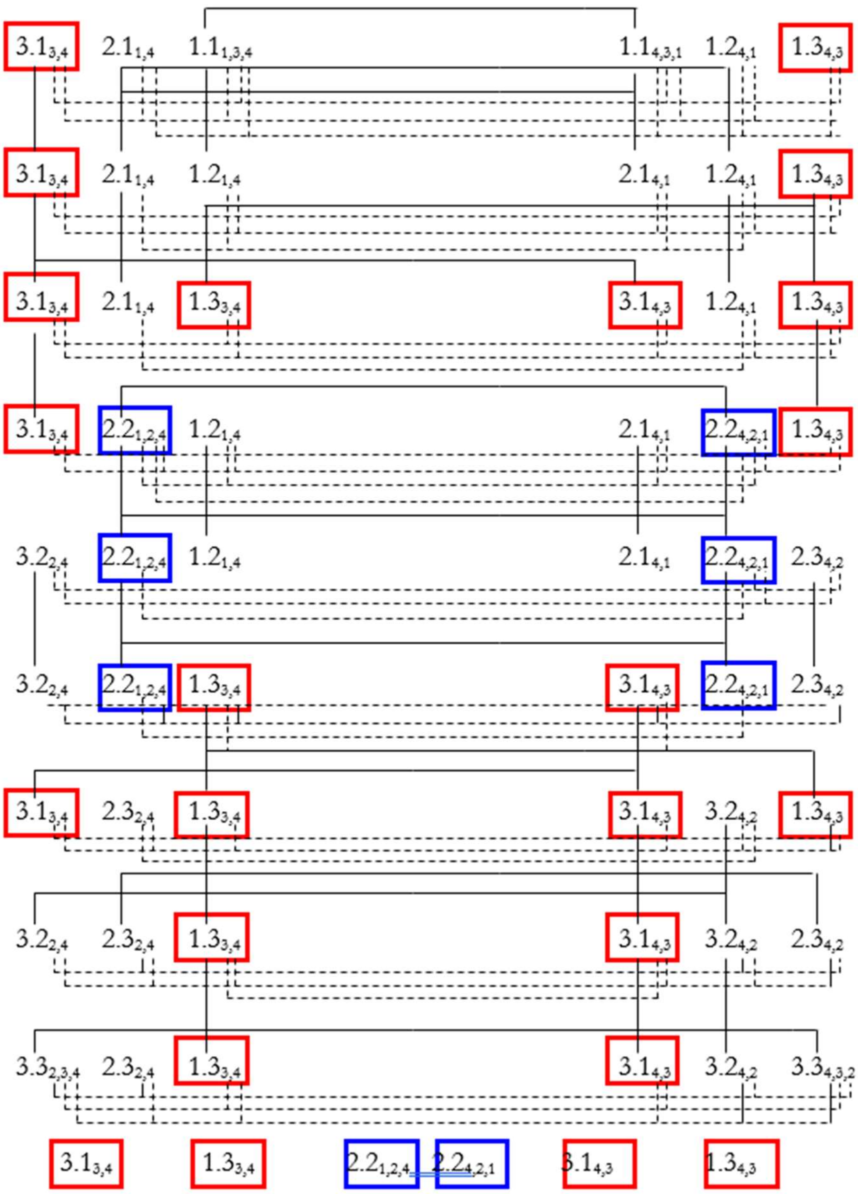
Prof. Dr. Alfred Toth

Semiotische Diamanten



STL

Vorwort

Das logische Tetralemma ist bekanntlich eine 4-stellige Relation zwischen A, nicht-A, sowohl A als auch nicht-A und weder A noch nicht-A, d.h. es umfaßt die Kategorien der Position, Opposition, Akzeption und Rejektion. Das Tetralemma enthält somit auch den sog. dialektischen Dreischritt, auf dessen Relevanz für die Semiotik Max Bense bereits 1975 in seinem Buch «Semiotische Prozesse und Systeme» aufmerksam gemacht hatte, als er die Akzeption als «Thesen-Repertoire» definierte und zum Schluß kam, «daß es sich bei einem dialektischen Dreischritt nicht um ein logisches Folgerungsschema, sondern um ein semiotisches Darstellungsschema handelt». Allerdings blieb die Rejektion in der Semiotik weiterhin undefiniert. Das änderte sich erst mit dem Übergang von der monokontexturalen zur polykontexturalen Semiotik, die lange nach Benses Tod mit Rudolf Kaehrs «The Book of Diamonds» (2008) einsetzte. Der diamond oder Diamant ist ein kontexturiertes Tetralemma, das in Sonderheit in der Semiotik zu bahnbrechenden Entdeckungen geführt hat, denen Kaehr ein eigenes Buch gewidmet hatte. Ab 2008 kam es dann auch zu enger Zusammenarbeit zwischen Kaehrs «ThinkArtLab» in Glasgow (UK) und meinem «SemTechLab» in Tucson, AZ (USA). Im vorliegenden Buch werden die zentralen, auf die Diamantentheorie beschränkten Beiträge zu einer polykontexturalen Semiotik in chronologischer Anordnung präsentiert.

Tucson, AZ, 6.4.2020 Prof. Dr. Alfred Toth

Semiotische Diamanten

1. Einführung

Die bedeutendste Neuerung innerhalb der von Gotthard Günther begründeten Polykontexturalitätstheorie stellt ohne Zweifel das erst kürzlich von Rudolf Kaehr gefundene Diamanten-Modell der Komposition kategorietheoretischer Morphismen dar, denn dieses erlaubt im Gegensatz zur herkömmlichen Kategorietheorie die Einführung einer retrograden Abbildung zwischen Objekten und Kategorien, von Rudolf Kaehr "Hetereo-Morphismen" genannt: "Finally, after 30 years of proemializing and chiastifying formal languages, the diamond of composition is introduced, which is accepting the rejectional aspect of chiastic compositions, too. It seems that the diamond concept of composition is building a complete holistic unit. With its radical closeness it is opening up unlimited, linear and tabular, repeatability and deployment" (Kaehr 2007, S. 43).

Im vorliegenden Aufsatz werde ich zeigen, dass es auch semiotische Diamanten gibt; eine Tatsache, welche die theoretische Semiotik einmal mehr in die Nähe der Polykontexturalitätstheorie rückt. Da die Einführung semiotischer Diamanten jedoch eine semiotische Operation voraussetzt, welche bisher noch nicht definiert wurde (vgl. Toth 2007, S. 31 ff.), werden semiotische Diamanten hier Schritt für Schritt, ausgehend von den verschiedenen möglichen Zeichenmodellen, eingeführt.

2. Graphentheoretische Zeichenmodelle

Zeichenklassen werden normalerweise in der abstrakten Form (3.a 2.b 1.c) mit a, b, c $\in$ {1, 2, 3} und a $\leq$ b $\leq$ d definiert:

1. (I $\rightarrow$ O $\rightarrow$ M)
   Beispiel: Zeichenklassen, degenerativer Graph (Bense 1971, S. 37)

Dass diese Anordnung nicht die einzige ist, zeigen die folgenden Fälle:

2. (M $\rightarrow$ O $\rightarrow$ I)
   Beispiel: Realitätsthematiken, generativer Graph (Bense 1971, S. 37)

3. (I $\rightarrow$ M $\rightarrow$ O)
   Beispiel: thetischer Graph (Bense 1971, S. 37)

4. (O $\rightarrow$ M $\rightarrow$ I)
   Beispiel: kommunikativer Graph (Bense 1971, S. 40 f.)

5. (I $\rightarrow$ M $\rightarrow$ O)
   (M $\rightarrow$ I $\rightarrow$ O)
   Beispiel: kreativer Graph (Bense 1971, S. 102)

6. (O $\rightarrow$ I $\rightarrow$ M)
   Beispiel: ? (bisher kein Fall bekannt)

3. Die 10 Zeichenklassen gemäss den 6 graphentheoretischen Zeichenmodellen

Im folgenden ordnen wir die 10 Zeichenklassen, die bekanntlich durch die Prinzipien der Triadizität und der semiotischen Inklusion beschränkt sind (vgl. Toth 2008a), gemäss den kombinatorisch möglichen graphentheoretischen Zeichenmodellen:

3.1. (I → O → M)

(3.1 2.1 1.1)	(3.1 2.3 1.3)
(3.1 2.1 1.2)	(3.2 2.2 1.2)
(3.1 2.1 1.3)	(3.2 2.2 1.3)
(3.1 2.2 1.2)	(3.2 2.3 1.3)
(3.1 2.2 1.3)	(3.3 2.3 1.3)

3.2. (M → O → I)

(1.1 2.1 3.1)	(1.3 2.3 3.1)
(1.2 2.1 3.1)	(1.2 2.2 3.2)
(1.3 2.1 3.1)	(1.3 2.2 3.2)
(1.2 2.2 3.1)	(1.3 2.3 3.2)
(1.3 2.2 3.1)	(1.3 2.3 3.3)

3.3. (M → I → O)

(1.1 3.1 2.1)	(1.3 3.1 2.3)
(1.2 3.1 2.1)	(1.2 3.2 2.2)
(1.3 3.1 2.1)	(1.3 3.2 2.2)
(1.2 3.1 2.2)	(1.3 3.2 2.3)
(1.3 3.1 2.2)	(1.3 3.3 2.3)

3.4. (O → M → I)

(2.1 1.1 3.1)	(2.3 1.3 3.1)
(2.1 1.2 3.1)	(2.2 1.2 3.2)
(2.1 1.3 3.1)	(2.2 1.3 3.2)
(2.2 1.2 3.1)	(2.3 1.3 3.2)
(2.2 1.3 3.1)	(2.3 1.3 3.3)

3.5. (O → I → M)

(2.1 3.1 1.1)	(2.3 3.1 1.3)
(2.1 3.1 1.2)	(2.2 3.2 1.2)
(2.1 3.1 1.3)	(2.2 3.2 1.3)
(2.2 3.1 1.2)	(2.3 3.2 1.3)
(2.2 3.1 1.3)	(2.3 3.3 1.3)

3.6. (I → M → O)

(3.1 1.1 2.1)	(3.1 1.3 2.3)
(3.1 1.2 2.1)	(3.2 1.2 2.2)
(3.1 1.3 2.1)	(3.2 1.3 2.2)

(3.1 1.2 2.2) (3.2 1.3 2.3)
(3.1 1.3 2.2) (3.3 1.3 2.3)

4. Transformationsoperationen zwischen den 6 Zeichenschemata

Es ist klar, dass die 6 Zeichenschemata durch Transformationen ineinander überführt werden können. Wir schauen sie uns hier genauer an.

4.1. (IOM) → (MOI)

Definition: (3.1 2.1 1.3) → (1.3 2.1 3.1):≡ INV
(3.1 2.1 1.3) → (3.1 1.2 1.3):≡ DUAL

Es gibt also zwei Möglichkeiten der Umkehrung: Wir bezeichnen reine Umkehrung der Reihenfolge der Subzeichen durch den Operator INV und Umkehrung sowohl der Reihenfolge der Subzeichen als auch der Primzeichen durch den Operator DUAL; dieser ist natürlich mit dem von Max Bense eingeführten Operator "×" der Dualisation identisch (vgl. Walther 1979, S. 106 ff.).

Im folgenden müssen wir zusätzlich die 15 möglichen Übergänge zwischen den 6 Zeichenschemata speziell definieren, und zwar am besten so, dass wir mit einem einzigen Operator auch INV und DUAL definieren können. Dies geschieht am besten mit einem Transpositions-Operator. Da eine vollständige Transposition eine Permutation ist, lassen sich auch die Operationen INV und DUAL durch einen einfachen Operator mit Indizes erfassen:

Definition: T_{ik}:≡ Transposition von w_i und w_k, wobei i = k = {1, 2, 3} gemäss den 3 Subzeichen pro Zeichenschema

Definition: $T_{1,3}$(3.1 2.1 1.3) → (1.3 2.1 3.1) ≡ INV

Der Transpositionsoperator vertauscht hier also zuerst das erste mit dem dritten und hernach das zweite mit dem dritten Subzeichen; er arbeitet also sukzessiv.

