Prof. Dr. Alfred Toth

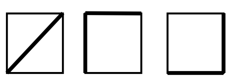
Semiotic Delannoy paths

1. Delannoy numbers describe the number of paths from the southwest corner of a rectangular grid to the northeast corner, using only single steps north, northeast, or east. Delannoy numbers can be computed recursively using the formula

$$D(a, b) = D(a-1, b) + D(a, b-1) + D(a-1, b-1),$$

where D(0, 0) = 1 (Weisstein 1999). We will use the term "Delannoy paths" for the paths through rectangular grids as described above.

2. For a 1 × 1 grid, there are 3 paths:



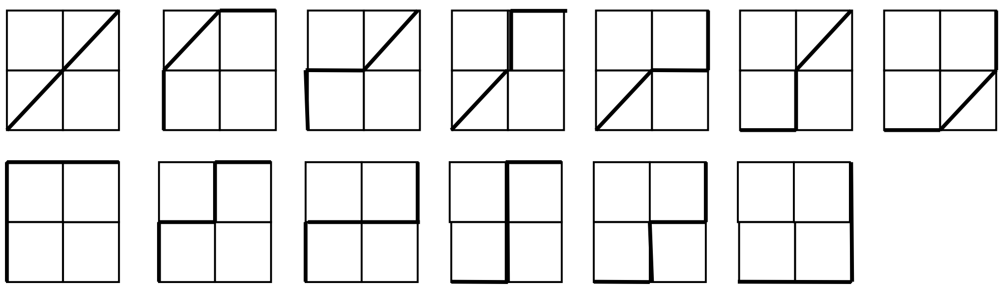
We can now consider a semiotic matrix being mad up of grids and the Cartesian products (sub-signs) being assigned to their corners. Therefore, a 1 × 1 grid has four corners which correspond to the sub-signs (1.1), (1.2), (2.1), (2.2) of the sub-matrix of the dyadic or "pre-semiotic" sign relation $DS_{2,2}$ (cf. Ditterich 1990, pp. 29, 81):

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

By using now "dynamic" category theoretic notation (Toth 2008b, pp. 159 ss.), we can thus analyze the 3 possible Delannoy paths in $DS_{2,2}$ as follows:

1. (2.1, 1.2) ≡ $[\alpha°, \alpha]$
2. ((2.1, 1.1), (1.1, 1.2)) ≡ $[[\alpha°, id1], [id1, \alpha]]$
3. ((2.1, 2.2), (2.2, 1.2)) ≡ $[[id2, \alpha], [\alpha°, id2]]$

