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Categorial and saltatorial sign classes

1. Categories and saltatories are dual notions from diamond theory (cf. Kaehr 2008, 2009b). Categories are dealing with objects and morphisms, while saltatories are dealing with (co-)objects and hetero-morphisms. Together, they form bi-objects. Kaehr (2009a) has shown that amongst the bi-objects, there are bi-signs.

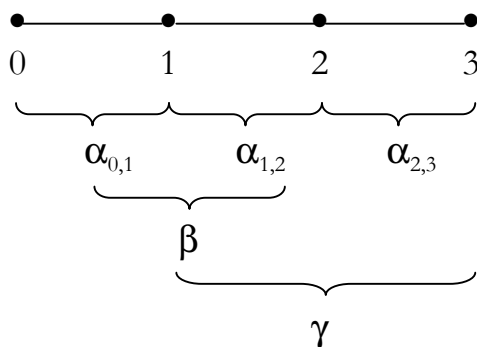
2. In this study, based on Toth (2009a, b), I introduce a “neutral” categorial notation system for sign classes and reality thematics. “Neutral” means that although contextures do not belong to the definition of these sign classes and reality thematics, they can be introduced without change in the notation system. The categorial system introduced here differs considerably from former semiotic categorial systems (cf. Toth 1997, pp. 21 ss.; Toth 2008a, pp. 159 ss.) insofar as it is based on dynamic and not on static prime-signs. This means that instead of starting with the usual static introduction of prime-signs

$$PS = \{.1., .2., .3.\},$$

we suggest the following dynamic introduction of prime-signs:

$$PS = \langle [[[0, 1], [[0, 2], [0, 3]]]] \rangle$$

with



Therefore, we can rewrite PS as follows

$$PS = [[\alpha_{0,1}], [[\alpha_{1,2}], [\alpha_{2,3}]]],$$

which allows us to generate the following categorial matrix

	$\alpha_{0,1}$	$\alpha_{1,2}$	$\alpha_{2,3}$
$\alpha_{0,1}$	$\alpha_{0,1}\alpha_{0,1}$	$\alpha_{0,1}\alpha_{1,2}$	$\alpha_{0,1}\alpha_{2,3}$
$\alpha_{1,2}$	$\alpha_{1,2}\alpha_{0,1}$	$\alpha_{1,2}\alpha_{1,2}$	$\alpha_{1,2}\alpha_{2,3}$
$\alpha_{2,3}$	$\alpha_{2,3}\alpha_{0,1}$	$\alpha_{2,3}\alpha_{1,2}$	$\alpha_{2,3}\alpha_{2,3}$

Now, in Toth (2009b), we have shown that Medads are trichotomically split into (0.1), (0.2) and (0.3). Further, we obtained

$$(0.1) = (\alpha_0 \alpha_{0,1})$$

$$(0.2) = (\alpha_0 \alpha_{1,2})$$

$$(0.3) = (\alpha_0 \alpha_{2,3})$$

Therefore, we can change the above 3x3 matrix into a 4x3, i.e. a tetradic-trichotomic matrix which corresponds exactly to the “pre-semiotic matrix” introduced in Toth (2008b)

	$\alpha_{0,1}$	$\alpha_{1,2}$	$\alpha_{2,3}$
α_0	$\alpha_0 \alpha_{0,1}$	$\alpha_0 \alpha_{1,2}$	$\alpha_0 \alpha_{2,3}$
$\alpha_{0,1}$	$\alpha_{0,1}\alpha_{0,1}$	$\alpha_{0,1}\alpha_{1,2}$	$\alpha_{0,1}\alpha_{2,3}$
$\alpha_{1,2}$	$\alpha_{1,2}\alpha_{0,1}$	$\alpha_{1,2}\alpha_{1,2}$	$\alpha_{1,2}\alpha_{2,3}$
$\alpha_{2,3}$	$\alpha_{2,3}\alpha_{0,1}$	$\alpha_{2,3}\alpha_{1,2}$	$\alpha_{2,3}\alpha_{2,3}$

Of course, this matrix is monocontextural. We can see that by comparing the sub-sign relations with their corresponding inverted relations:

$$\begin{aligned}
(1.2) &\equiv (\alpha_{0,1}\alpha_{1,2}) & (1.2)^\circ &= (2.1) \equiv (\alpha_{1,2}\alpha_{0,1}) \\
(1.3) &\equiv (\alpha_{0,1}\alpha_{2,3}) & (1.3)^\circ &= (3.1) \equiv (\alpha_{2,3}\alpha_{0,1}) \\
(2.3) &\equiv (\alpha_{1,2}\alpha_{2,3}) & (2.3)^\circ &= (3.2) \equiv (\alpha_{2,3}\alpha_{1,2})
\end{aligned}$$

In short: Morphism stays morphism. Therefore, in order to introduce heteromorphisms, we proceed as we did in Toth (2009c) and introduce the reflector R, which turns around not only the order of morphisms but also their indices:

$$\begin{aligned}
(1.2) &\equiv (\alpha_{0,1}\alpha_{1,2}) & R(1.2) &= (\alpha_{2,1}\alpha_{1,0}) \\
(1.3) &\equiv (\alpha_{0,1}\alpha_{2,3}) & R(1.3) &= (\alpha_{3,2}\alpha_{1,0}) \\
(2.3) &\equiv (\alpha_{1,2}\alpha_{2,3}) & R(2.3) &= (\alpha_{2,1}\alpha_{3,2})
\end{aligned}$$

