Prof. Dr. Alfred Toth

Elements of a recursive semiotics

1. In Toth (2008c), we have shown that mathematical semiotics can be based on Zermelo-Fraenkel set theory with anti-foundation axiom (cf. also Toth 2007, pp. 14 ss.), thus explicitly allowing recursivity (cf. Mirimanoff 1917; Barwise/Etchemendy 1987; Aczel 1988). In the present study, we introduce a complete substitution for the sign relation as ordered relation over relations (cf. Toth 2008b) by unordered sets in continuation of Wiener (1914). On this basis, we further make explicit the extremely intricate semiotic relations between sign classes and their transpositions as well as other recursive semiotic functions.

2. A sign (S) is an ordered relation between three objects x, y, z:

$S = \langle x, y, z \rangle$

Yet, these objects x, y and z are considered relations themselves, and x is a monadic, y a dyadic and z a triadic relation:

$y = \langle x, y \rangle$
$z = \langle x, y, z \rangle$

Therefore, we have

$S = \langle x, \langle x, y \rangle, \langle x, y, z \rangle$

Now, we can substitute ordered relations by unordered sets. With $\langle x, y \rangle = \{x, \{x, y\}\}$ (Wiener 1914), we get

$y = \{x, \{x, y\}\}$
$z = \{x, \{x, \{x, y\}\}, \{x, \{y, z\}, \{y, \{y, z\}\}\}$

$S = \{x, \{x, \{x, y\}\}, \{x, \{x, \{x, y\}\}, \{x, \{y, z\}, \{y, \{y, z\}\}\}\}$

As an example, we take the sign class (3.1 2.1 1.3) which we thus may rewrite as unordered set over unordered sets:

$S = \{\langle 1.3 \rangle, \{\langle 1.3 \rangle, \{\langle 1.3 \rangle, \langle 2.1 \rangle\}\}, \{\langle 1.3 \rangle, \{\langle 1.3 \rangle, \{\langle 1.3 \rangle, \langle 2.1 \rangle\}\}, \{\langle 1.3 \rangle,$
$\{\langle 2.1 \rangle, \langle 3.1 \rangle\}, \{\langle 2.1 \rangle, \{\langle 2.1 \rangle, \langle 3.1 \rangle\}\}\}\}$

Further, we can reduce the sub-signs to their constitutive prime-signs by the same method:

$x = \langle a.b \rangle = \{a, \{a, b\}\}$
$y = \langle c.d \rangle = \{c, \{c, d\}\}$
$z = \langle e.f \rangle = \{e, \{e, f\}\}$