Für die Dualisation muss der Transpositionsoperator jedoch auf den Primzeichen neu definiert werden, d.h. seine Indexmengen reichen von 1 bis 6. Zur Vermeidung von Verwechslung verwenden wir hier a, b, c, ..., f:

Definition: $T_{a,f;\, b,e;\, c,d}$(3.1 2.1 1.3) → (3.1 1.2 1.3) ≡ DUAL

4.2. (IOM) → (MIO)

Definition: $T_{1,3;\, 2,3}$(3.1 2.1 1.3) → (1.3 3.1 2.1)

4.3. (IOM) → (OMI)

Definition: $T_{1,2;\, 2,3}$(3.1 2.1 1.3) → (2.1 1.3 3.1)

4.4. (IOM) → (OIM)

Definition: $T_{1,2}$(3.1 2.1 1.3) → (2.1 3.1 1.3)

4.5. (IOM) → (IMO)

Definition: $T_{2,3}$(3.1 2.1 1.3) → (3.1 1.3 2.1)

4.6. (MOI) → (MIO)

Definition: $T_{2,3}$(1.3 2.1 3.1) → (1.3 3.1 2.1)

4.7. (MOI) → (OMI)

Definition: $T_{1,2}$(1.3 2.1 3.1) → (2.1 1.3 3.1)

4.8. (MOI) → (OIM)

Definition: $T_{1,3;\,1,2}$(1.3 2.1 3.1) → (2.1 3.1 1.3)

4.9. (MOI) → (IMO)

Definition: $T_{1,2;\,1,3}$(1.3 2.1 3.1) → (3.1 1.3 2.1)

4.10. (MIO) → (OMI)

Definition: $T_{1,3;\,2,3}$(1.3 3.1 2.1) → (2.1 1.3 3.1)

4.11. (MIO) → (OIM)

Definition: $T_{1,3}$(1.3 3.1 2.1) → (2.1 3.1 1.3)

4.12. (MIO) → (IMO)

Definition: $T_{1,2}$(1.3 3.1 2.1) → (3.1 1.3 2.1)

4.13. (OMI) → (OIM)

Definition: $T_{2,3}$(2.1 1.3 3.1) → (2.1 3.1 1.3)

4.14. (OMI) → (IMO)

Definition: $T_{1,3}$(2.1 1.3 3.1) → (3.1 1.3 2.1)

4.15. (OIM) → (IMO)

Definition: $T_{1,3;\,1,2}$(2.1 3.1 1.3) → (3.1 1.3 2.1)

5. Transpositionen und Dualisationen bei den 6 Zeichenschemata

Wir stellen nun alle möglichen Transpositionen und Dualisationen der Ausgangszeichenklasse (3.1 2.1 1.3) dar und bestimmen die Strukturtypen:

Zeichenklasse	Transpositionen	Dualisationen	Strukturtypen
(3.1 2.1 1.3)		(3.1 1.2 1.3)	I
	(1.3 2.1 3.1)	(1.3 1.2 3.1)	II
	(1.3 3.1 2.1)	(1.2 1.3 3.1)	III
	(2.1 1.3 3.1)	(1.3 3.1 1.2)	IV
	(2.1 3.1 1.3)	(3.1 1.3 1.2)	V
	(3.1 1.3 2.1)	(1.2 3.1 1.3)	VI
	(1.3 3.1 2.1)	(1.2 1.3 3.1)	III
	(2.1 1.3 3.1)	(1.3 3.1 1.2)	IV
	(2.1 3.1 1.3)	(3.1 1.3 1.2)	V
	(3.1 1.3 2.1)	(1.2 3.1 1.3)	VI
	(2.1 1.3 3.1)	(1.3 3.1 1.2)	IV
	(2.1 3.1 1.3)	(3.1 1.3 1.2)	V
	(3.1 1.3 2.1)	(1.2 3.1 1.3)	VI
	(2.1 3.1 1.3)	(3.1 1.3 1.2)	V
	(3.1 1.3 2.1)	(1.2 3.1 1.3)	VI
	(3.1 1.3 2.1)	(1.2 3.1 1.3)	VI

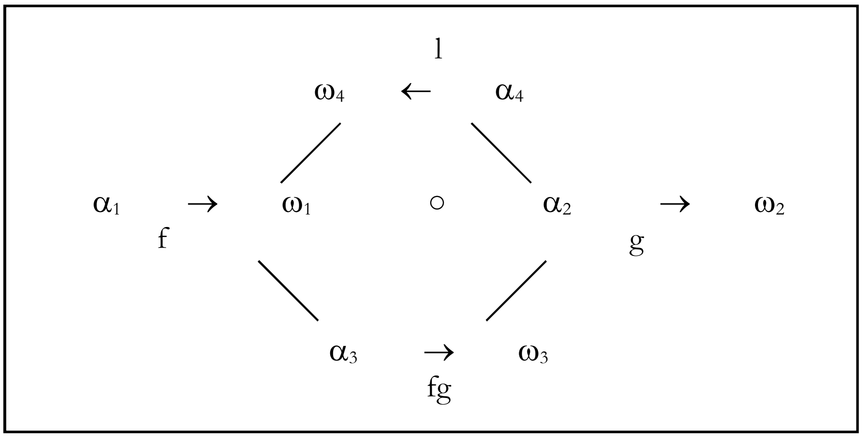
Wie man sieht, gibt es also nur 6 Strukturtypen und ihre Dualisate. Zu jeder Zeichenklasse (a.b c.d e.f) mit a, b, c, d, e, f ∈ {1, 2, 3} haben wir also die folgenden 12 Strukturschemata (links Transpositionen, rechts deren Dualisationen) gefunden:

1. (a.b c.d e.f) × (f.e d.c b.a)
2. (a.b e.f c.d) × (d.c f.e b.a)
3. (c.d e.f a.b) × (b.a f.e d.c)
4. (c.d a.b e.f) × (f.e b.a d.c)
5. (e.f c.d a.b) × (b.a d.c f.e)
6. (e.f a.b c.d) × (d.c b.a f.e)

Wir können also nun für (a.b c.d e.f) jede der 10 Zeichenklassen einsetzen und erhalten mit den zugehörigen Transpositionen und Dualisationen erstmals den ganzen der im semiotischen Zehnersystem eingeschlossenen Strukturreichtum, der von den Zeichenklassen bzw. den dualen Realitätsthematiken aus allein nicht erreichbar ist.

6. Das semiotische Diamanten-Modell

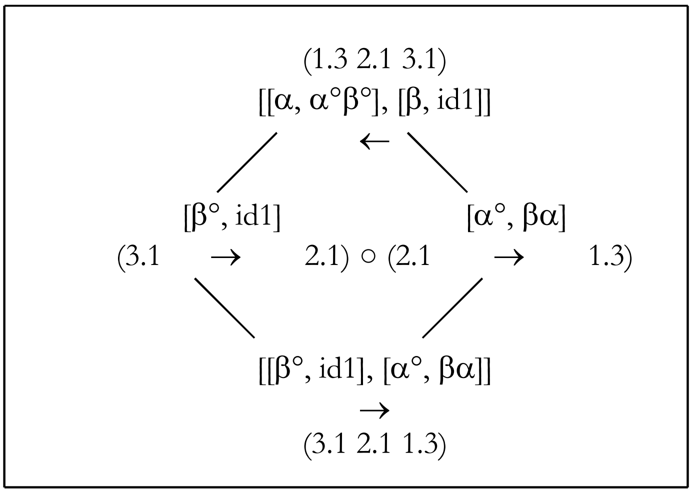
Das mathematische Diamantenmodell, das Kaehr (2007) eingeführt hatte, sieht wie folgt aus:



Das Besondere hier ist die Abbildung l: ω4 ← α4, die Kaehr als "saltisition" oder "jump operation" bestimmt: "Within Diamond theory, for the very first time, additional to category theory and in an interplay with it, the *gaps* and *jumps* involved are complementary to the connectedness of compositions. The counter-movements of compositions are generating jumps". Der Übergang von α4 → ω4 wird von Kaehr auch als "bridge", der Morphismus der Abbildung als "Hetero-Morphismus" bezeichnet (2007a, S. 12). Logisch entspricht die Abbildung α3 → ω3 der Akzeptanz und kybernetisch dem "System", und ω4 ← α4 entspricht logisch der Rejektion und kybernetisch der "Umgebung" (Kaehr 2007, S. 54).

Wenn wir nun unsere Zeichenklasse (3.1 2.1 1.3) in der Form eines semiotischen Diamanten schreiben, erkennen wir, dass die semiotische Rejektion dieser Zeichenklasse mit ihrer Inversion (INV(Zkl)) übereinstimmt. (1.3 2.1 3.1) ist damit kybernetisch interpretiert die semiotische Umgebung des semiotischen Systems (3.1 2.1 1.3).[1]

6.1. Semiotischer Diamant für (3.1 2.1 1.3):



Die semiotische Rejektionsfunktion ist nun aber keineswegs auf den Strukturtyp (e.f c.d a.b) wie im obigen semiotischen Diamanten beschränkt. Semiotische Inversion (INV) ist allgemein durch folgende zwei Anweisungsschritte erreichbar:

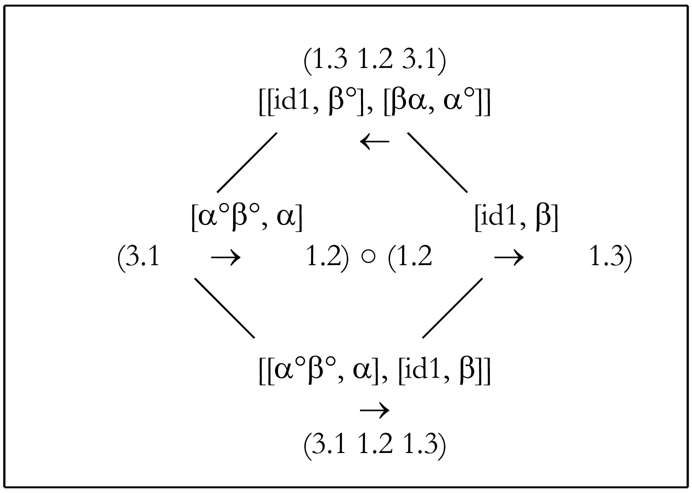
1. Kehre die Reihenfolge der konstituierenden Subzeichen einer Zeichenklasse (oder einer ihrer Transpositionen bzw. Dualisationen) um.

2. Vertausche alle semiotischen Morphismen mit ihren Inversen (wobei natürlich z.B. α°° = α, β°° = β und per definitionem (vgl. Toth 1993, S. 21 ff.) (βα)° = α°β° und (α°β°)° = βα gilt.

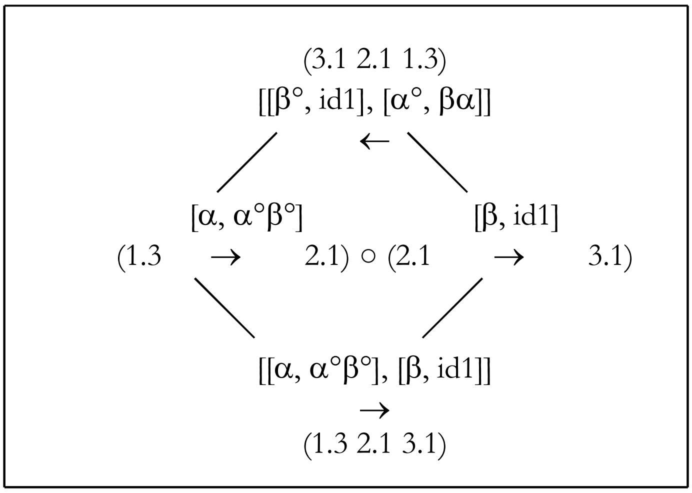
Mit anderen Worten bedeutet das, dass wir semiotische Diamanten für alle 12 Strukturtypen (und natürlich für sämtliche 10 Zeichenklassen und auch für die Genuine Kategorienklasse) angeben können. Wir beschränken uns im folgenden darauf, die semiotischen Diamanten für die 6 Typen von Transpositionen plus für die Dualisation der Ausgangs-Zeichenklasse (3.1 2.1 1.3) anzugeben.

