Prof. Dr. Alfred Toth

Outlines of a general model for a pre-semiotic space

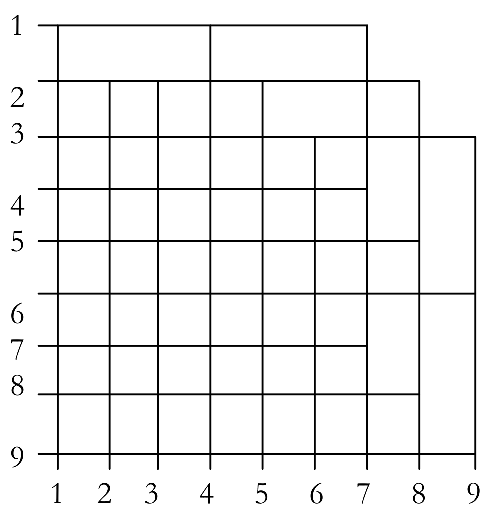
1. My "Semiotic Relational Grammar" (SRG), which had appeared in 1997, was the first attempt at constructing a topological semiotic space by aid of category theory (Toth 1997). SRG is a two-dimensional semiotic space in which only such structural realities are connected to one another, that present the same kind of sets of thematized realities. In the respective graph, the reality thematics that present the structural realities are the vertices and the connections of identical thematizates are the edges.

E.g., in SRG over $SR_{3,3}$, which we will abbreviate as $SRG_{3,3}$, the pair of structural realities to the left does not share the same thematizate, but the two pairs to the right do:

($\underline{3.1\ 3.2}$ 3.3)	($\underline{3.1\ 3.2}$ 2.3)	($\underline{3.1\ 3.2}$ 2.3)
	/	/
(1.1 $\underline{1.2\ 1.3}$)	(2.1 $\underline{2.2\ 2.3}$)	(2.1 $\underline{1.2\ 1.3}$)

The two structural realities to the left are (I-them. I) and (M-them. M), thus, the thematizates are in the first case an I and in the second case an M, hence they cannot be connected. However, in the two pairs to the right, we have (I-them. O) and (O-them. O) in the first case, and (I-them. O) and (M-them. O) in the second case, hence in both cases a thematized object (although the thematizing sub-signs are not the same), and thus both structural realities of both pairs will be connected.

Since only such structural realities are connected to one another, which show the same thematizates, the graph of SRG has an antimatroidal structure. An antimatroid is a family of sets closed under union, such that for every (nonempty) set in the family there is an element that can be removed to produce another set in the family. The antimatroid-character of SRG is what gives SRG an outer and inner "stairwell-like" appearance:



Antimatroids are also known as "learning spaces", whose structure can be made apparent by drawings in which all faces are quadrilaterals with the bottom and left sides parallel to the coordinate axis (and where the drawing has unique top-right and bottom-left vertices). "Such a drawing only exists for graphs coming from antimatroids" (Eppstein 2006a). It is even true that "each upright-quad drawing represents an st-planar learning space" (Eppstein 2006b, p. 11).

The nos. 1-9 of the vertices refer to the following structural realities:

1 := (3.1 3.2 3.3)	I-them. I	6 := (2.1 2.2 1.3)	O-them. M
2 := (3.1 3.2 2.3)	I-them. O	7 := (3.1 1.2 1.3)	M-them. I
3 := (3.1 3.2 1.3)	I-them. M	8 := (2.1 1.2 1.3)	M-them. O
4 := (3.1 2.2 2.3)	O-them. I	9 := (1.1 1.2 1.3)	M-them. M
5 := (2.1 2.2 2.3)	O-them. O		

So, if only thematized M, O, I are combined with thematized M, O, I, then $SRG_{3,3}$, as depicted above, has exactly 66 intersects of semiotic relations. For illustration, I show the sign connections of the first leftmost column of $SRG_{3,3}$, i.e. the connections between the subsets for ((1,1), (2,1), (3,1), ..., (9,1)). The left column beneath uses "static" morphisms, the right column "dynamic" morphisms (cf. Toth 2008a, pp. 159 ss., 259 ss.):