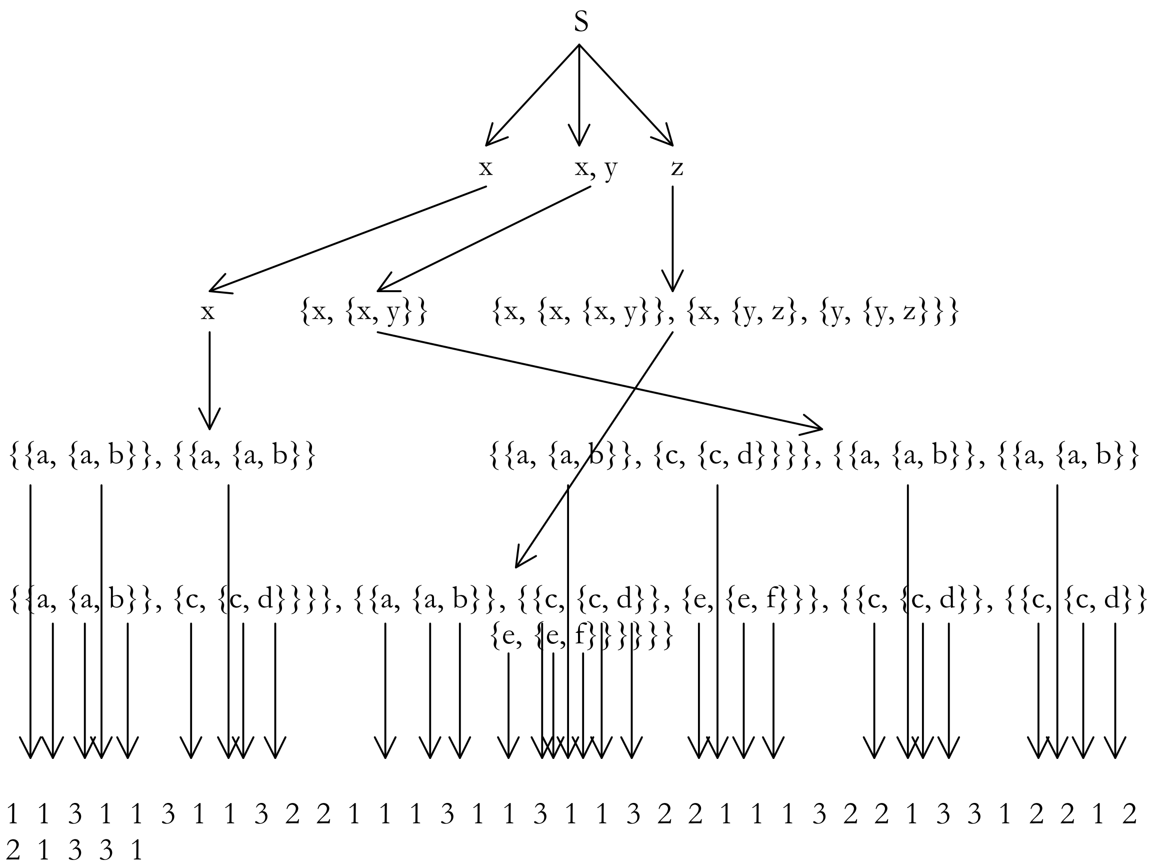
Thus,

S = {{a, {a, b}}, {{a, {a, b}}, {{a, {a, b}}, {c, {c, d}}}}, {{a, {a, b}}, {{a, {a, b}}, {{a, {a, b}}, {c, {c, d}}}}, {{a, {a, b}}, {{c, {c, d}}, {e, {e, f}}}, {{c, {c, d}}, {{c, {c, d}}, {e, {e, f}}}}}}

Since in the above example, x = <1.3>, y = <2.1>, and z = <3.1>, we get

S = {{1, {1, 3}}, {{1, {1, 3}}, {{1, {1, 3}}, {2, {2, 1}}}}, {{1, {1, 3}}, {{1, {1, 3}}, {{1, {1, 3}}, {2, {2, 1}}}}, {{1, {1, 3}}, {{2, {2, 1}}, {3, {3, 1}}}, {{2, {2, 1}}, {{2, {2, 1}}, {3, {3, 1}}}}}}

We may now visualize the sign relation as a triadic relation over a monadic, a dyadic and a triadic relation using strictly unordered sets, in the following diagram, showing again the example of the sign class (3.1 2.1 1.3):



3. We have already shown in earlier studies, that a sign class is nothing but a special case of six possible permutations or transpositions of the semiotic order structure (X. → Y. → Z.) with X, Y, Z ∈ {1, 2, 3}. Therefore, the following 6 possible transpositions of a sign class, obeying the 6 possible semiotic order structures, are possible. As an example, we use again the sign class (3.1 2.1 1.3):

(3.1 2.1 1.3) (3. → 2. → 1.)
(3.1 1.3 2.1) (3. → 1. → 2.)
(2.1 3.1 1.3) (2. → 3. → 1.)
(2.1 1.3 3.1) (2. → 1. → 3.)
(1.3 3.1 2.1) (1. → 3. → 2.)
(1.3 2.1 3.1) (1. → 2. → 3.)

If we now transform the transpositions into unordered sets of unordered sets, we get

1. (3.1 2.1 1.3)
1 3 3 1 2 2 1 2 2 1 3 3 1 2 2 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1 1 2 2 3 1 1 3
1 1 3 1 1

2. (1.3 2.1 3.1)
1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2
2 1 3 3 1

3. (3.1 1.3 2.1)
1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3 1 1 3 2 2 1 1
1 3 1 1 3

4. (2.1 1.3 3.1)
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2
2 1 3 3 1

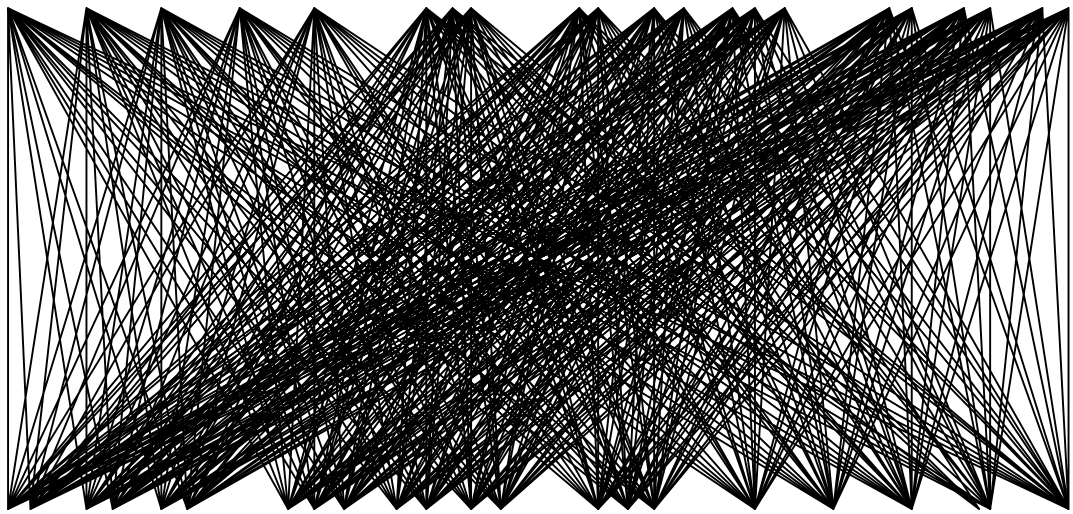
5. (2.1 3.1 1.3)
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1
1 3 1 1 3

