Prof. Dr. Alfred Toth

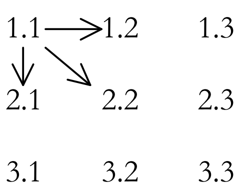
Polycontextural semiotic numbers

Hick, du siehst nur dich. Und ich bin in Wirklichkeit dein Schatten.
(Hick, you see only yourself. And I am in reality your shadow.)

Herbert Achternbusch, "Ab nach Tibet" (1994)

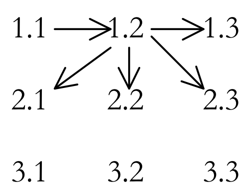
1. In a number of former publications, I have dealt with the interrelations of semiotic and polycontextural numbers (Toth 2003a, b, 2008a, pp. 85 ss.; 110 ss.; 155 ss.; 295 ss.; 2008c). Still a couple of years ago, in formal semiotics, Peirce's and Bense's idea that Peano's axiom system for natural numbers holds for the introduction of the sign relation as a relation over a triadic, a dyadic and a monadic relation, too (cf. Toth 2008b), was uncontroversial. However, if we have a look at the system of the antecedents and the successors of the Peirce-numbers as displayed in the semiotic matrix (Toth 2008d):

Quali-Sign (1.1):



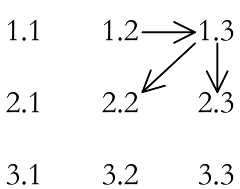
(1.1) has 0 antecedents and 3 successors.

Sin-Sign (1.2):



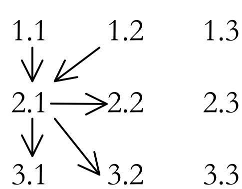
(1.2) has 1 antecedent and 4 successors.

Legi-Sign (1.3):



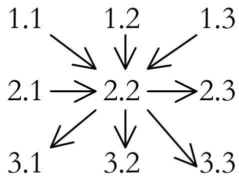
(1.3) has 1 antecedent and 2 successors.

Icon (2.1):



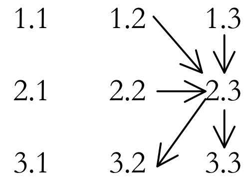
(2.1) has 2 antecedents and 3 successors.

Index (2.2):



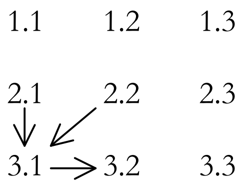
(2.2) has 4 antecedents and 4 successors.

Symbol (2.3):



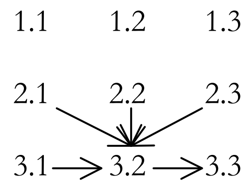
(2.3) has 3 antecedents and 2 successors.

Rhema (3.1):



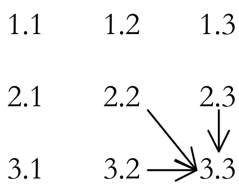
(3.1) has 2 antecedent and 1 successor.

Dicent (3.2):



(3.2) has 4 antecedents and 1 successor.

Argument (3.3):



(3.3) has 3 antecedents and 0 successors.

Then we see that each Peirce-number has a different (and characteristic) number of antecedents and successors:

	antec.	succ.
(1.1)	0	3
(1.2)	1	4
(1.3)	1	2
(2.1)	2	3
(2.2)	4	4
(2.3)	3	2
(3.1)	2	1
(3.2)	4	1
(3.3)	3	0,

and as we can also see, the system of the antecedents and the successors of Peirce-numbers is mirror-symmetric, the axis of symmetry being the part-system of (2.2):

(1.1)	0	3	(3.3)	3	0
(1.2)	1	4	(3.2)	4	1
(1.3)	1	2	(3.1)	2	1
(2.1)	2	3	(2.3)	3	2

Therefore, each sub-sign can be characterized unequivocally by a pair of the numbers of its antecedents and its successors; e.g., (1.1) = [0, 3] ≠ (3.3) = [3, 0].

From that it follows that the system of the sub-signs is not monocontextural (which is confirmed by the fact that it has antecedents and successors that are diagonal (cf. Kronthaler 1986, p. 137).