Since in monocontextual matrices, dual sub-signs are identical to converted sub-signs, they are in one and the same semiotic matrix. However, since dual sub-signs are not identical to reflected sub-signs in polycontextual matrices, we need another matrix to display them (and hence n matrices for n-valued reflections or one matrix per contexture):

	$\alpha_{1,0}$	$\alpha_{2,1}$	$\alpha_{3,2}$
α_0	$\alpha_0 \alpha_{1,0}$	$\alpha_0 \alpha_{2,1}$	$\alpha_0 \alpha_{3,2}$
$\alpha_{1,0}$	$\alpha_{1,0} \alpha_{1,0}$	$\alpha_{1,0} \alpha_{2,1}$	$\alpha_{1,0} \alpha_{3,2}$
$\alpha_{2,1}$	$\alpha_{2,1} \alpha_{1,0}$	$\alpha_{2,1} \alpha_{2,1}$	$\alpha_{2,1} \alpha_{3,2}$
$\alpha_{3,2}$	$\alpha_{3,2} \alpha_{1,0}$	$\alpha_{3,2} \alpha_{2,1}$	$\alpha_{3,2} \alpha_{3,2}$

Now, the identity law of classical logic is abolished, we have

$$\begin{aligned}
(2.2) &\equiv (\alpha_{1,2}\alpha_{1,2}) = \times(2.2) = (\alpha_{1,2}\alpha_{1,2}) \neq \\
R(2.2) &\equiv (\alpha_{2,1}\alpha_{2,1}),
\end{aligned}$$

thus, the indices referring in our notation to intervals of natural numbers and not to inner environments, behave like contextures, i.e. they point out the difference between morphisms and heteromorphisms or categories and saltatories.

Therefore, we get now two semiotic systems:

1. The monocontextural semiotic system consisting of the Peircean 10 sign classes and dual(ized) reality thematics:

1. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{0,1}] \times [\alpha_{0,1}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{2,3}\alpha_{0,1}]$
2. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{1,2}] \times [\alpha_{0,1}\alpha_{1,2}, \alpha_{1,2}\alpha_{0,1}, \alpha_{2,3}\alpha_{0,1}]$
3. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{0,1}, \alpha_{2,3}\alpha_{0,1}]$
4. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}] \times [\alpha_{0,1}\alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3}\alpha_{0,1}]$
5. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3}\alpha_{0,1}]$
6. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{2,3}\alpha_{0,1}]$
7. $[\alpha_{2,3} \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}] \times [\alpha_{0,1}\alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3} \alpha_{1,2}]$
8. $[\alpha_{2,3} \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3} \alpha_{1,2}]$
9. $[\alpha_{2,3} \alpha_{1,2}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{2,3} \alpha_{1,2}]$
10. $[\alpha_{2,3}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{2,3}\alpha_{2,3}]$

2. The polycontextural semiotic system consisting of the 10 Peircean sign classes and reflected sign/reality thematics.

1. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{0,1}] \text{ R } [\alpha_{1,0}\alpha_{1,0}, \alpha_{1,0}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
2. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{1,2}] \text{ R } [\alpha_{2,1}\alpha_{1,0}, \alpha_{1,0}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
3. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{2,3}] \text{ R } [\alpha_{3,2}\alpha_{1,0}, \alpha_{1,0}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
4. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}] \text{ R } [\alpha_{2,1}\alpha_{1,0}, \alpha_{2,1}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
5. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}] \text{ R } [\alpha_{3,2}\alpha_{1,0}, \alpha_{2,1}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
6. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \text{ R } [\alpha_{3,2}\alpha_{1,0}, \alpha_{3,2}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
7. $[\alpha_{2,3} \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}] \text{ R } [\alpha_{2,1}\alpha_{1,0}, \alpha_{2,1}\alpha_{2,1}, \alpha_{2,1} \alpha_{3,2}]$
8. $[\alpha_{2,3} \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}] \text{ R } [\alpha_{3,2}\alpha_{1,0}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3} \alpha_{1,2}]$
9. $[\alpha_{2,3} \alpha_{1,2}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \text{ R } [\alpha_{3,2}\alpha_{1,0}, \alpha_{3,2}\alpha_{2,1}, \alpha_{2,1} \alpha_{3,2}]$
10. $[\alpha_{2,3}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \text{ R } [\alpha_{3,2}\alpha_{1,0}, \alpha_{3,2}\alpha_{2,1}, \alpha_{3,2} \alpha_{3,2}]$

It is controversial, if an R(Scl) can be considered a reality thematics, like an \times (Scl) can; instead, we better use here the term of bi-sign, introduced into semiotics by Kaehr (2009a). However, the relation between (monocontextural) reality thematics) and (polycontextural) bi-signs or “saltatorial reality thematics” has still to be motivated.

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