3. For a 2 × 2 gird, there are 13 paths:



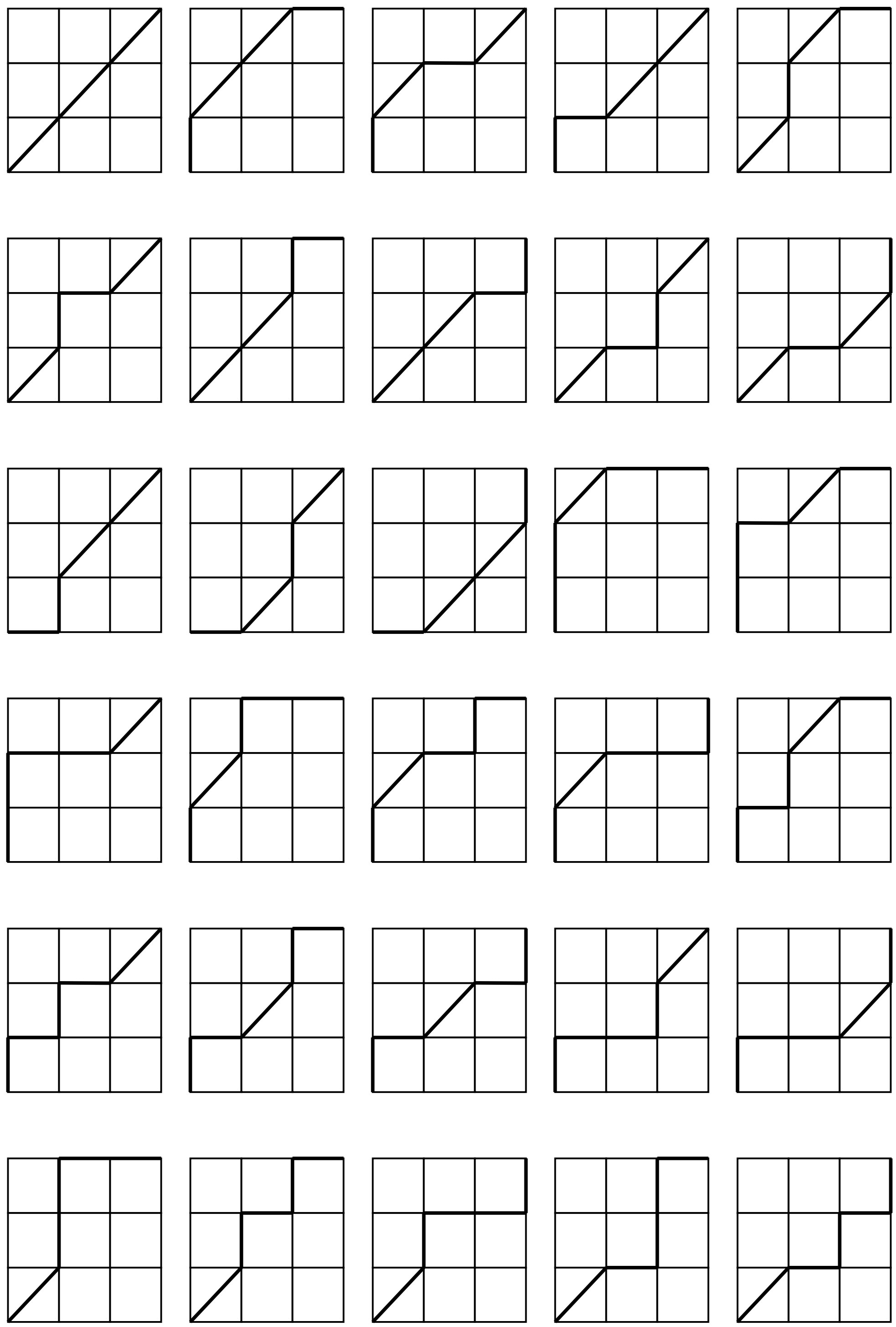
Since a 2 × 2 grid has 9 points of intersection, we can assign to it the 9 sub-signs of the semiotic matrix of $SR_{3,3}$:

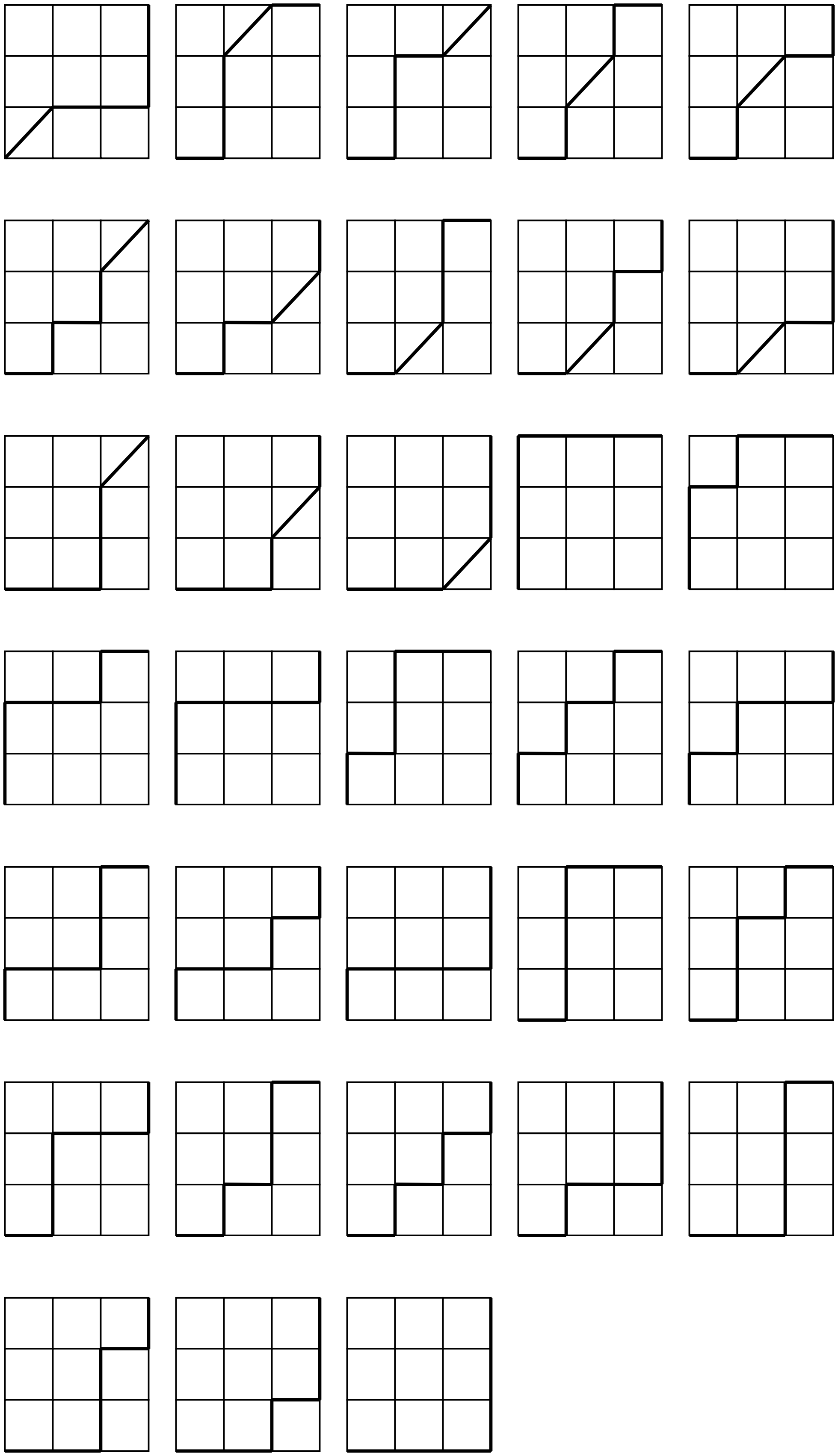
	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

The 13 possible Delannoy paths in $SR_{3,3}$ are:

1. ((3.1, 2.2), (2.2, 1.3)) ≡ [[β°, α], [α°, β]]
2. ((3.1, 2.1), (2.1, 1.2), (1.2, 1.3)) ≡ [[β°, id1], [α°, α]]
3. ((3.1, 2.1), (2.1, 2.2), (2.2, 1.3)) ≡ [[β°, id1], [id2, α], [α°, β]]
4. ((3.1, 2.2), (2.2, 1.2), (1.2, 1.3)) ≡ [[β°, α], [α°, id2], [id1, β]]
5. ((3.1, 2.2), (2.2, 2.3), (2.3, 1.3)) ≡ [[β°, α], [id2, β], [α°, id3]]
6. ((3.1, 3.2), (3.2, 2.2), (2.2, 1.3)) ≡ [[id3, α], [β°, id2], [α°, β]]
7. ((3.1, 3.2), (3.2, 2.3), (2.3, 1.3)) ≡ [[id3, α], [β°, β], [α°, id3]]
8. ((3.1, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 1.3)) ≡ [[β°, id1], [α°, id1], [id1, α]]
9. ((3.1, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 1.3)) ≡ [[β°, id1], [id2, α], [α°, id2], [id1, β]]
10. ((3.1, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 1.3)) ≡ [[β°, id1], [id2, α], [id2, β], [α°, id3]]
11. ((3.1, 3.2), (3.2, 2.2), (2.2, 1.2), (1.2, 1.3)) ≡ [[id3, α], [β°, id2], [α°, id2], [id1, β]]
12. ((3.1, 3.2), (3.2, 2.2), (2.2, 2.3), (2.3, 1.3)) ≡ [[id3, α], [β°, id2], [id2, β], [α°, id3]]
13. ((3.1, 3.2), (3.2, 3.3), (3.3, 2.3), (2.3, 1.3)) ≡ [[id3, α], [id3, β], [β°, id3], [α°, id3]]

