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Points in extensional sign-connections

1. The present study is a sequel of Toth (2008a), which is based on Clarke (1981). In another study, Clarke extended his "Calculus of individuals based on 'connection'" to "Individuals and points" (Clarke 1985), whose definitions, axioms and theorems we will follow here. "Although the predicate, 'x is connected with y', is taken as primitive and undefined, heuristically we would like it to be the case that two spatio-temporal regions are connected if, and only if, they a have a spatio-temporal point in common" (Clarke 1985, p. 62). As for Clarke (1981), for Clarke (1985), too, the basic logical theory is a classical first-order quantification theory with identity. The lower case letters x, y, z stand for individuals ranging over spatio-temporal regions, the upper case letters X, Y, Z are individual variables ranging over sets of spatio-temporal regions.

2. The traditional mereological predicates "x is part of y", "x is a proper part of y", "x overlaps y", and "x is discrete from y", as well as the mereological predicates "x is externally connected to y", "x is a tangential part of y", and "x is a non-tangential part of y", and "the interior of x" and "the closure of x" are defined as follows (Clarke 1985, p. 62):

D0.1 $Px,y := \forall z\ (Cz,x \rightarrow Cz,y)$
D0.2 $PPx,y := Px,y \wedge \neg Py,x$
D0.3 $Ox,y := \exists z\ Pz,x \wedge Pz,y$
D0.4 $DRx,y := \neg Ox,y$
D0.5 $ECx,y := Cx,y \wedge \neg Ox,y$
D0.6 $TPx,y := Px,y \wedge \exists z\ (ECz,x \wedge ECz,y)$
D0.7 $NTPx,y := Px,y \wedge \neg \exists z\ (ECz,x \wedge ECz,y)$

By the following definitions, we introduce f'X as "the fusion of a set of region" (which will itself be a region), x + y as "the union" or "sum of x and y", -x as "the complement of x", x ∧ y as "the intersect of x and y", and a* as "the universal individual":

D1.1 $x = f'X := \forall y\ [Cyx \equiv \exists z\ (z \in X \wedge Cy,z)]$
D1.2 $x + y := f'\{z: Pz,x \vee Pz,y\}$
D1.3 $-x := f'\{y: \neg Cy,x\}$
D1.4 $a^* := f'\{y: Cy,y\}$
D1.5 $x \wedge y := f'\{z: Pz,x \wedge Pz,y\}$
D2.1 $ix := f'\ \{y: NTPy,x\}$
D2.2 $cx := f'\ \{y: \neg Cy,i\text{-}x\}$

Moreover, we need the following mereological axioms:

A0.1 $\forall x\ [Cx,x \wedge \forall y\ (Cx,y \rightarrow Cy,x)]$
A0.2 $\forall x \forall y\ [\forall z\ (Cz,x \equiv Cz,y) \rightarrow x = y]$

A1.1 $\forall X (\neg X = \Lambda \rightarrow \exists x\ x = f'X)$

For semiotic examples for all definitions and axioms cf. Toth (2008a). At the hand of the above basis, Clarke (1985, p. 64) gives the following definition of "X is a point":

D3.1 $PT(X) := \forall x \forall y \{(x \in X \wedge y \in X) \rightarrow [ECx,y \vee (Ox,y \wedge (x \wedge y \in X)]\} \wedge \forall x \forall y [(x \in X \wedge Px,y) \rightarrow y \in X] \wedge \forall x \forall y [x + y \in X \rightarrow (x \in X \vee y \in X)] \wedge \neg X = \Lambda$

Thus, a semiotic point, like a logical point, can result either from external connection or overlapping (cf. Toth 2008a).

The following definition introduces the notion "point X is incident in region x":

D3.2 $IN(X,x) := PT(X) \wedge x \in X$

Thus, in semiotics, any sign relation can be a semiotic region; in a trivial sense, a point can be its own region. Note that sub-signs are introduced by Bense (1976, p. 123) as both static and dyanamic configurations, so that any sub-sign of the form (a.b) with a $\in$ {1., 2., 3.} (triadic values) and b $\in$ {.1, .2, .3} (trichotomic values) can be defined as point. The smallest regions are then the pairs of dyads of the general form ((a.b), (c.d)) and the triads of the general form (((a.b), (c.d)), (e.f) for sign classes, or ((a.b), ((c.d), (e.f)) for reality thematics. According to D3.2 we then have, e.g., in simplified notation: IN((3.1 2.2), (2.2)) or IN((3.1 2.2 1.3), (1.3)).

