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Relational and categorial numbers

1. In Bense (1975, p. 65), we find the astonishing sentence: "A medium which exists independently from each sign relation but as a potential medium M°, has the relational number r = 0". Bense's background is the construction of a topological (combinatorial) semiotic space: "The space with a 0-relational or 0-place semiotic structure would not be a semiotic space, but the ontological space of all available things O°, over which the semiotic space with r > 0 will be defined or introduced thetically". Thus, the relational number bridges the semiotic and the ontological space. However, there is another type of semiotic number, which bridges between the semiotic categories. This number, "which can be abstracted from the Peircean universal categories 'Firstness', 'Secondness' and 'Thirdness', is the so-called categorial number which refers to the fact that a sign process (of sign-generative or sign-degenerative type) is associated with each sign, and indicates which place value a certain sign with relational number r, thus as Z^r, possesses in the generative or degenerative semiosis such that a sign has not only to bear the relational number r but also the categorial number k as an index, if the semiotic space, in which it is embedded, is determined. Thus the complete notation of a sign Z would be $Z^r{}_k$" (Bense 1975, pp. 65 s.).

2. Semiotics is characterized by the three invariants of the medium relation (M), the denomination relation (M ⇒ O) and the designation relation (O ⇒ I), from which it follows, that the semiotic object and the semiotic interpretant are invariant, too. Medium, object and interpretant relation show, in their trichotomies, invariance of consistency (Firstness), invariance of identification (Secondness), and invariance of existence (Thirdness), cf. Toth (2008, pp. 166 ss.). By aid of the semiotic invariance schemes, presented objects are mapped onto "available" media. Bense (1975, pp. 45 s.) gives the following examples for this transition from the ontological to the semiotic space. (The superscript 0 indicates that the objects and media have relational number 0, because in this first state of transition, they are not yet embedded into a triadic relation; Bense 1975, p. 65):

$O^0 \Rightarrow M^0$:	**three available media**
$O^0 \Rightarrow M_1^0$:	qualitative substrate: heat
$O^0 \Rightarrow M_2^0$:	singular substrate: trail of smoke
$O^0 \Rightarrow M_3^0$:	nominal substrate: name

In a second level of transition, the available media are mapped onto relational media. In this process, the semiotic invariance scheme is "inherited":

$M^0 \Rightarrow M$:	**three relational media**
$M_1^0 \Rightarrow (1.1)$:	heat
$M_2^0 \Rightarrow (1.2)$:	trail of smoke
$M_3^0 \Rightarrow (1.3)$:	"fire"

With the three trichotomic sub-signs of Firstness, we are, of course, already inside of the semiotic space. But how can the three available media M_i^0 themselves be characterized? In order to do that, Matthias Götz (1982, p. 28) proposed a pre-semiotic level of "Zeroness" and its splitting into the following "pre-trichotomic" levels:

0.1 = secancy
0.2 = semancy
0.3 = selectancy,

whereby the neologisms are connected with Latin secare "to cut", with "semantics", and with "select": "Secancy as a diaphragmatic condition, which first of all must be designated as such, in order to enable semiotic mediation, since in-differentiated things cannot be represented; "semancy" as the condition for enabling form to be described as form"; selectancy as condition of posterior use, if this is conceived as selective process, or more generally as dealing with the object" (Götz 1982, p. 4).

If we sum up, we get the following scheme:

$$\underbrace{\text{presented objects}}_{\text{World of Objects}} \rightarrow \underbrace{\text{available media}}_{\text{Pre-semiotics}} \rightarrow \underbrace{\text{relational media}}_{\text{Semiotics}}$$

$$\underbrace{\text{World of Objects}}_{\text{Ontological Space}} \rightarrow \underbrace{\text{Pre-semiotics} \rightarrow \text{Semiotics}}_{\text{Semiotic Space}}$$

Scheme of Invariance: → Heredity of Invariance:

$$\left.\begin{array}{lcl} \text{1. Consistency/Secancy} & \rightarrow & 0.1 \\ \text{2. Identification/Semancy} & \rightarrow & 0.2 \\ \text{3. Existency/Selectancy} & \rightarrow & 0.3 \end{array}\right\} \text{Relational Numbers}$$

3. Therefore, by mapping the scheme of invariance onto Bense's relational number 0, we get a division of three relational numbers via the scheme of heredity of invariance. We then can construct the following pre-semiotic matrix by building the Cartesian products:

	0.1	0.2	0.3
0.1	0.1 0.1	0.1 0.2	0.1 0.3
0.2	0.2 0.1	0.2 0.2	0.2 0.3
0.3	0.3 0.1	0.3 0.2	0.3 0.3

as a basis for the following well-known semiotic matrix:

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3,

so that we thus have (0.1 0.1) → (1.1), (0.1 0.2) → (1.2), (0.1 0.3) → (1.3) by categorial reduction and (0.2. 0.1) → (2.1), (0.2 0.2) → (2.2), (0.2 0.3) → (2.3); (0.3 0.1) → (3.1), (0.3 0.2) → (3.2) and (0.3 0.3) → (3.3) by categorial reduction and heredity. In other words: The three-ness or pre-semiotic triad of the invariance scheme "consistency-identification-existence" is iterated for each of three invariances, whereby their features are just inherited, so that three pre-semiotic trichotomies are produced from the three pre-semiotic triads whose categorial structure has the same scheme of invariance:

Secancy-Consistency:	0.1 → 1.1 → 2.1 → 3.1
Semancy-Identification:	0.2 → 1.2 → 2.2 → 3.2
Selectancy-Existency:	0.3 → 1.3 → 2.3 → 3.3

However, in doing so, we have to transcend the triadic-trichotomic sign model that now has to be replaced by the following tetradic-tetratomic pre-semiotic sign-relation (PSR):

PSR = (.0., .1., .2., .3.),

which thus bridges the ontological and the semiotic space by integrating zeroness (.0.) into the sign relation SR = (.1., .2., .3.). In other words: PSR integrates SR by localizing it in the ontological space of objects to which the relational numbers refer. Since the ontological localization of numbers is a polycontextural phenomenon (cf. Kronthaler 1986, pp. 26 ss.), it follows that relational numbers belong both to the systems of polycontextural and to the system of monocontextural, i.e. both to qualitative and to quantitative numbers. From that, it also follows that the pre-semiotic sign-relation PSR is the most abstract common sign scheme of both qualitiative and quantitative mathematics (cf. Toth 2003, pp. 21 ss.).

Also remember, that, according to Bense (1975, pp. 65 ss.), the categorial number k is always greater than 0 ($k > 0$) and $r = 0$, thus $k = r$ holds only in SR, but not in PSR. Therefore, there is no identitive Cartesian product "0.0", and the pre-semiotic sign relation PSR leads to the following qualitative-quantitative matrix:

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

categorial numbers

relational numbers

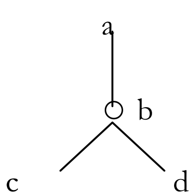
If we now take over Bense's symbol to characterize relational and categorial numbers in sign relations ($Z^r{}_k$), we may introduce $\mathbf{Z^r{}_k}$ as the set of dyadic qualitative-quantitative sub-signs as the Cartesian products from the above qualitative-quantitative matrix:

$\mathbf{Z^r{}_k}$ = (0.1, 0.2, 0.3, 1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3),

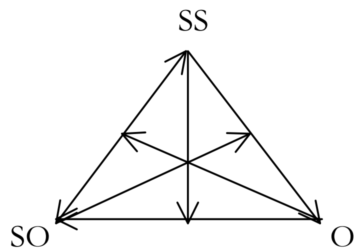
thus $Z^0{}_1$ = (0.1), $Z^1{}_2$ = (1.2), $Z^3{}_3$ = (3.3), etc.

We may add that these numbers are fiberings of what we have called "Peirce numbers" in some earlier studies (Toth 2008, pp. 151 ss., 155 ss., 295 ss.) and thus leading to quantitative-qualitative topological semiotic spaces.

4. We may also wonder if the following sign model given by Peirce (cf. Walther 1989, p. 298) that presupposes a tetradic-tetratomic sign-relation, is compatible to PSR:



If we draw the outer hull around this "sign-skeleton", we get a sign-model corresponding to the one that was introduced as a minimal triadic polycontextural model by Günther (1976, pp. 336 ss.), in which he differentiates between subjective subject (SS), objective subject (OS) and (objective) object (OO):



where SO corresponds to the semiotic medium relation (M), OO to the semiotic object relation (O) and SS to the semiotic interpretant relation (I), cf. Toth (2008, pp. 61 ss.). The polycontextural exchange and foundation relations (referring to the semiotic designation function (M $\Rightarrow$ O), the semiotic denomination function (O $\Rightarrow$ I) and the semiotic use function (I $\Rightarrow$ M)) meet exactly there where there is the category of zeroness (Z) in the corresponding Peircean tetradic sign model that guarantees the qualitative-quantitative and thus polycontextural localization of SR in PSR.

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