4. For a 3 × 3 grid, there are 63 paths:





Since a 3 × 3 grid has 16 points of intersection, we can assign to it the 16 sub-signs of the semiotic matrix of $SR_{4,4}$ (cf. Toth 2008a, pp. 179 ss.):

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3

The 63 possible Delannoy paths in $SR_{4,4}$ are:

1. ((3.0, 2.1), (2.1, 1.2), (1.2, 1.3)) = [[β°, γ], [α°, α], [id1, β]]
2. ((3.0, 2.0), (2.0, 1.1), (1.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, γ], [γ°, α], [id0, β]]
3. ((3.0. 2.0), (2.0, 1.1), (1.1, 1.2), (1.2, 0.3)) ≡ [[β°, id0], [α°, γ], [id1, α], [γ°, β]]
4. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.2), (1.2, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, α], [γ°, β]]
5. ((3.0, 2.1), (2.1, 1.1), (1.1, 0.2), (0.2, 0.3)) ≡ [[β°, γ], [α°, id1], [γ°, α], [id0, β]]
6. ((3.0, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 0.3)) ≡ [[β°, γ], [α°, id1], [id1, α], [γ°, β]]
7. ((3.0, 2.1), (2.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, γ], [α°, α], [γ°, id2], [id0, β]]
8. ((3.0, 2.1), (2.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, γ], [α°, α], [id1, β], [γ°, id3]]
9. ((3.0, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 0.3)) ≡ [[β°, γ], [id2, α], [α°, id2], [γ°, β]]
10. ((3.0, 2.1), (2.1, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[β°, γ], [id2, α], [α°, β], [γ°, id3]]
11. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.2), (1.2, 0.3)) ≡ [[id3, γ], [β°, id1], [α°, α], [γ°, β]]
12. ((3.0, 3.1), (3.1, 2.2), (2.2, 1.2), (1.2, 0.3)) ≡ [[id3, γ], [β°, α], [α°, id2], [γ°, β]]
13. ((3.0, 3.1), (3.1, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [β°, α], [α°, β], [γ°, id3]]
14. ((3.0, 2.0), (2.0, 1.0), (1.0, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, id0], [γ°, γ], [id0, α], [id0, β]]
15. ((3.0, 2.0), (2.0, 1.0), (1.0, 1.1), (1.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, id0], [id1, γ], [γ°, α], [id0, β]]
16. ((3.0, 2.0), (2.0, 1.0), (1.0, 1.1), (1.1, 1.2), (1.2, 0.3)) ≡ [[β°, id0], [α°, id0], [id1, γ], [id1, α], [γ°, β]]
17. ((3.0, 2.0), (2.0, 1.1), (1.1, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, γ], [γ°, id1], [id0, α], [id0, β]]
18. ((3.0, 2.0), (2.0, 1.1), (1.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, γ], [id1, α], [γ°, id2], [id0, β]]
19. ((3.0, 2.0), (2.0, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, id0], [α°, γ], [id1, α], [id1, β], [γ°, id3]]
20. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.1), (1.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, id1], [γ°, α], [id0, β]]
21. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, id1], [id1, α], [γ°, β]]

22. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, α], [γ°, id2], [id0, β]]
23. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, α], [id1, β], [γ°, id3]]
24. ((3.0, 2.0), (2.0, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 0.3)) ≡ [[β°, id0], [id2, γ], [id2, α], [α°, id2], [γ°, β]]
25. ((3.0, 2.0), (2.0, 2.1), (2.1, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[β°, id0], [id2, γ], [id2, α], [α°, β], [γ°, id]]
26. ((3.0, 2.1), (2.1, 1.1), (1.1, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[β°, γ], [α°, id1], [γ°, id1], [id0, α], [id0, β]]
27. ((3.0, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, γ], [α°, id1], [id1, α], [γ°, id2], [id0, β]]
28. ((3.0, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, γ], [α°, id1], [id1, α], [id1, β], [γ°, id3]]
29. ((3.0, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, γ], [id2, α], [α°, id2], [γ°, id2], [id0, β]]
30. ((3.0, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, γ], [id2, α], [α°, id2], [id1, β], [γ°, id3]]
31. ((3.0, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[β°, γ], [id2, α], [id2, β], [α°, id3], [γ°, id3]
32. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.1), (1.1, 0.2), (0.2, 0.3)) ≡ [[id3, γ], [β°, id1], [α°, id1], [γ°, α], [id0, β]]
33. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 0.3)) ≡ [[id3, γ], [β°, id1], [α°, id1], [id1, α], [γ°, β]]
34. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[id3, γ], [β°, id1], [α°, α], [γ°, id2], [id0, β]]
35. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [β°, id1], [α°, α], [id1, β], [γ°, id3]]
36. ((3.0, 3.1), (3.1, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 0.3)) ≡ [[id3, γ], [β°, id1], [id2, α], [α°, id2], [γ°, β]]
37. ((3.0, 3.1), (3.1, 2.1), (2.1, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [β°, id1], [id2, α], [α°, β], [γ°, id3]]
38. ((3.0, 3.1), (3.1, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[id3, γ], [β°, α], [α°, id2], [γ°, id2], [id0, β]]
39. ((3.0, 3.1), (3.1, 2.2), (2.2, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [β°, α], [α°, id2], [id1, β], [γ°, id3]]
40. ((3.0, 3.1), (3.1, 2.2), (2.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [β°, α], [id2, β], [α°, id3], [γ°, id3]]
41. ((3.0, 3.1), (3.1, 3.2), (3.2, 2.2), (2.2, 1.2), (1.2, 0.3)) ≡ [[id3, γ], [id3, α], [β°, id2], [α°, id2], [γ°, β]]
42. ((3.0, 3.1), (3.1, 3.2), (3.2, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [id3, α], [β°, id2], [α°, β], [γ°, id3]]

