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New elements of theoretical semiotics (NETS), based on the work of Rudolf Kaehr

1. Recently, Professor Rudolf Kaehr has published four papers (Kaehr 2008, 2009a, b, c) in which he applies some elements of polycontextural theory to selected fundamentals of mathematical semiotics introduced by me. I have to point out that Kaehr's work on semiotics surpasses in never seen dimensions almost everything that has been elaborated in the long history of semiotics. Therefore, I have no doubt that Kaehr's studies mark the beginning of a wholly new era of formal semiotics compared to which most of the writings of the last decades will look rather poor and provisory. In the present article, I will discuss some of the new theoretical fundamentals introduced into semiotics by Kaehr.

2. As Kaehr correctly sees, the so-called "Genuine Category Class"

(3.3 2.2 1.1)

is the only sign-relation that appears in Bense's "semiotic matrix" without being a defined sign class, since sign classes (SCl) must be built upon the relational form

SCl = (3.a 2.b 1.c) with a, b, c $\in$ {.1, .2, .3}

obeying the inclusive trichotomic order

(a $\leq$ b $\leq$ c),

but since (3.3 2.2 1.1) has the trichotomic order (a > b > c), is is not considered a sign class and therefore does not figure in the list of the 10 Peircean sign classes.

Nevertheless, the Genuine Category Class has given rise to speculations about its theoretical status as well as about its applications throughout the history of theoretical semiotics. F.ex., Bense (1975, p. 93) wrote:

> "Alle diese für die (dreistufige Hauptsemiose der (neunstufigen) semiotischen Matrix charakteristischen erkenntnistheoretischen und kommunikationstheoretischen, ersicht-

> lich auf Zeichenrelationen und Semiosen zurückführbaren Züge machen die Hauptsemiose (1.1, 2.2, 3.3) zu einer genuinen, die alle anderen möglichen Semiosen, die mit ihren stabilen Momenten in der semiotischen Matrix erkannt bzw. formuliert werden könen, **generiert** und **repräsentiert**. Sie kann daher in ihrer semiotischen Funktion, naheliegend und bei hinreichender Verallgemeinerung jenes Prinzips der Zustandsentwicklung, das Maxwell und Boltzmann für ihre Zwecke einführten, im Anschluss an die späteren Formulierungen von Planck, Takács, Lange, Chintschin u.a. als **ergodische Semiose** bezeichnet werden, um auszudrücken, dass ein bestimmter Abstraktionsfluss mit bestimmten relativ stabilen Abstraktionsmomenten existiert, der (relativ zur semiotischen Matrix der Gesamtheit der Semiosen und ihrer Subzeichen) als ergodischer Prozess zu beschreiben ist".

However, while there is no doubt that was Bense wrote, is true from a semantic standpoint, the formal side of generative and representative connections between the Genuine Category Class and the 10 regular sign classes is highly unclear. The Genuine Category Class is only connected to the following 6 sign classes:

(3.1 2.1 1.1), (3.1 2.2 1.2), (3.1 2.2 1.3), (3.2 2.2 1.2), (3.2 2.2 1.3), (3.3 2.3 1.3),

so that, unlike the eigenreal sign class (3.1 2.2 1.3), which is connected to all 10 sign classes and therefore induces a "determinant-theoretic duality system" (Walther 1982), the Genuine Category Class does not induce a discriminant-theoretic duality system.

However, in a new publication (Kaehr 2009c), Kaehr has shown that it is not sufficient to introduce the three fundamental categories of triadic semiotics as single objects or morphisms, but that they must be introduced as doublets, therein containing their "hetero-morphism" or "(inner) environment":

Firstness:	Peirce:	A
	Kaehr:	A **|** a
Secondness:	Peirce:	$A \to B$
	Kaehr	$A \to B$ **|** c
Thirdness:	Peirce:	$A \to C$
	Kaehr:	$A \to C$ **|** $b_1 \leftarrow b_2$

An informal approach to apply this so-called diamond-concept of defining the three semiotic fundamental categories not as single morphisms, but as doublets consisting of morphisms and their hetero-morphisms, can the derived from the correspondence between the fundamental categories and the so-called semiotic functions (cf. Walther 1979, pp. 113 ss.; Toth 1997, p. 33). Although Firstness is what stands for itself, it is also the domain of Thirdness in the semiotic "application function"

(M ⇒ I) or ((.1.) ⇒ (.3.)),

meaning that Firstness is what connects the whole (triadic) relation with itself (the monadic) relation, so that we can characterize Firstness with (1,3).[1]

On the other hand, Secondness is what connects Firstness with Thirdness in correspondence with the semiotic "designation function" (1,2)

(M ⇒ O) or ((.1.) ⇒ (.2.)),

and Thirdness is what connects Secondness with the whole (triadic) relation, thus with itself (2,3) in correspondence with the semiotic "denomination function"

(O ⇒ I) or ((.2.) ⇒ (.3.)).

