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The transpositions of sign classes as partially ordered sets

1. A partially ordered set or "poset" consists of a set together with a binary relation that describes, for certain pairs of elements in the set, the requirement that one of the elements must precede the other. More precisely, a partial order is a binary relation "≤" over a set P which is reflexive, antisymmetric, and transitive and thus fulfills the following requirements:

1.1. $a \leq a$ (reflexivity)
1.2. If $a \leq b$ and $b \leq a$ then $a = b$ (antisymmetry)
1.3. If $a \leq b$ and $b \leq c$ then $a \leq c$ (transitivity) (cf. Schröder 2003)

For a semiotic example, let us take the semiotic set of prime-signs

P = {.1., .2., .3.}.

Then, the respective poset is

$\underline{P}$ = {∅, {.1., .2., .3.}, {.1.}, {.2.}, {.3.}, {.1., .2.}, {.1., .3.}, {.2., .3.}},

i.e. the poset of the set P of prime-signs does **not** contain

1. the symmetric sub-signs {.1., .1.}, {.2., .2.}, {.3., .3.} and
2. the dual sub-signs {.2., .1.}, {.3., .1.}, {.3., .2.}

Therefore, posets are not appropriate for order structures that concern semiotic entities below the sign classes and reality thematics (cf. Toth 1996; 2007, pp. 77 ss.).

2. However, posets can be introduced for the semiotic set S of the 10 sign classes and their dual reality thematics:

S = {{3.1 2.1 1.1}, {3.1 2.1 1.2}, {3.1 2.1 1.3}, {3.1 2.2 1.2}, {3.1 2.2 1.3}, {3.1 2.3 1.3}, {3.2 2.2 1.2}, {3.2 2.2 1.3}, {3.2 2.3 1.3}, {3.3 2.3 1.3}}

As it has been shown (Toth 1996; 2007, pp. 64 ss.), the 10 sign classes can be ordered in several ways. Here, we shall introduce in addition the semiotic set TS of the transpositions of the sign classes. Since each sign class has 6 transpositions (cf. Toth 2008a, pp. 223 ss.), the set T looks as follows:

TS = {{3.1 2.1 1.1}, {3.1 1.1 2.1}, {2.1 3.1 1.1}, {2.1 1.1 3.1}, {1.1 3.1 2.1}, {1.1 2.1 3.1},{3.1 2.1 1.2}, {3.1 1.2 2.1}, {2.1 3.1 1.2}, {2.1 1.2 3.1}, {1.2 3.1 2.1}, {1.2 2.1 3.1}, {3.1 2.1 1.3}, {3.1 1.3 2.1}, {2.1 3.1 1.3}, {2.1 1.3 3.1}, {1.3 3.1 2.1}, {1.3 2.1 3.1}, {3.1 2.2 1.2}, {3.1 1.2 2.2}, {2.2 3.1 1.2}, {2.2 1.2 3.1}, {1.2 3.1 2.2}, {1.2 2.2 3.1}, {3.1 2.2 1.3}, {3.1 1.3 2.2}, {2.2 3.1 1.3}, {2.2 1.3 3.1}, {1.3 3.1 2.2}, {1.3 2.2 3.1}, {3.1 2.3 1.3}, {3.1 1.3 2.3}, {2.3 3.1 1.3}, {2.3 1.3 3.1}, {1.3 3.1 2.3}, {1.3 2.3 3.1}, {3.2 2.2 1.2}, {3.2 1.2 2.2}, {2.2 3.2 1.2}, {2.2 1.2 3.2}, {1.2 3.2 2.2}, {1.2 2.2 3.2}, {3.2 2.2 1.3}, {3.2 1.3 2.2}, {2.2 3.2 1.3}, {2.2 1.3 3.2}, {1.3 3.2 2.2}, {1.3 2.2 3.2}, {3.2 2.3 1.3}, {3.2 1.3 2.3}, {2.3 3.2 1.3}, {2.3 1.3 3.2}, {1.3 3.2 2.3}, {1.3 2.3 3.2}, {3.3 2.3 1.3}, {3.3 1.3 2.3}, {2.3 3.3 1.3}, {2.3 1.3 3.3}, {1.3 3.3 2.3}, {1.3 2.3 3.3}}

More generally, if we define a sign class as

SCl = {a.b c.d e.f} with a, ..., f $\in$ {1, 2, 3} and b $\leq$ d $\leq$ f,

thus the trichotomic values obeying a partial order, we get for each of the 10 sign classes the following respective set T of transpositions

T(SCl) = {{a.b c.d e.f}, {a.b e.f c.d}, {c.d a.b e.f}, {c.d e.f a.b}, {e.f c.d a.b}, {e.f a.b c.d}}

