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Towards a reality theory of pre-semiotics

1. In this study, I will make some further considerations following Bense (1975, pp. 64 ss.) and Toth (2008b, 2008c). A pre-semiotic tetradic sign class has the following general form:

(3.a 2.b 1.c 0.d) with $a \leq b \leq c \leq d$,

while the semiotic triadic sign class is noted as

(3.a 2.b 1.c).

Thus, from a purely quantitative standpoint, (3.a 2.b 1.c) $\subset$ (3.a 2.b 1.c 0.d). However, since zeroness (0.) cannot appear in the trichotomic positions of the sign classes, this inclusion does not hold. Moreover, unlike firstness (.1.), secondness (.2.) and thirdness (.3.), which are purely quantitative categories (cf. Bense 1975, pp. 168 ss.), zeroness integrates the triadic sign relations as parts of the semiotic space (Bense 1975, p. 65) and the ontological space out of which objects are selected in order to be thetically introduced as sign classes, from which follows that zeroness localizes the triadic semiotic sign relation in the tetradic sign relation and is thus a qualitative category (cf. Kronthaler 1992, p. 293). Hence, since the tetradic pre-semiotic relation PSR contains a category of quality, which is absent in the triadic semiotic relation SR, it follows that

SR $\not\subset$ PSR (cf. Toth 2003, pp. 27 ss.).

Therefore, the 15 pre-semiotic sign classes of PSR are both quantitative and qualitative representation schemes:

1	(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)
2	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)
3	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)
9	(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3),

They build the basic tool for pre-semiotics as the discipline to analyze and describe the complex semiotic-ontological networks in the never-land between sign and object or ontological and semiotic space. The 15 sign classes thus mediate between two contextures, which are normally separated from one another by a polycontextural border that cannot be bridged by classical semiotics, since classical semiotics does not deal with structures that lie beneath the semiotic representation schemes (Bense 1986, pp. 64 ss.), and neither can such polycontextural borders be bridged by polycontextural theory, since the concept of sign is explicitly denied and refused both for kenogrammatics and morphogrammatics (cf. Kaehr 2004, pp. 2 ss.). Without pre-semiotics, the connection of polycontextural theory, semiotics and mathematics stays vague and unclear (cf. Kronthaler 1992, p. 296). Pre-semiotics thus starts there where my books “Outlines of a Mathematical Semiotics” (Toth 2007, pp. 246 ss.) and “The Marriage of Semiotics and Structure” (Toth 2003) end. The decisive hint to formalization of pre-semiotics was given by Bense himself (1975, p. 65).

2. Let us take as an example the pre-semiotic sign class (3.1 2.1 1.2 0.3). Its dual reality thematic (3.0 2.1 1.2 1.3) has the following 4! = 24 permutations:

$(\underline{1.3\ 1.2}\ 2.1\ 3.0)$	$(1^{2,>} \rightarrow 2^1 \leftrightarrow 3^1)$	$(2.1\ 3.0\ \underline{1.2\ 1.3})$	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,<})$
$(\underline{1.2\ 1.3}\ 2.1\ 3.0)$	$(1^{2,<} \rightarrow 2^1 \leftrightarrow 3^1)$	$(\underline{1.3}\ 3.0\ \underline{1.2}\ 2.1)$	$(1^{1,>} \rightarrow 3^1 \leftarrow 1^1 \rightarrow 2^1)$
$(\underline{1.3}\ 2.1\ \underline{1.2}\ 3.0)$	$(1^{1,>} \rightarrow 2^1 \leftarrow 1^1 \rightarrow 3^1)$	$(\underline{1.2}\ 3.0\ \underline{1.3}\ 2.1)$	$(1^{1,<} \rightarrow 3^1 \leftarrow 1^1 \rightarrow 2^1)$
$(2.1\ \underline{1.3\ 1.2}\ 3.0)$	$(2^1 \leftarrow 1^{2,>} \rightarrow 3^1)$	$(2.1\ 3.0\ \underline{1.3\ 1.2})$	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,>})$
$(\underline{1.2}\ 2.1\ \underline{1.3}\ 3.0)$	$(1^{1,<} \rightarrow 2^1 \leftarrow 1^1 \rightarrow 3^1)$	$(\underline{1.3}\ 3.0\ 2.1\ \underline{1.2})$	$(1^{1,>} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$
$(2.1\ \underline{1.2\ 1.3}\ 3.0)$	$(2^1 \leftarrow 1^{2,<} \rightarrow 3^1)$	$(\underline{1.2}\ 3.0\ 2.1\ \underline{1.3})$	$(1^{1,<} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$