Therefore, we obain that the monocontextural set of prime-signs

PS = {.1., .2., .3.}

corresponds to the following polycontextural set of prime-signs

$PS^* = \{(.1.)_{1,3}, (.2.)_{1,2}, (.3.)_{2,3}\}$.

When we now have a look at Kaehr's "polycontextural semiotic 3-matrix" (Kaehr 2009c)

1 If we define a sign relation as SR = (M, (M⇒O),(O⇒I)) in Peirce's sense (followed by Walther 1979, pp. 113 ss.), consisting of a monadic, a dyadic and a triadic (part-)relation, then we omit the **fourth** part-relation (I⇒M) or (M⇒I), resp.! Therefore, the graph of SR would not be closed.

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix}$$

we recognize immediately that the genuine (identitive) sub-signs

$(3.3_{2,3})$, $(2,.2_{1,2})$, $(1.1_{1,3})$

are the only sub-signs whose "indices" are identical with the "indices" of the prime-signs. Thus, the polycontextural Genuine Category Class

$(3.3_{2,3}\ 2,.2_{1,2}\ 1.1_{1,3})$

is the **generating sign relation for all the sub-signs of the semiotic matrix and therefore for all the 10 (regular) Peircean sign classes**. This astonishing and extremely important result could not be achieved before the introduction of semiotic environments based on the doublet-definition of the semiotic fundamental categories ascribing to each semiotic morphism its hetero-morphism by Kaehr (2009c).

This generating function of the polycontextural Genuine Category Class can also be shown in the polycontextural 3-matrix itself:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ \downarrow & \uparrow & \uparrow \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ \downarrow & \downarrow & \uparrow \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix}$$

This means that the "index" (1,3) of (1.1) generates (downwards) both the "index" 1 of (2.1) and the "index" 3 of (3.1). The "index" (1,2) generates (upwards) the "index" 1 of (1.2) and (downwards) the "index" of (3.2). And the "index" (2.3) generates (upwards) both the "index" 3 of (1.3) and the "index" (2,3) of (3.3).

A more "impressionistic" characterization of the sub-signs is:

(1.2), or Secondness of Firstness, is what both connects itself and the whole and Firstness with itself, i.e. (1,3) □ (1,2) = 1.

(1.3), or Thirdness of Firstness, is what both connects itself and the whole and Secondness with itself, i.e. (1,3) □ (2,3) = 3.

(2.3), or Thirdness of Secondness, is what both connects Firstness with itself and Secondness with itself, i.e. (1,2) □ (2,3) = 2.

(2.1), (3.1), and (3.2) have the same "indices", since they are dual to the three above defined sub-signs. As already shown, the indices of the genuine or identitive (self-dual) sub-signs are identical with those of the prime-signs.

3. Polycontexturality is based on the abolition of the four basic Laws of Thinking: The Law of Identity, the Law of the Excluded Middle, The Law of Non-Contradiction and the Law of Double Negation. However, when the Law of Identity is abolished, for semiotics, it is to expect that one of its central theories, the theory of eigenreality (Bense 1992), disappears, too. Already Kaehr (2009c) has shown that the monocontextural eigenreal dual system

$\times(3.1\ 2.2\ 1.3) = (3.1\ 2.2\ 1.3); (3.1\ 2.2\ 1.3) = (3.1\ 2.2\ 1.3)$

does not hold anymore in the polycontextural semiotic framework based on the above polycontextural semiotic 3-matrix:

$\times(3.1_3\ 2.2_{1,2}\ 1.3_3) = (3.1_3\ 2.2_{2,1}\ 1.3_3); (3.1_3\ 2.2_{1,2}\ 1.3_3) \neq (3.1_3\ 2.2_{2,1}\ 1.3_3)$,

since through dualization, not only the sub-signs, but their "indices" are inverted as well. It follows that the 10 Peircean sign classes do not form anymore Walthers (monocontextural) "determinant-symmetric duality system" which says that each of the 10 sign classes/reality thematics is connected with every other sign class/reality thematics by at least 1 sub-sign. Since the theory of semiotic connections is based fundamentally on the concept of eigenreality, it has to be redefined, too.

