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Semiotic cube and tesseract

1. In Toth (2008c), it was shown that the 6 transpositions of each sign class and reality thematic of the system of the 10 sign classes and their dual reality thematics can be ordered in three pairs of transpositions, so that each two transpositions can be considered semiotic mirror-functions of one another:

1 (3.1 2.1 1.3)	3 (1.3 3.1 2.1)	5 (2.1 1.3 3.1)
2 (1.3 2.1 3.1)	4 (2.1 3.1 1.3)	6 (3.1 1.3 2.1)

Thus, M(1) = 2; M(2) = 1; MM(1) = 1; MM(2) = 2; MMM(1) = 2; MMM(2) = 1, etc.

Moreover, it was also shown that these 6 transpositions can be assigned pairwise to two mirroring sides of a semiotic cube:
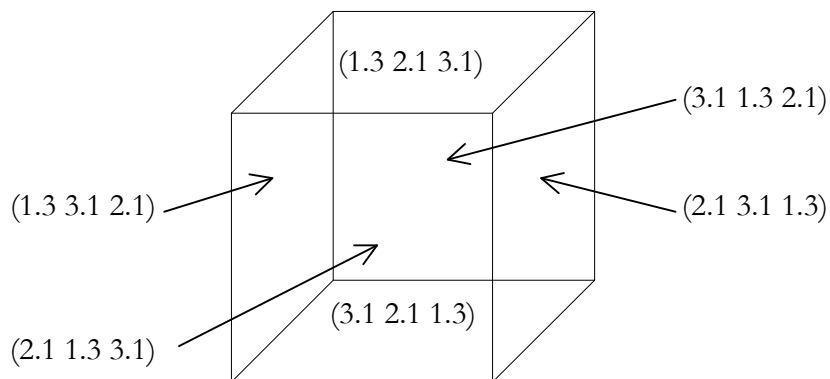

Thus, the pairs of transpositions (3.1 2.1 1.3) / (1.3 2.1 3.1) can be assigned to the sides on the bottom and on the top, (1.3 3.1 2.1) / (2.1 3.1 1.3) to left and right side, and the pair (2.1 1.3 3.1) / (3.1 1.3 2.1) to the front and the back side or to different pairs of mirroring sides as long as they are opposite to one another.

2. However, as it was shown already in Toth (2007b, pp. 82 ss.), the above system of 6 transpositions of a sign class or reality thematic is only a fragment of the complete representational system of this sign class or reality thematic, since it is possible to use negative besides positive semiotic categories and thus positive and negative prime-signs, out of which complex sub-signs, dyads, sign classes and reality thematics can be constructed (cf. Toth 2007a, pp. 52 ss.). Therefore, the complete representational system of a sign class contains 4 semiotic contextures and 24 transpositions. We show this using again our sign class (3.1 2.1 1.3):

(3.1 2.1 1.3)   (-3.1 -2.1 -1.3) (3.-1 2.-1 1.-3) (-3.-1 -2.-1 -1.-3)
(1.3 2.1 3.1)   (-1.3 -2.1 -3.1) (1.-3 2.-1 3.-1) (-1.-3 -2.-1 -3.-1)

(1.3 3.1 2.1)   (-1.3 -3.1 -2.1) (1.-3 3.-1 2.-1) (-1.-3 -3.-1 -2.-1)
(2.1 3.1 1.3)   (-2.1 -3.1 -1.3) (2.-1 3.-1 1.-3) (-2.-1 -3.-1 -1.-3)

(2.1 1.3 3.1)   (-2.1 -1.3 -3.1) (2.-1 1.-3 3.-1) (-2.-1 -1.-3 -3.-1)
(3.1 1.3 2.1)   (-3.1 -1.3 -2.1) (3.-1 1.-3 2.-1) (-3.-1 -1.-3 -2.-1)

However, in order to represent the complete semiotic system of 24 transpositions for each of the 10 sign classes and the 10 dual reality thematics, the semiotic cube is not sufficient anymore. Although triadic sign classes can without problems be represented in 2-dimensional as well as in 3-dimensional spaces (Toth 2007a, pp. 127 ss.), for transpositions, we need a 4-dimensional semiotic space that has hitherto never been introduced into semiotics, but already stipulated in connection with the introduction of semiotic quaternions (Toth 2006; 2007a, pp. 62 s.). The simplest geometrical model to represent a semiotic cell of 24 transpositions for a complex sign class is the tesseract:
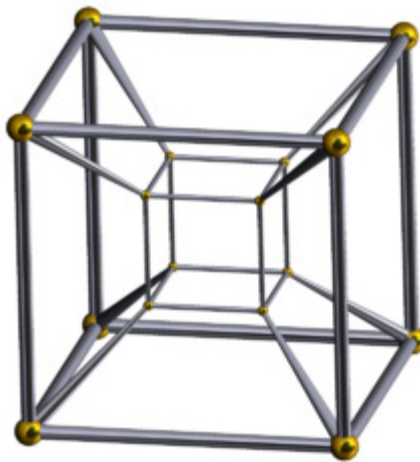
Schlegel diagramm of a tesseract (octachoron)
http://en.wikipedia.org/wiki/Tesseract

The tesseract is to the cube as the cube is to the square, it is a regular convex 4-polytope whose boundary consists of eight cubical cells and whose name is referring to the four lines from each vertex to other vertices. Thus, since each vertex of a tesseract is adjacent to four edges, these four edges of the regular tetrahedron can be assigned to the 4 sign classes of each semiotic contexture. Since each of these 4 complex sign classes has 6 transpositions, the 24 faces of the tesseract must correspond to the 24 complex transpositions of each sign class or reality thematic.

