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Partitions of semiotic sets

1. The number of partitions of a set with n elements is indicated by the Bell numbers. The Bell numbers are the sums of the Stirling numbers of the second kind which count the number of ways to partition a set of n elements into k nonempty subsets. The inverses of the Stirling numbers of the second kind are called Stirling numbers of the first kind and count the number of ways to permute a set of n elements into k orbits or cycles. Generally, a partition of a set X is a set of nonempty subsets of X such that every element x in X is in exactly one of these subsets. Therefore, the union of the elements x of X is equal to X, and the intersection of any two elements of X is empty, i.e. the elements of X are pairwise disjoint (Brualdi 2004).

2. In this study, we first show the semiotic relevance of the Stirling numbers of the first kind. s(n, k) determines the number of permutations of n elements into k cycles:

n \ k	0	---1	2	3	4	5	6
0	1						
1	0	---1					
2	0	----1	1				
3	0	---2	-3	1			
4	0	----6	11	-6	1		
5	0	---24	-50	35	-10	1	
6	0	----120	274	-225	85	-15	1

If we take $SR_{2,2}$ = {(1.1), (1.2), (2.1), (2.2)} (cf. Ditterich 1990, pp. 29, 81; Toth 2008b), we get f. ex.:

s(4, 2) = 11:

1. {{(1.1), (2.1), (1.2)}, {(2.2)}}
2. {{(1.1), (1.2), (2.1)}, {(2.2)}}
3. {{(1.1), (2.2), (1.2)}, {(2.1)}}
4. {{(1.1), (1.2), (2.2)}, {(2.1)}}
5. {{(1.1), (1.2)}, {(2.1), (2.2)}}
6. {{(1.1), (2.2), (2.1)}, {(1.2)}}
7. {{(1.1), (2.1), (2.2)}, {(1.2)}}
8. {{(1.1), (2.1)}, {(1.2), (2.2)}}
9. {{(1.1), (2.2)}, {(1.2), (2.1)}}
10. {{(1.1)}, {(1.2), (2.2), (2.1)}}
11. {{(1.1)}, {(1.2), (2.1), (2.2))}

Since $SR_{3,3}$ = {(1.1), (1.2), (1.3), (2.1), (2.2), (2.3), (3.1), (3.2), (3.3)} has 9 sub-signs, s(9, 2) = 109’584 gives the number of the possible cycles of pairs of dyads of a semiotic set with 9 dyadic sub-signs. The Stirling numbers of the first kind also predict that there are no less than s(9, 1) = 40’320 cycles of dyadic sub-signs and s(9, 3) = 118’124 cycles of triads for a set with 9 dyadic sub-signs. Since s(6, 1) = 120, s(6, 2) = 274, s(6, 3) = 225, s(6, 4) = 85, s (6, 5) = 15, and s(6, 6) = 1, we obtain totally an amount of 720 cycles of the set of the 6 transpositions of each sign class and each reality thematic (cf. Toth 2008a, pp. 159 ss.), thus 1'440 cycles for the respective partition of the hexadic semiotic sets, and thus for all 10 sign classes and reality thematics no less than 14'400. How important semiotic cycles are, can be seen in some of my previous studies (e.g., Toth 2008c-i).

2. Second, we will have a look at the Stirling numbers of the second kind S(n, k), which count the number of ways to partition a set of n elements into k nonemtpy subsets. The sum

$$B_n = \sum_{k=1}^{n} S(n, k)$$

is the nth Bell number. The following table gives the first Stirling numbers of the second kind together with their Bell numbers:

n \ k	0	---1	2	3	4	5	6	B_n
0	1	---						1
1	0	---1						1
2	0	---1	1					2
3	0	---1	3	1				5
4	0	---1	7	6	1			15
5	0	---1	15	25	10	1		52
6	0	---1	31	90	65	15	1	203