43. ((3.0, 3.1), (3.1, 3.2), (3.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [id3, α], [β°, β], [α°, id3], [γ°, id3]]
44. ((3.0, 2.0), (2.0, 1.0), (1.0, 0.0), (0.0, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, id0], [γ°, id0], [id0, γ], [id0, α], [id0, β]]
45. ((3.0, 2.0), (2.0, 1.0), (1.0, 1.1), (1.1, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, id0], [id1, γ], [γ°, id1], [id0, α], [id0, β]]
46. ((3.0, 2.0), (2.0, 1.0), (1.0, 1.1), (1.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [α°, id0], [id1, γ], [id1, α], [γ°, id2], [id0, β]]
47. ((3.0, 2.0), (2.0, 1.0), (1.0, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, id0], [α°, id0], [[id1, γ], [id1, α], [id1, β], [γ°, id3]]
48. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.1), (1.1, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, id1], [γ°, id1], [id0, α], [id0, β]]
49. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [id2, γ], [[α°, id1], [id1, α], [γ°, id2], [id0, β]]
50. ((3.0, 2.0), (2.0, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, id1], [id1, α], [id1, β], [γ°, id3]]
51. ((3.0, 2.0), (2.0, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, id0], [id2, γ], [id2, α], [α°, id2], [γ°, id2], [id0, β]]
52. ((3.0, 2.0), (2.0, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, id0], [id2, γ], [α°, id2], [α°, id2], [id1, β], [γ°, id3]]
53. ((3.0, 2.0), (2.0, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[β°, id0], [id2, γ], [id2, α], [id2, β], [α°, id3], [γ°, id3]]
54. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.1), (1.1, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[id3, γ], [β°, id1], [[α°, id1], [γ°, id1], [id0, α], [id0, β]]
55. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[id3, γ], [β°, id1], [α°, id1], [id1, α], [γ°, id2], [id0, β]]
56. ((3.0, 3.1), (3.1, 2.1), (2.1, 1.1), (1.1, 1.2), (1.1, 1.2), (1.2, 0.3)) ≡ [[id3, γ], [β°, id1], [[α°, id1], [id1, α], [id1, α], [γ°, β]]
57. ((3.0, 3.1), (3.1, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[id3, γ], [β°, id1], [[id2, α], [α°, id2], [γ°, id2], [id0, β]]
58. ((3.0, 3.1), (3.1, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [β°, id1], [id2, α], [α°, id2], [id1, β], [γ°, id3]]
59. ((3.0, 3.1), (3.1, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [β°, id1], [id2, α], [id2, β], [α°, id3], γ°, id3]]
60. ((3.0, 3.1), (3.1, 3.2), (3.2, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[id3, γ], [id3, α], [β°, id2], [α°, id2], [γ°, id2], [id0, β]]
61. ((3.0, 3.1), (3.1, 3.2), (3.2, 2.2), (2.2, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [id3, α], [β°, id2], [α°, id2], [id1, β], [γ°, id3]]
62. ((3.0, 3.1), (3.1, 3.2), (3.2, 2.2), (2.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [id3, α], [β°, id2], [id2, β], [α°, id3], [γ°, id3]]
63. ((3.0, 3.1), (3.1, 3.2), (3.2, 3.3), (3.3, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[id3, γ], [id3, α], [id3, β], [β°, id3], [α°, id3], [γ°, id3]]

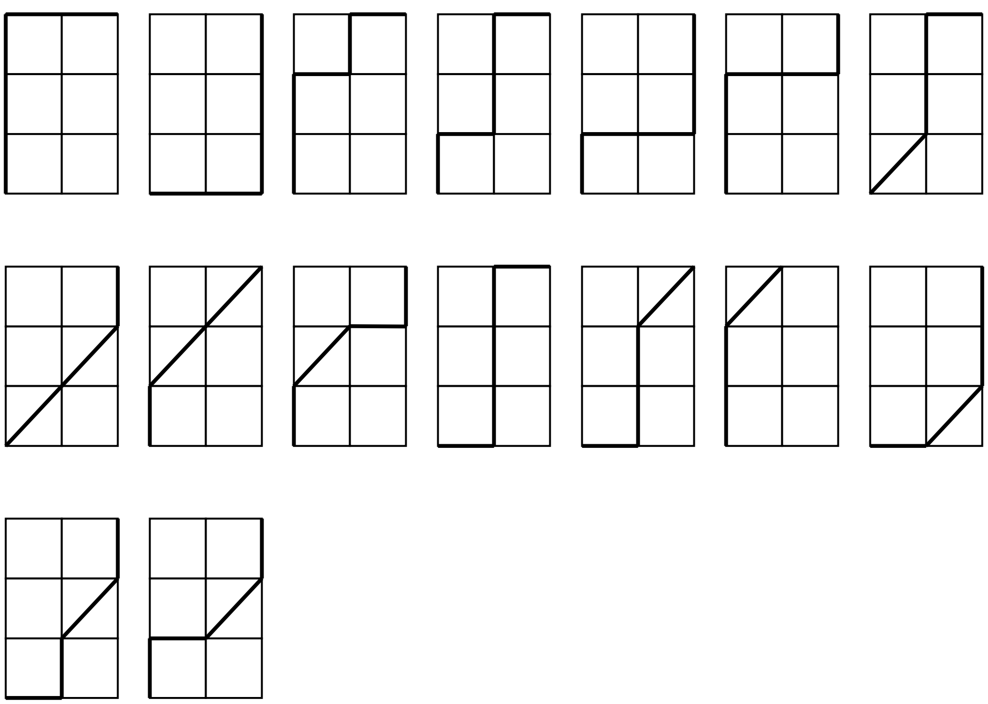
5. There exist no Delannoy numbers for a 4 × 3 grid. However, we will find out the possible paths from the SW corner of this rectangular grid to the NE corner, using again only single steps N, NE, or E. This 4 × 3 grid is a model for the tetradic-trichotomic pre-semiotic sign relation, that I had introduced in Toth (2008c):