The following axiom establishes the existence of points:

A3.1 $\forall x \forall y [Cx,y \rightarrow \exists X (PT(X) \wedge x \in X \wedge y \in X)]$

3. In displaying the following 47 theorems built over the definitions and axioms, we follow Clarke (1985, pp. 64 ss.).

T3.1 $\forall x \forall y \forall X\ \{PT(X) \wedge x \in X \wedge y \in X) \rightarrow \{ECx,y \vee (Ox,y \wedge (x \wedge y) \in X)]\}$

Cf. D3.1.

T3.2 $\forall x \forall y \forall X [PT(X) \wedge x \in X \wedge Px,y) \rightarrow y \in X]$

E.g., if there is a semiotic point X, and if a sign x is an element of that point, and if x is a part of the sign y, then y is an element of the semiotic point X, too.

T3.3 $\forall x \forall y \forall X [(PT(X) \wedge x + y \in X) \rightarrow (x \in X \vee y \in X)]$

E.g, if there is a semiotic point X, and the intersect of two signs x and y is and element of X, then either x or y is an element of X.

T3.4 $\forall x \forall y \forall X\ [(PT(X) \wedge x \in X \wedge y \in X) \rightarrow Cx,y]$

E.g., If there is a semiotic point X, and if both a sign x and a sign y are elements of X, then x is connected with y.

T3.5 $\forall x \forall y \forall X\ [(PT(X) \wedge x \in X) \rightarrow x + y \in X]$

E.g., If there is a semiotic point X, and the sign x is element of X, then the intersect of the signs x and y is (also) element of X,

T3.6 $\forall x \forall y \forall X\ \{PT(X) \rightarrow [(x \in X \vee y \in X) \equiv x + y \in X]\}$

E.g., If either a sign x or a sign y are element of a semiotic point X, then the intersect of x and y is necessarily an element of X.

T3.7 $\forall X\ (PT(X) \rightarrow \exists x\ x \in X$

E.g., a semiotic point cannot be empty.

T3.8 $\forall X\ (PT(X) \rightarrow a^* \in X)$

E.g., if there is a semiotic point X, it follows that the semiotic universal individual is element of X.

T3.9 $\forall x \forall X\ [PT(X) \rightarrow \neg(x \in X \wedge -x \in X)]$

E.g., if there is a semiotic point X, then it is not possible that both a sign x and its complement are element of X.

T3.10 $\forall x \forall X\ [PT(X) \rightarrow (x \in X \vee -x \in X)]$

E.g., if there is a semiotic point X, then either a sign x or its complement are element of X.

T3.11 $\forall x \forall X\ [PT(X) \rightarrow (x \in X \equiv \neg\neg x \in X)]$

E.g., the complement of the complement of a sign x is identical to the sign x.

T3.12 $\forall x \forall X\ \{PT(X) \rightarrow [\forall z\ z \in X \rightarrow Cz,x) \equiv x \in X]\}$

E.g., if a sign z is an element of a semiotic point X, and if z is connected to a sign x, then x is an element of X.

T3.13 $\forall x\ \exists X\ IN(X,x)$

E.g., for each sign x there is a semiotic point X, so that X is incident in region x.

T3.14 $\forall x \forall y\ [Cxy \equiv \exists X\ (PT(X) \wedge IN(X,x) \wedge IN(X,y))]$

E.g., if a sign x is connected to a sign y, then there is necessarily a semiotic point X, and X is incident both in x and in y.

T3.15 $\forall x \forall y\ [Ox,y \equiv \exists X\ (PT(X) \wedge IN(X,x) \wedge IN(X,y) \wedge \neg ECx,y)]$

E.g., if a sign x overlaps a sign y, then there is necessarily a semiotic point X, and X is incident both in x and in y, and x is not externally connected to y.

T3.16 $\forall x \forall y\ [ECx,y \equiv \exists X\ (PT(X) \wedge IN(X,x) \wedge IN(X,y) \wedge \neg Ox,y)]$

E.g., if a sign x is externally connected to a sign y, then there is necessarily a semiotic point X, and X is incident both in x and in y, and x does not overlap y.

T3.17 $\forall x \forall y\ \{Px,y \equiv \forall X\ [(PT(X) \wedge IN(X,x)) \rightarrow IN(X,y)\}$

E.g., if a sign x is part of a sign y, then there is necessarily a semiotic point X, and X is incident both in x and in y.