$(\underline{1.3\ 1.2}\ 3.0\ 2.1)$	$(1^{2,>} \rightarrow 3^1 \leftrightarrow 2^1)$	$(3.0\ 2.1\ \underline{1.2\ 1.3})$	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,<})$
$(\underline{1.2\ 1.3}\ 3.0\ 2.1)$	$(1^{2,<} \rightarrow 3^1 \leftrightarrow 2^1)$	$(3.0\ \underline{1.2\ 1.3}\ 2.1)$	$(3^1 \leftarrow 1^{2,<} \rightarrow 2^1)$
$(\underline{1.3}\ 2.1\ 3.0\ \underline{1.2})$	$(1^{1,>} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	$(3.0\ \underline{1.3}\ 2.1\ \underline{1.2})$	$(3^1 \leftarrow 1^{1,>} \rightarrow 2^1 \leftarrow 1^1)$
$(2.1\ \underline{1.3}\ 3.0\ \underline{1.2})$	$(2^1 \leftarrow 1^{1,>} \rightarrow 3^1 \leftarrow 1^1)$	$(3.0\ 2.1\ \underline{1.3\ 1.2})$	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,>})$
$(\underline{1.2}\ 2.1\ 3.0\ \underline{1.3})$	$(1^{1,<} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	$(3.0\ \underline{1.2}\ 2.1\ \underline{1.3})$	$(3^1 \leftarrow 1^{1,<} \rightarrow 2^1 \leftarrow 1^1)$
$(2.1\ \underline{1.2}\ 3.0\ \underline{1.3})$	$(2^1 \leftarrow 1^{1,<} \rightarrow 3^1 \leftarrow 1^1)$	$(3.0\ \underline{1.3\ 1.2}\ 2.1)$	$(3^1 \leftarrow 1^{2,>} \rightarrow 2^1)$

Now, since each reality thematic can appear in the 4 semiotic contextures (cf. Toth 2008a, pp. 82 ss.), we get a total amount of no less than 96 main permutational tetradic pre-semiotic reality thematics and thus presented structural realities:

$(\underline{1.3\ 1.2}\ 2.1\ 3.0)$	$(1^{2,>} \rightarrow 2^1 \leftrightarrow 3^1)$	$(2.1\ 3.0\ \underline{1.2\ 1.3})$	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,<})$
$(-\underline{1.3\ -1.2}\ -2.1\ -3.0)$	$(-1^{2,>} \rightarrow -2^1 \leftrightarrow -3^1)$	$(-2.1\ -3.0\ -\underline{1.2\ -1.3})$	$(-2^1 \leftrightarrow -3^1 \leftarrow -1^{2,<})$
$(\underline{1.-3\ 1.-2}\ 2.-1\ 3.-0)$	$(1^{-2,>} \rightarrow 2^{-1} \leftrightarrow 3^{-1})$	$(2.-1\ 3.-0\ \underline{1.-2\ 1.-3})$	$(2^{-1} \leftrightarrow 3^{-1} \leftarrow 1^{-2,<})$
$(-\underline{1.-3\ -1.-2}\ -2.-1\ -3.-0)$	$(-1^{-2,>} \rightarrow -2^{-1} \leftrightarrow -3^{-1})$	$(-2.-1\ -3.-0\ -\underline{1.-2\ -1.-3})$	$(-2^{-1} \leftrightarrow -3^{-1} \leftarrow -1^{-2,<})$

$(\underline{1.2\ 1.3}\ 2.1\ 3.0)$	$(1^{2,<} \rightarrow 2^1 \leftrightarrow 3^1)$	$(\underline{1.3}\ 3.0\ \underline{1.2}\ 2.1)$	$(1^{1,>} \rightarrow 3^1 \leftarrow 1^1 \rightarrow 2^1)$
$(-\underline{1.2\ -1.3}\ -2.1\ -3.0)$	$(-1^{2,<} \rightarrow -2^1 \leftrightarrow -3^1)$	$(-\underline{1.3}\ -3.0\ -\underline{1.2}\ -2.1)$	$(-1^{1,>} \rightarrow -3^1 \leftarrow -1^1 \rightarrow -2^1)$
$(\underline{1.-2\ 1.-3}\ 2.-1\ 3.-0)$	$(1^{-2,<} \rightarrow 2^{-1} \leftrightarrow 3^{-1})$	$(\underline{1.-3}\ 3.-0\ \underline{1.-2}\ 2.-1)$	$(1^{-1,>} \rightarrow 3^{-1} \leftarrow 1^{-1} \rightarrow 2^{-1})$
$(-\underline{1.-2\ -1.-3}\ -2.-1\ -3.-0)$	$(-1^{-2,<} \rightarrow -2^{-1} \leftrightarrow -3^{-1})$	$(-\underline{1.-3}\ -3.-0\ -\underline{1.-2}\ -2.-1)$	$(-1^{-1,>} \rightarrow -3^{-1} \leftarrow -1^{-1} \rightarrow -2^{-1})$

