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Semiotics as point-free geometry

1. According to Gerla and Miranda (2008), point-free geometry is a geometry, whose primitive ontological notion is region rather than point. Point-free geometry was founded by Alfred N. Whitehead (1919, 1920), not as a space-time geometry, but as a theory of events and on the extension relations between events. Semiotics can be connected with the theory of point-free geometry, because "the most essential consideration is, that a sign as a triadic relation is not only a static configuration, but, at the same time, fixes a semiotic process, the so-called semiosis" (Bense 1986, p. 123).

2. Point-free geometry is based on the fundamental primitive binary relation ≤, which is known from mereology as "parthood" (cf. Toth 2008b) and which means in semiotics "is of lower and even semioticity". Using the semiotic matrix, we can show the system of the sub-signs of triadic-trichotomic semiotics, the semioses between them and their inclusion relations. Notice, that the binary semiotic relation ≤ is defined here not only between trichotomies, but also between triads:

$$
\begin{array}{l|ccccc}
 & .1 & & .2 & & .3 \\
\hline
1. & 1.1 & < & 1.2 & < & 1.3 \\
 & \wedge & & \wedge & & \wedge \\
2. & 2.1 & < & 2.2 & < & 2.3 \\
 & \wedge & & \wedge & & \wedge \\
3. & 3.1 & < & 3.2 & < & 3.3
\end{array}
$$

3. In the following we give the seven axioms G1 – G7, which hold for point-free geometry, from Gerla and Miranda (2008):

3.1. Inclusion partially orders the domain:

G1: $x \leq x$ (reflexive)
G2: $(x \leq z \wedge z \leq y) \rightarrow x \leq y$ (transitive)
G3: $(x \leq y \wedge y \leq x) \rightarrow x = y$ (anti-symmetric)

For semiotic examples, take any $x \in \{1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3\}$.

3.2. Given any two regions, there exists a region that includes both of them:

G4: $\exists z\, [x \leq z \wedge y \leq z]$

E.g., $(1.3) \geq (1.1) \wedge (1.3) \geq (1.2)$.

3.3. Proper Part densely orders the domain:

G5: $x < y \rightarrow \exists z\ [x < z < y]$

E.g., x = (3.1 2.1 1.1), y = (3.2 2.2 1.2); then, there is a z = (3.3 2.3 1.3), where (3.1 2.1 1.3) < (3.2 2.2 1.2) < (3.3 2.3 1.3).

3.4. Both atomic regions and a universal region do not exist. Hence the domain has neither an upper nor a lower bound:

G6: $\exists yz\ [y < x \wedge x < z]$

Although lower and upper bound can be easily introduced to semiotics (cf. Berger 1976), the existence of atomic regions is controversial, because each "atomic" monadic prime-sign must occur in a dyadic sub-sign, and each dyadic sub-sign must occur in a triadic sign class or reality thematic. As for a semiotic universal region, this notion is controversial, too, since no sign occurs as a single one, due to the auto-reproduction of the interpretant-relation as a triadic sign-relation by itself (cf. Bense 1976, pp. 163 s.).

3.5. Proper Parts Principle: If all the proper parts of x are proper parts of y, then x is included in y:

G7: $\forall z\ [z < x \rightarrow z < y] \rightarrow x \leq y$

E.g., x = (3.1 2.2), then all proper parts of x are proper parts of y = (3.1 2.2 1.3), and x is included in y.

Gerla and Miranda call a model of G1 – G7 an inclusion space. Since G1 – G7 hold both for prime-signs, for sub-signs and for sign classes and reality thematics, we can speak of monadic, dyadic and triadic semiotic inclusion spaces. Most important is the following definition by Gerla and Miranda (2008, def. 4.1):

Definition: Given some inclusion space, an abstractive class is a class G of regions such that G is totally ordered by inclusion. Moreover, there does not exist a region included in all of the regions included in G.

Since all sign classes (3.a 2.b 3.c) and all reality thematics (c.3 b.2 a.3) are totally ordered by ≤ (a ≤ b ≤ c), it follows that each sign class is such an abstractive class G. In Toth (2008a, p. 28), it was shown that not all sign classes are connected to all sign classes (and not all reality thematics are connected to all reality thematics). It follows that there are dyadic part-relations of the triadic-trichotomic sign relations that are not included in all of the regions included in the semiotic region G. As abstractive classes define geometrical entities whose dimensionalities are less than that of the inclusion space, in semiotics, monadic relations can be defined as 1-dimensional semiotic relations, dyadic relations as 2-dimensional semiotic relations, and triadic relations as 3-dimensional semiotic relations (cf. Toth 2007, pp. 11).

4. In his 1929 book, Whitehead tried to build up another approach for a point-free geometry, using the topological notions of "contact" and "connect relation" between two regions as basic items. The primitive binary relation "connection" is abbreviated by C; therefore, $x \leq y \leftrightarrow \forall z\ [Czx \rightarrow Czy]$ means that x is included in y. Unlike the case with inclusion spaces, connection theory enables defining non-tangential inclusion, a total order that enables the construction of abstractive classes, and thus also of sign classes and reality thematics.

Connection theory is based on the following six axioms C1 – C6, shown in Gerla and Miranda (2008):

4.1. C is reflexive:

C1: Cxx

For semiotic examples, take any $x \in \{1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3\}$.

4.2. C is symmetric:

C2: $Cxy \rightarrow Cyx$

E.g., if (2.1) and (1.3) are connected, f.ex. in the dyadic relation (1.3 2.1), then (1.3) and (2.1) are connected, too, f.ex. in the dyadic relation (2.1 1.3).

4.3. C is extensional

C3: $\forall z\ [Czx \leftrightarrow Czy] \rightarrow x = y$

E.g., if the sub-sign z = (3.) is connected with the sub-sign x = (.1), and this connection (3.1) $\leftrightarrow$ C(3.y), then it follows that (.1) = y.

4.4. All regions have proper parts, so that C is an atomless theory:

C4: $\exists y\ [y < x]$

For semiotics, C4 makes only sense if one is aware that no monadic relation can occur outside of a dyadic relation, and no dyadic relation can occur outside of a triadic relation, so that for each x that is a dyadic sub-sign, there is always a prime-sign which is a proper part of it, and if x is a triadic sign-class or reality thematics, then there is always a sub-sign which is a proper part of it.

4.5. Given any two regions, there is a region connected to both of them:

C5: $\exists z\ [Czx \wedge Czy]$

Since not all sign classes and not all reality thematics are conncted to one another, e.g., the three (homogeneous) main sign classes and their main reality thematics are not connected to one another: (3.1 2.1 1.1) ∩ (3.2 2.2 1.2) = ∅, (3.1 2.1 1.1) ∩ (3.3 2.3 1.3) = ∅, (3.2 2.2 1.2) ∩ (3.3 2.3 1.3) = ∅, in semiotics, C5 is universally valid only on the level of the monadic prime-signs.

4.6. All regions have at least two unconnected parts:

C6: $\exists yz\ [(y \leq x) \wedge (z \leq x) \wedge \neg Cyz]$

E.g., if y = (2.), x = (3.), and z = (1.), then (2.) ≤ (3.) and (1.) ≤ (3.), and there is no connection (2.) ≤ (1.). Note, however, that C6 does not hold for the sign classes, since there are sign classes which are connected by two sub-signs (e.g., (3.2 2.2 1.2) and (3.2 2.2 1.3)).

Gerla and Miranda (2008) call a model of C1 – C6 a connection space. As we have shown under the single axioms, there are monadic and dyadic semiotic connection spaces, but there is no triadic connection space.

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