[1] Dass mit dem semiotischen Diamanten-Modell erstmals seit Ditterich (1990, S. 54) operable und mit der Kybernetik kompatible Definitionen des semiotischen "Systems" und der semiotischen "Umgebung" erreicht sind, sei hier vorläufig bloss angedeutet.

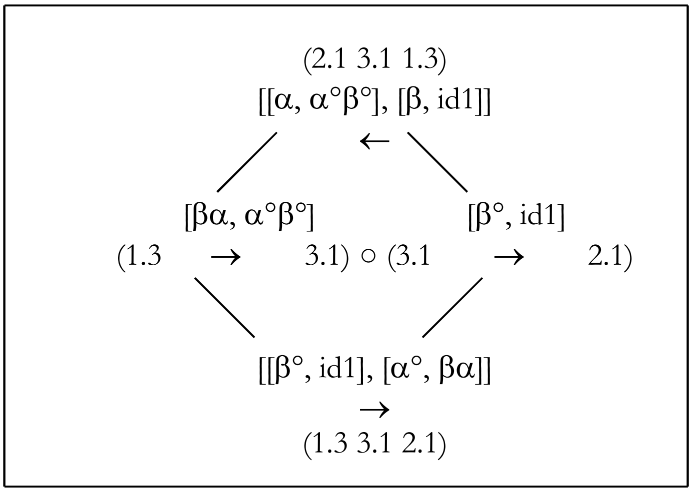
6.2. Semiotischer Diamant für (3.1 1.2 1.3):



6.3. Semiotischer Diamant für (1.3 2.1 3.1):



6.4. Semiotischer Diamant für (1.3 3.1 2.1):



6.5. Semiotischer Diamant für (2.1 1.3 3.1):

(3.1 1.3 2.1)
[[α°β°, βα], [α, α°β°]]
←

[α°, βα]　　　　[βα, α°β°]
(2.1 → 1.3) ○ (1.3 → 3.1)

[[α°, βα], [βα, α°β°]]
→
(2.1 1.3 3.1)

6.6. Semiotischer Diamant für (2.1 3.1 1.3):

(1.3 3.1 2.1)
[[βα, α°β°], [β°, id1]]
←

[β, id1]　　　　[α°β°, βα]
(2.1 → 3.1) ○ (3.1 → 1.3)

[[β, id1], [α°β°, βα]]
→
(2.1 3.1 1.3)

6.7. Semiotischer Diamant für (3.1 1.3 2.1):

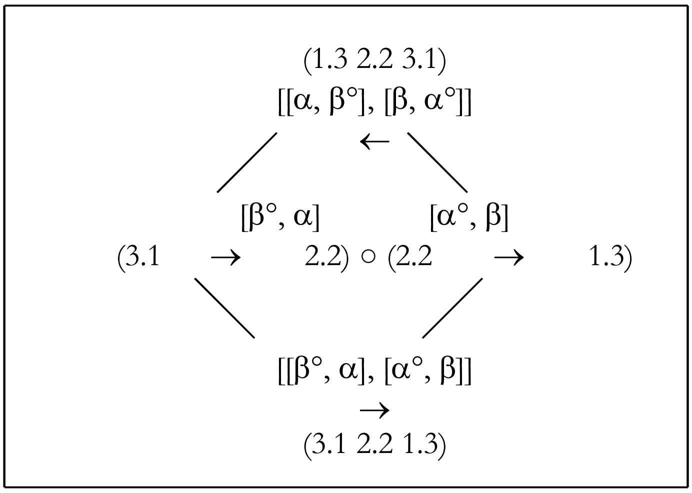
(2.1 1.3 3.1)
[[α°, βα], [βα, α°β°]]
←

[α°β°, βα]　　　　[α, α°β°]
(3.1 → 1.3) ○ (1.3 → 2.1)

[[α°β°, βα], [α, α°β°]]
→
(3.1 1.3 2.1)

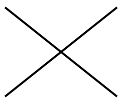
Nun schauen wir uns den semiotischen Diamanten für die dual-identische Zeichenklasse (3.1 2.2 1.3) an:

6.8. Semiotischer Diamant für (3.1 2.2 1.3):



Diese Zeichenklasse der “Eigen-Realität” (vgl. Bense 1992) weist also neben vielen, bereits von Bense verzeichneten strukturellen Besonderheiten auch den semiotischen Chiasmus auf, der ohne das semiotische Diamanten-Modell nicht erkennbar ist:

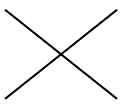
[[α, β°], [β, α°]]



[[β°, α], [α°, β]]

In den anderen Zeichenklassen ist der semiotische Chiasmus quasi durch die Notation der komponierten Morphismen “verdeckt”; das allgemeine kategorietheoretische Schema für semiotischen Chiasmus lautet:

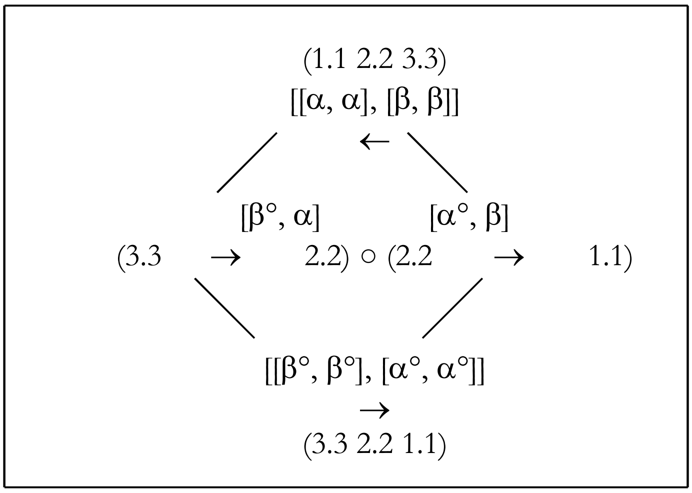
[[V°, W°], [X°, Y°]]



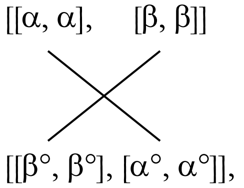
[[X, Y], [V, W]]

Eine weitere besondere semiotische Klasse ist die “Genuine Kategorienklasse”, auf deren strukturelle Besonderheiten Bense ebenfalls bereits hingewiesen (Bense 1992, S. 39 f., 43) und die er als “ergodische Semiose” bezeichnet hatte (Bense 1975, S. 93). Wenn wir uns ihren semiotischen Diamanten anschauen:

6.9. Semiotischer Diamant für (3.3 2.2 1.1):



so sieht hier der semiotische Chiasmus wie folgt aus:



wobei diese semiotische Klasse die einzige ist, in der die Morphismen und Hetero-Morphismen pro Unterkategorie kategoriell homogen sind; [α°, α°] und [β°, β°] spiegeln hier also die "Autoreproduktivität" der identitiven Subzeichen (1.1), (2.2) und (3.3) im Sinne der Genuinen Kategorienklasse "als normierter Führungssemiose aller Zeichenprozesse überhaupt" (Bense 1975, S. 89).

7. Semiotische Diamanten der Komposition

Man kann Zeichenklassen und Realitätsthematiken mit Hilfe der kategorietheoretischen Semiotik auf zwei Arten analysieren: Entweder man weist sowohl den Objekten – d.h. den Subzeichen – als auch den Abbildungen, d.h. den Semiosen, semiotische Morphismen zu, oder man beschränkt sich auf Semiosen, wobei man in diesem Fall sowohl die triadischen wie die trichotomischen Abbildungen, d.h. die semiosischen Morphismen zwischen den semiotischen Haupt- und Stellenwerten berücksichtigt.

Für unsere Zeichenklasse (3.1 2.1 1.3) erhält man also im ersten Falle:

(3.1 2.1 1.3) → [α°β°, α°, βα]

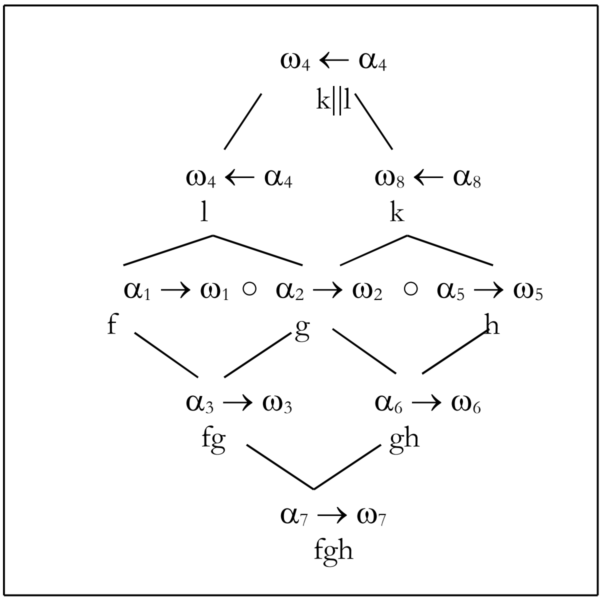
und im zweiten Falle:

(3.1 2.1 1.3) ≡ [[β°, id1], [α°, βα]].

Nur die zweite Analysemethode bildet Zeichenklassen bzw. Realitätsthematiken eineindeutig auf semiotische Kategorien ab, denn [α°β°, α°, βα] liesse sich z.B. auch als (3.2 1.1), (1.3) interpretieren. Die zweite Methode trägt also der Beobachtung Walthers Rechnung, dass triadische Zeichenrelationen aus der verbandstheoretischen Vereinigung der beiden dyadischen Relationen

(M ⇒ O) und (M ⇒ I) konstruiert werden können ((M ⇒ O) (O ⇒ I)) = (M ⇒ O. O ⇒ I), vgl. Walther (1979, S. 79).