T3.18 $\forall x \forall X\ [(PT(X) \wedge IN(X,ix)) \rightarrow IN(X,x)]$

E.g., if there is a semiotic point X, and if X is incident in the interior of a sign x, then X is also incident in x.

T3.19 $\forall x \forall X\ [(PT(X) \wedge IN(X,x)) \rightarrow (\exists z\ z = -x \rightarrow IN(X,cx)]$

E.g., if there is a semiotic point X, and if X is incident in a sign x, then X is incident with the closure of x for the complement of x.

T3.20 $\forall x \forall X\ \{[PT(X) \wedge IN(X,x) \wedge \neg\exists z\ z \in X \wedge ECz,x] \rightarrow IN(X,ix)\}$

E.g., if there is a semiotic point X, and if X is incident in a sign x, and if there is no sign z ∈ X, so that z is externally connected to x, then X is incident in the interior of x.

T3.21 $\forall x \forall X\ \{[PT(X) \wedge IN(X,x) \wedge \exists z\ (z \in X \wedge ECz,x)] \rightarrow \neg IN(X,ix)\}$

E.g., if there is a semiotic point X, and if X is incident in a sign x, and if a sign z is externally connected to x, then X is not incident in the interior of x.

4. In order to enlighten the relation between regions and sets of points incident in particular regions, Clarke (1985, p. 65 ss.) next introduces X°, Y°, Z° as variables ranging over sets of regions as well as over sets of points; V° stands for the set of all points, ⌉X° for the complement of X° restricted to the set of all points, and P(x) for the set of all the points incident in the region x:

D3.3 $V^{\circ} := \{X: PT(X)\}$
D3.4 $\rceil X^{\circ} := V^{\circ} \cap -X^{\circ}$
D3.5 $P(x) := \{X: PT(X) \wedge x \in X\}$

The following theorems based on these additional definitions, are again numbered in the order of Clarke (1985, p. 66). Most of them need no semiotic example, since their semiotic validity is clear:

T3.22 $\rceil V^{\circ} = \Lambda$
T3.23 $V^{\circ} = P(a^*)$
T3.24 $\forall x\ P(x) \subseteq P(a^*)$
T3.25 $\rceil P(a^*) = \Lambda$
T3.26 $\forall x\ \rceil P(x) = P(-x)$
T3.27 $\forall y \forall y\ P(x \wedge y) \subseteq P(x) \cap P(y)$
T3.28 $\forall x \forall y\ (\neg ECx,y \rightarrow P(x) \cap P(y) = P(x \wedge y))$

E.g., if a sign x is not externally connected to a sign y, then x and y intersect.

T3.29 $\forall x \forall y\ P(ix) \cap P(iy) = P(ix \wedge iy)$
T3.30 $\forall x \forall y\ P(x) \cup P(y) = P(x + y)$

Clarke now introduces an interior operator I, on the subsets of V°, which associates with each set of points the set of all its interior points (1985, p. 66):

D3.6 $IX^{\circ} = Y^{\circ} := \exists x\ \exists y\ (X^{\circ} = P(x) \cap P(y) \wedge Y^{\circ} = P(ix) \cap P(iy) \vee$
$[Y^{\circ} = \Lambda \wedge \neg \exists x\ \exists y\ (X^{\circ} = P(x) \cap P(y) \wedge Y^{\circ} = P(ix) \cap P(iy))]$

Therefore, the interior of a set of semiotic boundary points is identical to the semiotic null set.

T3.31 $\forall x\ IP(x) = P(ix)$

E.g., the interior of a set of semiotic points incident in semiotic region x is even to the set of points incident in the interior of the region x.

T3.32 $IV^{\circ} = V^{\circ}$

E.g., the interior of the set of all semiotic points is this set itself.

T3.33 $\forall x \forall y\ I(P(x) \cap P(y)) = IP(x) \cap IP(y)$

E.g., the interior of the intersection of two semiotic sets of the points incident in the region x is even to the intersection of the interior of x and the interior of y.

T3.34 $\forall x\ IP(x) \subseteq P(x)$

E.g., the interior of the semiotic set of points incident in x is contained or identical to this set itself.

T3.35 $\forall x\ IIP(x) = IP(x)$

E.g., the interior of the interior of a semiotic set of points incident in x is identical to the (simple) interior of this set.

T3.36 $I\Lambda = \Lambda$

T3.37 $\forall x \forall y\ (ECx,y \rightarrow I(Px) \cap P(y)) = \Lambda)$

E.g., if a sign x is externally connected to a sign y, then the intersection of the two semiotic sets of points (incident in x and in y, respectively), is even to Λ.