$(\underline{1.3}\ 2.1\ \underline{1.2}\ 3.0)$	$(1^{1,>} \rightarrow 2^{1} \leftarrow 1^{1} \rightarrow 3^{1})$	$(\underline{1.2}\ 3.0\ \underline{1.3}\ 2.1)$	$(1^{1,<} \rightarrow 3^{1} \leftarrow 1^{1} \rightarrow 2^{1})$
$(-\underline{1.3}\ -2.1\ -\underline{1.2}\ -3.0)$	$(-1^{1,>} \rightarrow -2^{1} \leftarrow -1^{1} \rightarrow -3^{1})$	$(-\underline{1.2}\ -3.0\ -\underline{1.3}\ -2.1)$	$(-1^{1,<} \rightarrow -3^{1} \leftarrow -1^{1} \rightarrow -2^{1})$
$(\underline{1.-3}\ 2.-1\ \underline{1.-2}\ 3.-0)$	$(1^{-1,>} \rightarrow 2^{-1} \leftarrow 1^{-1} \rightarrow 3^{-1})$	$(\underline{1.-2}\ 3.-0\ \underline{1.-3}\ 2.-1)$	$(1^{-1,<} \rightarrow 3^{-1} \leftarrow 1^{-1} \rightarrow 2^{-1})$
$(-\underline{1.-3}\ -2.-1\ -\underline{1.-2}\ -3.-0)$	$(-1^{-1,>} \rightarrow -2^{-1} \leftarrow -1^{-1} \rightarrow -3^{-1})$	$(-\underline{1.-2}\ -3.-0\ -\underline{1.-3}\ -2.-1)$	$(-1^{-1,<} \rightarrow -3^{-1} \leftarrow -1^{-1} \rightarrow -2^{-1})$

$(2.1\ \underline{1.3\ 1.2}\ 3.0)$	$(2^{1} \leftarrow 1^{2,>} \rightarrow 3^{1})$	$(2.1\ 3.0\ \underline{1.3\ 1.2})$	$(2^{1} \leftrightarrow 3^{1} \leftarrow 1^{2,>})$
$(-2.1\ -\underline{1.3\ -1.2}\ -3.0)$	$(-2^{1} \leftarrow -1^{2,>} \rightarrow -3^{1})$	$(-2.1\ -3.0\ -\underline{1.3\ -1.2})$	$(-2^{1} \leftrightarrow -3^{1} \leftarrow -1^{2,>})$
$(2.-1\ \underline{1.-3\ 1.-2}\ 3.-0)$	$(2^{-1} \leftarrow 1^{-2,>} \rightarrow 3^{-1})$	$(2.-1\ 3.-0\ \underline{1.-3\ 1.-2})$	$(2^{-1} \leftrightarrow 3^{-1} \leftarrow 1^{-2,>})$
$(-2.-1\ -\underline{1.-3\ -1.-2}\ -3.-0)$	$(-2^{-1} \leftarrow -1^{-2,>} \rightarrow -3^{-1})$	$(-2.-1\ -3.-0\ -\underline{1.-3\ -1.-2})$	$(-2^{-1} \leftrightarrow -3^{-1} \leftarrow -1^{-2,>})$