(1,1)	[id3, id3, id3]	[[id3, α], [id3, β]]
(2,1)	[id3, id3, β]	[[id3, α], [β°, β]]
(3,1)	[id3, id3, βα]	[[id3, α], [α°β°, β]]
(4,1)	[id3, β, β]	[[β°, α], [id2, β]]
(5,1)	[β, β, β]	[[id2, α], [id2, β]]
(6,1)	[β, β, βα]	[[id2, α], [α°, β]]
(7,1)	[id3, βα, βα]	[[α°β°, α], [id1, β]]
(8,1)	[β, βα, βα]	[[α°, α], [id1, β]]
(9,1)	[βα, βα, βα]	[[id1, α], [id1, β]]

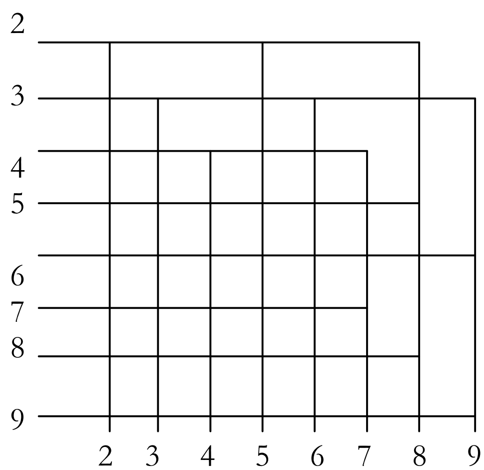
2. As one recognizes, the structure of the connections of $SRG_{3,3}$ is the same from top to bottom and from left to the right, so that the graph is symmetric for rotation. This allows to consider $SRG_{3,3}$ a topologically stratified space. Generally, an n-dimensional topological stratification of a topological space X is a filtration

$$\varnothing = X_{-1} \subset X_0 \subset X_1 \subset ... \subset X_n = X$$

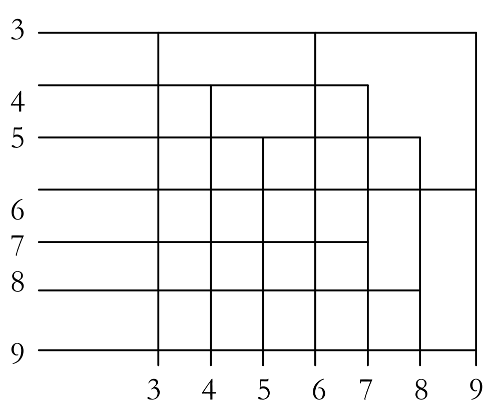
of X by closed subspaces such that for each i and for each point x of $X_i \setminus X_{i-l}$, there exists a neighborhood $U \subset X$ of x in X, a compact n-i-l-dimensional stratified space L, and a filtration-preserving homeomorphism $U \cong \mathbb{R}^i \times CL$. Here, CL is the open cone on L. If X is a topologically stratified space, the i-dimensional stratum of X is the space $X_i \setminus X_{i-l}$ (Goresky 1983).

In the case of $SRG_{3,3}$, the stratified spaces are simply the sub-spaces, and there are as many nonempty subspaces as there are nonempty subsets of its carrier set. However, for SRG as a semiotic space, it is senseless to construct 8 subspaces, because then we would get only identical thematizates at the end. Since $SRG_{3,3}$ is constructed from 3 blocks of 3 reality thematics, according to the Trichotomic Triads (cf. Toth 1997, pp. 36 ss.), we obtain the following 6 subspaces, whose last one consists of the self-thematizations of M-them. I, M-them. O, and M-them. M. Therefore, according to the antimatroidal structure of $SRG_{3,3}$, we can construct the following subspaces by letting away step by step one thematization while proceeding downward and rightward from one stratum to the next:

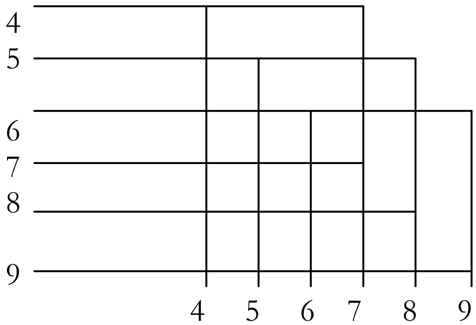
SRG\ (I-I)