However, the loss of eigenreality due to introduction of environment-contextuated sub-signs is not so unexpected as it might seem to be. Even without knowledge of the different contextures involved in the index (2.2), it is clear that in

×(3.1 2.2 1.3) = (3.1 2.2 1.3),

the rhema (3.1) of the second sign class is not identical with the rhema (3.1) of the first sign class, but is identical with the dualized legi-sign of the first sign class (1.3). The same holds for the legi-sign of the second sign class which is the dualized rhema of the first sign class and not its legi-sign. This means: The "identity" between (3.1) and x(1.3) and (1.3) and x(3.1) is a pure formal one. However, this purely formal identity stands in contradiction with the assertion of semiotics that the two rhemata

×(3.1 x y) = (3.1 x y)

are in fact rhemata and the two legi-signs

×(x y 1.3) = (x y 1.3)

are in fact legi-signs and thus semantically identical, which is, as we have just shown, not true. If this would be true, than sign-sign (1.2) and icon (2.1) and symbol (2.3) and dicent (3.2) would be identical, too.

For the set of the semiotic dual-systems, the abolishment of eigenreality implicates that there is no longer a partition into the eigenreal dual-system and the one side and the other 9 dual-systems on the other side. As it is show, dualization inverts all 10 sign classes or reality thematics in exactly the same way, i.e. through inversion of not only their sub-signs but also of their environmental contextures. Thus, all 10 sign classes and reality thematics need **two dualizations** in order to regain their original structure:

$(3.1_3\ 2.1_1\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$	×	$(3.1_3\ 2.1_1\ 1.1_{1,3})$
$(3.1_3\ 2.1_1\ 1.2_1)$	×	$(2.1_1\ 1.2_1\ 1.3_3)$	×	$(3.1_3\ 2.1_1\ 1.2_1)$
$(3.1_3\ 2.1_1\ 1.3_3)$	×	$(3.1_3\ 1.2_1\ 1.3_3)$	×	$(3.1_3\ 2.1_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$	×	$(3.1_3\ 2.2_{1,2}\ 1.2_1)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	×	$(3.1_3\ 2.2_{1,2}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 1.3_3)$	×	$(3.1_3\ 2.3_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$	×	$(3.2_2\ 2.2_{1,2}\ 1.2_1)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$	×	$(3.2_2\ 2.2_{1,2}\ 1.3_3)$
$(3.2_2\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 2.3_2)$	×	$(3.2_2\ 2.3_2\ 1.3_3)$
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 3.3_{3,2})$	×	$(3.3_{2,3}\ 2.3_2\ 1.3_3)$

The same holds for the polycontextural Genuine Category Class:

$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3}) \times \quad (3.1_{3,1}\ 3.2_{2,1}\ 3.3_{3,2}) \quad \times \quad (3.3_{2,3}\ 2.3_{1,2}\ 1.3_{1,3})$

So, in the third row, every sub-sign and every environment is not only formally, but also semantically identical with the respective sub-sign and environment in the first row.

4. In chapter 2., I had already mentioned that regular sign classes are restricted through obeying the inclusive semiotic order

(3.a 2.b 1.c) with $a \leq b \leq c$.

Thus, every other order of the trichotomic values a, b, c leads to irregular sign classes. However, this restriction is one of those not so rare semiotic restrictions, which have no theoretical basis at all. Moreover, the special restriction in discussion here has not even a semantic motivation, since there is no reason, why a sign relation like, e.g.,

(3.2 2.1 1.3)

is not to be considered a (regular) sign class. An example for (2.1 1.3) is a literary metaphor, which as a metaphor is iconic (2.1) and by use of letters, i.e. conventional media, is a legi-sign (1.3). So, why should our metaphor (2.1 1.3) not be able to figure as part of a dicentic sentence, i.e. a sentence, which can be judged concerning its truth or falseness? The arbitrarily chosen German sentence

Der Zahn der Zeit hat an diesem Gebäude genagt

can surely be stated as true or false when uttered about a specific building. Generally, is does not need much fantasy to find counter-evidence against the "forbidden" (irregular) sign classes which are constructed just by the rule

(3.a 2.b 1.c) with $a, b, c \in \{.1, .2, .3\}$

If we construct them, we get $3 \cdot 3 \cdot 3 = 27$ sign classes. We will note them as polycontextural sign classes, i.e. together with their contextural "indices"