$(\underline{1.2}\ 2.1\ \underline{1.3}\ 3.0)$	$(1^{1,<} \rightarrow 2^{1} \leftarrow 1^{1} \rightarrow 3^{1})$	$(\underline{1.3}\ 3.0\ 2.1\ \underline{1.2})$	$(1^{1,>} \rightarrow 3^{1} \leftrightarrow 2^{1} \leftarrow 1^{1})$
$(-\underline{1.2}\ -2.1\ -\underline{1.3}\ -3.0)$	$(-1^{1,<} \rightarrow -2^{1} \leftarrow -1^{1} \rightarrow -3^{1})$	$(-\underline{1.3}\ -3.0\ -2.1\ -\underline{1.2})$	$(-1^{1,>} \rightarrow -3^{1} \leftrightarrow -2^{1} \leftarrow -1^{1})$
$(\underline{1.-2}\ 2.-1\ \underline{1.-3}\ 3.-0)$	$(1^{-1,<} \rightarrow 2^{-1} \leftarrow 1^{-1} \rightarrow 3^{-1})$	$(\underline{1.-3}\ 3.-0\ 2.-1\ \underline{1.-2})$	$(1^{-1,>} \rightarrow 3^{-1} \leftrightarrow 2^{-1} \leftarrow 1^{-1})$
$(-\underline{1.-2}\ -2.-1\ -\underline{1.-3}\ -3.-0)$	$(-1^{-1,<} \rightarrow -2^{-1} \leftarrow -1^{-1} \rightarrow -3^{-1})$	$(-\underline{1.-3}\ -3.-0\ -2.-1\ -\underline{1.-2})$	$(-1^{-1,>} \rightarrow -3^{-1} \leftrightarrow -2^{-1} \leftarrow -1^{-1})$

$(2.1\ \underline{1.2\ 1.3}\ 3.0)$	$(2^{1} \leftarrow 1^{2,<} \rightarrow 3^{1})$	$(\underline{1.2}\ 3.0\ 2.1\ \underline{1.3})$	$(1^{1,<} \rightarrow 3^{1} \leftrightarrow 2^{1} \leftarrow 1^{1})$
$(-2.1\ -\underline{1.2\ -1.3}\ -3.0)$	$(-2^{1} \leftarrow -1^{2,<} \rightarrow -3^{1})$	$(-\underline{1.2}\ -3.0\ -2.1\ -\underline{1.3})$	$(-1^{1,<} \rightarrow -3^{1} \leftrightarrow -2^{1} \leftarrow -1^{1})$
$(2.-1\ \underline{1.-2\ 1.-3}\ 3.-0)$	$(2^{-1} \leftarrow 1^{-2,<} \rightarrow 3^{-1})$	$(\underline{1.-2}\ 3.-0\ 2.-1\ \underline{1.-3})$	$(1^{-1,<} \rightarrow 3^{-1} \leftrightarrow 2^{-1} \leftarrow 1^{-1})$
$(-2.-1\ -\underline{1.-2\ -1.-3}\ -3.-0)$	$(-2^{-1} \leftarrow -1^{-2,<} \rightarrow -3^{-1})$	$(-\underline{1.-2}\ -3.-0\ -2.-1\ -\underline{1.-3})$	$(-1^{-1,<} \rightarrow -3^{-1} \leftrightarrow -2^{-1} \leftarrow -1^{-1})$

$(\underline{1.3\ 1.2}\ 3.0\ 2.1)$	$(1^{2,>} \rightarrow 3^{1} \leftrightarrow 2^{1})$	$(3.0\ 2.1\ \underline{1.2\ 1.3})$	$(3^{1} \leftrightarrow 2^{1} \leftarrow 1^{2,<})$
$(-\underline{1.3\ -1.2}\ -3.0\ -2.1)$	$(-1^{2,>} \rightarrow -3^{1} \leftrightarrow -2^{1})$	$(-3.0\ -2.1\ -\underline{1.2\ -1.3})$	$(-3^{1} \leftrightarrow -2^{1} \leftarrow -1^{2,<})$
$(\underline{1.-3\ 1.-2}\ 3.-0\ 2.-1)$	$(1^{-2,>} \rightarrow 3^{-1} \leftrightarrow 2^{-1})$	$(3.-0\ 2.-1\ \underline{1.-2\ 1.-3})$	$(3^{-1} \leftrightarrow 2^{-1} \leftarrow 1^{-2,<})$
$(-\underline{1.-3\ -1.-2}\ -3.-0\ -2.-1)$	$(-1^{-2,>} \rightarrow -3^{-1} \leftrightarrow -2^{-1})$	$(-3.-0\ -2.-1\ -\underline{1.-2\ -1.-3})$	$(-3^{-1} \leftrightarrow -2^{-1} \leftarrow -1^{-2,<})$