T3.38 $CP(a^*) = P(a^*)$

E.g, the closure of the semiotic set of points incident in a* is identical to this set.

T3.39 $\forall x\ CP(x) = P(cx)$

E.g., the closure of the semiotic set of points incident in x is identical to the semiotic set of points incident in the closure of x.

T3.40 $C\Lambda = C\Lambda$

E.g., since the interior of Λ is identical to Λ (cf. T3.36), the closure of Λ is identical to Λ, too.

T3.41 $\forall x\ P(x) \subseteq CP(x)$

E.g., the set of all the semiotic points incident in the region x is contained in or identical with the closure of this set.

T3.42 $\forall x \forall y\ C(P(x) \cup P(y)) = CP(x) \cup CP(y)$

E.g., the closure of the union of the semiotic set of the points incident in x and the set of the points incident in y is even to the union of the closures of the two sets.

T3.43 $\forall x\ CCP(x) = CP(x)$

E.g., the closure of the closure of a semiotic set of points incident in x is identical to the (simple) closure of this set; cf. T3.35.

The following theorems deal with the relationship between the regions and their mereological relations and the sets of points incident in regions and their topological operators (Clarke 1985, pp. 67 s.):

T3.44 $\forall x \forall y$ (Cxy $\equiv \neg$P(x) $\cap$ P(y) = Λ)
T3.45 $\forall x \forall y$ (Oxy $\equiv$ IP(x) $\cap$ IP(y) = Λ)
T3.46 $\forall x \forall y$ [ECx,y $\equiv$ ($\neg$P(x) $\cap$ P(y) = Λ $\wedge$ IP(x) $\cap$ IP(y) = Λ)]
T3.47 $\forall x \forall y$ (Px,y $\equiv$ P(x) $\subseteq$ P(y)))

E.g., if a sign x is part of a sign y, then the semiotic set of points incident in x is necessarily contained in or identical with the set of points incident in y.

D2.6 SPx,y := $\neg$Ccx,y $\wedge$ $\neg$Cx,cy

E.g., a sign x is separated from a sign y, iff there is neither the closure of x connected to y, nor is x connected to the closure of y.

D2.7 CONx := $\neg(\exists z)$ $(\exists y)$ (z + y = x $\wedge$ SPz,y)

E.g., x is a connected individual means that the union of two signs z and y cannot hold, if the two signs are separated.

5. In a next step, Clarke (1985) establishes a time-order analogous to the topological order. To respective semiotic attempts cf. Toth (2008b and 2008c): "In the beginning of the present paper [Clarke 1985, A.T.], we allowed our lower case variables to range over spatio-temporal regions. The interesting question arises: Can the temporal ordering of regions be mirrored in the ordering of points somewhat analogous to the way in which we have seen the topological properties mirrored? In order to examine this possibility, let us add to our calculus of individuals another two-place primitive predicate, 'Bx,y', to be taken as a rendering of 'x is wholly before y'" (Clarke 1985, p. 69); cf. the following axioms:

A4.1 $\forall x$ {$\neg$Bx,x $\wedge$ $\forall y \forall z$ [(Bx,y $\wedge$ Byz) $\rightarrow$ Bx,z]}

E.g., the reflexivity and transitivity, already shown for semiotics in Toth (1996).

A4.2 $\forall x \forall y$ (Bx,y $\rightarrow$ {$\neg$Cx,y $\wedge$ $\forall z \forall w$ [Pz,x $\wedge$ Pw,y) $\rightarrow$ Bz,w]})

This axiom "relates the new primitive relation to the mereological relations in such a way as to characterize the relation as *wholly* before, rather than *partially* before" (Clarke 1985, p. 70). With Bx,y, we can also define "x is after y", "x is contemporaneous with y", "x is partially contemporaneous with y", "x is partially before y", and "x is partially after y":

D4.1 Ax,y := By,x
D4.2 COx,y := $\forall z$ [Pz,x $\rightarrow$ $\neg$(Bz,y $\vee$ Az,y)] $\wedge$ [Pz,y $\rightarrow$ $\neg$(Bz,x $\vee$ Az,x)]
D4.3 PCx,y := $\exists z \exists w$ (Pz,x $\wedge$ Pw,y $\wedge$ COz,w)
D4.4 PBx,y := $\exists z$ (Pz,x $\wedge$ Bz,y)

D4.5 $PAx,y := \exists z\ (Pz,x \wedge Az,y)$

The following theorems are based again on Clarke (1985, pp. 70 ss.):

