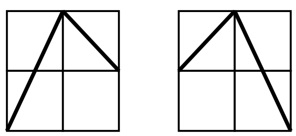
Prof. Dr. Alfred Toth

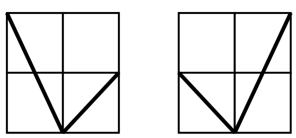
Euler Zigzag numbers in semiotics

1. After having shown three different kinds of numbers that enable to count the number of different paths through semiotic grids (Toth 2008a, 2008b), we will here briefly show the application of Euler Zigzag number for semiotics. "The numbers of alternating permutations (permutations in which the difference between each successive pair of adjacent elements changes sign – that is, each 'rise' is followed by a 'fall', and vice versa) for n elements are sometimes called the Euler Zigzag numbers. For the set {1, 2, 3}, the permutation {1, 3, 2} is an alternating permutation, while {3, 2, 1} is not. Determining the number of alternative permutations of the elements {1, 2, ..., n} is called André's Problem" (Dickau 2008).

2. For the set {1, 2, 3} there are 2 alternating permutations starting with "rise":



and 2 more alternating permutations starting with "fall":



If these 2×2 grids are representing $SR_{3,3}$ or $SR_{4,3}$, we get for the semiotic Zigzag paths:

1. ((3.1, 1.2), (1.2, 2.3)) ≡ [[α°β°, α], [α, β]]
2. ((2.1, 1.2), (1.2, 3.3)) ≡ [[α°, α], [βα, β]]
3. ((1.1, 3.2), (3.2, 2.3)) ≡ [[βα, α], [β°, β]]
4. ((2.1, 3.2), (3.2, 1.3)) ≡ [[β, α], [α°β°, β]]

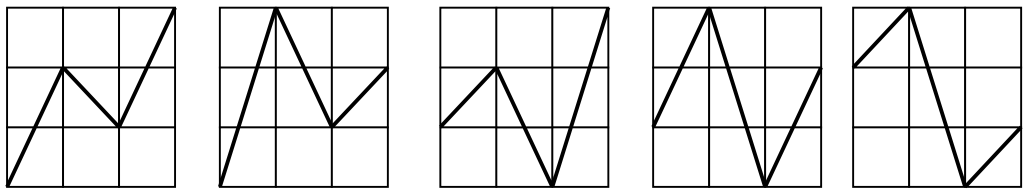
In opposition to Delannoy, Motzkin and Schröder paths (Toth 2008a, b), we meet here for the first time semiotic paths that do not only connect the squares of the grids step by step, but also by diagonal paths in $n \times m$ sub-grids with $n \neq m$. Since these paths do not intersect the angles of the middle squares between the squares they connect, but their sides, we have here for the first time the semiotic morphisms (1.3) ≡ [βα] and (3.1) ≡ [α°β°].

If these 2×2 grids represent $SR_{4,4}$, we have:

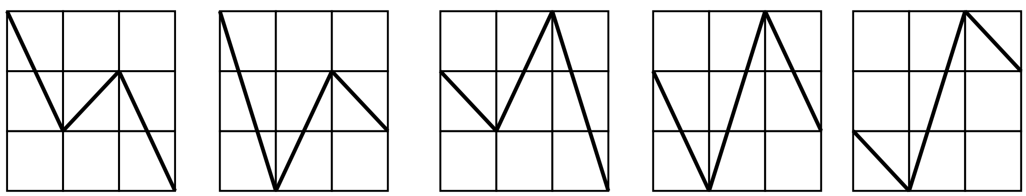
5. ((3.0, 1.1), (1.1, 2.2)) ≡ [[α°β°, γ], [α, α]]
6. ((2.0, 1.1), (1.1, 3.2)) ≡ [[α°, γ], [βα, α]]
7. ((1.0, 3.1), (3.1, 2.2)) ≡ [[βα, γ], [β°, α]]

8. ((2.0, 3.1), (3.1 1.2)) ≡ [[β, γ], [α°β°, α]]

For the set {1, 2, 3, 4}, there are 5 alternating permutations starting with “rise”



and 5 more starting with “fall”:



Since these 3×3 grids are representing $SR_{4,4}$, we get for the semiotic Zigzag paths:

9. ((3.0, 1.1), (1.1, 2.2), (2.2, 0.3)) ≡ [[α°β°, γ], [α, α], [δ°, β]]
10. ((3.0, 0.1), (0.1, 2.2), (2.2, 1.3)) ≡ [[γ°δ°, γ], [δ, α], [α°, β]]
11. ((2.0, 1.1), (1.1, 3.2), (3.2, 0.3)) ≡ [[α°, γ], [βα, α], [γ°δ°, β]]
12. ((2.0, 0.1), (0.1, 3.2), (3.2, 1.3)) ≡ [[δ°, γ], [δγ, α], [α°β°, β]]
13. ((1.0, 0.1), (0.1, 3.2), (3.2, 2.3)) ≡ [[γ°, γ], [δγ, α], [β°, β]]

14. ((0.0, 2.1), (2.1, 1.2), (1.2, 3.3)) ≡ [[δ, γ], [α°, α], [βα, β]]
15. ((0.0, 3.1), (3.1, 1.2), (1.2, 2.3)) ≡ [[δγ, γ], [α°β°, α], [α, β]]
16. ((1.0, 2.1), (2.1, 0.2), (0.2, 3.3)) ≡ [[α, γ], [δ°, α], [δγ, β]]
17. ((1.0, 3.1), (3.1, 0.2), (0.2, 2.3)) ≡ [[βα, γ], [γ°δ°, α], [δ, β]]
18. ((2.0, 3.1), (3.1, 0.2), (0.2, 1.3)) ≡ [[β, γ], [γ°δ°, α], [γ, β]]

Using the category theoretic notation, we also recognize that there is no semiotic conversion between the respective pairs of paths starting with “rise” and with “fall”, although the images are won by simple vertical flipping:

9. [[α°β°, γ], [α, α], [δ°, β]] 10. [[γ°δ°, γ], [δ, α], [α°, β]] 11. [[α°, γ], [βα, α], [γ°δ°, β]]
14. [[δ, γ], [α°, α], [βα, β]] 15. [[δγ, γ], [α°β°, α], [α, β]] 16. [[α, γ], [δ°, α], [δγ, β]]

12. [[δ°, γ], [δγ, α], [α°β°, β]] 13. [[γ°, γ], [δγ, α], [β°, β]]
17. [[βα, γ], [γ°δ°, α], [δ, β]] 18. [[β, γ], [γ°δ°, α], [γ, β]]

In the category theoretic triples of morphisms, we first met δ, $\delta\gamma$, $\gamma°\delta°$. The reason is the "thorough diagonality" like in the case of $\beta\alpha$ and $\alpha°\beta°$ mentioned above.

The present study as well as the former ones (Toth 2008a, b) can be seen as investigations of paths through the hitherto most complex semiotic network, called SRG (Toth 1997).

Bibliography

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