$(\underline{1.2\ 1.3}\ 3.0\ 2.1)$	$(1^{2,<} \rightarrow 3^{1} \leftrightarrow 2^{1})$	$(3.0\ \underline{1.2\ 1.3}\ 2.1)$	$(3^{1} \leftarrow 1^{2,<} \rightarrow 2^{1})$
$(-\underline{1.2\ -1.3}\ -3.0\ -2.1)$	$(-1^{2,<} \rightarrow -3^{1} \leftrightarrow -2^{1})$	$(-3.0\ -\underline{1.2\ -1.3}\ -2.1)$	$(-3^{1} \leftarrow -1^{2,<} \rightarrow -2^{1})$
$(\underline{1.-2\ 1.-3}\ 3.-0\ 2.-1)$	$(1^{-2,<} \rightarrow 3^{-1} \leftrightarrow 2^{-1})$	$(3.-0\ \underline{1.-2\ 1.-3}\ 2.-1)$	$(3^{-1} \leftarrow 1^{-2,<} \rightarrow 2^{-1})$
$(-\underline{1.-2\ -1.-3}\ -3.-0\ -2.-1)$	$(-1^{-2,<} \rightarrow -3^{-1} \leftrightarrow -2^{-1})$	$(-3.-0\ -\underline{1.-2\ -1.-3}\ -2.-1)$	$(-3^{-1} \leftarrow -1^{-2,<} \rightarrow -2^{-1})$

$(\underline{1.3}\ 2.1\ 3.0\ \underline{1.2})$	$(1^{1,>} \rightarrow 2^{1} \leftrightarrow 3^{1} \leftarrow 1^{1})$	$(3.0\ \underline{1.3}\ 2.1\ \underline{1.2})$	$(3^{1} \leftarrow 1^{1,>} \rightarrow 2^{1} \leftarrow 1^{1})$
$(-\underline{1.3}\ -2.1\ -3.0\ -\underline{1.2})$	$(-1^{1,>} \rightarrow -2^{1} \leftrightarrow -3^{1} \leftarrow -1^{1})$	$(-3.0\ -\underline{1.3}\ -2.1\ -\underline{1.2})$	$(-3^{1} \leftarrow -1^{1,>} \rightarrow -2^{1} \leftarrow -1^{1})$
$(\underline{1.-3}\ 2.-1\ 3.-0\ \underline{1.-2})$	$(1^{-1,>} \rightarrow 2^{-1} \leftrightarrow 3^{-1} \leftarrow 1^{-1})$	$(3.-0\ \underline{1.-3}\ 2.-1\ \underline{1.-2})$	$(3^{-1} \leftarrow 1^{-1,>} \rightarrow 2^{-1} \leftarrow 1^{-1})$
$(-\underline{1.-3}\ -2.-1\ -3.-0\ -\underline{1.-2})$	$(-1^{-1,>} \rightarrow -2^{-1} \leftrightarrow -3^{-1} \leftarrow -1^{-1})$	$(-3.-0\ -\underline{1.-3}\ -2.-1\ -\underline{1.-2})$	$(-3^{-1} \leftarrow -1^{-1,>} \rightarrow -2^{-1} \leftarrow -1^{-1})$

$(2.1\ \underline{1.3}\ 3.0\ \underline{1.2})$	$(2^{1} \leftarrow 1^{1,>} \rightarrow 3^{1} \leftarrow 1^{1})$	$(3.0\ 2.1\ \underline{1.3\ 1.2})$	$(3^{1} \leftrightarrow 2^{1} \leftarrow 1^{2,>})$
$(-2.1\ -\underline{1.3}\ -3.0\ -\underline{1.2})$	$(-2^{1} \leftarrow -1^{1,>} \rightarrow -3^{1} \leftarrow -1^{1})$	$(-3.0\ -2.1\ -\underline{1.3\ -1.2})$	$(-3^{1} \leftrightarrow -2^{1} \leftarrow -1^{2,>})$
$(2.-1\ \underline{1.-3}\ 3.-0\ \underline{1.-2})$	$(2^{-1} \leftarrow 1^{-1,>} \rightarrow 3^{-1} \leftarrow 1^{-1})$	$(3.-0\ 2.-1\ \underline{1.-3\ 1.-2})$	$(3^{-1} \leftrightarrow 2^{-1} \leftarrow 1^{-2,>})$
$(-2.-1\ -\underline{1.-3}\ -3.-0\ -\underline{1.-2})$	$(-2^{-1} \leftarrow -1^{-1,>} \rightarrow -3^{-1} \leftarrow -1^{-1})$	$(-3.-0\ -2.-1\ -\underline{1.-3\ -1.-2})$	$(-3^{-1} \leftrightarrow -2^{-1} \leftarrow -1^{-2,>})$

