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Semiotic antimatroids

1. According to Korte, Lovász and Schrader (1991), an antimatroid is a family of sets F, called feasible sets, which obey the following two properties:

1.1. The union of any two feasible sets is also feasible. That is, F is closed under unions.

1.2. If S is a nonempty feasible set, then there exists some x in S such that S\{x} is also feasible. That is, F is an accessible set system.

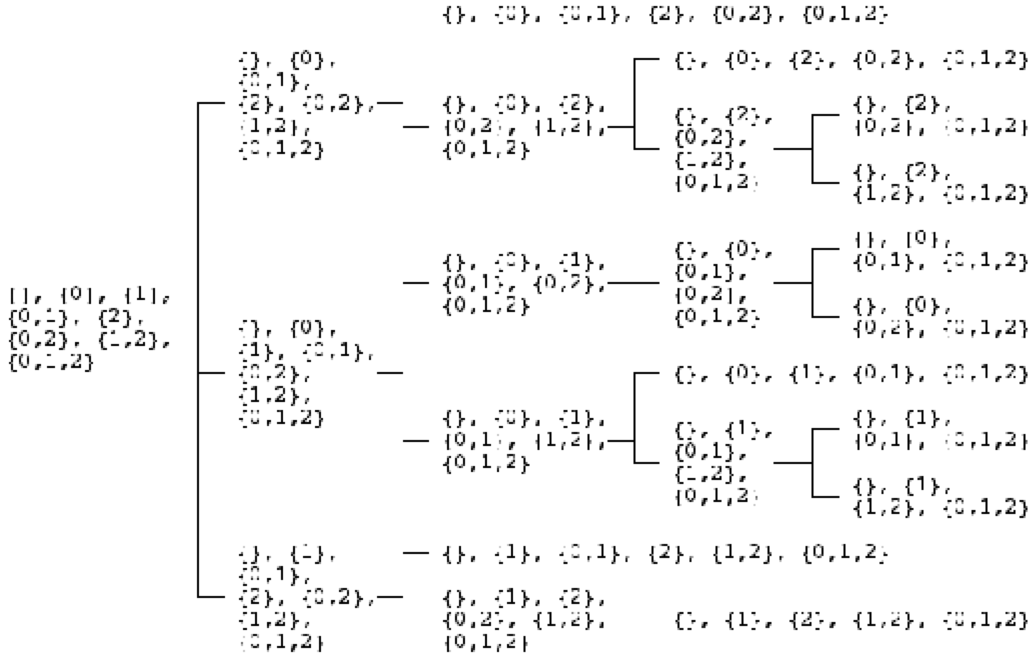
An element x that can be removed from a feasible set S to form another feasible set is called an endpoint of S, and a feasible set that has only one endpoint is called a path of the antimatroid. The subset ordering of the paths forms a partially ordered set, called the path poset of the antimatroid. Each feasible set in an antimatroid is the union of its path subsets. The number of possible antimatroids on a set of elements grows rapidly with the number of elements in the set. For sets with 1, 2, 3, ... elements, the number of possible antimatroids is

1, 3, 22, 485, 59'386 ... (Eppstein 2008).

Thus, the triadic-trichotomic sign relation $SR_{3,3}$ = has 22 possible antimatroids. If we make the following substitutions in the beneath diagram taken from Eppstein (2008)

$\{0\} \rightarrow \{1\}$
$\{1\} \rightarrow \{2\}$
$\{2\} \rightarrow \{3\}$

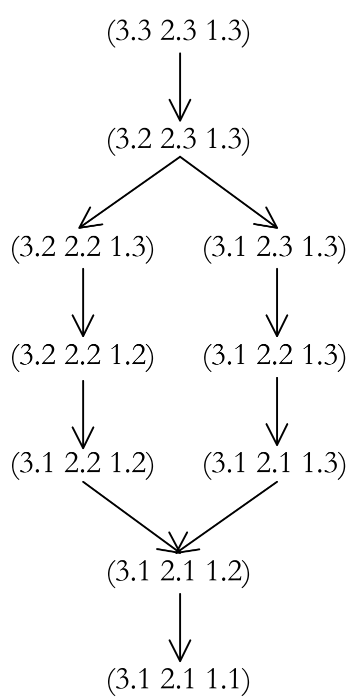
assume the existence of the empty sign as in Toth (2007, pp. 14 ss.), then we get the following diagram showing the 22 antimatroids of $SR_{3,3}$:



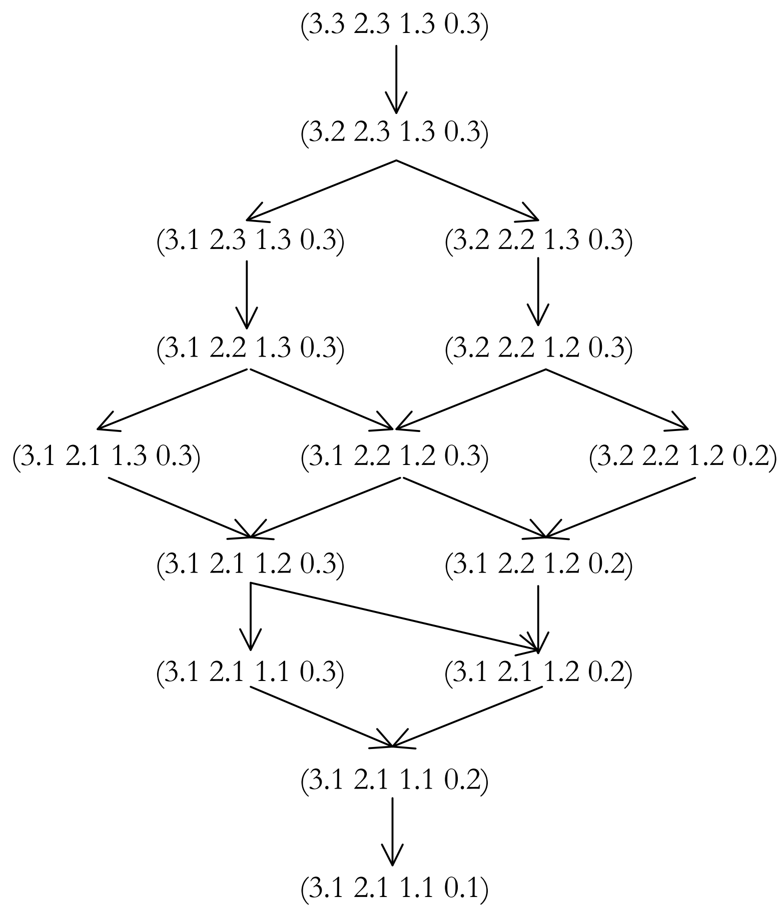
2. A greedoid (F, E) is an accessible set system that satisfies the exchange property:

2.1. For all X,Y □ F with |X| > |Y|, there is some x □ X such that Y □ {x} □ F.

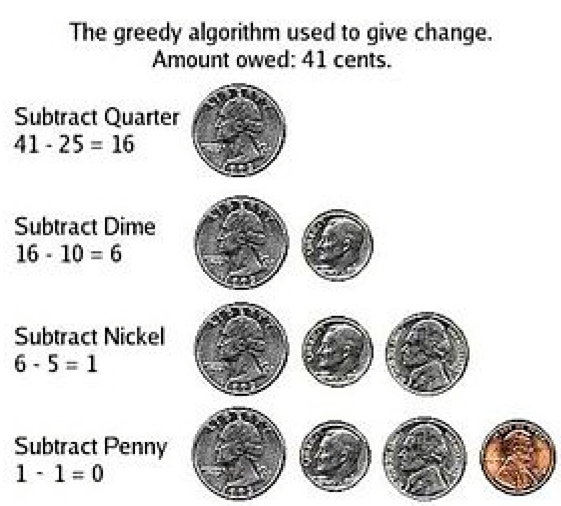
The following semiotic structure shows that the set of the 10 sign classes over $SR_{3,3}$ is not only an antimatroid, but also a greedoid:



The same is true for the set of the 15 sign classes over $SR_{4,3}$:



Therefore, the above partially ordered sets of sign classes over $SR_{3,3}$ and $SR_{4,3}$ can be constructed by a greedy algorithm, that "is just an iterative process in which a locally best choise, usually in input of minimum weight, is chosen each round until all available choices have been exhausted" (Björner and Ziegler 1992):



The above greedy algorithm "determines the minimum number of US coins to give while making change. These are the steps a human would take to emulate a greedy algorithm. The coin of the highest value, less than the remaining change owed, is the local optimum. (Note that in general the change-making problem requires dynamic programming to find an optimal solution; US and other currencies are special cases where the greedy strategy works.)" (http://en.wikipedia.org/wiki/Greedy_algorithm).

Thus, the family of semiotic sets of SS10 and SS15 can be described by antimatroids and greedoids. However, since antimatroids can be viewed as a special case of semimodular lattices, the idea of arranging the sign classes of SS10 in decreasing order by exchanging in each step one of the three sub-signs of a triadic-trichotomic sign class by another sub-sign whose trichotomic representation value is by Rpv = 1 lower than the one in the sign class or sign classes just above, has been applied before; cf., e.g., Walther (1979, pp. 137 s.) for SS10 as a category theoretical lattice, Herrmann 1990) for SS10 as a system of replicas, and Toth (1997, pp. 43 ss.) for a category theoretical antimatroidal network based on trichotomic triads.

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