Diese zweite Analysemethode, die wir schon in den vorherigen Kapiteln sowie in früheren Arbeiten angewandt haben, entspricht nun umgekehrt exakt der Methode der Komposition semiotischer Diamanten. Das allgemeine mathematische Schema für die Komposition von Morphismen und Hetero-Morphismen in einem Diamanten lautet nach Kaehr (2007, S. 44):

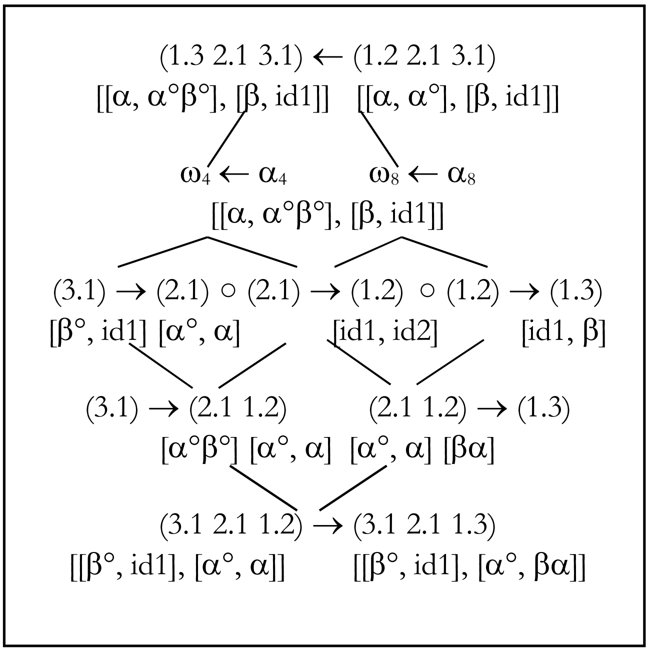


Mit Hilfe komponierter Diamanten können nun Zusammenhänge von Zeichenklassen (vgl. Toth 2008b) analysiert werden. Voraussetzung ist allerdings, dass je 2 Zeichenklassen bzw. Realitätsthematiken paarweise, d.h. in je 2 Subzeichen, zusammenhängen.[2]

Als Beispiel wählen wir unsere Zeichenklasse (3.1 2.1 1.3) und die Zeichenklasse (3.1 2.1 1.2); ihr verbandstheoretischer Durchschnitt ist (3.1 2.1):

[2] Da gemäss dem Prinzip der Trichotomischen Triaden alle 10 Zeichenklassen und Realitätsthematiken entweder in (3.1), in (2.2), in (1.3) oder in zwei von diesen drei Subzeichen miteinander zusammenhängen, muss nach Lösungen gesucht werden, um verbandstheoretische Durchschnitte von nur einem Subzeichen pro Paar von Zeichenklassen bzw. Realitätsthematiken mit Hilfe von semiotischen Diamanten-Kompositionen darzustellen.

7.1. Komponierter semiotischer Diamant für den Zeichenzusammenhang (3.1 2.1 1.2 – 3.1 2.1 1.3)



Literatur

Bense, Max, Zeichen und Design. Baden-Baden 1971

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Ditterich, Joseph, Selbstreferentielle Modellierungen. Klagenfurt 1990

Kaehr, Rudolf, Towards Diamonds. Glasgow 2007

Toth, Alfred, Entwurf einer semiotisch-relationalen Grammatik. Tübingen 1993

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

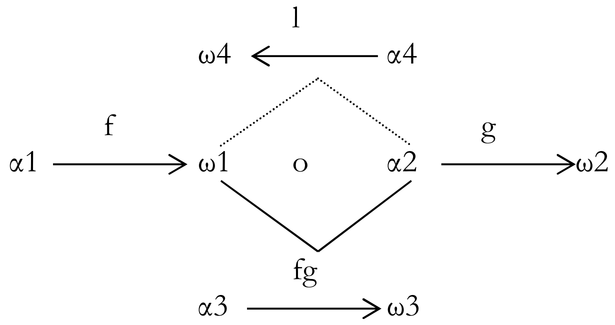
Toth, Alfred, Kenogrammatik, Präsemiotik und Semiotik. In: Electronic Journal for Mathematical Semiotics, 2008a

Toth, Alfred, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008 (= 2008b)

Walther, Elisabeth, Allgemeine Zeichenlehre. 2. Aufl. Stuttgart 1979

Präsemiotische Diamanten

1. Diamanten wurden von Kaehr (2007) in die Polykontexturalitätstheorie eingeführt: "Diamonds may be thematized as 2-categories where two mutual antidromic categories are in an interplay" (Kaehr 2007, S. 20). Ein polykontexturaler Diamant "consists on a simultaneity of a category and a jumpoid, also called a saltatory. If the category is involving m arrows, its antidromic saltatory is involving m-1 inverse arrows" (2007, S. 20). Kaehr (2007, S. 2) gibt folgendes Beispiel:



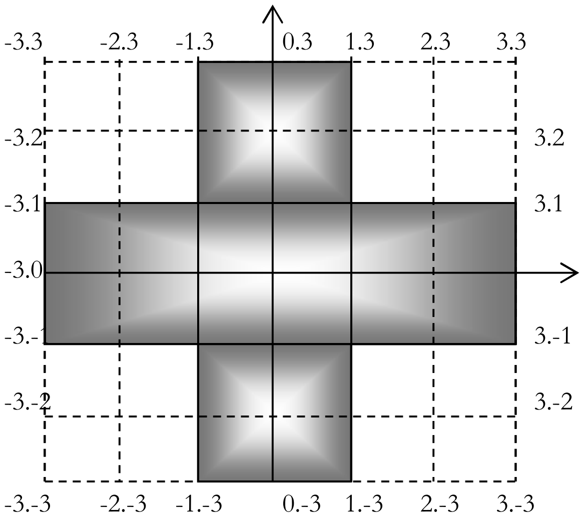
In der Semiotik hatte ich Diamanten in Toth (2008a) eingeführt. In Toth (2008b, S. 177 ff. und S. 282 ff.) sowie in einigen Aufsätzen wurde die semiotische Diamantentheorie weiterentwickelt. Nachdem ich in Toth (2008c, d) und einigen weiteren Arbeiten nachgewiesen hatte, dass der präsemiotische Raum, der durch die folgenden Funktionswerte innerhalb des semiotischen Koordinatensystems definiert wird

x	-3	-2	-1	0	1	2	3
y	±1	±1	±1	±1	±1	±1	±1

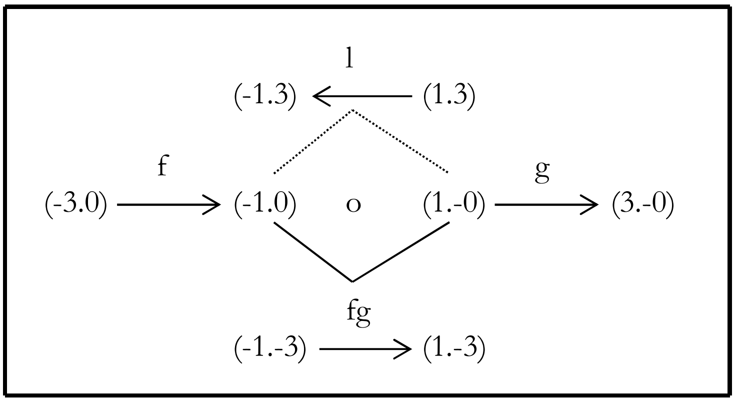
y	-3	-2	-1	0	1	2	3
x	±1	±1	±1	±1	±1	±1	±1

ein polykontexturaler Raum ist, ist es nötig, auf die Konzeption semiotischer Relationen als Diamanten zurückzukommen. Ziel der vorliegenden Arbeit ist es, die Grundtypen sowie die Anzahl präsemiotischer Diamanten zu bestimmen.

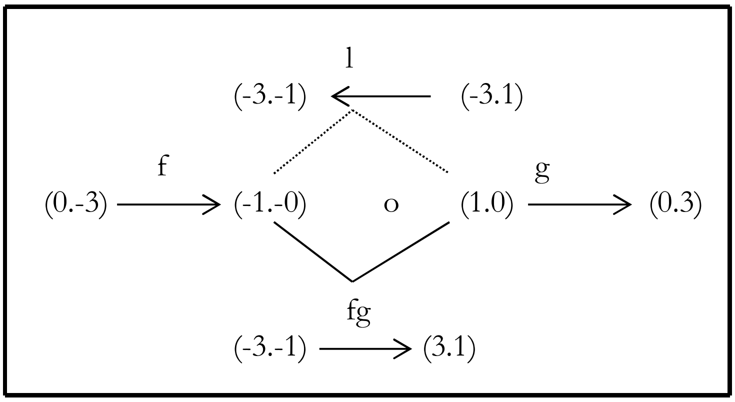
2. Der präsemiotische Raum entspricht also dem grau schraffierten Teilraum des semiotischen Koordinatensystems:



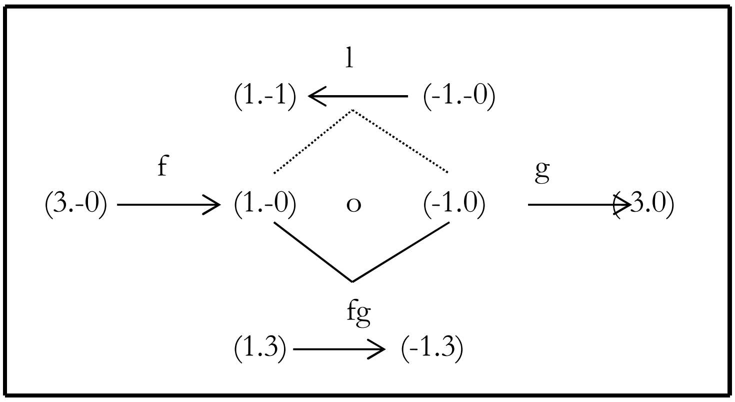
2.1. Wenn wir vom präsemiotischen Raum in seiner obigen, ungedrehten Position ausgehen, bekommen wir den ersten präsemiotischen Diamanten:



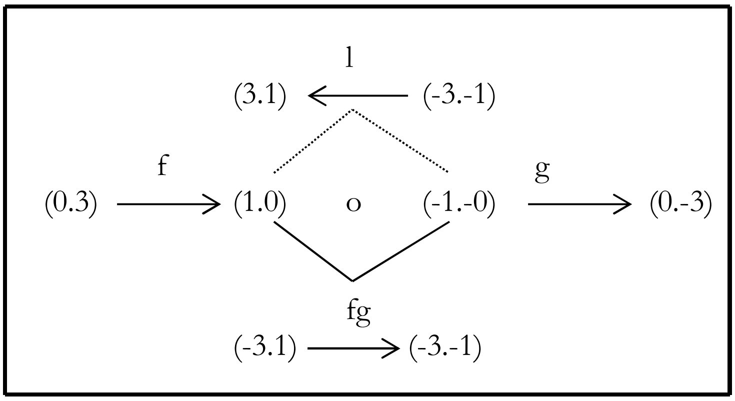
2.2. Wenn wir den präsemiotischen Raum um 90° im Uhrzeigersinn drehen, bekommen wir den zweiten präsemiotischen Diamanten:



2.3. Wenn wir den präsemiotischen Raum um 180° im Uhrzeigersinn drehen, bekommen wir den dritten präsemiotischen Diamanten:

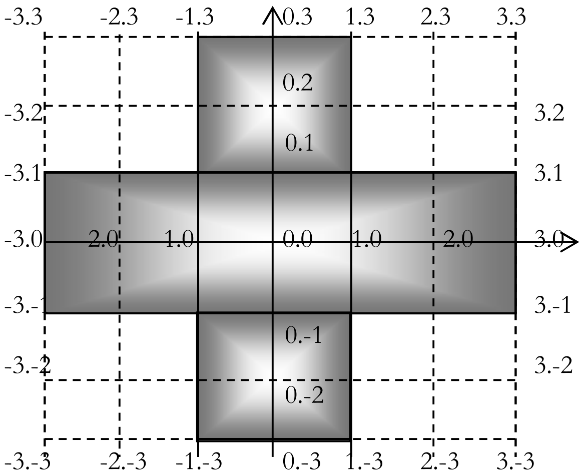


2.4. Wenn wir den präsemiotischen Raum um 270° im Uhrzeigersinn drehen, bekommen wir den vierten präsemiotischen Diamanten:



3. Wir erkennen also, das in den obigen vier präsemiotischen Diamanten die mit l bezeichneten Heteromorphismen die Brücken über die semiotischen Morphismen f und g bauen. Diese sind also nach Kaehrs Unterscheidung von Kategorien als Saltatorien oder Jumpoids aufzufassen, weil sie nämlich den "Spagat" über die Kontexturengrenzen bewerkstelligen. Semiotische Spagate sind in unseren semiotischen Diamanten einfach überall dort zu finden, wo Morphismen oder Heteromorphismen Subzeichen miteinander verbinden, die verschiedene Vorzeichen haben und daher in verschiedenen Kontexturen liegen. Kaehr unterscheidet ferner in einer an Heidegger angelehnten Terminologien zwischen "Schritt" und "Sprung" (2007, S. 27). Bei präsemiotischen Diamanten möchte ich semiotische "Schritte" so definieren, dass sie (semiosische oder retrosemiosische) Prozesse zwischen Subzeichen der gleichen tetradischen Hauptwerte darstellen. Semiotische "Sprünge" dagegen sind dann (semiosische oder retrosemiosische) Prozesse zwischen Subzeichen mit verschiedenen tetradischen Hauptwerten. Im letzten präsemiotischen Diamenten liegen also Schritte bei dem komponierten Morphismus fg und dem Heteromorphismus l, Sprünge dagegen bei den simplizialen Morphismen f und g vor.