$(\underline{1.2}\ 2.1\ 3.0\ \underline{1.3})$	$(1^{1,<} \rightarrow 2^{1} \leftrightarrow 3^{1} \leftarrow 1^{1})$	$(3.0\ \underline{1.2}\ 2.1\ \underline{1.3})$	$(3^{1} \leftarrow 1^{1,<} \rightarrow 2^{1} \leftarrow 1^{1})$
$(-\underline{1.2}\ -2.1\ -3.0\ -\underline{1.3})$	$(-1^{1,<} \rightarrow -2^{1} \leftrightarrow -3^{1} \leftarrow -1^{1})$	$(-3.0\ -\underline{1.2}\ -2.1\ -\underline{1.3})$	$(-3^{1} \leftarrow -1^{1,<} \rightarrow -2^{1} \leftarrow -1^{1})$
$(\underline{1.-2}\ 2.-1\ 3.-0\ \underline{1.-3})$	$(1^{-1,<} \rightarrow 2^{-1} \leftrightarrow 3^{-1} \leftarrow 1^{-1})$	$(3.-0\ \underline{1.-2}\ 2.-1\ \underline{1.-3})$	$(3^{-1} \leftarrow 1^{-1,<} \rightarrow 2^{-1} \leftarrow 1^{-1})$
$(-\underline{1.-2}\ -2.-1\ -3.-0\ -\underline{1.-3})$	$(-1^{-1,<} \rightarrow -2^{-1} \leftrightarrow -3^{-1} \leftarrow -1^{-1})$	$(-3.-0\ -\underline{1.-2}\ -2.-1\ -\underline{1.-3})$	$(-3^{-1} \leftarrow -1^{-1,<} \rightarrow -2^{-1} \leftarrow -1^{-1})$

$(2.1\ \underline{1.2}\ 3.0\ \underline{1.3})$	$(2^{1} \leftarrow 1^{1,<} \rightarrow 3^{1} \leftarrow 1^{1})$	$(3.0\ \underline{1.3\ 1.2}\ 2.1)$	$(3^{1} \leftarrow 1^{2,>} \rightarrow 2^{1})$
$(-2.1\ -\underline{1.2}\ -3.0\ -\underline{1.3})$	$(-2^{1} \leftarrow -1^{1,<} \rightarrow -3^{1} \leftarrow -1^{1})$	$(-3.0\ -\underline{1.3\ -1.2}\ -2.1)$	$(-3^{1} \leftarrow -1^{2,>} \rightarrow -2^{1})$
$(2.-1\ \underline{1.-2}\ 3.-0\ \underline{1.-3})$	$(2^{-1} \leftarrow 1^{-1,<} \rightarrow 3^{-1} \leftarrow 1^{-1})$	$(3.-0\ \underline{1.-3\ 1.-2}\ 2.-1)$	$(3^{-1} \leftarrow 1^{-2,>} \rightarrow 2^{-1})$
$(-2.-1\ -\underline{1.-2}\ -3.-0\ -\underline{1.-3})$	$(-2^{-1} \leftarrow -1^{-1,<} \rightarrow -3^{-1} \leftarrow -1^{-1})$	$(-3.-0\ -\underline{1.-3\ -1.-2}\ -2.-1)$	$(-3^{-1} \leftarrow -1^{-2,>} \rightarrow -2^{-1})$

We can recognize the following 10 possible types of thematization in the above structural realities. A, B, C, D $\in$ (0.1, 0.2, 0.3, 1.1, 1.2, 1.3, ..., 3.3), A $\neq$ B (refers to trichotomic difference). If C and D are different in their triadic value, the thematization is triadic, otherwise dyadic:

1. (A, B), C, D
2. (B, A), C, D
3. (A), C, (B), D
4. (B), C, (A), D
5. C, (A, B), D
6. C, (B, A), D
7. C, (A), D, (B)
8. C, (B), D, (A)
9. C, D, (A, B)
10. C, D, (B, A)

Besides these basic 96 structural realities, each reality thematic can appear with "mixed" algebraic signs, i.e. it can lie in 1, 2, 3, or 4 semiotic contextures according to the fact that pre-semiotic sign classes are tetradic.

Examples for pre-semiotic reality thematics in 1 contexture: all 96 above reality thematics.