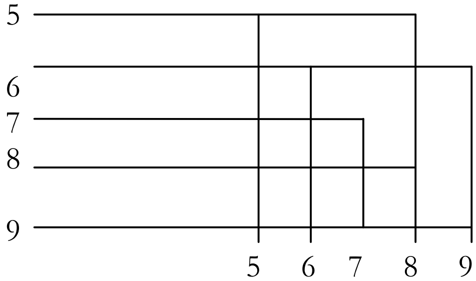
SRG \ (I-I ∧ I-O)



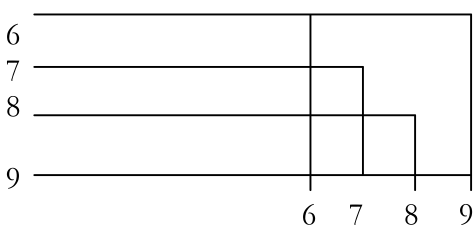
SRG \ (I-I ∧ I-O ∧ I-M)



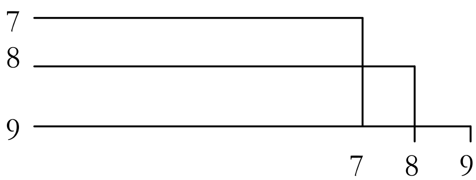
SRG \ (I-I ∧ I-O ∧ I-M ∧ O-I)



SRG \ (I-I ∧ I-O ∧ I-M ∧ O-I ∧ O-O)



SRG \ (I-I ∧ I-O ∧ I-M ∧ O-I ∧ O-O ∧ O-M)



3. We now turn to the set SS15 of 15 sign classes and reality thematics over the pre-semiotic sign relation $SR_{4,3}$:

1	(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)	M-them. M
2	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)	M-O
3	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)	M-them. I
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)	M-them. O / O-them. M (2)
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)	M-them. O / M-them. I (2)
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)	M-them. I / I-them. M (2)
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)	O-them. M
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)	O-them. M / O-them. I (2)
9	(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)	I-them. O / I-them. M (2)
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)	I-them. M
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)	O-them. O
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)	O-them. I
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)	I-them. O / O-them. I (2)
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)	I-them. O
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)	I-them. I

As we recognize, SS15 cannot be written as blocks of n-tomic n-ads (cf. Toth 2008b). Moreover, while in $SRG_{3,3}$ the dual-identical sign class (3.1 2.2 1.3) × (3.1 2.2 1.3) determines the two blocks of three trichotomic triads, in $SRG_{4,3}$, there is no dual-identical sign class. It follows that there is no symmetric $SRG_{4,3}$ model. Nevertheless, a maximal model for $SRG_{4,3}$ displays even amounts of M, O and I thematizates:

Maximal $SRG_{4,3}$max:

Thematized M: 7 (nos. 1, 4, 6, 7, 8, 9, 10)
Thematized O: 7 (nos. 2, 4, 5, 9, 11, 13, 14)
Thematized I: 7 (nos. 3, 5, 6, 8, 12, 13, 15)

As for minimal $SRG_{4,3}$ models, we get two variants. In the first model, we restrict thematizates to the cases appearing after the slash in the thematization alternatives of the above list. In the second model, we restrict thematizates to the cases appearing before the slash in the above thematization alternatives. As it turns out, in both minimal $SRG_{4,3}$ models, we get (2n : n : 2n) correlations of the amounts of M, O and I thematizates:

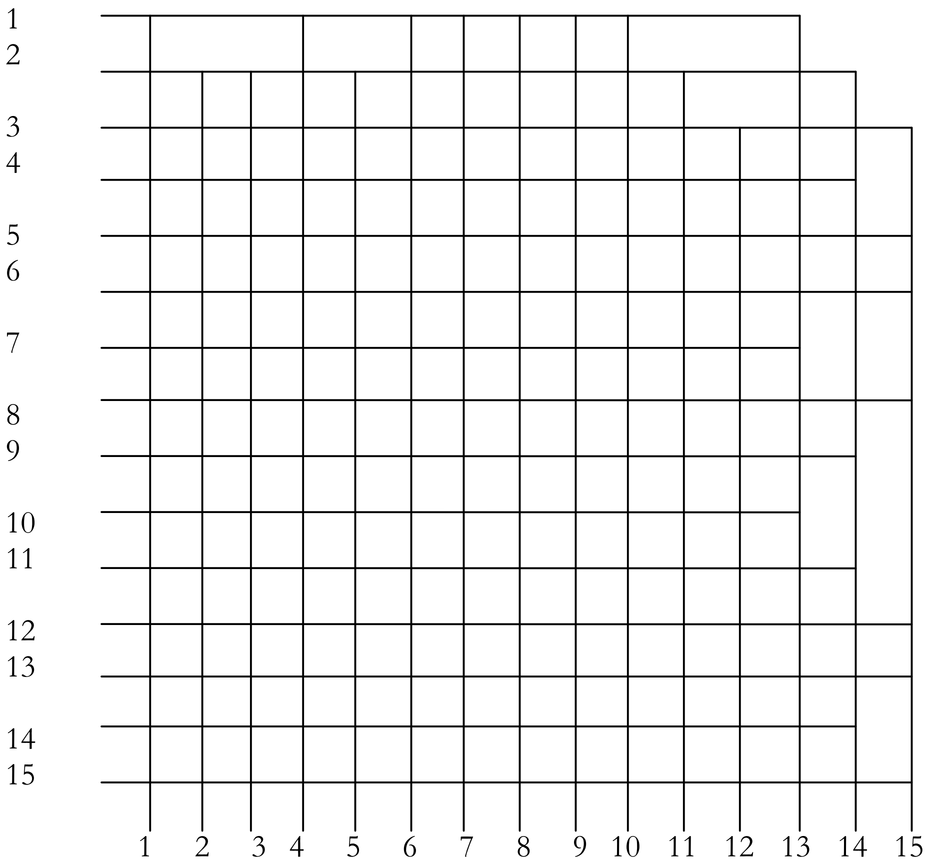
Minimal $SRG_{4,3}$min1:

Thematized M: 6 (nos. 1, 4, 6, 7, 9, 10)
Thematized O: 3 (nos. 2, 11, 14)
Thematized I: 6 (nos. 3, 5, 8, 12, 13, 15)

Minimal $SRG_{4,3}$min2:

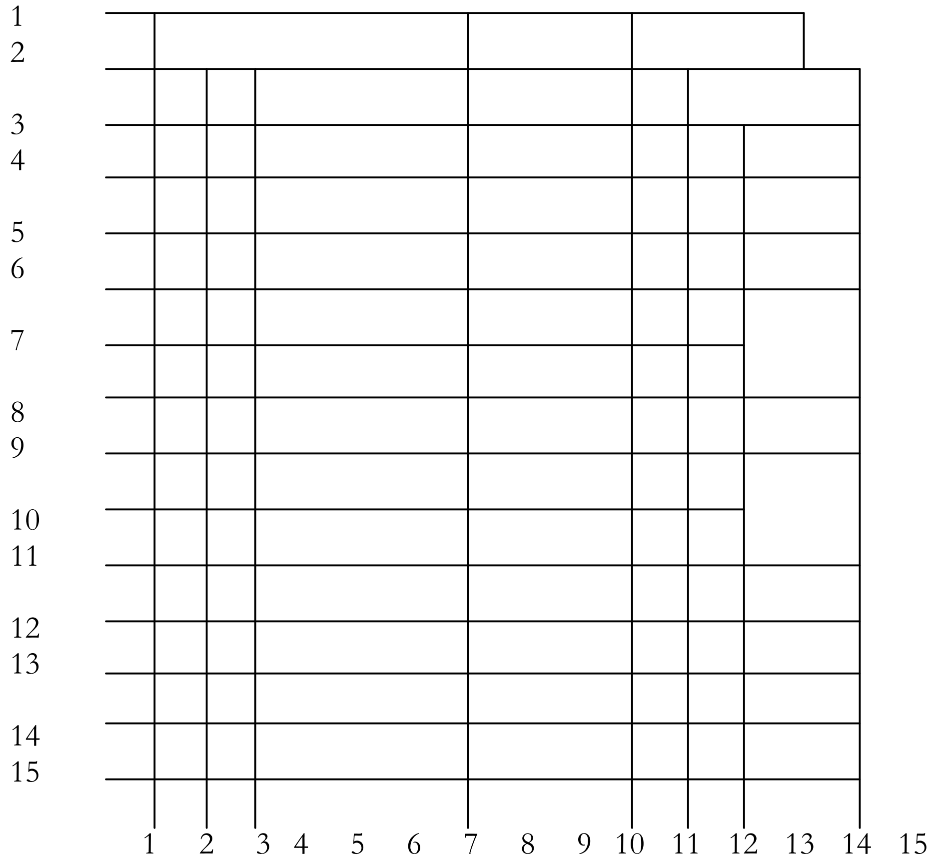
Thematized M: 4 (nos. 1, 7, 8, 10)
Thematized O: 8 (nos. 2, 4, 5, 9, 11, 12, 13, 14)
Thematized I: 4 (nos. 3, 6, 12, 15)