2. In a triadic semiotics, there are three basic kinds of "Peirce-numbers": the monads or prime-signs, the dyads or sub-signs, and the triads or sign classes and reality thematics. Since we have already had a look at the dyads, let us now turn to pairs of dyads (out of which sign classes can be constructed by concatenation; cf. Walther 1979, p. 79). As an example we take a part-system of the set of all possible combinations of pairs of dyads, and we choose those that have been called "pre-semiotic sign relations" by Ditterich (1990, pp. 29, 81), consisting of the sub-signs (1.1), (1.2) and (2.1), (2.2) from the semiotic matrix. Then, the following 8 combinations are possible:

(1.1 1.1)	(1.2 1.1)
(1.1 1.2)	(1.2 1.2)
(1.1 2.1)	(1.2 2.1)
(1.1 2.2)	(1.2 2.2)

We can now assign each of these pairs of dyads a polycontextural number. Since there are 2 places with (a.b c.d) ≠ (c.d a.b) and 4 dyads, we need trito-numbers of the contexture T_4 (cf. Kronthaler 1986, p. 34):

(1.1 1.1) ≈ 0000

(1.1 1.2) ≈ 0001

(1.1 2.1) ≈ 0010

(1.1 2.2) ≈ 0011

Up to this point, the correspondence between Peirce numbers and trito-numbers is unequivocal. But for the next place, the following trito-numbers has no corresponding Peirce-number (*):

*(1.1 2.3) ≈ 0012

(1.2 1.1) ≈ 0100

(1.2 1.2) ≈ 0101

Now, again a Peirce-number is lacking:

*(1.2 1.3) ≈ 0102
(1.2 2.1) ≈ 0110
(1.2 2.2) ≈ 0111

And finally, the last 5 corresponding Peirce-numbers are lacking, too:

*(1.2 2.3) ≈ 0112
*(1.2 3.1) ≈ 0120
*(1.2 3.2) ≈ 0121
*(1.2 3.3) ≈ 0122
*(1.2 3.4) ≈ 0123

In other words: The system of the Peirce-numbers built from pairs of dyads is defective concerning its corresponding polycontextural system of trito-numbers of contexture 4, since it contains 8 numbers, while T_4 contains 15. However, it is remarkable that the system of the Peirce-numbers does not contain any numbers that are not contained in T_4. However, a polycontextural system with 4 places needs 4 and not only 2 kenograms. On the other side, a polycontextural system with 2 kenograms has only the two morphograms 00 and 01 and is thus not even sufficient for presenting the system of prime-numbers, i.e. the system of monadic Peirce-numbers which requires 3 kenograms and thus T_3.

3. If we write now the system of the 10 sign classes as morphograms (kenogram sequences) and assign again natural numbers to the different kenograms, we recognize that for a triadic semiotics with 3 semiotic values and 6 places, we need trito-numbers of the contexture T_6:

(3.1 2.1 1.1) × (1.1 1.2 1.3) ≈ 20 10 00 ≈ 012111
(3.1 2.1 1.2) × (2.1 1.2 1.3) ≈ 20 10 01 ≈ 012112
(3.1 2.1 1.3) × (3.1 1.2 1.3) ≈ 20 10 02 ≈ 012110
(3.1 2.2 1.2) × (2.1 2.2 1.3) ≈ 20 11 01 ≈ 012212
(3.1 2.2 1.3) × (3.1 2.2 1.3) ≈ 20 11 02 ≈ 012210
(3.1 2.3 1.3) × (3.1 3.2 1.3) ≈ 20 12 02 ≈ 012010
(3.2 2.2 1.2) × (2.1 2.2 2.3) ≈ 21 11 01 ≈ 022212 ≈ 011121
(3.2 2.2 1.3) × (3.1 2.2 2.3) ≈ 211102 ≈ 022210 ≈ 011120
(3.2 2.3 1.3) × (3.1 3.2 2.3) ≈ 21 12 02 ≈ 022012 ≈ 011021
(3.3 2.3 1.3) × (3.1 3.2 3.3) ≈ 22 12 02 ≈ 002010 ≈ 001020

As we see, we have to apply one or two times the normal-form operator that is a vector operator with fixed positions and brings equivalent trito-numbers into lexicographic order (cf. Kronthaler 1986, pp. 26 s.; Toth 2003, pp. 14 ss.). Then, we can order the triadic Peirce-numbers, written as semiotic trito-numbers, in the following table:

001020
011021
011120
011121
012010
012110
012111
012112
012210
012212

Since it is quite clear, that these 10 triadic Peirce numbers alias semiotic trito-numbers are only a small fragment of the number of trito-numbers of the contexture T_6, we do here without indicating the several lacunae. First, the above 10 trito-sign classes are a **semiotic fragment** of the total of $3 \times 3 \times 3 = 27$ possible combinations of sign classes, restricted by the semiotic trichotomic inclusion order (3.a 2.b 1.c) with $a \leq b \leq c$. Second, the contexture T_6 has totally 203 trito-numbers. The latter number can be calculated by summing up the Stirling numbers of the second kind for T_6, which numbers are also known as Bell numbers and give the number of partitions of a set with n members (cf. Andrew 1965). Therefore, the above 10 trito-sign classes are also a **polycontextural fragment** of the total of 203 trito numbers of T_6.