$(3.1_3\ 2.1_1\ 1.1_{1,3})$	$(3.2_2\ 2.1_1\ 1.1_{1,3})$	$(3.3_{2,3}\ 2.1_1\ 1.1_{1,3})$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.2_2\ 2.1_1\ 1.2_1)$	$(3.3_{2,3}\ 2.1_1\ 1.2_1)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.2_2\ 2.1_1\ 1.3_3)$	$(3.3_{2,3}\ 2.1_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.1_{1,3})$	$(3.2_2\ 2.2_{1,2}\ 1.1_{1,3})$	~~$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3})$~~
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	**$(3.2_2\ 2.2_{1,2}\ 1.2_1)$**	$(3.3_{2,3}\ 2.2_{1,2}\ 1.2_1)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	**$(3.2_2\ 2.2_{1,2}\ 1.3_3)$**	$(3.3_{2,3}\ 2.2_{1,2}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.1_{1,3})$	$(3.2_2\ 2.3_2\ 1.1_{1,3})$	$(3.3_{2,3}\ 2.3_2\ 1.1_{1,3})$
$(3.1_3\ 2.3_2\ 1.2_1)$	$(3.2_2\ 2.3_2\ 1.2_1)$	$(3.3_{2,3}\ 2.3_2\ 1.2_1)$
$(3.1_3\ 2.3_2\ 1.3_3)$	**$(3.2_2\ 2.3_2\ 1.3_3)$**	**$(3.3_{2,3}\ 2.3_2\ 1.3_3)$**

In bold are the "regular" sign classes. **Simply by looking at the positions of the regular 10 sign classes, we recognize that they build only a sub-set or perhaps better: a fragment of the set of the 27 sign classes.** If we look at the system of the contextural "indices", this gets even clearer:

3-1-(1,3) **3-1-1** **3-1-3**	2-1-(1,3) 2-1-1 2-1-3	(2,3)-1-(1,3) (2,3)-1-1 (2,3)-1-3
3-(1,2)-(1.3) **3-(1,2)-1** **3-(1,2)-3**	2-(1,2)-(1,3) **2-(1,2)-1** **2-(1,2)-3**	~~(2,3)-(1,2)-(1,3)~~ (2,3)-(1,2)-1 (2,3)-(1,2)-3
3-2-(1,3) 3-2-1 **3-2-3**	2-2-(1,3) 2-2-1 **2-2-3**	(2,3)-2-(1,3) (2,3)-2-1 **(2,3)-2-3,**

since we recognize that each of the 3 horizontal squares has the following double-structure:

3-x-y	2-x-y	(2,3)-x-y

with

$$x = \left\{ \begin{matrix} 1 \\ (1,2) \\ 2 \end{matrix} \right\} \downarrow \qquad y = \overrightarrow{\left\{ \begin{matrix} (1,3) \\ 1 \\ 3 \end{matrix} \right\}}$$

whereby, in x, (1,2) mediates between 1 and 2, and in y, (1,3) is unfolded into 1 and 3.

5. Semiotics belongs to the oldest scientific branches, although it never became so popular like, e.g., logic. However, while logic has been thoroughly formalized in the last two millennia, in semiotics, hardly anything more has been done than to produce endless and senseless discussions about the reality status of the sign (physei or thesei). Then, since the 60ies, Bense introduced formal concepts into semiotics, but he mainly saw in semiotics a branch of metamathematics rather than mathematics. The "mathematical turn" of semiotics was left for me to achieve. Although I have started in the early 80ies to try to elevate semiotics on the formal level of at least elementary mathematics, the bigger part of this work I could only publish in the last years, due to other scientific obligations. Included in these studies was the adaptation of some basic notions of Günther's polycontextural theory, which I had studied only in the 90ies. However, most semioticians - me included - have long time overseen that Günthers work has been expanded into a whole new scientific branch by his student Rudolf Kaehr. Since Kaehr's work surpasses Günther's work both in formal accuracy and in metaphysical depth, an approximation between semiotics and polycontextural theory can only be achieved from Kaehr's and not directly from Günther's work. I am convinced that the future of semiotics lies in big parts in this common semiotic-polycontextural basis. The very few examples given in this study may be sufficient to show the enormous power that emerges from this common basis. The present author has titled this article "New elements of theoretical semiotics" and even invented the acronym "NETS" in the hope that this study will not remain alone but continued in many sequels.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, Diamond semiotics. In: http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (Kaehr 2008)

Kaehr, Rudolf, Toth's semiotic diamonds. In: http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf (2009a)

Kaehr, Rudolf, XANADU's textemes. In: http://www.thinkartlab.com/pkl/lola/Xanadu-textemes/Xanadu-textemes.pdf (2009b)

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009c)
Toth, Alfred, Entwurf einer Semiotisch-Relationalen Grammatik. Tübingen 1997
Walther, Elisabeth, Nachtrag zu Trichotomischen Triaden. In: Semiosis 27, 1982, S. 15-20
Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

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