T4.1 $\forall x\ \neg Bx,x$

T4.2 $\forall x \forall y\ [(Bx,y \wedge By,z) \rightarrow Bx,z]$

E.g., if (1.1) is before (1.2), and (1.2) is before (1.3), than (1.1) is before (1.3)

T4.3 $\forall x \forall y\ (Bx,y \rightarrow \neg By,x)$

E.g., if (1.1) is before (1.2), then (1.2) is not before (1.1)

T4.4 $\forall x \forall y\ (Bx,y \equiv \{\neg Cx,y \wedge \forall z \forall w\ [Pz,x \wedge Pw,y) \rightarrow Bz,w]\})$

T4.5 $\forall x \forall y \forall z\ [(Px,y \wedge Bz,y) \rightarrow Bz,x]$

E.g., if a sign x is a part of a sign y, and a sign z is before y, then z is (also) before x.

T4.6 $\forall x \forall y \forall z\ [(Px,y \wedge By,z) \rightarrow Bx,z]$

E.g., if a sign x is a part of a sign y, and the sign y is before a sign z, then x is (also) before z.

T4.7 $\forall x \forall\ (Bx,y \rightarrow \neg Px,y)$

E.g., if a sign x is before a sign y, it follows that x is not a part of y.

T4.8 $\forall x \forall y\ [\forall z\ (Pz,x \rightarrow Bz,y) \equiv Bx,y]$

E.g., that a sign x is before a sign y means, that whenever a sign z is a part of x, then z is before y.

T4.9 $\forall x\ \neg Ax,x$

Cf. T4.1.

T4.10 $\forall x \forall y \forall z\ [(Ax,y \wedge Ay,z) \rightarrow Ax,z]$

Cf. T4.2.

T4.11 $\forall x \forall y\ (Ax,y \rightarrow \neg Ay,x)$

Cf. T4.3.

T4.12 $\forall x \forall y\ (Ax,y \equiv \{\neg Cx,y \wedge \forall z \forall w\ [(Pz,x \wedge Px,y) \rightarrow Az,w]\})$

T4.13 $\forall x \forall y\ (Ax,y \rightarrow Px,y)$

E.g., if a sign x is before a sign y, then x is a part of y.

T4.14 $\forall x \forall y \forall z\ [(Px,y \wedge Ay,z) \rightarrow Ax,z]$

E.g., if a sign x is a part of a sign y, and the sign y is before the sign z, then x is before z.

T4.15 $\forall x \forall y \forall z\ [(Px,y \wedge Az,y) \rightarrow Az,x]$

E.g., if a sign x is a part of a sign y, and if a sign z is before y, then z is before x, too.

T4.16 $\forall x \forall y\ [\forall z\ (Pz,x \rightarrow Az,y) \equiv Ax,y]$

E.g., a sign x is before a sign y, whenever a sign z is a part of x implies that z is before y.

T4.17 $\forall x\ COx,x$
T4.18 $\forall x \forall y\ (COx,y \equiv COy,x)$
T4.19 $\forall x \forall y\ (Px,y \rightarrow PCx,y)$

E.g., if a sign x is a part of a sign y, then x is partially contemporaneous with y.

T4.20 $\forall x\ PCx,x$
T4.21 $\forall x \forall y\ (PCx,y \equiv PCy,x)$

E.g., if a sign x is partially contemporaneous with a sign y, then y is also partially contemporaneous with x.

T4.22 $\forall x \forall y\ (COx,y \rightarrow PCx,y)$

E.g., if a sign x is contemporaneous to a sign y, then x is also partially contemporaneous to y.

T4.23 $\forall x \forall y\ [\neg COx,y \equiv (PBx,y \vee PAx,y)]$

E.g., if a sign x is not contemporaneous to a sign y, then x is either partially before of partially after y.

T4.24 $\forall x \forall y \forall z\ [Bx+y,z \rightarrow (Bx,z \wedge By,z)]$

E.g., if the union of two signs x and y are before a sign z, then both x and y are before z.

T4.25 $\forall x \forall y \forall z\ [Bz,x+y \rightarrow (Bz,x \wedge Bz,y)]$

E.g., if a sign z is before the union of two signs x and y, then z is before x as well as before y.

T4.26 $\forall x \forall y \forall z \{\exists w\ w = x \wedge z \rightarrow [Bx,y \rightarrow Bx{\wedge}z,y]\}$

E.g., if a sign w is the intersect of two signs x and z, then both x is before y and as well as the intersect of x and z-

T4.27 $\forall x \forall y \forall z \{(\exists w\ w = y \wedge z \rightarrow [Bx,y \rightarrow Bx,y{\wedge}z]\}$

E.g., if a sign w is the intersection of two signs y and z, then x is both before y and before the intersection of y and z.

