

Ordered sets and cofinality in semiotics

1. A **directed set** is a nonempty set A together with a preorder, i.e. a reflexive and binary relation \leq having the additional property that every pair of elements has an upper bound. Thus, for any two elements a and b in A , there exists an element c in A (not necessarily distinct from a, b) with $a \leq b$ and $b \leq c$ (directedness). Therefore, a directed set obeys the following axioms:

- 1.1. $\forall x \in X: x \triangleleft x$ (reflexivity)
- 1.2. $\forall x, y, z \in X: (x \triangleleft y) \wedge (y \triangleleft z) \Rightarrow (x \triangleleft z)$ (transitivity)
- 1.3. $\forall x, y \in X \exists z \in X: (x \triangleleft z) \wedge (y \triangleleft z)$ (directedness)

For a directed set (I, \triangleleft) and a set X , a **net** is a mapping $x: I \rightarrow X$. (I, \triangleleft_I) and (J, \triangleleft_J) be directed sets, $(x_i)_{i \in I}$ a net in X and $\varphi: J \rightarrow I$ a mapping that satisfies the following condition:

- 1.4. $\forall i_0 \in I \exists j_0 \in J: \forall j \triangleright_J j_0 \varphi(j) \triangleright_I i_0$,

then such a mapping is called **cofinal**, and the net $(x_{\varphi(j)})_{j \in J}$ a **sub-net** of the net $(x_i)_{i \in I}$ (von Querenburg 2001).

2. The semiotic system of the 10 sign classes is ruled by the two following laws:

- 2.1. Law of Triadicity: In a sign-relation (a.b c.d e.f), the triadic sub-set (a., c., e.) must be mapped to the set {1., 2., 3.} so that this mapping is bijective.
- 2.2. Law of Inclusive Trichotomic Order: In a sign-relation (a.b c.d e.f), the trichotomic sub-set (.b .d .f) must obey the preorder ($b \leq d \leq f$).

For each sign class we can now calculate their respective representation values, i.e. the sum of the values of the triadic and the trichotomic prime signs (cf. Bense 1981, pp. 17 ss.). Therefore we get the following network:

3.1 2.1 1.1	Rpw = 9	
3.1 2.1 1.2	Rpw = 10	
3.1 2.1 1.3	Rpw = 11	┌───┐
3.1 2.2 1.2	Rpw = 11	└───┘
3.1 2.2 1.3	Rpw = 12	┌───┐
3.1 2.3 1.3	Rpw = 13	└───┘
3.2 2.2 1.2	Rpw = 12	┌───┐
3.2 2.2 1.3	Rpw = 13	└───┘
3.2 2.3 1.3	Rpw = 14	
3.3 2.3 1.3	Rpw = 15	

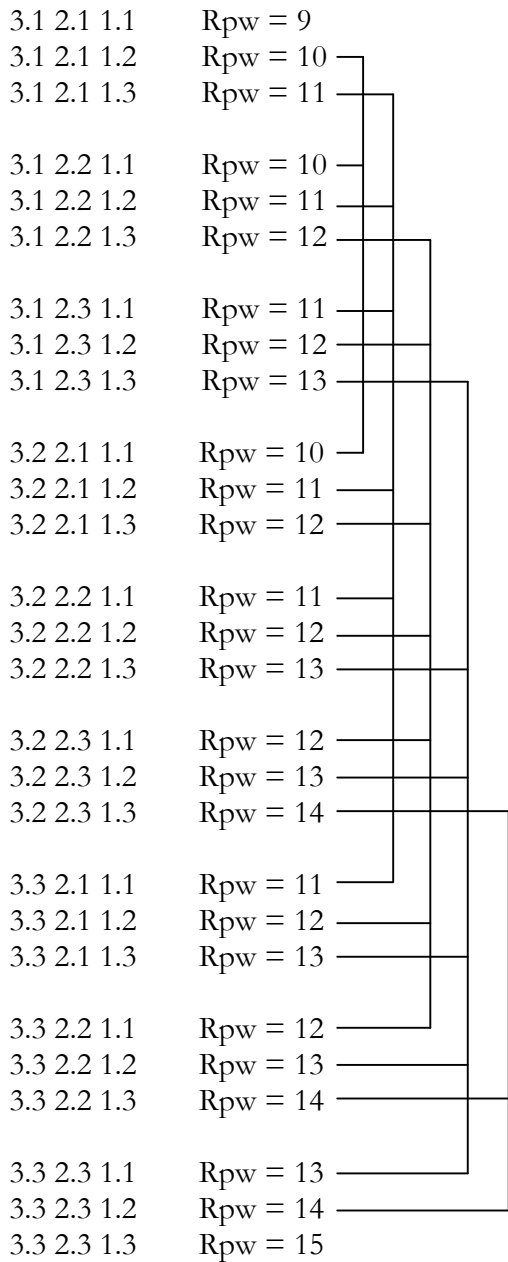
Thus, although the Law of Inclusive Trichotomic Order is based on the preorder that is required for directed sets, the system of the 10 sign classes shows only partial cofinality and does not form a net as a whole. If we write the sign classes both in horizontal and in vertical direction and connect all sign classes, which share at least one sub-sign, we get the following net that shows in its “holes” the lack of cofinality:

	1.1	1.2	1.3	1.2	1.3	1.3	1.2	1.3	1.3	1.3
	2.1	2.1	2.1	2.2	2.2	2.3	2.2	2.2	2.3	2.3
	3.1	3.1	3.1	3.1	3.1	3.1	3.2	3.2	3.2	3.3
3.1 2.1 1.1										
3.1 2.1 1.2										
3.1 2.1 1.3										
3.1 2.2 1.2										
3.1 2.2 1.3										
3.1 2.3 1.3										
3.2 2.2 1.2										
3.2 2.2 1.3										
3.2 2.3 1.3										
3.3 2.3 1.3										

3. If we now abolish the Law of Inclusive Trichotomic Order (cf. Toth 2008), thus allowing the following additional semiotic orders

- (.b < .d > .f) (.b > .d < .f)
- (.b > .d > .f) (.b = .d > .f)
- (.b > .d = .f),

we get a system of 27 sign classes, which can be connected in the following network:



Thus, unlike the system of the 10 sign classes, the system of 27 sign classes not obeying the Law of Inclusive Trichotomic Order is cofinal, however, the first and the last sign-class, i.e. the representation schemes with the lowest and the highest representation value, excluded. We can now construct the same kind of network as we did above for the system of the 10 sign classes:

In conclusion: Although the lack of the Law of Inclusive Trichotomic Order allows types of semiotic order that are not preorders in the system of the 27 sign classes, this system show cofinality in the largest part of its network, while the system of the 10 sign classes whose semiotic order is preorder nevertheless lacks cofinality but in a sub-net of two of its sign classes. This result is another hint towards using both the smaller of the larger semiotic system in order to get more general results about the formal structure of semiotics.

Bibliography

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Toth, Alfred, Semiotic symmetry and the question of identity. Ch. 1 (2008)

von Querenburg, Boto, Mengentheoretische Topologie. 3rd ed. Berlin 2001

©2008, Prof. Dr. Alfred Toth