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Polycontextural matrices

1. Kaehr (2008, p. 8) has proposed the following 3-contextural 3-adic matrix:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

From the standpoint of logic and mathematics, the existence of a matrix is per se enough; nobody has ever tried to interpret, e.g. the Sylvester-Matrix in using sense and meaning. And this is good so, since traditional logic and mathematics handle signs as tokens. However, in semiotics, we use a mathematical sign which carries sense and meaning, and therefore we must try to give the motivation of every mathematical concept that is introduced in semiotics.

2. The above semiotic matrix is interesting first, because the contextural indices hang on sub-signs which are dyads, and these dyads consist of monads or what Bense (1980) called "prime-sign" in analogy to the prime-numbers. That the monads and not the dyads are basic in semiotics, we see, e.g., then, when we dualize a dyad

$\times$(a.b) = (b.a)

and realize, that its constituents, the prime-signs, are turned around. Therefore, it is necessary to ascribe contextures not only to the sub-signs, but also to the prime-signs. And here, we are free at least from a purely formal standpoint. E.g., in a 3-contextural semiotics, we have the choice:

a $\rightarrow$ 1; 2; 3; (1,2); (2,3); (1.3) (a $\in$ {.1., .2., .3.})

However, a Secondness (M$\rightarrow$O) is a relation that combines a Firstness with itself, that means (1,2). And a Thirdness (O$\rightarrow$I), consequently, is a relation that combines a Secondness with itself, that means (2,3). Now, we realize that a Firstness – quite different form Peirce's concept – is not something that stands for itself, since, for the sake of closure of the sign as a triadic relation, the Firstness is a relation, which combines itself with the whole triadic relation (1,3). Or in other words: (M→O) and (O→I) need a third mapping (M→I) for closure, so that it is impossible that Firstness as a monad stands alone, just being included in Secondness, and with Secondness in Thirdness. This has been constantly overseen in Theoretical Semiotics until Kaehr (2008) introduced the prime-signs by aid of doublets. However, since the mapping (M→I) has been known in semiotics since decades (cf. Walther 1979, p. 73), one could have seen it.

Therefore, we can introduce the 3-contextural prime-signs as follows:

$PS = \{.1._{1,3}, .2._{1,2}, .3._{2,3}\}$.

Since dyads are nothing else than Cartesian products of the prime-signs onto themselves, we get

I	$.1._{1,3}$	$.2._{1,2}$	$.3._{2,3}$
$.1._{1,3}$	$1.1_{1,3}$	1.2_{1}	1.3_{3}
$.2._{1,2}$	2.1_{1}	$2.2_{1,2}$	2.3_{2}
$.3._{2,3}$	3.1_{3}	3.2_{2}	$3.3_{2,3}$

Now, we obviously have a very special law of multiplication in this matrix. The rules are:

$(a,b) ♣ (a,b) = (a,b)$
$(a,b) ♣ (a,c) = (a,b) ♣ (c,a) = a$

However, since we are free, at least from a formal standpoint, to assign any contextures to the sub-signs, it follows that the above matrix is not the only one and that we can calculate the contextural values of any semiotic matrix. Let us look at the following "alternative" matrices:

II	$.1._{1,3}$	$.2._{1,2}$	$.3._{2,3}$
$.3._{2,3}$	1.1_3	1.2_2	$1.3_{2,3}$
$.2._{1,2}$	2.1_1	$2.2_{1,2}$	2.3_2
$.1._{1,3}$	$3.1_{1,3}$	3.2_1	3.3_3

III	$.1._{1,3}$	$.2._{1,2}$	$.3._{2,3}$
$.2._{1,2}$	1.1_1	$1.2_{1,2}$	1.3_2
$.1._{1,3}$	$2.1_{1,3}$	2.2_1	2.3_3
$.3._{2,3}$	3.1_3	3.2_2	$3.3_{2,3}$

IV	$.3._{2,3}$	$.1._{1,3}$	$.2._{1,2}$
$.2._{1,2}$	1.1_2	1.2_1	$1.3_{1,2}$
$.3._{2,3}$	$2.1_{2,3}$	2.2_3	2.3_2
$.1._{1,3}$	3.1_3	$3.2_{1,3}$	3.3_1

Up to now, the following law for converse dyadic relations held:

$(a.b_{i,j}) = (b.a_{1,j})$,

as long as both sub-signs are in the same matrix. (This restriction excludes $\times(a.b_{i,j}) = (b.a_{j,i})$.) However, there is no formal reason either, why this law can not be abolished like in the 3 matrices above.