Examples for pre-semiotic reality thematics in 2 contextures:

$(-2.-1\ -\underline{1.-2}\ 3.-0\ -\underline{1.-3})$, $(-2.-1\ -\underline{1.-2}\ -3.0\ -\underline{1.-3})$, $(2.1\ -\underline{1.-2}\ -3.-0\ -\underline{1.-3})$, etc.

Examples for pre-semiotic reality thematics in 3 contextures:

$(2.-1\ -\underline{1.-2}\ -3.0\ -\underline{1.-3})$, $(-2.-1\ \underline{1.2}\ 3.-0\ -\underline{1.-3})$, $(2.1\ \underline{1.-2}\ -3.0\ -\underline{1.-3})$, etc.

Examples for pre-semiotic reality thematics in 4 contextures:

$(-2.-1\ -\underline{1.2}\ 3.-0\ \underline{1.3})$, $(2.1\ -\underline{1.-2}\ -3.0\ \underline{1.-3})$, $(2.-1\ \underline{1.2}\ -3.0\ -\underline{1.-3})$, etc.

In addition to the mathematical basics presented in Toth (2001; 2008a, pp. 97 ss.), we want underline that tetradic pre-semiotic sign classes and reality thematics need a 3-dimensional space in order to be displayed. Since SR can be displayed in a 2-dimensional semiotic space, and since (3.a 2.b 1.c) $\not\subset$ (3.a 2.b 1.c 0.d), the 2-dimensional semiotic space cannot be embedded in the 3-dimensional pre-semiotic space! The construction of the latter is a big challenge for future mathematical semiotic inquiry.

3. We shall now have a look at the general abstract structure of the 96 structural realities presented by each reality thematic of the 15 pre-semiotic sign classes. But first, we want to outline that the category of zeroness that guarantees the quality of these sign classes and reality thematics and integrates SR into PSR by connecting the semiotic with the ontological space can appear in all 4 positions of the tetradic sign relations:

(0.d 1.c 2.b 3.a) × (a.3 b.2 c.1 d.0)
(0.d 1.c 3.a 2.b) × (b.2 a.3 c.1 d.0)
(0.d 2.b 1.c 3.a) × (a.3 c.1 b.2 d.0)
(0.d 2.b 3.a 1.c) × (c.1 a.3 b.2 d.0)
(0.d 3.a 1.c 2.b) × (b.2 c.1 a.3 d.0)
(0.d 3.a 2.b 1.c) × (c.1 b.2 a.3 d.0)

(3.a 2.b 0.d 1.c) × (c.1 d.0 b.2 a.3)
(1.c 2.b 0.d 3.a) × (a.3 d.0 b.2 c.1)
(1.c 3.a 0.d 2.b) × (b.2 d.0 a.3 c.1)
(2.b 3.a 0.d 1.c) × (c.1 d.0 a.3 b.2)
(2.b 1.c 0.d 3.a) × (a.3 d.0 c.1 b.2)
(3.a 1.c 0.d 2.b) × (b.2 d.0 c.1 a.3)

(1.c 0.d 2.b 3.a) × (a.3 b.2 d.0 c.1)
(1.c 0.d 3.a 2.b) × (b.2 a.3 d.0 c.1)
(2.b 0.d 1.c 3.a) × (a.3 c.1 d.0 b.2)
(2.b 0.d 3.a 1.c) × (c.1 a.3 d.0 b.2)
(3.a 0.d 1.c 2.b) × (b.2 c.1 d.0 a.3)
(3.a 0.d 2.b 1.c) × (c.1 b.2 d.0 a.3)

(3.a 2.b 1.c 0.d) × (d.0 c.1 b.2 a.3)
(1.c 3.a 2.b 0.d) × (d.0 b.2 a.3 c.1)
(2.b 1.c 3.a 0.d) × (d.0 a.3 c.1 b.2)
(2.b 3.a 1.c 0.d) × (d.0 c.1 a.3 b.2)
(3.a 1.c 2.b 0.d) × (d.0 b.2 c.1 a.3)
(3.a 2.b 1.c 0.d) × (d.0 c.3 b.2 a.1)

The following table presents the abstract structures of all 2 × 24 permutations of each of the 15 pre-semiotic sign classes and their dual reality thematics:

1.	(±3.±a	±2.±b	±1.±c	±0.±d)	×	(±d.±0	±c.±1	±b.±2	±a.±3)
2.	(±2.±b	±1.±c	±3.±a	±0.±d)	×	(±d.±0	±a.±3	±c.±1	±b.±2)
3.	(±1.±c	±2.±b	±0.±d	±3.±a)	×	(±a.±3	±d.±0	±b.±2	±c.±1)
4.	(±2.±b	±1.±c	±0.±d	±3.±a)	×	(±a.±3	±d.±0	±c.±1	±b.±2)
5.	(±1.±c	±3.±a	±2.±b	±0.±d)	×	(±d.±0	±b.±2	±a.±3	±c.±1)
6.	(±2.±b	±3.±a	±1.±c	±0.±d)	×	(±d.±0	±c.±1	±a.±3	±b.±2)
7.	(±1.±c	±3.±a	±0.±d	±2.±b)	×	(±b.±2	±d.±0	±a.±3	±c.±1)
8.	(±2.±b	±3.±a	±0.±d	±1.±c)	×	(±c.±1	±d.±0	±a.±3	±b. ±2)
9.	(±1.±c	±0.±d	±2.±b	±3.±a)	×	(±a.±3	±b.±2	±d.±0	±c.±1)
10.	(±2.±b	±0.±d	±1.±c	±3.±a)	×	(±a.±3	±c.±1	±d.±0	±b.±2)
11.	(±1.±c	±0.±d	±3.±a	±2.±b)	×	(±b.±2	±a.±3	±d.±0	±c.±1)
12.	(±2.±b	±0.±d	±3.±a	±1.±c)	×	(±c.±1	±a.±3	±d.±0	±b.±2)
13.	(±3.±a	±1.±c	±2.±b	±0.±d)	×	(±d.±0	±b.±2	±c.±1	±a.±3)
14.	(±0.±d	±1.±c	±2.±b	±3.±a)	×	(±a.±3	±b.±2	±c.±1	±d.±0)
15.	(±3.±a	±1.±c	±0.±d	±2.±b)	×	(±b.±2	±d.±0	±c.±1	±a.±3)
16.	(±0.±d	±1.±c	±3.±a	±2.±b)	×	(±b.±2	±a.±3	±c.±1	±d.±0)
17.	(±3.±a	±2. ±b	±1.±c	±0.±d)	×	(±d.±0	±c.±1	±b.±2	±a.±3)
18.	(±0.±d	±2.±b	±1.±c	±3.±a)	×	(±a.±3	±c.±1	±b.±2	±d.±0)
19.	(±3.±a	±2.±b	±0.±d	±1.±c)	×	(±c.±1	±d.±0	±b.±2	±a.±3)

20.(±0.±d	±2.±b	±3.±a	±1.±c)	×	(±c.±1	±a.±3	±b.±2	±d.±0)
21.(±3.±a	±0.±d	±1.±c	±2.±b)	×	(±b.±2	±c.±1	±d.±0	±a.±3)
22.(±0.±d	±3.±a	±1.±c	±2.±b)	×	(±b.±2	±c.±1	±a.±3	±d.±0)
23.(±3.±a	±0.±d	±2.±b	±1.±c)	×	(±c.±1	±b.±2	±d.±0	±a.±3)
24.(±0.±d	±3.±a	±2.±b	±1.±c)	×	(±c.±1	±b.±2	±a.±3	±d.±0)

Thus the 48 permutational structures of pre-semiotic representative systems can appear in 4 contextures, which yields 192 homogeneous pre-semiotic sign classes and reality thematics. Since the pre-semiotic system has 15 representative systems, we obtain totally 2'880 homogeneous pre-semiotic sign classes and reality thematics. If we also allow heterogenous representation systems, i.e. sign classes and reality thematics that lie in more than one semiotic contexture, we get a mathematical-semiotic instrument that is fully adequate to describe and analyze the enormous complexity of the networks between semiotic and ontological spaces and thus between semiotics and polycontextural theory.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Kaehr, Rudolf, Skizze eines Gewebes rechnender Räume in denkender Leere. Glasgow 2004. Digital version: www.vordenker.de

Kronthaler, Engelbert, Zahl – Zeichen – Begriff. In: Semiosis 65-68, 1992, pp. 282-310

Toth, Alfred, Monokontexturale und polykontexturale Semiotik. In: Bernard, Jeff/Withalm, Gloria (eds.), Myths, Rites, Simulacra. Vol. I. Vienna 2001, pp. 117-134

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2008 (2008a)

Toth, Alfred, Relational and categorial numbers. Ch. 40 (2008b)

Toth, Alfred, Tetradic sign classes from relational and categorial numbers. Ch. 41 (2008c)