4. Die Unterscheidung von semiotischem Schritt und semiotischem Sprung führt uns nun zu weiteren als den oben vorgestellten 4 Grundtypen präsemiotischer Diamanten. Wenn wir uns die beiden Achsen des semiotischen Koordinatensystems anschauen:



dann stellen wir fest, dass es auf der Abszisse in dieser ungedrehten Form neben dem in den 4 Haupttypen vorausgesetzten Morphismus

1. (-3.0) → (-1.0)

noch die folgenden 3 weiteren Morphismen gibt, die ebenfalls semiotische Sprünge sind:

2. (-2.0) → (-1.0)
3. (-3.0) → (-2.0).

Ferner sehen wir, dass der Morphismus Nr. 1 ein aus den Morphismen 2 und 3 komponierter Morphismus ist:

1.' (-3.0) → (-1.0) = ((-3.0) → (-2.0) o (-2.0) → (-1.0)),

worin also 2 semiotische Sprünge involviert sind.

In derselben Weise können wir nun an allen 4 äusseren Ecken des präsemiotischen Raumes vorgehen und bekommen dann die folgenden weiteren Nebentypen:

4. (0.3) → (0.1)	7. (3.0) → (1.0)	10. (0.-3) → (0.-1)
5. (0.2) → (0.1)	8. (2.0) → (1.0)	11. (0.-2) → (0.-1)
6. (0.3) → (0.2)	9. (3.0) → (2.0)	12. (0.-3) → (0.-2)

Damit ergeben sich also 4 mal 4 = 16 Typen präsemiotischer Morphismen, nämlich die 4 Haupt- und die 3 mal 4 = 12 Nebentypen.

Bibliographie

Kaehr, Rudolf, Towards Diamonds. Glasgow 2007
Toth, Alfred, In Transit. Klagenfurt 2008 (2008a)
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008b)

Toth, Alfred, Die präsemiotischen Strukturbereiche. In: Electronic Journal for Mathematical Semiotics, 2008c
Toth, Alfred, Der präsemiotische Transit-Raum. In: Electronic Journal for Mathematical Semiotics, 2008d

Semiotische Kategorien und Saltatorien

1. In Toth (2008) hatte ich gezeigt, dass man aus Zeichenklassen der Form (3.a 2.b 1.c) als Kategorien und invertierten Zeichenklassen der Form (1.c 2.b 3.a) als Saltatorien semiotische Diamanten komponieren kann, wobei die hetero-morphismische Komposition der zur Kategorie der triadischen Zeichenrelation retrosemiotischen Relation korrespondiert. Meine diesbezüglichen Erkenntnisse stützten sich auf Kaehr (2007). Nun ist in der Zwischenzeit ein weiteres Paper von Kaehr erschienen, in welchem die Interaktion von Kategorien und Saltatorien in Diamanten und von Diamanten untereinander fokussiert wird (Kaehr 2008).

2. Eine Zeichenklasse hat allgemein die Form

(a.b c.d e.f)

und ihre durch Dualisierung gewonnene Realitätsthematik hat die Form

(f.e d.c b.a)

Neben der in Toth (2008a) als "Inversion" bezeichneten Transposition

(e.f c.d a.b)

gibt es jedoch weitere 5 Transpositionen für jede Zeichenklasse, also total 6:

6 Transpositionen: (a.b c.d e.f), (a.b e.f c.d), (c.d a.b e.f), (c.d e.f a.b), (e.f a.b c.d), (e.f c.d a.b)

Diese 6 Transpositionen können nun auch dualisiert werden:

6 Dualisationen: (f.e d.c b.a), (d.c f.e b.a), (f.e b.a d.c), (b.a f.e d.c), (d.c b.a f.e), (b.a d.c f.e)

3. Wie bislang üblich (Bense 1981, S. 124 ff., Leopold 1990, Toth 1997, S. 21 ff.), definieren wir eine Zeichenklasse als semiotische Kategorie:

Semiotische Kategorie :≡ Cat_{sem} = (a.b c.d e.f)

und ihre duale Realitätsthematik als duale semiotische Kategorie:

Duale Semiotische Kategorie :≡ $Cat_{sem}^{\circ\circ}$ = (f.e d.c b.a)

Die Inversion und die übrigen 4 Transpositionen können dann im Einklang mit Toth (2008) als semiotische Saltatorien definiert werden. Wir bekommen:

$\mathbf{Salt}_{sem}$ = {(a.b e.f c.d), (c.d a.b e.f), (c.d e.f a.b), (e.f a.b c.d), (e.f c.d a.b)}

Entsprechend erhalten wir auch die dualen semiotischen Saltatorien:

$\mathbf{Salt}^{\circ\circ}_{sem}$ = {(d.c f.e b.a), (f.e b.a d.c), (b.a f.e d.c), (d.c b.a f.e), (b.a d.c f.e)}

4. In semiotischen Diamanten und Diamanten-Kompositionen können daher semiotische Kategorien und Saltatorien wie folgt miteinander kombiniert werden:

Cat$_{sem}$ □ Salt$_{sem}$:	Cat$_{sem}$ □ Salt$_{sem}$°°:
(a.b c.d e.f), (a.b e.f c.d)	(a.b c.d e.f), (d.c f.e b.a)
(a.b c.d e.f), (c.d a.b e.f)	(a.b c.d e.f), (f.e b.a d.c)
(a.b c.d e.f), (c.d e.f a.b)	(a.b c.d e.f), (b.a f.e d.c)
(a.b c.d e.f), (e.f a.b c.d)	(a.b c.d e.f), (d.c b.a f.e)
(a.b c.d e.f), (e.f c.d a.b)	(a.b c.d e.f), (b.a d.c f.e)

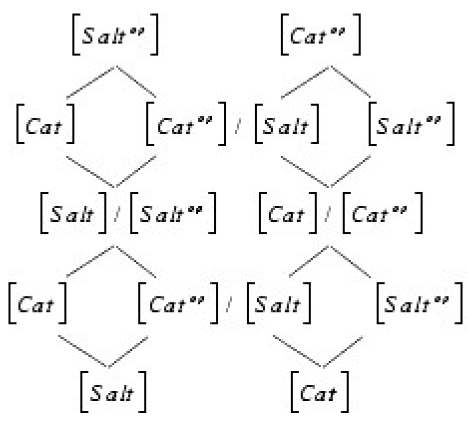
Cat$_{sem}$°° □ Salt$_{sem}$:	Cat$_{sem}$°° □ Salt$_{sem}$°°:
(f.e d.c b.a), (a.b e.f c.d)	(f.e d.c b.a), (d.c f.e b.a)
(f.e d.c b.a), (c.d a.b e.f)	(f.e d.c b.a), (f.e b.a d.c)
(f.e d.c b.a), (c.d e.f a.b)	(f.e d.c b.a), (b.a f.e d.c)
(f.e d.c b.a), (e.f a.b c.d)	(f.e d.c b.a), (d.c b.a f.e)
(f.e d.c b.a), (e.f c.d a.b)	(f.e d.c b.a), (b.a d.c f.e)

Für das formale Grundschema (a.b c.d e.f) kann nun jede der zehn Zeichenklassen eingesetzt werden:

(3.1 2.1 1.1)	(3.1 2.3 1.3)
(3.1 2.1 1.2)	(3.2 2.2 1.2)
(3.1 2.1 1.3)	(3.2 2.2 1.3)
(3.1 2.2 1.2)	(3.2 2.3 1.3)
(3.1 2.2 1.3)	(3.3 2.3 1.3)

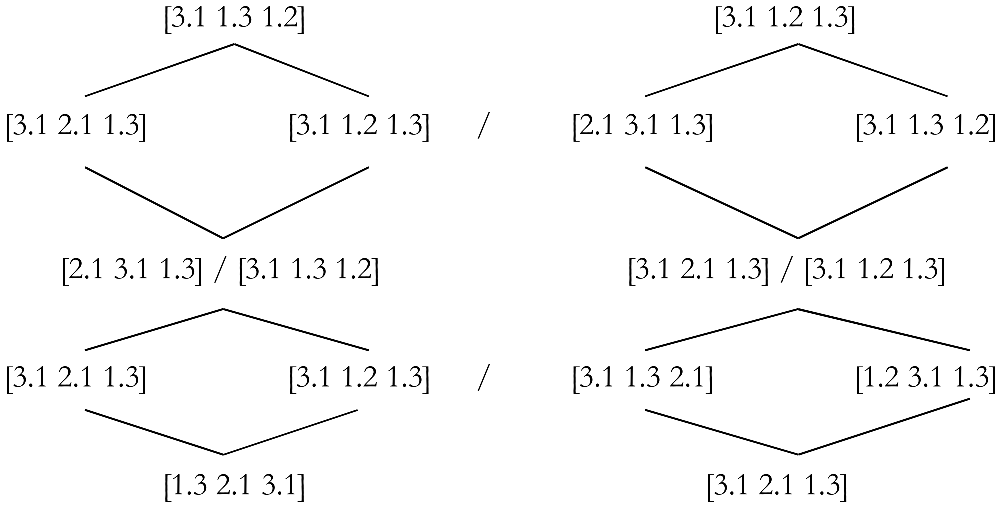
und ebenfalls die Genuine Kategorienklasse (3.3 2.2 1.1), die als Determinante der kleinen semiotischen Matrix eine semiotische Realität ist.

5. Wir zeigen nun anhand der Zeichenklasse (3.1 2.1 1.3), wie eine semiotische Diamantenkomposition ausschaut. Zunächst folgt das allgemeine Kaehrsche Modell:



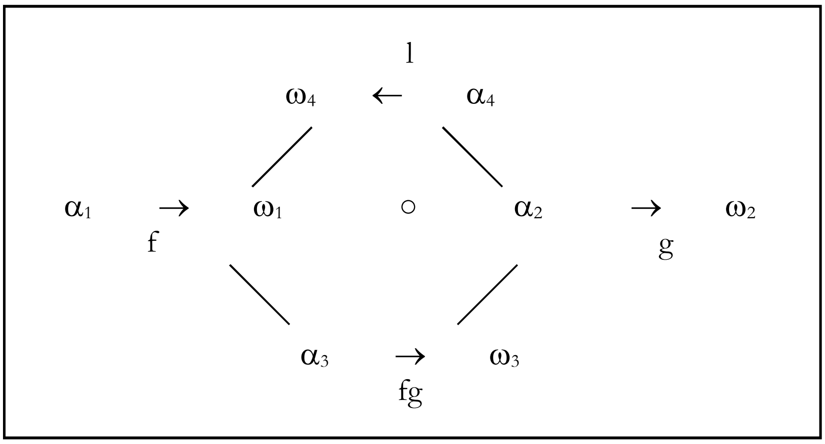
Quelle: http://www.rudys-diamond-strategies.blogspot.com/

Die Zeichenklasse (3.1 2.1 1.3), ihre Transpositionen und Dualisationen lassen sich dann kompositionstheoretisch wie folgt darstellen:



Es sei jedoch betont, dass die vorstehende Diamantenkomposition nur ein Repräsentant einer grösseren Klasse von zu einander semiotisch-diamantentheoretisch isomorpher Kompositionen ist.