3. Up to now, sign connections have been based on shared (static) sub-signs or (dynamic) semioses, i.e. morphims between n-tuples of sign classes or reality thematics (cf. Toth 2008), e.g.

(3.1 2.1 1.1)
  | |
(3.1 2.1 1.3),

i.e. (3.1 2.1 1.1) ∩ (3.1 2.1 1.3) = (3.1 2.1).

However, what if the two sign classes do not lie in the same contextures? Cf., e.g.,

$(3.1_3\ 2.1_1\ 1.1_{1,3}) \cap (3.1_2\ 2.1_3\ 1.3_2) = ??$

In a monocontextural world, this intersection is as senseless as Günther's famous addition of his mother's toothache, a crocodile and the Silesian church-tower is.

We therefore have to learn to apply arithmetic operations beyond the contexture-borders. For sign connections, this means that we must give up the common sub-signs and semioses and connect only such sub-signs, which lie in the same contexture(s). Thus, we no longer connect the same sub-signs or semioses, but the same contextures:

$(3.1_3 \quad 2.1_1 \quad 1.1_{1,3})$

$(3.1_2 \quad 2.1_3 \quad 1.3_2)$

From our three matrices above, we may guess what an enormous amount of different sign connections result from the free ascription of contextures to sub-signs.

4. If we take our above matrix I, we can distribute the sub-signs in the following manner to the contextures:

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)
K2	—	—	—	—	(2.2)	(2.3)	—	(3.2)	—
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—

However, if we take matrix II, the distribution looks like that:

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)
K2	—	(1.2)	(1.3)	—	(2.2)	(2.3)	—	—	—
K1	—	—	—	(2.1)	(2.2)	—	(1.3)	(3.2)	—

For matrix III, we get:

K3	—	—	—	(2.1)	—	(2.3)	(3.1)	—	(3.3)
K2	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.2)
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—

And for matrix IV:

K3	—	—	—	(2.1)	(2.2)	—	(3.1)	(3.2)	—
K2	(1.1)	—	(1.3)	(2.1)	—	(2.3)	—	—	—
K1	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.3)

5. Whichever matrix we use, already in 3-contextural sign relations, there are sub-signs that lie in 2 contextures, f. ex.

1. $(3.1_3\ 2.1_1\ 1.3_3)$
2. $(3.1_{1,3}\ 2.1_1\ 1.3_{2,3})$
3. $(3.1_3\ 2.1_{1,3}\ 1.3_3)$
4. $(3.1_3\ 2.1_{2,3}\ 1.3_{1,2})$

However, strictly speaking, such sign relations contain 2 sign classes, which we shall call “twin” or “multiple” sign classes:

2. $(3.1_{1,3}\ 2.1_1\ 1.3_{2,3}) \rightarrow (3.1_1\ 2.1_1\ 1.3_2) \mid (3.1_3\ 2.1_1\ 1.3_2) \mid (3.1_1\ 2.1_1\ 1.3_3) \mid$
   $(3.1_3\ 2.1_1\ 1.3_3)$
3. $(3.1_3\ 2.1_{1,3}\ 1.3_3) \rightarrow (3.1_3\ 2.1_1\ 1.3_3) \mid (3.1_3\ 2.1_3\ 1.3_3)$
4. $(3.1_3\ 2.1_{2,3}\ 1.3_{1,2}) \rightarrow (3.1_3\ 2.1_2\ 1.3_1) \mid (3.1_2\ 2.1_3\ 1.3_1) \mid (3.1_3\ 2.1_3\ 1.3_1) \mid$
   $(3.1_3\ 2.1_3\ 1.3_2)$

Another solution how to handle this "multi-ordinality", is by embedding the "ambiguous" fundamental categories into the sign relation, therefore getting to sign relations which are

tetradic: $(3.1_3\ 2.1_1\ 2.1_3\ 1.3_3)$, or

pentadic: $(3.1_3\ 2.1_2\ 2.1_3\ 1.3_1\ 1.3_2)$.

In the case of the "Genuine Category Class" we even get a

hexadic sign relation: (3.3 3.3 2.2 2.2 1.1 1.1).

If we chose this solution, we would not have to calculate with twin or multiple sign classes, but with different types of embeddings and hence besides 3-adic with 4-, 5- and 6-adic sign relations.

Bibliography

Bense, Max, Die Einführung der Primzeichen. In: Ars Semeiotica 3/3, 1980, pp. 287-294

Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008

Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

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