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Rough-fuzzy hybridization in semiotics

1. In a former study (Toth 2008), I had introduced rough sets into mathematical semiotics. As one recalls, a rough set is a formal approximation of a crisp set in terms of a pair of sets, which give the lower and the upper approximation of the original set. The lower and upper approximation sets themselves are crisp sets in the standard version of rough set theory (Pawlak 1991), but in other variations, the approximating sets may be fuzzy sets as well. The latter theory is called "rough-fuzzy hybridization" and is the topic of the present study.

2. Sign classes and reality thematics can be compared respecting their representation values (cf. Bense 1981, pp. 86 ss.), respecting their thematized and respecting their thematizing realities (cf. Bense 1981, pp. 111 ss.). All three criteria are ambiguous, since there is no bijective mapping between sign classes or reality thematics onto either representation values, thematized or thematizing realities. In Toth (2008), we had shown that the mapping of the 10 classical and the 27 trans-classical sign classes onto the system of the representation values leads to rough sets. In this study, we will map the system of the representation values onto the thematized and realities of both the classical and the trans-classical semiotic systems and show that we have here a case of rough-fuzzy semiotic hybridization.

3. The following table gives the 10 sign classes (SS10), their representation values and their structural realities, whereby the thematized realities are focussed:

SS10:

1. (3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	Rpv = 9	M-them. M
2. (3.1 2.1 1.2) × (2.1 $\underline{1.2\ 1.3}$)	Rpv = 10	M-them. O
3. (3.1 2.1 1.3) × (3.1 $\underline{1.2\ 1.3}$)	Rpv = 11	M-them. I
4. (3.1 2.2 1.2) × ($\underline{2.1\ 2.2}$ 1.3)	Rpv = 11	O-them. M
5. (3.1 2.2 1.3) × ($\underline{3.1}$ $\underline{2.2}$ $\underline{1.3}$)	Rpv = 12	(triadic reality*)
6. (3.1 2.3 1.3) × ($\underline{3.1\ 3.2}$ 1.3)	Rpv = 13	I-them. M
7. (3.2 2.2 1.2) × (2.1 $\underline{2.2\ 2.3}$)	Rpv = 12	O-them. O
8. (3.2 2.2 1.3) × (3.1 $\underline{2.2\ 2.3}$)	Rpv = 13	O-them. I
9. (3.2 2.3 1.3) × ($\underline{3.1\ 3.2}$ 2.3)	Rpv = 14	I-them. O
10. (3.3 2.3 1.3) × (3.1 $\underline{3.2\ 3.3}$)	Rpv = 15	I-them. I

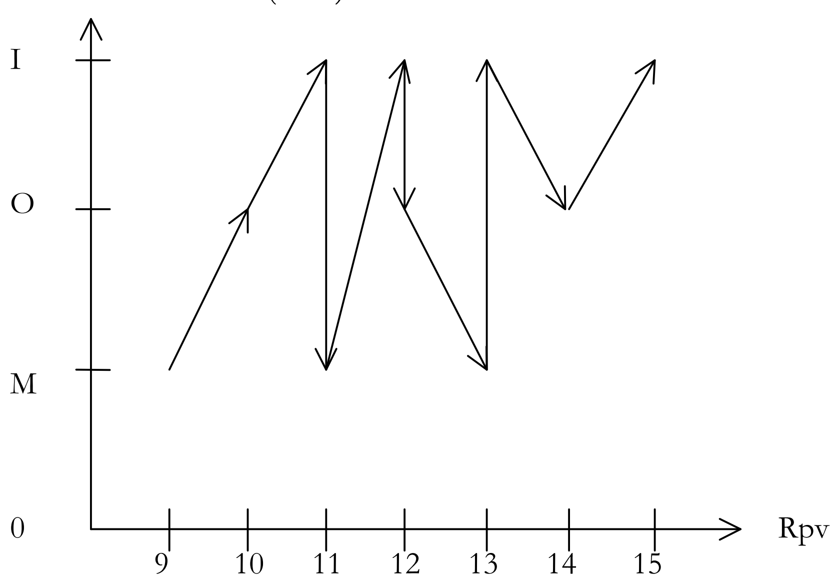
Let therefore DS1 ... DS 10 be the set of objects $X = \{O_1, ..., O_{10}\}$, i.e. the elements of SS10, Rpv = 9 ... Rpv = 15 the first set of attributes $P_i = \{P_1, ..., P_7\}$, and the structural realities the second set of $P_j = \{M, O, I\}$. By combining these two sets of attributes we thus have an instance of a combination of fuzzy and rough sets (cf. Dubois and Prader 1992). For the sake of simplification, we will choose the structural reality of (M, O)-them. I from the triadic reality marked above by asterisk (*). Then we can build the following equivalence classes of SS10:

7 equivalence classes of P_i: 3 equivalence classes of P_j:

$$\begin{cases} \{O_1\} \\ \{O_2\} \\ \{O_3, O_4\} \\ \{O_5, O_7\} \\ \{O_6, O_8\} \\ \{O_9\} \\ \{O_{10}\} \end{cases} \qquad \begin{cases} \{O_1, O_4, O_6\} \\ \{O_2, O_7, O_9\} \\ \{O_3, O_5, O_8, O_{10}\} \end{cases}$$

Thus, in the attribute set P_i, the two objects in the third, fourth and fifth equivalence classes, and in P_j, the three objects in the first and in the second and the four objects in the third equivalence classes are indiscernible. We may visualize this recognition in the following rough-fuzzy semiotic graph:

Thematized realities (SS10)



We shall now have a look at the 27 sign classes of SS27, their representation values and their structural realities, whereby again the thematized realities are focussed:

SS27:

1. (3.1 2.1 1.1) × (1.1 $\underline{1.2\ 1.3}$)	Rpv = 9	M-them. M
2. (3.1 2.1 1.2) × (2.1 $\underline{1.2\ 1.3}$)	Rpv = 10	M-them. O
3. (3.1 2.1 1.3) × (3.1 $\underline{1.2\ 1.3}$)	Rpv = 11	M-them. I
4. *(3.1 2.2 1.1) × *($\underline{1.1}$ 2.2 $\underline{1.3}$)	Rpv = 10	M-them. O
5. (3.1 2.2 1.2) × ($\underline{2.1\ 2.2}$ 1.3)	Rpv = 11	O-them. M
6. (3.1 2.2 1.3) × ($\underline{3.1}$ $\underline{2.2}$ $\underline{1.3}$)	Rpv = 12	(triadic reality*)
7. *(3.1. 2.3 1.1) × *($\underline{1.1}$ 3.2 $\underline{1.3}$)	Rpv = 11	M-them. I

8. *(3.1 2.3 1.2) × *($\underline{2.1}$ $\underline{3.2}$ $\underline{1.3}$)	Rpv = 12	(triadic reality*)
9. (3.1 2.3 1.3) × ($\underline{3.1\ 3.2}$ 1.3)	Rpv = 13	I-them. M
10. *(3.2 2.1 1.1) × *($\underline{1.1\ 1.2}$ 2.3)	Rpv = 10	M-them. O
11. *(3.2 2.1 1.2) × *($\underline{2.1}$ 1.2 $\underline{2.3}$)	Rpv = 11	O-them. M
12. *(3.2 2.1 1.3) × *($\underline{3.1}$ $\underline{1.2}$ $\underline{2.3}$)	Rpv = 12	(triadic reality *)
13. *(3.2 2.2 1.1) × *(1.1 $\underline{2.2\ 2.3}$)	Rpv = 11	O-them. M
14. (3.2 2.2 1.2) × (2.1 $\underline{2.2\ 2.3}$)	Rpv = 12	O-them. O
15. (3.2 2.2 1.3) × (3.1 $\underline{2.2\ 2.3}$)	Rpv = 13	O-them. I
16. *(3.2 2.3 1.1) × *($\underline{1.1}$ $\underline{3.2}$ $\underline{2.3}$)	Rpv = 12	(triadic reality*)
17. *(3.2 2.3 1.2) × *($\underline{2.1}$ 3.2 $\underline{2.3}$)	Rpv = 13	O-them. I
18. (3.2 2.3 1.3) × ($\underline{3.1\ 3.2}$ 2.3)	Rpv = 14	I-them. O
19. *(3.3 2.1 1.1) × *($\underline{1.1\ 1.2}$ 3.3)	Rpv = 11	M-them. I
20. *(3.3 2.1 1.2) × *($\underline{2.1}$ $\underline{1.2}$ $\underline{3.3}$)	Rpv = 12	(triadic reality*)
21. *(3.3 2.1 1.3) × *($\underline{3.1}$ 1.2 $\underline{3.3}$)	Rpv = 13	I-them. M
22. *(3.3 2.2 1.1) × *($\underline{1.1}$ $\underline{2.2}$ $\underline{3.3}$)	Rpv = 12	(triadic reality*)
23. *(3.3 2.2 1.2) × *($\underline{2.1\ 2.2}$ 3.3)	Rpv = 13	O-them. I
24. *(3.3 2.2 1.3) × *($\underline{3.1}$ 2.2 $\underline{3.3}$)	Rpv = 14	I-them. O
25. *(3.3 2.3 1.1) × *(1.1 $\underline{3.2\ 3.3}$)	Rpv = 13	I-them. M
26. *(3.3 2.3 1.2) × *(2.1 $\underline{3.2\ 3.3}$)	Rpv = 14	I-them. O
27. (3.3 2.3 1.3) × (3.1 $\underline{3.2\ 3.3}$)	Rpv = 15	I-them. I