T4.28 $\forall x\ (\neg Bx,a^* \wedge \neg Ax,a^*)$

E.g., there is no sign before a*, nor after a*.

T4.29 $\forall x\ PCx,a^*$

E.g., all signs are partially contemporaneous with a*

T4.30 $\forall x \forall y\ [Bx,y \rightarrow (Bx,iy \wedge Bix,y \wedge Bix,iy)]$

E.g., if a sign x is before a sign y, then x is before the interior of y, the interior of x is before y, and the interior of x is before the interior of y.

The following definition establishes a temporal ordering relation between points (Clarke 1985, p. 71):

D5.1 $B(X,Y) := PT(X) \wedge PT(Y) \wedge \exists x \exists y\ (x \in X \wedge y \in Y \wedge Bx,y)$

E.g., a semiotic set X is before a semiotic set Y iff X and Y are semiotic points, and if there is an x and a y (e.g., x and y are signs) such that x is element of X and y is element of Y, and x is before y.

With D5.1, we can also define "point X is after point Y" and "point X is contemporary with point Y":

D5.2 $A(X,Y) := B(Y,X)$
D5.3 $C(X,Y) := PT(X) \wedge PT(Y) \wedge \neg B(X,Y) \wedge \neg A(X,Y)$

Together with these definitions, we will formulate the following two new axioms:

A5.1 $\forall x \forall y\ (\neg Bx,y \rightarrow \exists X \exists Y\ \{PT(X) \wedge PT(Y) \wedge x \in X \wedge y \in Y \wedge$
$\forall z \forall w\ [z \in X \wedge w \in Y) \rightarrow Bz,w]\})$
A5.2 $\forall x \forall y \forall X \forall Y\ \{PT(X) \wedge PT(Y) \wedge x \in X \wedge y \in Y \wedge Bx,Y) \rightarrow$
$\forall z \forall w\ [(z \in X \wedge w \in Y) \rightarrow \exists u \exists v\ (Pu,z{\wedge}u \in X \wedge Pv,w{\wedge}v \in Y \wedge Bu,v)]\}$

Cf. A2.1 and A3.1.

Furthermore, we give the following 6 theorems according to Clarke (1985, p. 72).

T5.1 $\forall X \forall Y\ [(PT(X) \wedge PT(Y)) \rightarrow B(X,Y) \vee C(X,Y) \vee A(X,Y)))]$

E.g., in the intersection of two semiotic points X and Y, X is either before Y, or contemporaneous with X, or after Y.

T5.2 $\forall X \forall Y\ (B(X,Y) \equiv \{PT(X) \wedge PT(Y) \wedge \forall x \forall y\ [x \in X \wedge y \in Y) \rightarrow$
$\exists z \exists w\ (Pz,x \wedge z \in X \wedge Pw,y \wedge w \in Y \wedge Bz,w)]\})$

T5.3 $\forall x \forall y\ \{Bx,y \equiv \forall X \forall Y\ \{(PT(X) \wedge PT(Y) \wedge x \in X \wedge y \in Y) \rightarrow B(X,Y)]\}$

T5.4 $\forall X\ \neg B(X,X)$

T5.5 $\forall X \forall Y \forall Z\ [(B(X,Y) \wedge B(Y,Z)) \rightarrow B(X,Z)]$

E.g., no semiotic set can be before itself. To transitivity in semiotics cf. Toth (1996).

T5.6 $\forall X \forall Y\ (B(X,Y) \rightarrow \neg B(Y,X))$

E.g., if the semiotic set X is before the semiotic set Y, then Y is not before X.

With the following three additional definitions, Clarke (1985, p. 73) introduces Minkowski cones into his space-time topology. "CP" stands for "the causal past of", "CF" for "the causal future of", and CO "the causal contemporaries of":

D5.4 $X° = CP'Y := X° = \{X: B(X,Y)\}$

D5.5 $X° = CF'Y := X° = \{X: A(X,Y)\}$

D5.6 $X° = CO'Y := X° = \{X: C(X,Y)\}$

For related temporal notions in connections with semiotic posets cf. Toth (2007, pp. 83 s.). For sets analogous to Carnap's (1958) world lines cf. Clarke (1985, p. 73, D5.7 and D5.8).

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