3. If we have a look at the 24 complex transpositions of each sign class, we recognize that each mirrored transposition is orthogonal to its original transposition:

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        3.1                   -3.1                   3.-1                    -3.-1
        2.1                   -2.1                   2.-1                    -2.-1
3.1  2.1  1.3       -3.1  -2.1  -1.3      3.-1  2.-1  1.-3      -3.-1  -2.-1  -1.-3

        1.3                   -1.3                   1.-3                    -1.-3
        3.1                   -3.1                   3.-1                    -3.-1
1.3  3.1  2.1       -1.3  -3.1  -2.1      1.-3  3.-1  2.-1      -1.-3  -3.-1  -2.-1

        2.1                   -2.1                   2.-1                    -2.-1
        1.3                   -1.3                   1.-3                    -1.-3
2.1  1.3  3.1       -2.1  -1.3  -3.1      2.-1  1.-3  3.-1      -2.-1  -1.-3  -3.-1
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Since the outer cube of the above tesseract model contains 6 faces like a regular cube does, the remaining 18 faces must belong to the inside of this hypercube. We thus will probably not fail in assuming that the 6 outer faces are identical with the 3 pairs of transpositions that we had already assigned to the 6 sides of the semiotic cube and thus with the reel semiotic transpositions:

```
        3.1
        2.1
3.1  2.1  1.3

        1.3
        3.1
1.3  3.1  2.1

        2.1
        1.3
2.1  1.3  3.1
```

while the 18 inner faces are identical with the following 9 pairs of complex semiotic transpositions:

```
        -3.1                   3.-1                   -3.-1
        -2.1                   2.-1                   -2.-1
-3.1  -2.1  -1.3      3.-1  2.-1  1.-3      -3.-1  -2.-1  -1.-3

        -1.3                   1.-3                   -1.-3
        -3.1                   3.-1                   -3.-1
-1.3  -3.1  -2.1      1.-3  3.-1  2.-1      -1.-3  -3.-1  -2.-1

        -2.1                   2.-1                   -2.-1
        -1.3                   1.-3                   -1.-3
-2.1  -1.3  -3.1      2.-1  1.-3  3.-1      -2.-1  -1.-3  -3.-1
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Therefore, since each of the 480 transpositions of the complete semiotic representational system can be connected 1. to a transposition of the reel semiotic contexture and 2. to a transposition of one of the three complex semiotic contextures, they are connected to 4 complex semiotic transpositions which can be assigned to the 4 lines from each vertex to other vertices, for example:

(3.1 2.1 1.3)	(3.1 1.3 2.1)	(2.1 1.3 3.1)	(1.3 3.1 2.1)
|	|	|	|
(3.1 1.3 2.1)	(-2.1 1.3 –3.1)	(2.-1 1.-3 3.1)	(1.3 –3.-1 –2.-1)
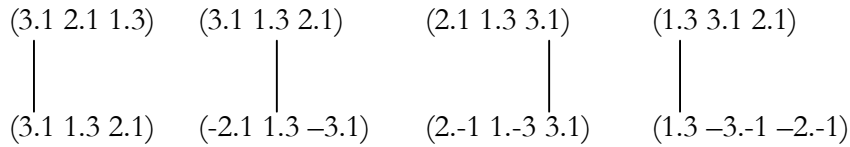
Moreover, since the 6 sides of the outer cube have been assigned to the 6 transpositions of a reel sign class or reality thematic, it follows that the 18 inner faces must be ascribed to the complex transpositions. But from that is follows, too, that in a semiotic tesseract model, the system of complex transpositions is inside of the system of reel transpositions, and from the 4 semiotic contextures, the three consisting of negative prime-signs are a part of the system of positive prime-signs. Therefore, the lines connecting the outer cube with the inner vertices and edges must be assigned to those transpositions and pairs of transpositions with mixed positive and negative categories such as in second links of the above pairs.

4. The results obtained here may indicate the long-searched way out of the semiotic transit-torus as depicted in Toth (2008a), since with the semiotic tesseract model we have, for the first time, a 4-dimensional semiotic space for the reel and complex systems of transpositions and thus also for semiotic diamond theory that forms the base of the semiotic transit-model (cf. Toth 2008a, pp. 32 ss.; 2008b, pp. 177 ss.). As a matter of fact, since there is no basic need to embed sign classes and reality thematics into a 4-dimensional semiotic space up to the point when we are dealing with complete semiotic representational systems, the fourth dimension needed in order to handle transpositional semiotic systems provides the liberty to escape the prison of the transit corridor (cf. Toth 2008a, p. 55 ss.) But still, this does not mean that somebody will be able to escape his mode of being represented. As it was stated in Toth (2008, pp. 304 ss.), once born, an individuum enters the corridor of representation and cannot leave it anymore even after his death. Therefore, semiotics proves again his Kafkaesque status as an "eschatology of hopelessness" (Bense 1952, p. 100) even in the complete system of complex representation. However, what the implications are to have the liberty of escaping from the positive into the negative semiotic spaces is subject to intense further inquiry.

Bibliography

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