6. Das mathematische Diamantenmodell, das Kaehr (2007) eingeführt hatte, sieht wie folgt aus:



Im obigen Beispiel semiotischer Diamantenkomposition haben wir folgende semiotische Kategorien und Saltatorien verwendet:

Cat_{sem}: [3.1 2.1 1.3] $Cat_{sem}^{°°}$: [3.1 1.2 1.3]

$Salt_{sem}3$: [2.1 3.1 1.3] $Salt^{°°}_{sem}3$: [3.1 1.3 1.2]
$Salt_{sem}2$: [3.1 1.3 2.1] $Salt^{°°}_{sem}2$: [1.2 3.1 1.3]
$Salt_{sem}1$: [1.3 2.1 3.1]

Deren Komposition sieht also wie folgt aus:

Cat_{sem}: [3.1 2.1 1.3] = (3.1 → 2.1) ○ (2.1 → 1.3) ≡ [[β°, id1], [α°, βα]]
$Cat_{sem}^{°°}$: [3.1 1.2 1.3] = (3.1 → 1.2) ○ (1.2 → 1.3) ≡ [[α°β°, α], [id1, β]]

$Salt_{sem}1$: [1.3 2.1 3.1] = (1.3 ← 2.1) ○ (2.1 ← 3.1) ≡ [[α, α°β°], [β, id1]]
$Salt_{sem}2$: [3.1 1.3 2.1] = (3.1 ← 1.3) ○ (1.3 ← 2.1) ≡ [[α°β°, βα], [α, α°β°]]

$Salt^{\circ\circ}{}_{sem}2$: [1.2 3.1 1.3] = (1.2 ← 3.1) ○ (3.1 ← 1.3) ≡ [[βα, α°], [α°β°, βα]]
$Salt_{sem}3$: [2.1 3.1 1.3] = (2.1 ← 3.1) ○ (3.1 ← 1.3) ≡ [[β, id], [α°β°, βα]]
$Salt^{\circ\circ}{}_{sem}3$: [3.1 1.3 1.2] = (3.1 ← 1.3) ○ (1.3 ← 1.2) ≡ [[α°β°, βα], [id1, β°]]

Die im allgemeinen Diamantenschema durch Striche angedeuteten Transitionen ("⇒") zwischen Cat_{sem} und $Cat_{sem}^{\circ\circ}$ sowie $Salt_{sem}i$ sind also die folgenden:

Cat ⇒ Cat°°: [3.1 2.1 1.3] ⇒ [3.1 1.2 1.3] ≡ [[β°, id1], [α°, βα]] ⇒ [[α°β°, α], [id1, β]]
Cat°° ⇒ Cat: [3.1 1.2 1.3] ⇒ [3.1 2.1 1.3] ≡ [[α°β°, α], [id1, β]] ⇒ [[β°, id1], [α°, βα]]
Cat ⇒ Salt1: [3.1 2.1 1.3] ⇒ [1.3 2.1 3.1] ≡ [[β°, id1], [α°, βα]] ⇒ [[α, α°β°], [β, id1]]
Salt1 ⇒ Cat: [1.3 2.1 3.1] ⇒ [3.1 2.1 1.3] ≡ [[α, α°β°], [β, id1]] ⇒ [[β°, id1], [α°, βα]]
Cat ⇒ Salt2: [3.1 2.1 1.3] ⇒ [3.1 1.3 2.1] ≡ [[β°, id1], [α°, βα]] ⇒ [[α°β°, βα], [α, α°β°]]
Salt2 ⇒ Cat: [3.1 1.3 2.1] ⇒ [3.1 2.1 1.3] ≡ [[α°β°, βα], [α, α°β°]] ⇒ [[β°, id1], [α°, βα]]
Cat ⇒ Salt3: [3.1 2.1 1.3] ⇒ [2.1 3.1 1.3] ≡ [[β°, id1], [α°, βα]] ⇒ [[β, id1], [α°β°, βα]]
Salt3 ⇒ Cat: [2.1 3.1 1.3] ⇒ [3.1 2.1 1.3] ≡ [[β, id1], [α°β°, βα]] ⇒ [[β°, id1], [α°, βα]]

Cat°° ⇒ Salt1: [3.1 1.2 1.3] ⇒ [1.3 2.1 3.1] ≡ [[α°β°, α], [id1, β]] ⇒ [[α, α°β°], [β, id1]]
Salt1 ⇒ Cat°°: [1.3 2.1 3.1] ⇒ [3.1 1.2 1.3] ≡ [[α, α°β°], [β, id1]] ⇒ [[α°β°, α], [id1, β]]
Cat°° ⇒ Salt2: [3.1 1.2 1.3] ⇒ [3.1 1.3 2.1] ≡ [[α°β°, α], [id1, β]] ⇒ [[α°β°, βα], [α, α°β°]]
Salt2 ⇒ Cat°°: [3.1 1.3 2.1] ⇒ [3.1 1.2 1.3] ≡ [[α°β°, βα], [α, α°β°]] ⇒ [[α°β°, α], [id1, β]]
Cat°° ⇒ Salt3: [3.1 1.2 1.3] ⇒ [2.1 3.1 1.3] ≡ [[α°β°, α], [id1, β]] ⇒ [[β, id1], [α°β°, βα]]
Salt3 ⇒ Cat°°: [2.1 3.1 1.3] ⇒ [3.1 1.2 1.3] ≡ [[β, id1], [α°β°, βα]] ⇒ [[α°β°, α], [id1, β]]
Salt1 ⇒ Salt2: [1.3 2.1 3.1] ⇒ [3.1 1.3 2.1] ≡ [[α, α°β°], [β, id1]] ⇒ [[α°β°, βα], [α, α°β°]]
Salt2 ⇒ Salt1: [3.1 1.3 2.1] ⇒ [1.3 2.1 3.1] ≡ [[α°β°, βα], [α, α°β°]] ⇒ [[α, α°β°], [β, id1]]
Salt2 ⇒ Salt3: [3.1 1.3 2.1] ⇒ [2.1 3.1 1.3] ≡ [[α°β°, βα], [α, α°β°]] ⇒ [[β, id1], [α°β°, βα]]
Salt3 ⇒ Salt2: [2.1 3.1 1.3] ⇒ [3.1 1.3 2.1] ≡ [[β, id1], [α°β°, βα]] ⇒ [[α°β°, βα], [α, α°β°]]
Salt1 ⇒ Salt3: [1.3 2.1 3.1] ⇒ [2.1 3.1 1.3] ≡ [[α, α°β°], [β, id1]] ⇒ [[β, id1], [α°β°, βα]]
Salt3 ⇒ Salt1: [2.1 3.1 1.3] ⇒ [1.3 2.1 3.1] ≡ [[β, id1], [α°β°, βα]] ⇒ [[α, α°β°], [β, id1]].

Literatur

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981
Kaehr, Rudolf, Towards Diamonds. Glasgow 2007. http://www.thinkartlab.com/pkl/lola/Towards_Diamonds.pdf
Kaehr, Rudolf, Double Cross Playing Diamonds. 2008. www.rudys-diamond-strategies.blogspot.com
Leopold, Cornelie, Kategorietheoretische Konzeption der Semiotik. In: Semiosis 57/58, 1990, S. 93-100
Toth, Alfred, Entwurf einer Semiotisch-Relationalen Grammatik. Tübingen 1997
Toth, Alfred, Semiotische Diamanten. In: Electronic Journal for Mathematical Semiotics, 2008

Schritt und Sprung in der Semiotik

(...) ob nicht überhaupt die Dialektik der Qualitäten eine andere ist; ob nicht 'der Übergang' hier eine andere Rolle spielt.

Søren Kierkegaard, *Die Krankheit zum Tode* (1984, S. 93)

Die neue Qualität entsteht mit der ersten, mit dem Sprunge, mit der Plötzlichkeit des Rätselhaften.

Søren Kierkegaard, *Der Begriff Angst* (1984, S. 30)

Die Sünde kommt also hinein als das Plötzliche, d.h. durch einen Sprung; aber dieser Sprung setzt zugleich die Qualität; doch indem die Qualität gesetzt ist, ist im selben Augenblick der Sprung in die Qualität hineinverflochten und von der Qualität vorausgesetzt und die Qualität vom Sprunge.

Søren Kierkegaard, *Der Begriff Angst* (1984, S. 32)

Die äusserste quantitierende Bestimmtheit erklärt den qualitativen Sprung ebenso wenig wie die geringste.

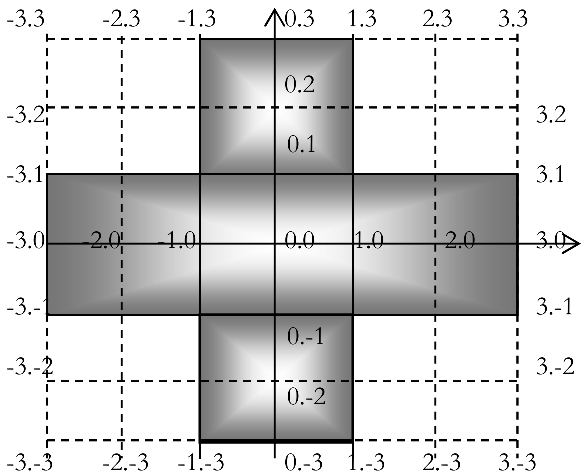
Søren Kierkegaard, *Der Begriff Angst* (1984, S. 37)

Da tut sie einen Sprung mitten in diesen Lichtstrahl hinein und beginnt sich von nun an selbst zuzusehen.

Unica Zürn, *Der Mann im Jasmin* (1977, S. 80)

1. Rudolf Kaehr (2007) hatte das Begriffspaar Schritt und Sprung in die polykontexturale Logik eingeführt, um die mathematische Unterscheidung zwischen Morphismen und den von Kaehr entdeckten Hetero-Morphismen bzw. von Kategorien und "Saltatorien" (oder "Jumpoids") in Anlehnung an die Terminologie Heideggers metaphysisch zu untermauern. Wie die obigen Zitate belegen, geht die Idee, den "Schritt" mit dem "Gänsemarsch" der Peanozahlen und das heisst mit der Nachfolge-Konzeption der vollständigen Induktion auf die quantitative Mathematik, dagegen den "Sprung" auf die qualitative Mathematik, genauer: auf die Überbrückung des kontexturalen Abgrundes zwischen den Peano-Zahlen einerseits und den polykontexturalen Strukturbereichen der Proto-, Deutero- und Trito-Zahlen anderseits anzuwenden, bereits auf Kierkegaard zurück. Auch Kronthaler, der Schöpfer der qualitativen Mathematik, spricht von einem Sprung: "Die von rechts nach links zunehmende Quantität von Ausdifferenzierungen zeigt u.a. einen Qualitätssprung von Proto → Deutero → Trito" (Kronthaler 1986, S. 35), dazu Anm. 116: "Hier im Sinne von: Quantität schlägt in Qualität um, verstanden" (1986, S. 187). Kronthaler benutzt dann die Unterscheidung von Schritt und Sprung dazu, die flächige Zählstruktur der qualitativen Zahlen darzustellen (1986, S. 31).

2. Wenn wir das semiotische Koordinatensystem ansehen, wie es in Toth (2008b) dargestellt wurde, können wir zwischen externen und internen Übergängen unterscheiden.