4. In Toth (2008e) and in a few other papers, I have made a first sketch of a polycontextural semiotics based on the sign-relation

$SR_{4,3}$ = (0., .1., .2., .3.); $SR_{4,3}$ (3.a 2.b 1.c 0.d)

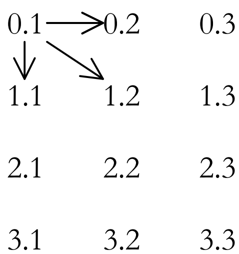
with the corresponding trichotomic inclusion order

$(a \geq b \geq c)$,

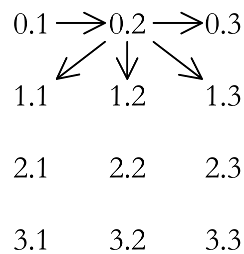
whose corresponding semiotic structure is thus 4-adic, but 3-otomic, since in $Z^r{}_k$, the categorial number $k \neq 0$, but since the relational number is allowed to be $r = 0$, this sign relation integrates pre-semiotic objects and thus connects the triadic sign relation SR_3 with the ontological space (Bense 1975, p. 65):

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

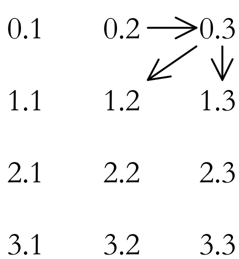
As we did for the Peirce-numbers contained in the semiotic matrix of SR_3, we will now show the systems of antecedents and successors of each Peirce-number in the above semiotic matrix of $SR_{4,3}$:



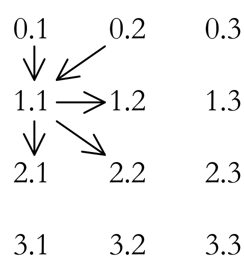
(0.1) has 0 antecedent and 3 successors.



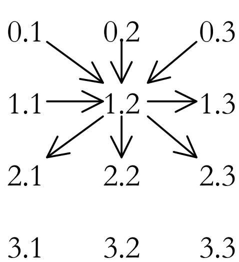
(0.2) has 1 antecedent and 4 successors.



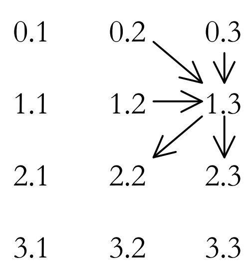
(0.3) has 1 antecedent and 2 successors.



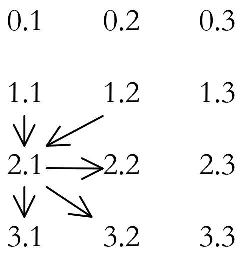
(1.1) has 2 antecedents and 3 successors



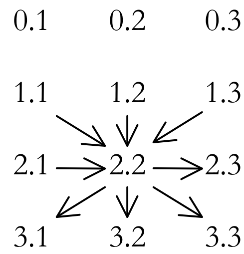
(1.2) has 4 antecedents and 4 successors.



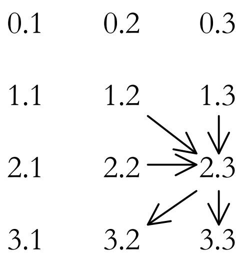
(1.3) has 3 antecedent and 2 successors.



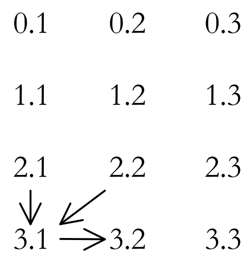
(2.1) has 3 antecedents and 3 successors.



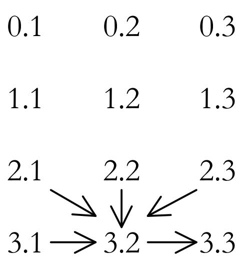
(2.2) has 4 antecedents and 4 successors.



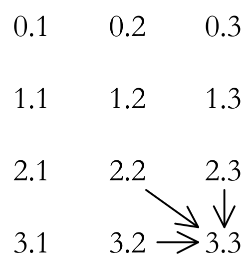
(2.3) has 3 antecedents and 2 successors.



(3.1) has 2 antecedents and 1 successor.



(3.2) has 4 antecedents and 1 successor.



(3.3) has 3 antecedents and 0 successors.

