1. According to Walther (1979, pp. 113 ss.), in the Peircean triangle-model three sign relations can be differentiated:

- the denomination function \( M \Rightarrow O \),
- the designation function \( O \Rightarrow I \),
- the application function \( I \Rightarrow M \)

Not differentiated and labeled are the three dual functions

- \( O \Rightarrow M \)
- \( I \Rightarrow O \)
- \( M \Rightarrow I \),

although they have perfect category theoretic equivalent morphisms (cf. Toth 1997, pp. 21 ss.).

2. For a minimal, i.e. three-valued, polycontextural logic, Günther (1976, pp. 336 ss.) differentiates between the three reflectional categories subjective subject (SS), objective subject (OS) and (objective) object (OO) and displays them, as Peirce did with his relational categories for his minimal sign model, as follows:
However, as one can easily see, the fundamental difference between the Peircean and the Guntherian triadic models is that in the Guntherian graph, relations that start in vertices end in edges, while in the Peircean graph, relations solely connect vertices. Furthermore, Gunther differentiates between three kinds of relations:

1. the order relations \((SS \rightarrow O)\) and \((O \rightarrow OS)\)
2. the exchange relation \((SS \leftrightarrow OS)\)
3. the foundational relations \((OS \rightarrow (SS \rightarrow O)), (SS \rightarrow (O \rightarrow OS))\) and \((O \rightarrow (SS \leftrightarrow OS))\)

Thus, the Peircean denomination and designation functions correspond with the Guntherian order relations \((OS \rightarrow O)\) and \((O \rightarrow SS)\), and the Peircean application function corresponds to the Guntherian exchange relation \((SS \rightarrow OS)\). As one sees, in the Guntherian model, also the reversal of the arrows, i.e. the dual semiotic functions, can be defined.

Therefore, the Guntherian model is superior to the Peircean model insofar as the existence of the foundational relations is concerned. Moreover, while the order relations are hierarchical-asymmetric, the exchange relation is heterarchical-symmetric. About the foundational relations, Günther says: “We call this the founding relation because by it, and only by it, a self-reflective subject separates itself from the whole Universe which thus becomes the potential contents of the consciousness of a Self gifted with awareness” (1976, p. 339). Therefore, the foundational relations are in the case of \((OS \rightarrow (SS \rightarrow O))\) and \((SS \rightarrow (O \rightarrow OS))\) order relations over order relations and in the case of \((O \rightarrow (SS \leftrightarrow OS))\) an order relation over an exchange relation.

3. Ditterich (1990, pp. 91 ss., 123 ss.) had already remarked the correspondences between OS and M, O and O, and SS and I. While there are no problems to identify the semiotic and the polycontextural object and the semiotic interpretant relation and the polycontextural category of subjective subject, the identification of the semiotic medium relation with the polycontextural category of objective subject needs some explanation. In his last lecture at the University of Stuttgart, Bense mentioned that “the legi-sign (1.3) is the lowest interpretant” (winter-semester 1989/90). This is of special interest, because we have \((1.3) \times (3.1)\), i.e. the “smallest” interpretant (3.1) is the dualization of the legi-sign (1.3), and vice versa, and at the same time, this dualization expresses again the Guntherian exchange relation (OS \(\leftrightarrow SS\)). Contrarily, the polycontextural and the semiotic order relations \((SS \rightarrow O)\) vs. \((I \Rightarrow O)\) and \((O \rightarrow OS)\) vs. \((O \Rightarrow M)\) are not dual to one another. However, of particular interest are the foundational relations that are not present in semiotics; the correspondences are: \((OS \rightarrow (SS \rightarrow O))\) corresponds to \((M \Rightarrow (I \Rightarrow O))\), \((SS \rightarrow (O \rightarrow OS))\) corresponds to \((I \Rightarrow (O \Rightarrow M))\), and \((O \rightarrow (SS \leftrightarrow OS))\) corresponds to \((O \Rightarrow (I \leftrightarrow M))\).
From the logical standpoint, the latter means that the “Thou” founds the order relation between an “I” and an “It” (OS → (SS → O)), that an “I” founds the order relation between an “It” and a “Thou” (SS → (O → OS)), and finally, that an “It” founds the exchange relation between an “I” and a “thou” (O → (SS ↔ OS)).

4. Starting with the geometric model of a sign class or reality thematic as an (equilateral) triangle, we notice that the semiotic foundational relations (FR) are orthogonal relations between the categories and the sign relations:

\[
\begin{align*}
FR_1 & := I \leftrightarrow (M \rightarrow O) \equiv (.3.) \leftrightarrow ((.1.) \rightarrow (.2.)) \\
FR_2 & := M \leftrightarrow (O \rightarrow I) \equiv (.1.) \leftrightarrow ((.2.) \rightarrow (.3.)) \\
FR_3 & := O \leftrightarrow (M \rightarrow I) \equiv (.2.) \leftrightarrow ((.1.) \rightarrow (.3.))
\end{align*}
\]

If we use the framework of the “General Sign Grammar” (Toth 2008a), we can display the semiotic foundational relations like that:

[Diagram]

where X, Y, Z ∈ \{.1., .2., .3.\} and X, Y, Z are pairwise different, which means that for Z any of the three prime-signs can be chosen, so that for FR_i the following 6 relations are possible:

\[
\begin{align*}
(3.a & 2.b \ldots 1.c) (3.a & 2.b & 1.c) (3.a & 2.b & 1.c) (3.a & 2.b & 1.c) (3.a & 2.b & 1.c) (3.a & 2.b & 1.c) \\
1.c & 1.c & 2.b & 2.b & 1.c & 1.c \\
2.b & 3.a & 3.a & 1.c & 3.a & 3.a \\
3.a & 2.b & 1.c & 3.a & 2.b & 3.a
\end{align*}
\]

And, of course, instead of (3.a 2.b 1.c), we may also set all other 5 transpositions of the general sign relation, so that we come to a total amount of 30 possible combinations for each i of FR_i (i = 1, 2, 3), thus together 90 combinations.
In a case like the following, taken from (Toth 2008b):

we have a semiotic superization system, in which the superizative steps are not based on the semiotic “coincidence” of categories (cf. Bense 1971, p. 54), but on the “concidence” of categories and relations like in the examples above. In accordance with Günther (1991), these superizations are based on semiotic orthogonality. In the above example, we thus obtain a system of 20 complex semiotic relations, namely monadic, dyadic and triadic relations, relations over relations and relations over categories:

\begin{align*}
1 &= M \\
2 &= O \equiv O' \\
3 &= I \\
4 &= M'' \\
5 &= I'' \\
6 &= M' \\
7 &= O'' \equiv O''' \\
8 &= I'' \leftrightarrow (M \to O) \\
9 &= M''' \\
10 &= I''' \leftrightarrow (O' \leftrightarrow M') \\
11 &= M'''' \leftrightarrow (M' \to I') \\
12 &= O'''' \equiv O''''
\end{align*}

\begin{align*}
13 &= O''''' \\
14 &= M'''''
\end{align*}

\begin{align*}
16 &= M'''''' \leftrightarrow (M''' \to I'') \\
17 &= O'''''' \equiv O'''''''
\end{align*}

\begin{align*}
18 &= I''''' \leftrightarrow (M''' \to O'''') \\
19 &= M''''''
\end{align*}
20 = I'''''' ↔ (O'''''' → M''''')

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