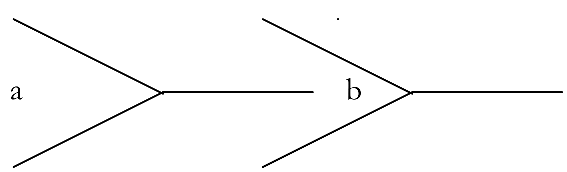
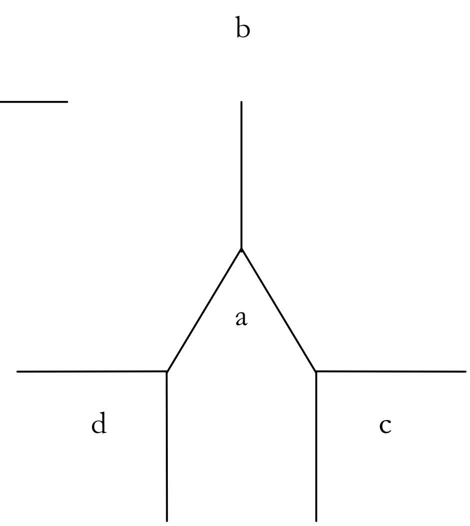
Prof. Dr. Alfred Toth

Notes on Peirce's reduction theorem

1. Peirce asserts in many passages of this work that each n-adic relation with n > 3 be dissolved into a combination of triadic relations: "In his 3rd Lowell lecture (partly to be found in CP. 1.343-349), Peirce has given such a proof, too, namely by aid of his existential graphs by starting from a triadic relation and noticing that a graph with three 'tails' cannot be built from graphs with two or one tails each, but graphs with three tails are sufficient to build graphs with an arbitrarily high number of tails:





And our analysis will show that each relation which is tetradic, pentadic or has any higher number of correlates, is nothing else than a combination of triadic relations. Thus it does not surprise that one finds that besides the three elements of Firstness, Secondness and Thirdness there is noting else to be found in the phenomenon' (CP. 1.347)" (Walther 1989, p. 298).

In a similar manner the modern proof of Marty (1980) is constructed. According to Marty, a tetradic relation R(A, B, C, D) can be displayed as a concatenation of two triadic relations R(A, B, (C, D)) and R((A, B), C, D). The indices 1 and 2 refer to monadic and dyadic part-relations, respectively:

$$R(A, B, C, D) := R_{1,1,2}(A, B, (C, D)) * R_{2,2}\, R_{2,1,1}((A, B), C, D)$$

The respective scheme looks as follows:

$$\begin{array}{ccccccccccccc} & & B & & & & & & & & C & & \\ & & | & & & & & & & & | & & \\ A & — & R_{1,1,2} & — & (C, D) & — & R_{2,2} & — & (A, B) & — & R_{2,1,1} & — & D \end{array}$$

The special case for n = 4 can be expanded to any n-adic relation:

$R(A_1, A_2, \ldots, A_n) :=$
$R_{1,1,\ldots,2}(A_1, A_2, \ldots, (A_{n-1}, A_n)) * R_{n-2,2}\, R_{n-2,1,1}((A_1, \ldots, A_{n-2}), A_{n-1}, A_n),$

and we get the following respective scheme:

A_2 ---------- A_{n-2} $\qquad\qquad\qquad\qquad$ A_{n-1}

A_1 — $R_{1,1,\ldots,2}$ — (A_{n-1}, A_n) — $R_{n-2,2}$ — $(A_{1, \ldots, A n-2})$ — $R_{n-2,1,1}$ — A_n

Based on this, in theoretical semiotics, the sign is understood as „a triadic relation of three relational members whose first, the ‚medium' (M), is monadic, whose second, the ‚relation of the object' (O), is dyadic, and whose third, the ‚interpretant' (I), is triadic. Therefore, the complete sign is a hierarchic triadic relation of relations" (Bense 1979, p. 67).

2.1. Peirce's assertion, that "a graph with three 'tails' cannot be built from graphs with two or one tails each" is wrong, at least insofar as we speak about sign relations.