We see that also in $SR_{4,3}$, each Peirce-number can be characterized by a pair of antecedents and successors, although the following system is not symmetric:

	antec.	succ.
(0.1)	0	3
(0.2)	1	4
(0.3)	1	2
(1.1)	2	3
(1.2)	4	4
(1.3)	3	2
(2.1)	2	1
(2.2)	4	4
(2.3)	3	2
(3.1)	2	1
(3.2)	4	1
(3.3)	3	0,

Now, we proceed again by assigning polycontextural numbers to the 15 sign classes of $SR_{4,3}$. Since the 4 semiotic values are distributed over 8 places whose order is relevant, we need trito-numbers from the contexture T_8:

(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3) ≈ 00201030 ≈ 00102030

(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3) ≈ 01201030

(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3) ≈ 01211030

(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3) ≈ 01211130

(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3) ≈ 01211131

(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3) ≈ 01211132

(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3) ≈ 01211230

(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3) ≈ 01211232

(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3) ≈ 01221030

(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3) ≈ 01221230

(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3) ≈ 01221232

(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3) ≈ 02201030 ≈ 01102030

(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3) ≈ 02221230 ≈ 01112130

(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3) ≈ 02221232 ≈ 01112131

(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3) ≈ 02221030 ≈ 01112030

We recognize that the above 15 sign classes are a **semiotic fragment** of the total possible amount of 3 × 3 × 3 × 3 = 81 tetradic-trichotomic sign classes, restricted by the trichotomic semiotic inclusion order (3.a 2.b 1.c 0.d) with a ≤ b ≤ c ≤ d. Moreover, the above 15 trito-sign classes (which interestingly correspond to the number of trito-numbers of the contexture T_4), are a **polycontextural fragment** of the total number of 4'140 trito-numbers of T_8.

5. By comparing the 8 pairs of dyadic relations ($DR_{2,2}$), the 10 out of 27 triadic-trichotomic sign classes ($SR_{3,3}$) and the 15 out of 81 tetradic-trichotomic sign classes ($SR_{4,3}$) with the respective 15 trito-numbers of the contexture T_4, the respective 203 trito-numbers of the contexture T_6, and the respective 4'140 trito-numbers of the contexture T_8, we come to the conclusion that there are no Peirce-numbers which are not presented in the respective contextures of trito-numbers. Generally, the index of a contexture depends only on the n-adic (and not on the n-otomic) semiotic value, whereby we found that the trito-contexture (TC) has double the index of the respective triadic value (TV) of a sign class, i.e. $TV_i = TC_{2i}$. On the other side, all sign classes (including the dyadic relations) are fragments of the respective systems of trito-numbers, whereby we have that the higher the index of a contexture increases, the smaller the number of the corresponding sign classes becomes:

Sign Classes		Number of	Corresp.	Contexture
triad. value	trich. value	sign classes	Trito-nos.*	
2	2	8**	15	T_4
3	3	10 / 27	203	T_6
4	3	15 / 81	4'140	T_8

(* = Bell numbers; ** totally 20 pairs of dyads by restriction of trichotomic semiotic inclusion)

However, if we simply would consider, e.g., the full amount of the 203 trito-numbers of T_6 as "sign relations", we would have to abolish all relational conditions for any relation to be defined as a sign relation. Moreover, in this case, there would be no difference anymore between a kenogram sequence and a sign-relation. Since kenograms are defined by abolishment of all definitory tools of what turns a relation into a sign relation (cf. Kaehr 2004, pp. 2 ss.), it follows that it is simply impossible to define any sign relations on polycontextural level. Nevertheless, we have shown that it is possible to lay the fundaments deeper than they are on the level of triadic-trichotomic semiotics of $SR_{3,3}$, whose sign classes and reality thematics exclusively belong to what Bense called the "semiotic space" (1975, pp. 64 ss.). Therefore, $SR_{4,3}$, which bridges between the semiotic and the ontological spaces by integrating the category of zeroness or quality into $SR_{3,3}$, seems to be the deepest possible level on which the sign still can be defined, the area between semiotic and ontological space, representation and presentation, subject and object. Thus $SR_{4,3}$ includes the representational-presentational bridge over the contextural border between semiotic and ontological space and hence between sign and object. "More object" and "less sign" cannot be represented in a sign relation whose minimal condition is that it be triadic and the triadic values be pairwise different (3.a 2.b 1.c). Even if we abolish the condition that a sign relation must have the trichotomic inclusion order ($a \leq b \leq c$), and thus expand the system of the 15 sign classes to the system of the 81 sign classes, the latter is still a relatively small polycontextural fragment of the contexture T_6 with its 203 trito-numbers, representing a bit more than a third of the structural complexity of their respective trito-numbers.

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