In SS27, we can build the following equivalence classes:

7 equivalence classes of P_i:

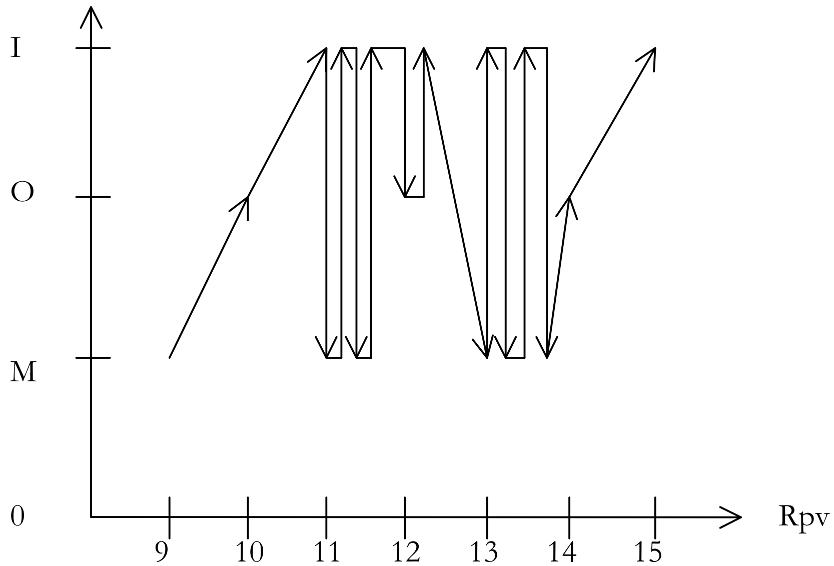
$$\begin{cases} \{O_1\} \\ \{O_2, O_4, O_{10}\} \\ \{O_3, O_5, O_7, O_{11}, O_{13}, O_{19}\} \\ \{O_6, O_8, O_{12}, O_{14}, O_{16}, O_{20}, O_{22}\} \\ \{O_9, O_{15}, O_{17}, O_{21}, O_{23}, O_{25}\} \\ \{O_{18}, O_{24}, O_{26}\} \\ \{O_{27}\} \end{cases}$$

3 equivalence classes of P_j:

$\{O_1, O_5, O_9, O_{11}, O_{13}, O_{21}, O_{25}\}$
$\{O_2, O_4, O_{10}, O_{14}, O_{18}, O_{24}, O_{26}\}$
$\{O_3, O_6, O_7, O_8, O_{12}, O_{15}, O_{16}, O_{17}, O_{19}, O_{20}, O_{22}, O_{23}, O_{27}\}$

Thus, in the attribute set P_i, the three objects in the second and sixth, the six objects in the third and fifth and the eight objects in the fourth equivalence classes, and in P_j, the seven objects in the first and second, and the thirteen objects in the third equivalence class are indiscernible. We may again visualize this in the following rough-fuzzy semiotic graph:

Thematized realities (SS27)



From this graph, in which we "unfolded" identical thematized realities in function of the same representation values, it clearly follows that graph(SS10) $\subset$ graph(SS27), i.e. that the rough-fuzzy set of SS10 is included in the rough-fuzzy set of SS27.