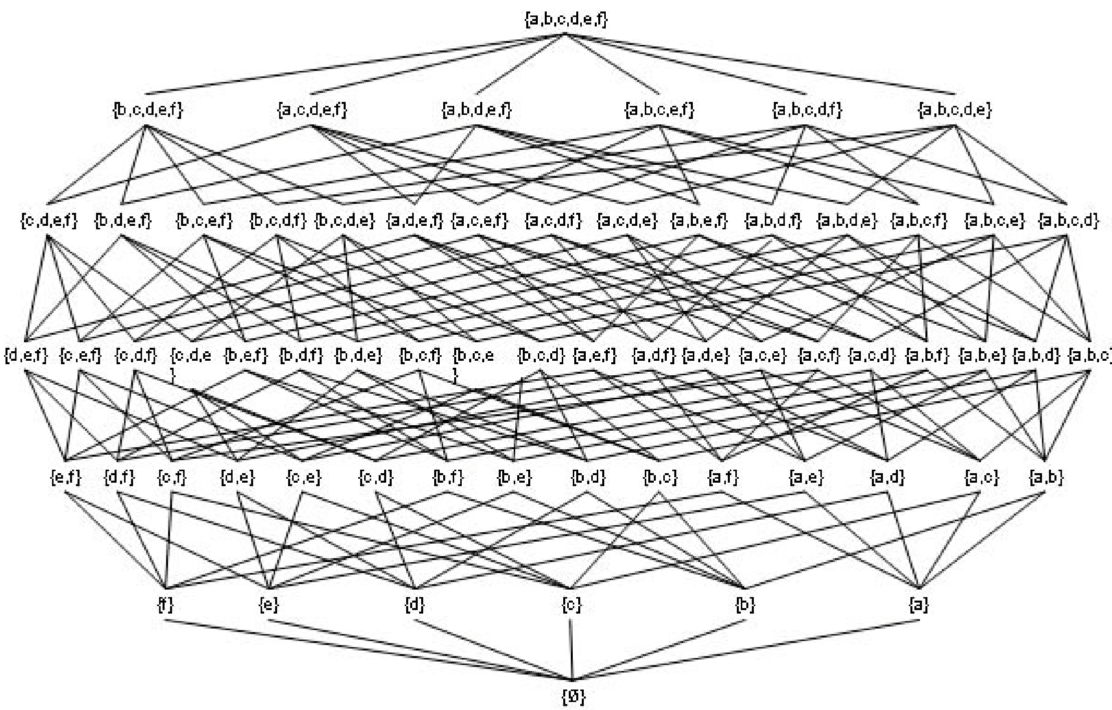
and for the dual set of a reality thematic

RTh(f.e d.c b.a)

T(RTh) = {{f.e d.c b.a}, {f.e b.a d.c}, {d.c f.e b.a}, {d.c b.a f.e}, {b.a f.e d.c}, {b.a d.c f.e}}

The question is now: How many combinations of these transpositions per sign class or reality thematic are possible and which types of semiotic connections (cf. Toth 2008b, pp. 28 ss.) do they show?

3. From the following graph, we learn that there are exactly 64 combinations of a partially ordered set with 6 elements. Since we are dealing here with mathematical semiotics, it has, however, to be pointed out that we thereby must assume the existence of an empty semiotic set (cf. Toth 2007, p. 64 ss.) and that again (like in the case of the set P of prime-signs) both the symmetric and the dual combinations are excluded:



http://en.wikipedia.org/wiki/Partially_ordered_set

From this graph, we may also easily see that there are the following types of semiotic connections:

- the semiotic ∅-set has 6 connections to the 1-element sets of each transpositions
- the 1-element sets have each 5 connections to the 2-element sets of transpositions
- the 2-element sets have each 4 connections to the 3-element sets of transpositions
- the 3-element sets have each 3 connections to the 4-element sets of transpositions
- the 4-element sets have each 2 connections to the 5-element sets of transpositions
- the 5-element sets have each 1 connections to the 6-element set of transpositions,

thus, the complete set of all transpositions of a sign class or reality thematic has 6 connections to the 5-element sets of transpositions.

As an example, let us show the semiotic connections between the following sets of transpositions:

{e, f}

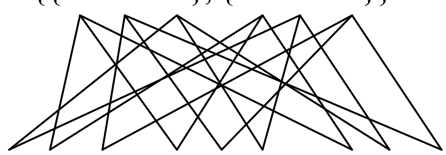
{d, e, f}, {c, e, f}, {b, e, f}, {a, e, f}

We will now assign a, ..., f to the elements of the set of transpositions in degenerative semiotic order. As an example, we choose the sign class (3.1 2.1 1.3):

a := (3.1 2.1 1.3)   d := (2.1 1.3 3.1)
b := (3.1 1.3 2.1)   e := (1.3 3.1 2.1)
c := (2.1 3.1 1.3)   f := (1.3 2.1 3.1)

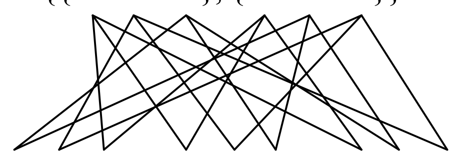
Then, we get the 4 sub-relations:

{{1.3 3.1 2.1}, {1.3 2.1 3.1}}



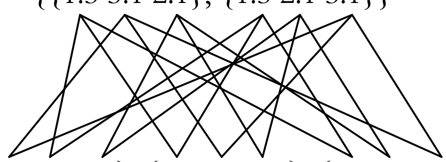
{{2.1 1.3 3.1}, {1.3 3.1 2.1}, {1.3 2.1 3.1}}

{{1.3 3.1 2.1}, {1.3 2.1 3.1}}



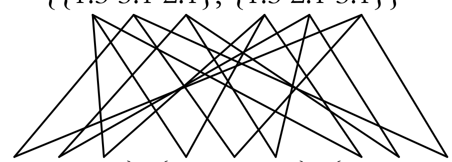
{{2.1 3.1 1.3}, {1.3 3.1 2.1}, {1.3 2.1 3.1}}

{{1.3 3.1 2.1}, {1.3 2.1 3.1}}



{{3.1 1.3 2.1}, {1.3 3.1 2.1}, {1.3 2.1 3.1}}

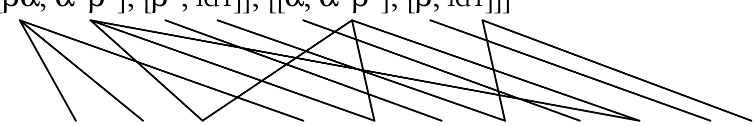
{{1.3 3.1 2.1}, {1.3 2.1 3.1}}



{{3.1 2.1 1.3}, {1.3 3.1 2.1}, {1.3 2.1 3.1}}

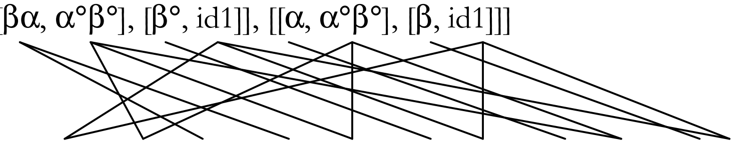
In order to display more intricate sign connections, we may use the category theoretic notation of dynamic morphisms introduced in Toth (2008a, pp. 139 ss.). In doing so, we obtain the following semiotic structures of the above 4 part-posets of the set of the transpositions of the sign class (3.1 2.1 1.3):

$[[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta^\circ, id1]], [[\alpha, \alpha^\circ\beta^\circ], [\beta, id1]]]$



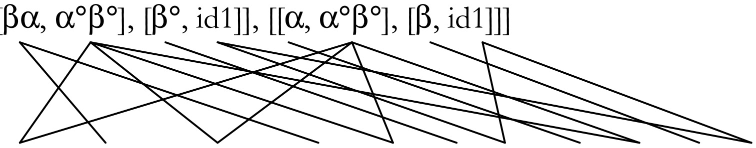
$[[[\alpha^\circ, \beta\alpha], [\beta\alpha, \alpha^\circ\beta^\circ]], [[\beta\alpha, \alpha^\circ\beta^\circ], [\beta^\circ, id1]], [[\alpha, \alpha^\circ\beta^\circ], [\beta, id1]]]$

$[[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta^\circ, id1]], [[\alpha, \alpha^\circ\beta^\circ], [\beta, id1]]]$



$[[[\beta, id1], [\alpha^\circ\beta^\circ, \beta\alpha]], [[\beta\alpha, \alpha^\circ\beta^\circ], [\beta^\circ, id1]], [[\alpha, \alpha^\circ\beta^\circ], [\beta, id1]]]$

$[[[\beta\alpha, \alpha^\circ\beta^\circ], [\beta^\circ, id1]], [[\alpha, \alpha^\circ\beta^\circ], [\beta, id1]]]$



$[[[\alpha^\circ\beta^\circ, \beta\alpha], [\alpha, \alpha^\circ\beta^\circ]], [[\beta\alpha, \alpha^\circ\beta^\circ], \beta^\circ, id1]], [[\alpha, \alpha^\circ\beta^\circ], [\beta, id1]]]$

$$[[[\beta\alpha, \alpha°\beta°], [\beta°, id1]], [[\alpha, \alpha°\beta°], [\beta, id1]]]$$

$$[[[\beta°, id1], [\alpha°, \beta\alpha]], [[\beta\alpha, \alpha°\beta°], [\beta°, id1]], [[\alpha, \alpha°\beta°], [\beta, id1]]]$$

All these relations – the ones using the numerical as well as the ones using the category theoretical notation – can now be calculated exactly, cf. Toth (2008b, pp. 41 ss.), offering a huge field of application for computational semiotics.

Bibliography

Schröder, Bernd S.W., Ordered Sets. Boston 2003

Toth, Alfred, Grundriss einer ordnungstheoretischen Semiotik. In: European Journal for Semiotic Studies 8, 1996, pp. 503-526

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)

Toth, Alfred, Semiotic Ghost Trains. Klagenfurt 2008 (2008b)