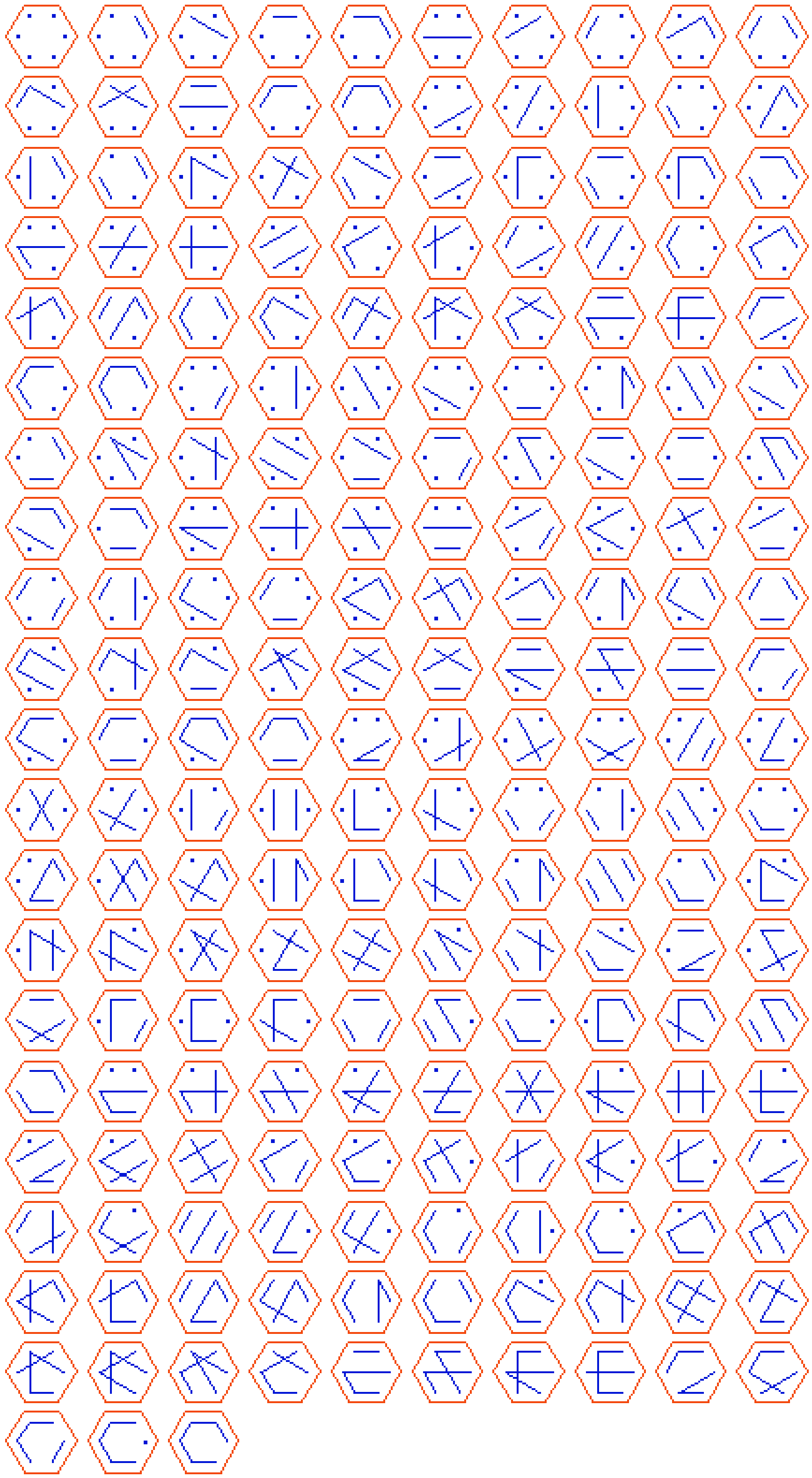
Thus, the set $SR_{2,2}$ = {(1.1), (1.2), (2.1), (2.2)} has S(4, 1) = 1 partition into a set with 1 element, S(4, 2) = 7 partitions into sets with 2 elements, S(4, 3) = 6 partitions into sets with 3 elements, and S(4, 4) = 1 partition into a set with 4 elements, thus together 15 partitions. The set $SR_{3,3}$ = {.1., .2., .3.}, or simplified {1, 2, 3}, has the following B_3 = 5 partitions:

S(3, 1) = 1 = {1, 2, 3}
S(3, 2) = 3 = {{1, 2}, {3}}, {{1, 3}, {2}}, {{2, 3}, {1}}
S(3, 3) = 1 = {{1}, {2}, {3}}

The sets $SR_{4,3}$ = (0., .1., .2., .3.) and $SR_{4,4}$ = (.0., .1., .2., .3.) or shortly {0, 1, 2, 3} have the same number B_4 = 15, because their partitions are exclusively defined over their triadic, but not over their trichotomic values! Thus, we get for both sign relations:

S(4, 1) = 1 = {0, 1, 2, 3}
S(4, 2) = 7 = {{0, 3}, {1, 2}}, {{0}, {1, 2, 3}}, {{0, 1, 3}, {2}}, {{0, 2}, {1, 3}},
{{0, 1, 2}, {3}}, {{0, 2, 3}, {1}}, {{0, 1}, {2, 3}}
S(4, 3) = 6 = {{0}, {1, 2}, {3}}, {{0, 3}, {1}, {2}}, {{0}, {1, 3}, 2}}, {{0, 2}, {1}, {3}},
{{0, 1}, {2}, {3}}, {{0}, {1}, {2, 3}}
S(4, 4) = 1 = {{1}, {2}, {3}, {4}}

The sets of the 6 transpositions of each sign class and their dual reality thematics from $SR_{3,.3}$ has Bell number B_6 = 203 (Dickau 2008):



Therefore, the complete system of triadic-trichotomic semiotics contains $10 \cdot 406 = 4'060$ partitions, amongst them 1 monadic, 31 dyadic, 90 triadic, 65 tetradic, 15 pentadic, and 1 hexadic partition for each of the 10 sign classes and 10 reality thematics.

Thus, about partitions, the same it to say like what we have remarked about derangements (Toth 2008j), namely that they increase enormously the traditionally known structures of theoretical semiotics (cf. Bense 1975).

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