If we define now, as usual, the **P-lower** ($\underline{P}X$) and the **P-upper approximations** ($\overline{P}X$) of X:

$\underline{P}X = \{x \mid [x]_p \subseteq X\}$
$\overline{P}X = \{x \mid [x]_p \cap X \neq \emptyset\}$,

and the ordered pair $<\underline{P}X, \overline{P}X>$, as the rough set (whose elements may be fuzzy, cf. Komorowski, Polkowski, and Skowron 2000, pp. 46 ss.), then we get for target sets $X_{i,\,j} \subseteq \mathbf{U}$, whereby i indicates attribute set P_i and j indicates attribute set P_j:

For X_i = SS10: $<\underline{P}X_i, \overline{P}X_i> = \{O_1\} \cup \{O_2\} \cup \{O_3, O_4\} \cup \{O_5, O_7\} \cup \{O_6, O_8\} \cup \{O_9\} \cup \{O_{10}\}$

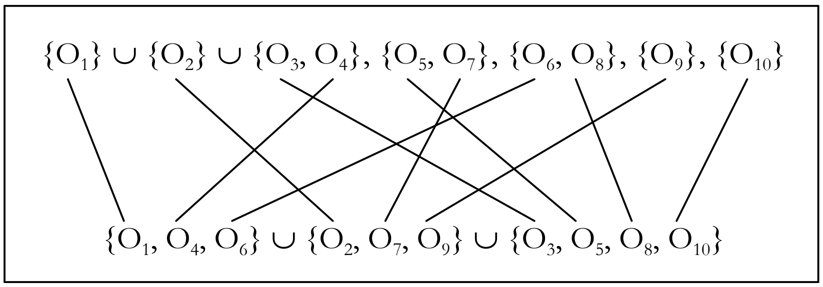
For X_j = SS10: $<\underline{P}X_j, \overline{P}X_j> = \{O_1, O_4, O_6\} \cup \{O_2, O_7, O_9\} \cup \{O_3, O_5, O_8, O_{10}\}$

For X_i = SS27: $<\underline{P}X_i, \overline{P}X_i> = <\{O_1\} \cup \{O_2, O_4, O_{10}\} \cup \{O_3, O_5, O_7, O_{11}, O_{13}, O_{19}\} \cup \{O_6, O_8, O_{12}, O_{14}, O_{16}, O_{20}, O_{22}\} \cup \{O_9, O_{15}, O_{17}, O_{21}, O_{23}, O_{25}\} \cup \{O_{18}, O_{24}, O_{26}\}, \{O_{27}\}>$

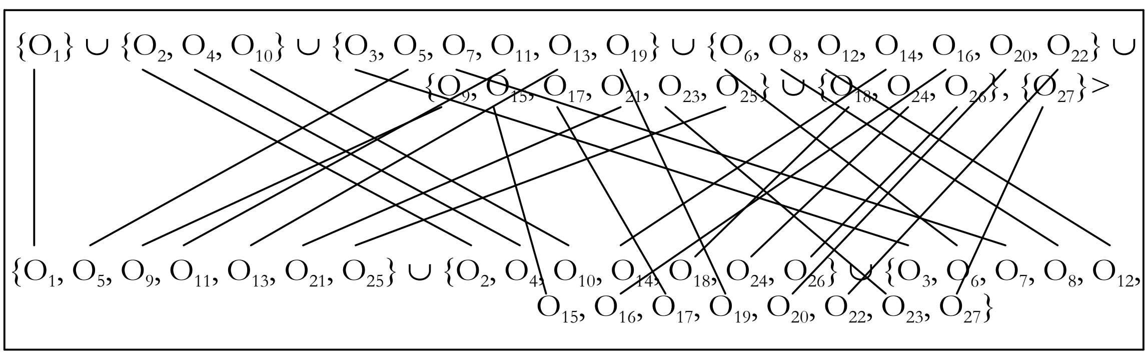
For X_j = SS27: $<\underline{P}X_j, \overline{P}X_j> = \{O_1, O_5, O_9, O_{11}, O_{13}, O_{21}, O_{25}\} \cup \{O_2, O_4, O_{10}, O_{14}, O_{18}, O_{24}, O_{26}\} \cup \{O_3, O_6, O_7, O_8, O_{12}, O_{15}, O_{16}, O_{17}, O_{19}, O_{20}, O_{22}, O_{23}, O_{27}\}$

If we have now a look at the four rough sets, we also recognize that in semiotics – at least as far as SS10 and SS27 are concerned – there is no way to construct reducts and cores from the sets of equivalences, and neither are there possibilities of feature extraction and construction of minimal sets of cuts (cf. Komorowski, Polkowski, and Skowron 2000, pp. 13ss.). However, the dependencies of the attributes of the sets P_i and P_j can be visualized as follows:

Dependencies of attributes in SS10:



Dependencies of attributes in SS27:



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