The following graph presents $SRG_{4,3}$max. In displays 208 points of intersecting pre-semiotic connections and is thus the maximal pre-semiotic learning space or antimatroid possible over the pre-semiotic sign relation $SR_{4,3}$:

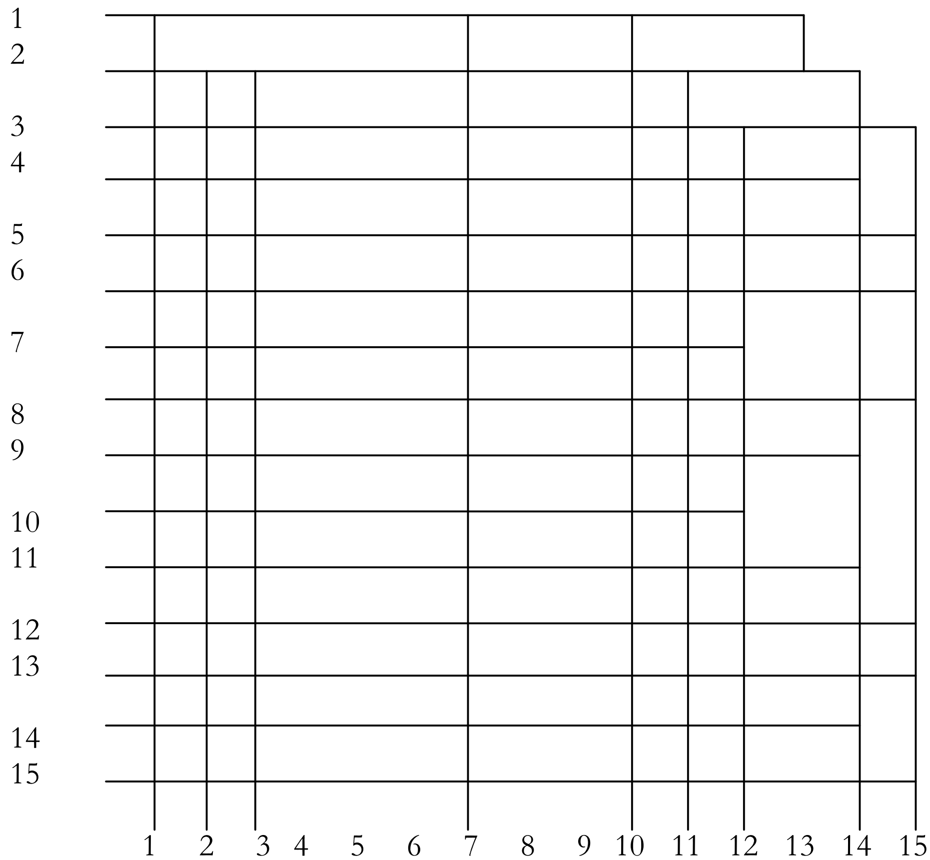


As one recognizes, the rotational "stairwell" structure of $SRG_{3,3}$ appears non-symmetric in $SRG_{4,3}$max.