6. (1.3 3.1 2.1)
1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 2 2 1 1
1 3 1 1 3

Totally, 15 combinations of transpositions are possible. A small fragment of the extremely intricate recursive structure between the two transpositions (3.1 2.1 1.3) and (1.3 2.1 3.1) is shown in the following diagram, omitting the relations between secondness (2) and thirdness (3) for the sake of avoiding even more complexity:

(Diagram of the combination of transpositions no. 1 and 2:)

1 3 3 1 2 2 1 2 2 1 3 3 1 2 2 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1



1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1

For the 14 remaining possible combinations of transpositions of a sign class, we restrict ourselves to indicate the order of the prime-signs. By “1-2”, “1-3”, etc., we denote combinations of the transpositions of the sign class (3.1 2.1 1.3) in the above numbering:

1-3

1 3 3 1 2 2 1 2 2 1 3 3 1 2 2 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1
1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3

1-4

1 3 3 1 2 2 1 2 2 1 3 3 1 2 2 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1

1-5

1 3 3 1 2 2 1 2 2 1 3 3 1 2 2 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3

1-6

1 3 3 1 2 2 1 2 2 1 3 3 1 2 2 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1 1 2 2 3 1 1 3 1 1 3 1 1
1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 2 2 1 1 1 3 1 1 3

2-3

1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1
1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3

2-4

1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1

2-5

1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3

2-6

1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1
1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 2 2 1 1 1 3 1 1 3

3-4

1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1

3-5

1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3

3-6

1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 2 2 1 1 1 3 1 1 3

4-5

1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3

4-6

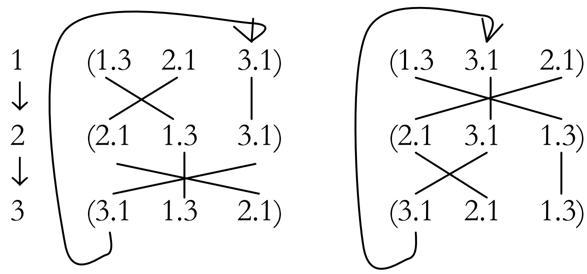
1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1
1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 2 2 1 1 1 3 1 1 3

5-6

1 1 3 2 2 1 1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3 2 2 1 1 1 3 2 2 1 3 3 1 2 2 1 2 2 1 3 3 1 1 1 3 2 2 1 1 1 3 1 1 3

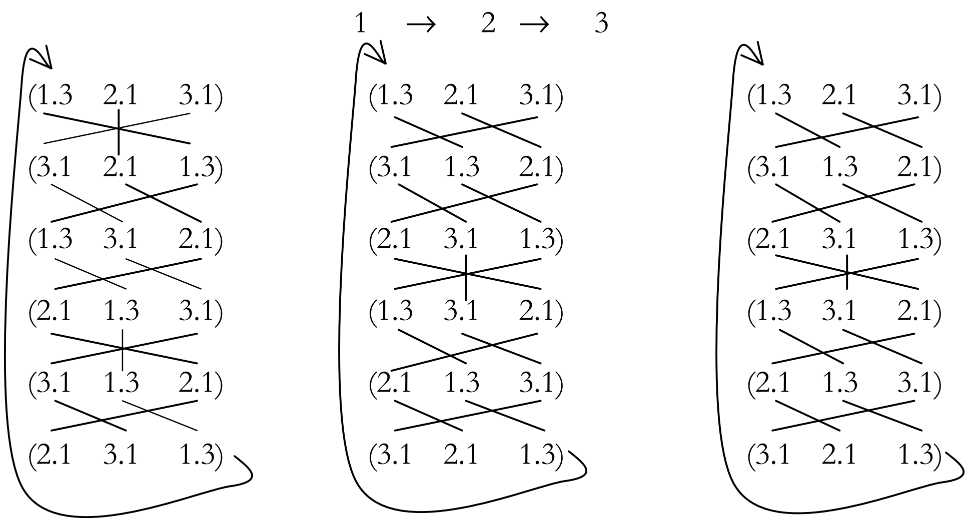
4. We shall now have a look at possible recursive structures built from transpositions of sign classes (or their dual reality thematics).