$SR_{4,3}$ (3.a 2.b 1.c 0.d)

with the corresponding trichotomic inclusion order (a ≥ b ≥ c), whose corresponding semiotic structure is thus 4-adic, but 3-otomic, since in $Z^{r}{}_{k}$, the categorial number k ≠ 0 (Bense 1975, p. 65), and therefore the pre-semiotic matrix is "defective" from the viewpoint of a the quadratic matrix of Cartesian products over (.0., .1., .2., .3.) as displayed above in chapter 4:

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

For this 4 × 3 grid, there are 16 paths:



The 16 possible "Delannoy" paths in $SR_{4,3}$ are:

1. ((3.1, 2.1), (2.1, 1.1), (1.1, 0.1), (0.1, 0.2), (0.2, 0.3)) ≡ [[β°, id1], [α°, id1], [γ°, id1], [id0, α], [id0, β]]
2. ((3.1, 3.2), (3.2, 3.3), (3.3, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[id3, α], [id3, β], [β°, id3], [α°, id3], [γ°, id3]]
3. ((3.1, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, id1], [α°, id1], [id1, α], [γ°, id2], [id0, β]]
4. ((3.1, 2.1), (2.1, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[β°, id1], [id2, α], [α°, id2], [γ°, id2], [id0, β]]
5. ((3.1, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[β°, id1], [id2, α], [id2, β], [α°, id3], [γ°, id3]]
6. ((3.1, 2.1), (2.1, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, id1], [α°, id1], [id1, α], [id1, β], [γ°, id3]]
7. ((3.1, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3) ≡ [[β°, α], [α°, id2], [γ°, id2], [id0, β]]
8. ((3.1, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[β°, α], [α°, β], [γ°, id3]]
9. ((3.1, 2.1), (2.1, 1.2), (1.2, 0.3)) ≡ [[β°, id1], [α°, α], [γ°, β]]
10. ((3.1, 2.1), (2.1, 1.2), (1.2, 1.3), (1.3, 0.3)) ≡ [[β°, id1], [α°, α], [id1, β], [γ°, id3]]
11. ((3.1, 3.2), (3.2, 2.2), (2.2, 1.2), (1.2, 0.2), (0.2, 0.3)) ≡ [[id3, α], [β°, id2], [α°, id2], [γ°, id2], [id0, β]]
12. ((3.1, 3.2), (3.2, 2.2), (2.2, 1.2), (1.2, 0.3)) ≡ [[id3, α], [β°, id2], [α°, id2], [γ°, β]]
13. ((3.1, 2.1), (2.1, 1.1), (1.1, 0.2), (0.2, 0.3)) ≡ [[β°, id1], [α°, id1], [γ°, α], [id0, β]]
14. ((3.1, 3.2), (3.2, 2.3), (2.3, 1.3), (1.3, 0.3)) ≡ [[id3, α], [β°, β], [α°, id3], [γ°, id3]]
15. ((3.1, 3.2), (3.2, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[id3, α], [β°, id2], [α°, β], [γ°, id3]]
16. ((3.1, 2.1), (2.1, 2.2), (2.2, 1.3), (1.3, 0.3)) ≡ [[β°, id1], [id2, α], [α°, β], [γ°, id3]]

There are many more paths through grids of semiotic networks; cf. also Toth (2008d). We will examine several of them in further publications.

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