toth (2007, pp. 14 ss.), we can presuppose its existence here. The absence of a sign is a sign, too. The number of antichains of a set with n elements is equal to the number of monotonic increasing Boolean functions of n variables, and also to the number of free distributive lattices with n generators (cf. Comtet 1974, p. 273). The number of antichains for n = 1, 2, 3, ..., n is determined by the Dedekind numbers:

1, 2, 5, 19, 167, ...

Thus, a semiotic set with 3 elements like set of the prime-signs S = {.1., .2., .3.} (Bense 1980) has the following 5 antichains:

∅, {{1}}, {{2}}, {{1}, {2}}, {{1}, {2}}

A semiotic set with 4 elements like the set of the pre-semiotic prim-signs S' = {0., .1., .2., .3.} (Toth 2008) has the following 19 antichains:

∅, {{1}}, {{2}}, {{3}}, {{1, 2}}, {{1, 3}}, {{2, 3}}, {{1}, {2}}, {{1}, {3}}, {{2}, {3}}, {{1, 2, 3}}, {{1}, {2, 3}}, {{1, 2}, {2, 3}}, {{1, 2}, {1, 3}}, {{1, 2}, {3}}, {{2}, {1, 3}}, {{2, 3}}, {{1, 3}}, {{1}, {2}, {3}}, {{1, 2}, {2, 3}, {1, 3}}

Bibliography

Bense, Max, Die Einführung der Primzeichen. In: Ars Semeiotica 3/3, 1980, pp. 287-294
Comtet, Louis, Advanced Combinatorics. Dordrecht 1974
Toth, Alfred, Grundriss einer ordnungstheoretischen Semiotik. In: European Journal for Semiotic Studies 8, 1996, pp. 503-526
Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007
Toth, Alfred, Relational and categorial numbers. Ch. 40 (2008)
Weisstein, Eric, Antichain. http://mathworld.wolfram.com/Antichain.html