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Embeddings of semiotic triangles, hexagons, and pentagons

1. In Toth (2008, pp. 159 ss.) it was stated that each sign class of the general form

(3.a 2.b 1.c)

has the following class of transpositions:

I	(3.a 2.b 1.c)	IV	(2.b 1.c 3.a)
II	(3.a 1.c 2.b)	V	(2.b 3.a 1.c)
III	(2.b 3.a 1.c)	VI	(2.b 1.c 3.a)

The same is true for each reality thematic of the general form

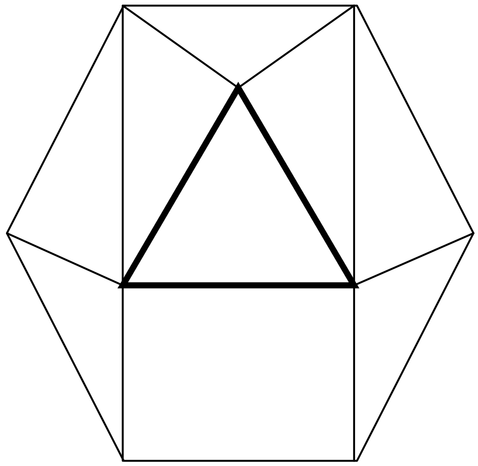
(c.1 b.2 a.3),

which has the following class of transpositions:

I'	(c.1 b.2 a.3)	IV'	(a.3 c.1 b.2)
II'	(b.2 c.1 a.3)	V'	(c.1 a.3 b.2)
III'	(c.1 a.3 b.2)	VI'	(a.3 c.1 b.2)

2. The classes of transpositions of each sign class and reality class (thematic) can be ordered either in the shape of a hexagon or a pentagon with a center point. We will agree that the mappings of the classes of the transpositions on a pentagon and hexagon should be clockwise, starting with the uppermost and rightmost vertex in the respective graphs. Further, the central vertex in a pentagon should be ascribed to the "unmarked" transpositions of the form (3.a 2.b 1.c) for a sign class, and (c.1 b.2 a.3) for its dual reality thematic, respectively. In doing so, we get the following four possibilities: 1. Embedding of the semiotic triangle into a semiotic hexagon. 2. Embedding of the semiotic triangle into a semiotic pentagon. 3. Embedding of the semiotic hexagon into a semiotic triangle. 4. Embedding of the semiotic pentagon into a semiotic triangle.

2.1. Embedding of the semiotic triangle into a semiotic hexagon:



If we number the vertices of the hexagon by Roman and the vertices of the triangle by Arabian numbers in the order given above, we get the following semiotic connections of this type of embedding:

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I-1:   (3.a 2.b 1.c)   II-1:  (3.a 1.c 2.b)   III-1: (2.b 3.a 1.c)
         |                      |                          |
       (3.a)                  (3.a)                      (3.a)

I-2:   (3.a 2.b 1.c)   II-2:  (3.a 1.c 2.b)   III-2: (2.b 3.a 1.c)
             |                          |              |
           (2.b)                      (2.b)          (2.b)

I-3:   (3.a 2.b 1.c)   II-3:  (3.a 1.c 2.b)   III-3: (2.b 3.a 1.c)
                 |                  |                          |
               (1.c)              (1.c)                      (1.c)

IV-1:  (2.b 1.c 3.a)   V-1:   (1.c 3.a 2.b)   VI-1:  (1.c 2.b 3.a)
                 |                  |                          |
               (3.a)              (3.a)                      (3.a)

IV-2:  (2.b 1.c 3.a)   V-2:   (1.c 3.a 2.b)   VI-2:  (1.c 2.b 3.a)
         |                              |                  |
       (2.b)                          (2.b)              (2.b)

IV-3:  (2.b 1.c 3.a)   V-3:   (1.c 3.a 2.b)   VI-3:  (1.c 2.b 3.a)
             |                  |                      |
           (1.c)              (1.c)                  (1.c)
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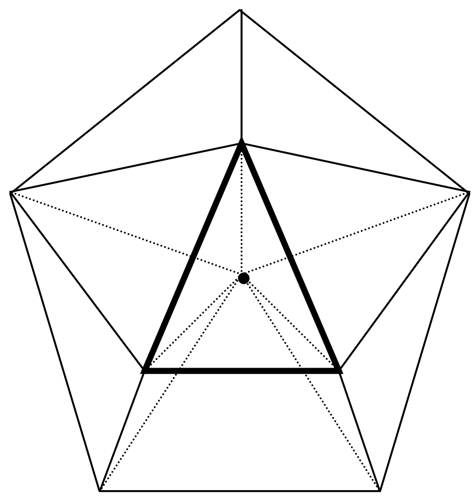
2.2. Embedding of the semiotic triangle into a semiotic pentagon:



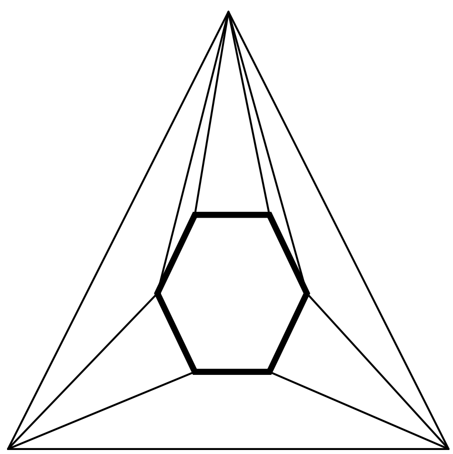
As we recognize easily, the semiotic connections are exactly the same. Thus, we only want to mention specially the semiotic connections starting or ending in the central point of the semiotic pentagon:

Part-system of the sign classes	**Part-system of the reality thematics**
1. Transpositions and sub-signs	
(3.a 2.b 1.c)	(c.1 b.2 a.3)
|	|
(3.a)	(a.3)
(3.a 2.b 1.c)	(c.1 b.2 a.3)
|	|
(2.b)	(b.2)
(3.a 2.b 1.c)	(c.1 b.2 a.3)
|	|
(1.c)	(c.1)

2. Transpositions and transpositions

(3.a 2.b 1.c)	(c.1 b.2 a.3)
(3.a 1.c 2.b)	(b.2 c.1 a.3)
(2.b 3.a 1.c)	(c.1 a.3 b.2)
(2.b 1.c 3.a)	(a.3 c.1 b.2)
(2.b 3.a 1.c)	(c.1 a.3 b.2)
(2.b 1.c 3.a)	(a.3 c.1 b.2)

2.3. Embedding of the semiotic hexagon into a semiotic triangle



With a little consideration, we recognize that in the embedding of the semiotic hexagon into the semiotic triangle, we have again 8 mappings like in the embedding of the semiotic triangle into the semiotic hexagon, but in their dual forms: In the first case, the mappings have the general form

(a.b c.d e.f) → (x.y), where a, b, ..., x, z ∈ {1, 2, 3}, i.e. the set of the prime-sign (cf. Bense 1980),

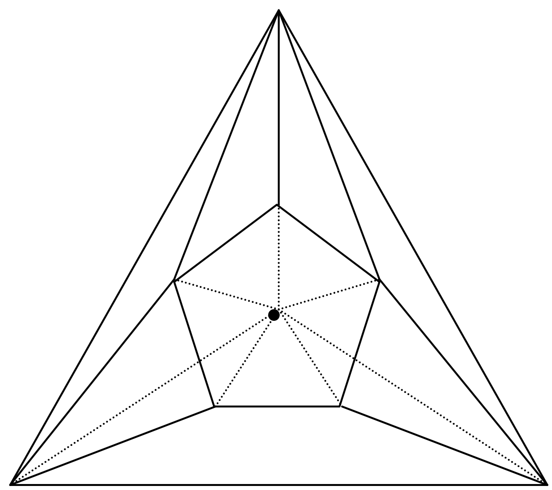
while in the second case, they have the general form

(x.y) → (a.b c.e e.f),

which means "reversal of the arrows", i.e., all category theoretic semiotic morphisms (cf. Toth 1997, pp. 21 ss.) of the embedding of the semiotic triangle into the hexagon are reversed, if the semiotic hexagon is embedded into the semiotic triangle.

Without giving a proof, we state that the latter is true for each exchange of embeddings of a semiotic n-gon into a semiotic m-gon ($n \neq m$). Thus, also the semiotic connections of the embedding of the semiotic pentagon into the semiotic triangle are dualized, if compared to the embedding of the semiotic triangle into the semiotic pentagon:

2.4. Embedding of the semiotic pentagon into a semiotic triangle



A list of non-planar graphs that may be useful for semiotic analysis is given in Weisstein.

Bibliography

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