Die externen Übergänge liegen am äusseren Rand des Koordinatensystems jeweils auf einer horizontalen Achse, wenn das Koordinatensystem schrittweise um 90° gedreht wird. Die internen Übergänge liegen auf Achsen, die zu den horizontalen Achsen orthogonal sind, d.h. sie gehen bei allen 90°-Drehungen des Koordinatensystems von "aussen nach innen", d.h. dem absoluten Nullpunkt zu:

2.1. Externe Übergänge

1. (-1.3) → (0.3) → (1.3)
2. (3.1) → (3.0) → (3.-1)
3. (-1.-3) → (0.-3) → (1.-3)
4. (-3.-1) → (-3.0) → (-3.1)

2.2. Interne Übergänge

5. (0.3) → (0.2) → (0.1) → (0.0)
6. (3.0) → (2.0) → (1.0) → (0.0)
7. (0.-3) → (0.-2) → (0.-1) → (0.0)
8. (-3.0) → (-2.0) → (-1.0) → (0.0)

Man erkennt sofort:

1. (1., 3.) ⊥ (2., 4.) sowie (5., 7.) ⊥ (6., 8.), d.h. diese externen und internen Paare von Übergängen sind orthogonal zueinander.

2. Die Orthogonalen der externen Übergänge verhalten sich wie Morphismen zu Hetero-Morphismen. Die Orthogonale der internen Übergänge verhalten sich wie Morphismen zu inversen Morphismen.

3. 1. bis 4. bzw. 5. bis 8. sind alternative Sprünge und Schritte bzw. Schritte und Sprünge.

Wir erinnern uns daran, dass in Toth (2008c) semiotische Schritte als semiosische oder retrosemiosische Prozesse zwischen triadischen bzw. tetradischen Hauptwerten und Sprünge als semiosische oder retrosemiosische Prozesse zwischen trichotomischen Stellenwerten definiert wurden. Wenn wir also die in Toth (2008d) eingeführten semiotischen Kontexturen, berücksichtigen, d.h. die Tatsache, dass man die tetradisch-trichotomische Zeichenrelation als parametrisierte Relation über parametrisierten Relationen einführen kann:

PZR = (±3.±a ±2.±b ±1.±c ±0.±d),

dann können wir semiotisch folgendermassen zwischen Schritten, Sprüngen und Kontexturen unterscheiden (die Beispiele sind willkürlich gewählt):

(2.1) → (2.2) Schritt ohne Kontexturübergang
(2.1) → (-2.2) Schritt mit Kontexturübergang

(2.1) → (3.2) Sprung ohne Kontexturübergang
(2.1) → (-3.2) Sprung mit Kontexturübergang

Aus dieser Unterscheidung geht hervor, dass die Begriffe Sprung und Kontextur also wenigstens in der Semiotik getrennt werden können bzw. müssen. Neue Qualitäten können sich daher auch ausserhalb kontextureller Überschreitungen einstellen. Da Kontexturübergänge durch negative Vorzeichen sofort erkennbar sind, führen wir für die beiden Operatoren Schritt und Sprung die Symbole S und Σ ein.

3. Wie bereits in Toth (2008a, S. 38 f.), führen wir hier im Anschluss an Kaehr (2007, S. 12 u. passim) zwei polykontextural-semiotische Operatoren ein:

- den Jump-Operator ||
- den Bridging-Operator ⋈

Damit können wir nun die externen und die internen Übergänge zwischen dem präsemiotischen und dem semiotischen Raum mit Hilfe der Begriffe Schritt, Sprung und Kontextur sowie mit beiden semiotischen Trans-Operatoren formal darstellen:

3.1. Externe Übergänge

1. Σ((-1.3) || (0.3) || (1.3))
2. S((3.1) ⋈ (3.0) ⋈ (3.-1))
3. Σ((-1.-3) || (0.-3) || (1.-3))
4. S((-3.-1) ⋈ (-3.0) ⋈ (-3.1))

3.2. Interne Übergänge

5. S((0.3) ⋈ (0.2) ⋈ (0.1) ⋈ (0.0))
6. Σ((3.0) || (2.0) || (1.0) || (0.0)
7. S((0.-3) ⋈ (0.-2) ⋈ (0.-1) ⋈ (0.0))
8. Σ((-3.0) || (-2.0) || (-1.0) || (0.0))

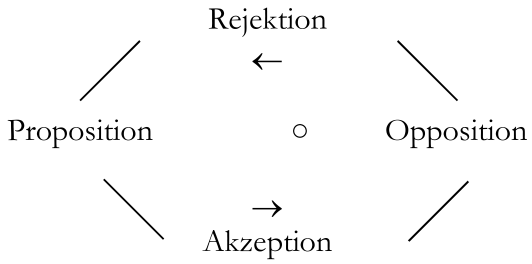
Damit haben wir also die grundlegenden polykontextural-semiotischen Operatoren des präsemiotischen Transit-Raumes formalisiert.

Bibliographie

Kaehr, Rudolf, Towards Diamonds. Glasgow 2008
Kierkegaard, Søren, Der Begriff Angst. Frankfurt am Main 1984
Kierkegaard, Søren, Die Krankheit zum Tode. Frankfurt am Main 1984
Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986
Toth, Alfred, In Transit. Klagenfurt 2008 (2008a)
Toth, Alfred, Die präsemiotischen Strukturbereiche. In: Electronic Journal for Mathematical Semiotics, 2008b
Toth, Alfred, Kompositionen präsemiotischer Diamanten. In: Electronic Journal for Mathematical Semiotics, 2008c
Toth, Alfred, Der präsemiotische Transit-Raum. In: Electronic Journal for Mathematical Semiotics, 2008d
Zürn, Unica, Der Mann im Jasmin. Frankfurt am Main 1977

Semiotische Diamanten aus symplerotischen Zeichenklassen

Im Anschluss an Toth (2008a, S. 177 ff.) wird in dieser Arbeit eine neue Methode zur Konstruktion semiotischer Diamanten eingeführt. Ein logischer Diamant hat nach Kaehr (2007, S. 55) folgende allgemeine Form:



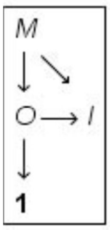
Nun wurde bereits in Toth (2008a, S. 183 ff.) gezeigt, dass die 6 Permutationen jeder Zeichenklasse in der Form von semiotischen Diamanten notiert werden können. Allerdings hat Kaehr in seiner bisher jüngsten Arbeit die Ansicht vertreten, die mathematische Semiotik sei "strictly monocontextural" (Kaehr 2008, S. 5 ff.):

Example $M \longrightarrow O \longrightarrow I$

Semiotic composition:

$$(M \longrightarrow O) \circ (O \longrightarrow I) \Longrightarrow (M \longrightarrow I).$$

Conceptual graph for signs



Semiotics (Peirce, Bense, Toth) is fundamentally mono – contextural and it is blind for its monocontexturality, *i.e.* the *uniqueness* property, **1**, is not part of the definition of semiotics.

Diamond composition:

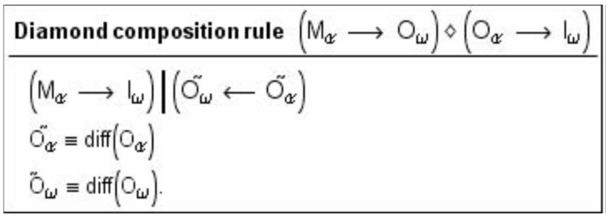
$$(M_\alpha \longrightarrow O_\omega) \diamond (O_\alpha \longrightarrow I_\omega) \Longrightarrow (M_\alpha \longrightarrow I_\omega) \,\Big|\, (O_\omega \longleftarrow O_\alpha)$$

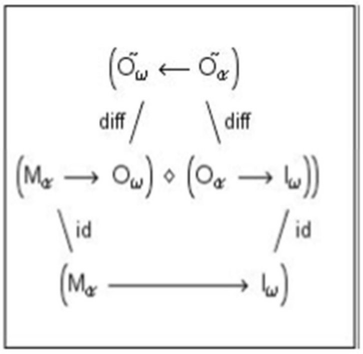
Diamond relations as rules

Semiotic composition rule

$$\frac{(M \longrightarrow O) \circ (O \longrightarrow I)}{M \longrightarrow I}$$

Null





Nun wurde aber in Toth (2008b) gezeigt, dass die gruppentheoretische Operation der Symplerosis zur Unterscheidung von Akzeption und Rejektion in (klassischen) semiotischen Systemen führt. Natürlich hat Kaehr recht, wenn er bemerkt, wegen des Bestehens des logischen Identitätssatzes bleibe die Semiotik monokontextural; allein, dies hindert sie nicht daran, einige Dutzend polykontexturaler Eigenschaften zu zeigen, dies im Einklang mit den Vermutungen Masers (1973, S. 29 ff.) und Benses (1980) sowie meiner auf der Webseite www.mathematical-semiotics.com erneut zugänglich gemachter Arbeiten. Aus philosophischer Sicht hatte Udo Bayer (1994) in seinem Aufsatz "Semiotik und Ontologie" im Detail aufgezeigt, dass die der theoretischen Semiotik zugrunde liegende Ontologie eine polykontexturale ist. Deshalb erstaunt nicht, dass man innerhalb der mathematischen Semiotik auch zahlreichen formalen polykontexturalen Strukturen begegnet.

In Toth (2008b) hatte ich gezeigt, dass man über der Menge der Primzeichen PZ = {.1., .2., .3.} genau drei abelsche Gruppen konstruieren kann, wobei in der ersten Gruppe die Drittheit (.3.), in der zweiten Gruppe die Zweitheit (.2.) und in der dritten Gruppe die Erstheit (.1.) zugleich als Einselement sowie als semiotisch-logischer Rejektionswert fungiert.

$(PZ, \circ_1)$:	$(PZ, \circ_2)$:	$(PZ, \circ_3)$:
$1 \rightarrow 2$	$1 \rightarrow 3$	$1 = 1$
$2 \rightarrow 1$	$2 = 2$	$2 \rightarrow 3$
$3 = 3$	$3 \rightarrow 1$	$3 \rightarrow 2$

Damit lassen sich nun aus den 10 Zeichenklassen je 3 symplerotische Zeichenklassen nach den drei abelschen Gruppen konstruieren:

Zkln	PZ, $\circ_1$	PZ, $\circ_2$	PZ, $\circ_3$
(3.1 2.1 1.1)	(3.2 1.2 2.2)	(1.3 2.3 3.3)	(2.1 3.1 1.1)
(3.1 2.1 1.2)	(3.2 1.2 2.1)	(1.3 2.3 3.2)	(2.1 3.1 1.3)
(3.1 2.1 1.3)	(3.2 1.2 2.3)	(1.3 2.3 3.1)	(2.1 3.1 1.2)
(3.1 2.2 1.2)	(3.2 1.1 2.1)	(1.3 2.2 3.2)	(2.1 3.3 1.3)
(3.1 2.2 1.3)	(3.2 1.1 2.3)	(1.3 2.2 3.1)	(2.1 3.3 1.2)
(3.1 2.3 1.3)	(3.2 1.3 2.3)	(1.3 2.1 3.1)	(2.1 3.2 1.2)
(3.2 2.2 1.2)	(3.1 1.1 2.1)	(1.2 2.2 3.2)	(2.3 3.3 1.3)
(3.2 2.2 1.3)	(3.1 1.1 2.3)	(1.2 2.2 3.1)	(2.3 3.3 1.2)
(3.2 2.3 1.3)	(3.1 1.3 2.3)	(1.2 2.1 3.1)	(2.3 3.2 1.2)
(3.3 2.3 1.3)	(3.3 1.3 2.3)	(1.1 2.1 3.1)	(2.2 3.2 1.2)

Damit ergeben sich also zu jeder Zeichenklasse der Form

(3.a 2.b 1.c)

mit ihren 6 Permutationen

(3.a 2.b 1.c)
(3.a 1.c 2.b)
(2.b 3.a 1.c)
(2.b 1.c 3.a)
(1.c 3.a 2.b)
(1.c 2.b 3.a)

1. heteromorphe Zeichenklassen der nicht-symplerotischen Formen

(1.c 2.b 3.a)
(2.b 1.c 3.a)
(1.c 3.a 2.b)
(3.a 1.c 2.b)
(2.b 3.a 1.c)
(3.a 2.b 1.c)