4.1. First, we may induce recursion by ordering the transpositions according to the natural numbers corresponding to prime-signs in triadic relations. In this case, there are 2 possibilities:



In this case, the recursive cycles contain 3 transpositions each.

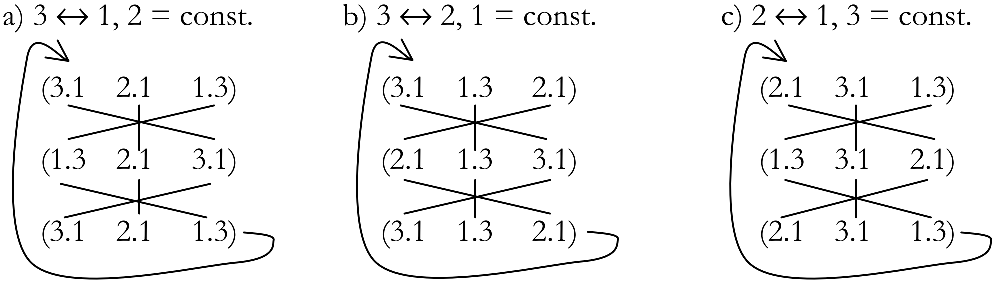
4.2. Second, we may also induce recursion by ordering the transpositions according to the natural numbers corresponding to prime-signs in trichotomic relations. In this case, there are 15 possibilities amongst which we show the following 3:



Here, the recursive cycles contain all 6 transpositions each.

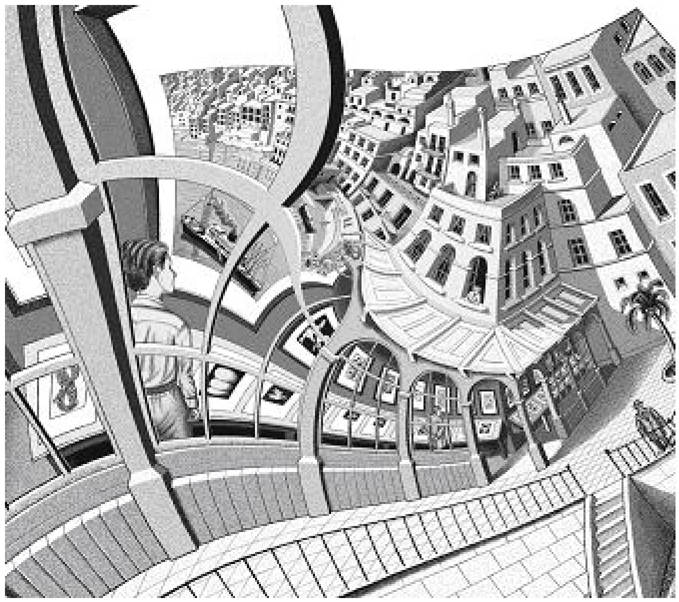
4.3. Using some results presented in Toth (2008a), recursive structures are often induced by ordering the transpositions according to their orthogonal (mirroring) counterparts, which behave like the three pairs of opposite sides of a cube. To these type also belongs our above diagram with its heavily complicated structure:

a) 3 ↔ 1, 2 = const. b) 3 ↔ 2, 1 = const. c) 2 ↔ 1, 3 = const.



In this case, we get recursive cycles of 3 transpositions each.

To conclude, we show M.C. Escher's famous lithography "Print Gallery" (1956) which is an artistic visualization of the recursive semiotic structures investigated in this present study, but hitherto neglected by the merely monadic mathematical works dedicated to Escher's oeuvre:



http://www.math.uni-konstanz.de/fb_seiten/contrib/startseite/JDM/vortraege/escher.php

Bibliography

Aczel, Peter, Non-Well-Founded Sets. Stanford 1988

Barwise, Jon/Etchemendy, John, The Liar: An Essay on Truth and Circular Propositions. Oxford 1987

Mirimanoff, Dmitry, Les antinomies de Russell et de Burali-Forti et le problème fondamental de la théorie des ensembles. In: L'Enseignement Mathématique 19, 1917, pp. 37-52

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Semiotic perspectives from Another World. Ch. 10 (2008a)

Toth, Alfred, The sign as relation over relations. Ch. 12 (2008b)

Toth, Alfred, The Droste effect in Semiotics. To appear in: Grundlagenstudien aus Kybernetik und Geisteswissenschaft 49/4, 2008 and ch. 15 (2008c)

Wiener, Norbert, A simplification of the logic of relations. In: Proceedings of the Cambridge Philosophical Society 17, 1914, pp. 387-390