2.1.1. The semiotic matrix contains the following 9 sub-signs: {(1.1), (1.2), (1.3), (2.1), (2.2), (2.3), (3.1), (3.2), (3.3)}. Since Peirce's "pragmatic maxim" (cf. Bense 1975, p. 165) requires that a triadic sign relation must be built from three monadic relations in the following order:

(3.a 2.b 1.c),

each of the 9 sub-signs can theoretically appear as trichotomy of the three triadic positions. However, the semiotic inclusion order requires in addition that

$(a \geq b \geq c)$,

so that the total possible amount of $3 \cdot 3 \cdot 3 = 27$ combinations is reduced to 10 triadic relations, which are called, following Peirce, "sign classes", thus excluding the following three types of possible semiotic orders

$(a > b > c)$
$(a > b < c)$
$(a < b > c)$

These two semiotic restrictions on general triadic relations – the pragmatic maxim and the inclusion order – allow now to generate all and exactly the 10 triadic sign relations from the 9 monadic relations displayed in the semiotic matrix.

2.1.2. In order to show that triadic relations can also be built from dyadic relations, it suffices to refer to Walther (1979, p. 79), who had shown that all and exactly the 10 triadic sign relations can be constructed by concatenations of two dyadic relations. Thereby, the 9 · 9 · 9 = 729 possible dyadic relations are reduced to 10 · 2 = 20 dyadic relations, namely the 2 dyadic relations of which each of the 10 sign classes consists.

2.2. The third assertion of Peirce, that each n-adic sign relation for n > 3 can be reduced to triadic relations, is true, however, only if we specify it massivly. Besides the fact that n-adic relations for n > 3 exhibit different semiotic structures not present in semiotic relations of lower degrees (cf. Toth 2008, pp. 173 ss.), from which results that tetradic, pentadic, hexadic, etc. relations are qualitatively not reducible to triadic relations, their purely quantitative reduction to triadic relation works only if one introduces a semiotic normal form operator that changes the triadic values for n > 3 in n-adic relations with n > 3.

In order to show that, let us take the tetradic sign relation

(4.1 3.1 2.1 1.3),

which can be reduced to the following two triadic relations:

(4.1 3.1 2.1) and (3.1 2.1 1.3).

Obviously, (4.1 3.1 2.1) contradicts to the above mentioned law which results from Peirce's own Pragmatic Maxim. So, to turn this triadic relation into a proper sign relation, we have to introduce a normal form operator that changes (4.) → (3.), (3.) → (2.) and (2.) → (1.) respecting the positions (i.e. non-cyclic) of the three correlates of the triadic sign relation, thus:

N(4.1 3.1 2.1) = (3.1 2.1 1.1).

As one sees, the normal form operator N leaves the trichotomic values alone. The second triadic relation as part of the tetradic relation is left unchanged by N.

Since Peirce did not state of how many triadic relations an n-adic relation for n > 3 consists, let us remember that each n-adic relation can be split up into n-2 dyadic relations, so a triadic relation consists of (3-2) = 1 dyadic relations. Thus, the following pentadic relation

(5.1 4.1 3.1 2.1 1.1)

can be split up into 5-2 = 3 triadic relations

1. (5.1 4.1 3.1)
2. (4.1 3.1 2.1)
3. (3.1 2.1 1.1),

which have then to undergo the normal form operator N:

1'. N(5.1 4.1 3.1) = (3.1 2.1 1.1)
2.' N(4.1 3.1 2.1) = (3.1 2.1 1.1)

3.‘ N(3.1 2.1 1.1) = (3.1 2.1 1.1),

that apparently puts out three times the same triadic relation (3.1 2.1 1.1). However, at this point, it has to be emphasized again that these three triadic relations are quantitatively, but not qualitatively identical (cf. Toth 2008, pp. 186 ss.). Moreover, N does not put out quantitatively identical triadic sign classes, if the higher-than-triadic sign classes are not trichotomically homogeneous, e.g. in the case of (5.1 4.2 3.3 2.3 1.3):

1“. N(5.1 4.2 3.3) = (3.1 2.2 1.3)
2.“ N(4.2 3.3 2.3) = (3.2 2.3 1.3)
3.“ N(3.3 2.3 1.3) = (3.3 2.3 1.3)

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975
Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979
Marty, Robert, Sur la réduction triadique. In: Semiosis 17/18, 1980, pp. 5-9
Peirce, Charles Sanders, Collected Papers, ed. by Charles Hartshorne and Paul Weiss. Cambridge, MA 1933-1958
Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2008
Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979
Walther, Elisabeth, Charles Sanders Peirce. Leben und Werk. Baden-Baden 1989