The graph of $SRG_{4,3}$min1 shows 114 points of intersecting pre-semiotic connections. Like in the graph of $SRG_{4,3}$max, the antimatroidal "stairwell" structure (3-2-1; 3-2-1; 3-2-1) is strongly disturbed. In $SRG_{4,3}$min1, there are also many pre-semiotic connections that bridge over undefined pre-semiotic intersection points:



The graph of $SRG_{4,3}$min2 shows 121 points of intersecting pre-semiotic connections:



Like in the graph of $SRG_{4,3}$max and $SRG_{4,min}$1, the antimatroidal "stairwell" structure

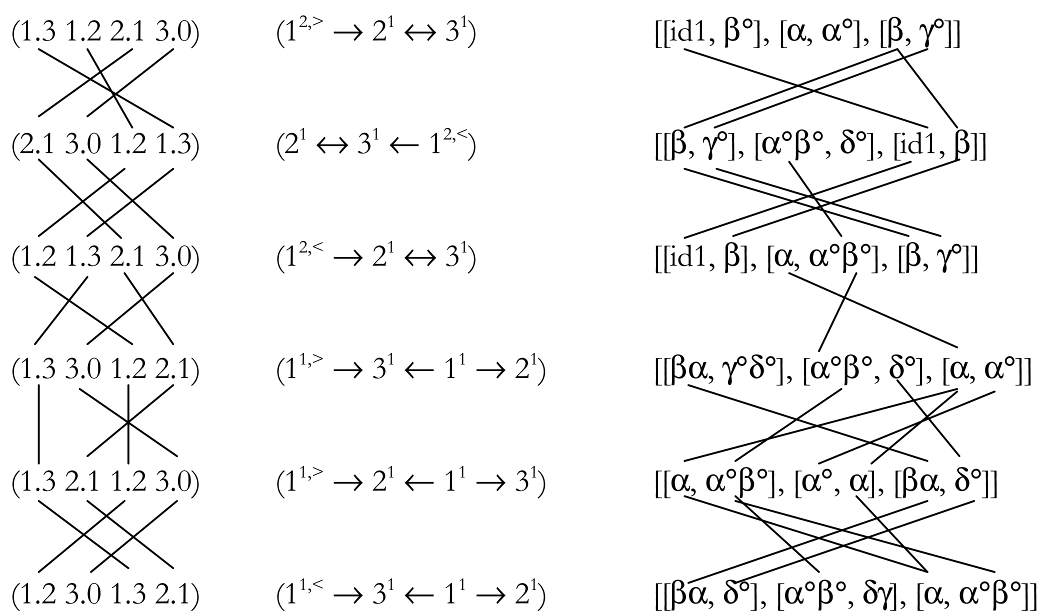
3-2-1; 3-2-1; 3-2-1
2-1-3; 2-1-3; 2-1-3
1-3-2; 1-3-2; 1-3-2
3-2-1; 3-2-1; 3-2-1
2-1-3; 2-1-3; 2-1-3
1-3-2; 1-3-2; 1-3-2
3-2-1; 3-2-1; 3-2-1
2-1-3; 2-1-3; 2-1-3
1-3-2; 1-3-2; 1-3-2

is strongly disturbed. As we see, $SRG_{4,3}$min2 differs from $SRG_{4,3}$min1 solely in preserving the thematization quadrant

3-1-2-3—7—10-11-12—14- 15
-
5
6
-
8
-
12
13
-
15

It goes without further demonstration, that none of the three SRG$_{4,3}$ models can be appropriately stratified, since there is not filtration like in SRG$_{3,3}$. Hence, in accordance with our above insights, SRG$_{4,3}$ contains sub-spaces, but the union of the sub-spaces of SRG$_{4,3}$min1 and SRG$_{4,3}$min2 does not yield SRG$_{4,,3}$max.

4. As we did above for the connections between the subsets for ((1,1), (2,1), (3,1), ..., (9,1)) in SRG$_{3,3}$, we will now show some possible pre-semiotic connections in SRG$_{4,3}$. Since permutations of sign relations are the most complex source for semiotic structures (cf. Toth 2008a, pp. 177 ss.; 2008c, pp. 28 ss.), and since the thematization structures are not changed by permutations of reality thematics (Toth 2008d), we show in the following the 4! = 24 possible permutations of the pre-semiotic sign class (3.1 2.1 1.2 0.3) with its dual reality thematic (3.0 2.1 <u>1.2 1.3</u>) and its two structural realities (M-them. O) / (M-them. I). It is thus possible to construct any SRG models and thus any learning spaces using permutated reality thematics instead of "non-permutated" ones. In the following table, the left column displays the permutated reality thematics, the middle column gives the respective structure of the structural reality, and the right column shows the categorial structure of the structural realities:



(2.1 1.3 1.2 3.0)	$(2^1 \leftarrow 1^{2,>} \rightarrow 3^1)$	[[α°, βα], [id1, β°], [βα, δ°]]
(2.1 3.0 1.3 1.2)	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,>})$	[[β, γ°], [α°β°, δγ], [id1, β°]] (unconnected!)
(1.2 2.1 1.3 3.0)	$(1^{1,<} \rightarrow 2^1 \leftarrow 1^1 \rightarrow 3^1)$	[[α, α°], [α°, βα], [βα, γ°δ°]]
(1.3 3.0 2.1 1.2)	$(1^{1,>} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$	[[βα, γ°δ°], [β°, γ], [α°, α]]
(2.1 1.2 1.3 3.0)	$(2^1 \leftarrow 1^{2,<} \rightarrow 3^1)$	[[α°, α], [id1, β], [βα, γ°δ°]]
(1.2 3.0 2.1 1.3)	$(1^{1,<} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$	[[βα, δ°], [β°, γ], [α°, βα]]
(1.3 1.2 3.0 2.1)	$(1^{2,>} \rightarrow 3^1 \leftrightarrow 2^1)$	[[id1, β°], [βα, δ°], [β°, γ]]
(3.0 2.1 1.2 1.3)	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,<})$	[[β°, γ], [α°, α], [id1, β]]
(1.2 1.3 3.0 2.1)	$(1^{2,<} \rightarrow 3^1 \leftrightarrow 2^1)$	[[id1, β], [βα, γ°δ°], [β°, γ]]
(3.0 1.2 1.3 2.1)	$(3^1 \leftarrow 1^{2,<} \rightarrow 2^1)$	[[α°β°, δ], [id1, β], [α, α°β°]]
(1.3 2.1 3.0 1.2)	$(1^{1,>} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	[[α, α°β°], [β, γ°], [α°β°, δ]]
(3.0 1.3 2.1 1.2)	$(3^1 \leftarrow 1^{1,>} \rightarrow 2^1 \leftarrow 1^1)$	[[α°β°, δγ], [α, α°β°], [α°, α]]
(2.1 1.3 3.0 1.2)	$(2^1 \leftarrow 1^{1,>} \rightarrow 3^1 \leftarrow 1^1)$	[[α°, βα], [βα, γ°δ°], [α°β°, δ]]
(3.0 2.1 1.3 1.2)	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,>})$	[[β°, γ], [α°, βα], [id1, β°]]
(1.2 2.1 3.0 1.3)	$(1^{1,<} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	[[α, α°], [β, γ°], [α°β°, δγ]]

(3.0 1.2 2.1 1.3) ($3^1 \leftarrow 1^{1,<} \rightarrow 2^1 \leftarrow 1^1$) [[α°β°, δ], [α, α°], [α°, βα]]

(2.1 1.2 3.0 1.3) ($2^1 \leftarrow 1^{1,<} \rightarrow 3^1 \leftarrow 1^1$) [[α°, α], [βα, δ°], [α°β°, δγ]]

(3.0 1.3 1.2 2.1) ($3^1 \leftarrow 1^{2,>} \rightarrow 2^1$) [[α°β°, δγ], [id1, β°], [α, α°]]

From the above fragment, we also recognize that full information about semiotic and pre-semiotic connections in any (semiotic or pre-semiotic spaces) between reality thematics and their permutations can only be won by using both numerical (or "static") and "dynamic" category theoretic analysis.

Bibliography

Eppstein, David, Reverse search for antimatroids. 2006a. http://11011110.livejournal.com/58994.html

Eppstein, David, Upright-quad-drawing of st-planar learning spaces. arXiv:cs/0607094v1, 20 Jul 2006

Goresky, Mark/MacPherson, Robert, Intersection homology II. In: Invent. Math. 72/1 (1983), pp. 77-129

Toth, Alfred, Entwurf einer semiotisch-relationalen Grammatik. Tübingen 1997

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)

Toth, Alfred, Tetradic, triadic, and dyadic sign classes. Ch. 44 (2008b)

Toth, Alfred, Semiotic Ghost Trains. Klagenfurt 2008 (2008c)

Toth, Alfred, Towards a reality theory of pre-semiotics. Ch. 42 (2008d)