2. heteromorphe Zeichenklassen der symplerotischen Formen nach (PZ, $\circ_1$)

(2.c 1.b 3.a)
(1.b 2.c 3.a)
(2.c 3.a 1.b)
(3.a 2.c 1.b) } sowie a, b, c = .1 ↔ a, b, c = .2
(1.b 3.a 2.c)
(3.a 1.b 2.c)

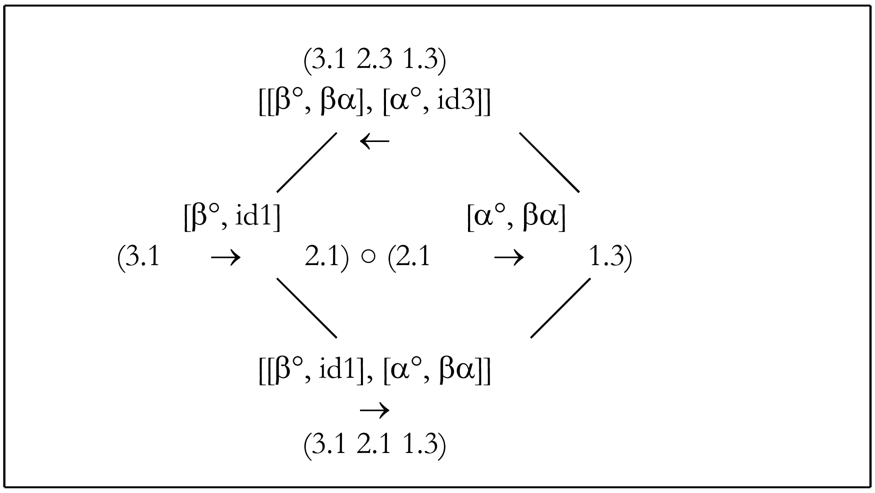
3. heteromorphe Zeichenklassen der symplerotischen Formen nach (PZ, $\circ_2$)

(3.c 2.b 1.a)
(2.b 3.c 1.a)
(3.c 1.a 2.b)
(1.a 3.c 2.b) sowie a, b, c = .1 ↔ a, b, c = .3
(2.b 1.a 3.c)
(1.a 2.b 3.c)

4. heteromorphe Zeichenklassen der symplerotischen Formen nach (PZ, $\circ_3$)

(2.a 3.b 1.c)
(2.a 1.c 3.b)
(3.b 2.a 1.c)
(3.b 1.c 2.a) sowie a, b, c = .2 ↔ a, b, c = .3
(1.c 2.a 3.b)
(1.c 3.b 2.a)

Da somit alle Permutationen aus den Zeichenklassen Nrn. 2., 3. und 4. als Heteromorphismen in Frage kommen, kann damit jede der 6 Permutationen jeder der 10 Zeichenklassen mit je einer der total 18 heteromorphen Zeichenklassen zu einem semiotischen Diamanten kombiniert werden. Diese grosse Anzahl semiotischer Diamanten verdoppelt sich ausserdem, wenn wir statt von Zeichenklassen von Realitätsthematiken ausgehen. Während also die Identitätsrelationen zwischen den beiden dyadischen Teilrelationen jeder triadischen Zeichenklasse und dieser Zeichenklasse bestehen, bestehen die Differenzrelationen zwischen den dyadischen Teilrelationen einer Zeichenklassen und ihren heteromorphen, d.h. durch eine der drei symplerotischen Operationen gewonnen Zeichenklassen. Da das Konstruktionprinzip dieser homomorph-heteromorphen Diamanten dem in Toth (2008a, S. 177 ff.) angegebenen folgt, begnügen wir uns abschliessend mit dem folgenden einen Beispiel: Gegeben sei die Zkl (3.1 2.1 1.3) sowie $\circ_2$. Dann erhalten wir also als 2-symplerotische Zkl (1.3 2.3 3.1) und daraus als heteromorphe (3.1 2.3 1.3) und daher den folgenden semiotischen Diamanten:



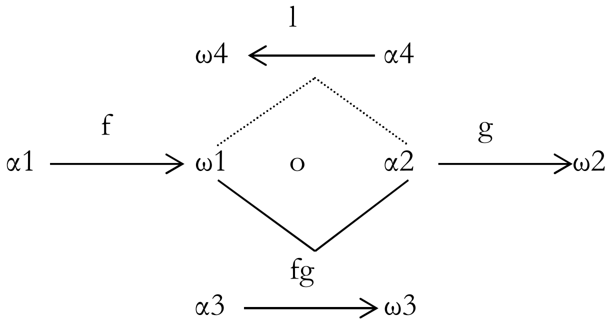
Bibliographie

Bayer, Udo, Semiotik und Ontologie. In: Semiosis 74-76, 1994, S. 3-34
Bense, Max, Gotthard Günthers Universal-Metaphysik. In: Neue Zürcher Zeitung 20./21.9.1980
Maser, Siegfried, Grundlagen der allgemeinen Kommunikationstheorie. 2. Aufl. Stuttgart 1973

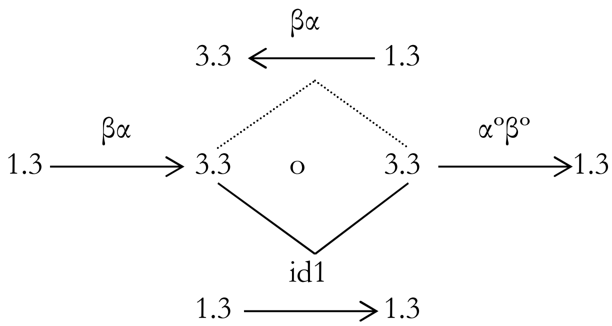
Kaehr, Rudolf, Towards Diamons. Glasgow 2007. Digitalisat:
http://www.thinkartlab.com/pkl/lola/Towards_Diamonds.pdf
Kaehr, Rudolf, Diamond Semiotics. Glasgow 2008. Digitalisat:
http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Heteromorphismen aus symplerotischen Zeichenklassen

1. Semiotische Diamanten wurden von mir (Toth 2008a, S. 32 ff.) im Anschluss an Kaehr (2007, S. 2) eingeführt. Sie haben nach Kaehr die folgende allgemeine Form



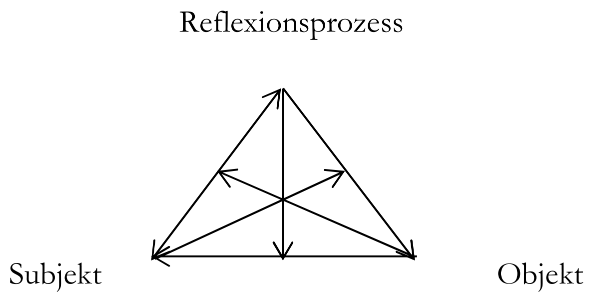
Setzt man nun (α1) = (1.3), α2 = (3.3), ω1 = (3.3), ω2 = (1.3), so bekommt man den folgenden semiotischen Diamanten.



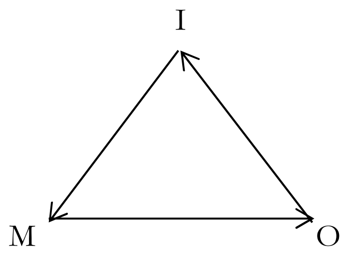
In einer kürzlich veröffentlichten Kritik bemerkte Rudolf Kaehr zurecht, dass in dergestalt eingeführten semiotischen Diamanten die Heteromorphismen nichts anderes seien als Spiegelungen dyadischer semiotischer Funktionen (Kaehr 2008, S. 3). Kaehr übersieht allerdings, dass die Umkehrungen dyadischer Funktionen nur formal, aber nicht inhaltlich Spiegelungen sind. Z.B. bedeutet (2.1 ⇒ 3.1) die rhematische Interpretation eines Abbildes, aber die umgekehrte Funktion (3.1 ⇒ 2.1) muss, wie bereits Bense (1981, S. 124 ff.) bemerkte, nicht zum selben Icon zurückführen. Es kann sich hier also um einen echten semiotischen Heteromorphismus handeln.[3]

2. In Toth (2008b, S. 61 ff.) hatte ich gezeigt, dass sich Günthers triadisches Schema einer dreiwertigen Logik (1976, S. 336 ff.)

[3] Dieses Missverständnis beruht, wie ich überzeugt bin, auf dem allgemeineren Missverständnis, das Kaehr mit vielen Logikern und Mathematikern teilt, dass nämlich die mathematische Semiotik eine "künstliche" (Kaehr 2008, S. 7 f. spricht von "artificial") Formalisierung sei. In Wahrheit besteht die Neuerung der mathematischen Semiotik über die quantitative ebenso wie über die qualitative Mathematik gerade darin, dass sie als einzige Mathematik mit Bedeutung und Sinn rechnet. Auch Kaehrs Überzeugung (a.a.O.), der mathematische Zahlbegriff sei monadisch, weshalb sich seine Semiotisierung a priori verbiete, ist unzutreffend, da bereits Bense (1980) gezeigt hatte, dass jeder bisher in der Mathematik verwandte Zahlbegriff eine triadische Relation im Sinne des peirceschen Zeichenmodells erfüllt.



mit den Entsprechungen Subjekt = objektives Subjekt, Objekt = (objektives) Objekt und Reflexionsprozess = subjektives Subjekt auf das bekannte triadische Peircesche Zeichenmodell



abbilden lässt, so dass wir also folgende logisch-semiotischen Korrespondenzen bekommen (zur Begründung vgl. Toth 2008b, S. 64 f.):

Subjekt = objektives Subjekt = Mittelbezug
Objekt = objektives Objekt = Objektbezug
Reflexionsprozess = subjektives Subjekt = Interpretantenbezug

Als weitere Korrespondenzen erhalten wir folgende logisch-semiotischen Prozesse (vgl. Günther 1963, S. 38):

(Subjekt ⇒ Objekt) ≡ (M ⇒ O) ≡ Transzendentalidentität
(Subjekt ⇒ Reflexionsprozess) ≡ (M ⇒ I) ≡ Reflexionsidentität
(Objekt ⇒ Reflexionsprozess) ≡ (O ⇒ I) ≡ Seinsidentität

Somit werden also bei der Transzendentalidentität die beiden semiotischen Werte (.1.) und (.2.) vertauscht, d.h. (.3.) = const. Bei der Reflexionsidentität werden die beiden semiotischen Werte (.1.) und (.3.) vertauscht, d.h. (.2.) = const. Schliesslich werden bei der Seinsidentität die beiden semiotischen Werte (.2.) und (.3.) vertauscht, d.h. (.1.) = const. Wie in Toth (2008c) gezeigt wurde, entsprechen diese Wertvertauschungen genau der Anwendung der drei möglichen abelschen gruppentheoretischen Operationen σ_1, σ_2 und σ_3. Diese drei symplerotischen Operationen erzeugen also aus den 10 Zeichenklassen eine erste Gruppe von transzendentalidentischen, eine zweite Gruppe von reflexionsidentischen und eine dritte Gruppe von seinsidentischen Zeichenklassen. Wir können diese Verhältnisse in dem folgenden Schema zusammenfassen:

Zkln	**3 = const** **Transzendental-identität**	**2 = const** **Reflexions-identität**	**1 = const** **Seins-identität**
(3.1 2.1 1.1)	(3.2 1.2 2.2)	(1.3 2.3 3.3)	(2.1 3.1 1.1)
(3.1 2.1 1.2)	(3.2 1.2 2.1)	(1.3 2.3 3.2)	(2.1 3.1 1.3)
(3.1 2.1 1.3)	(3.2 1.2 2.3)	(1.3 2.3 3.1)	(2.1 3.1 1.2)
(3.1 2.2 1.2)	(3.2 1.1 2.1)	(1.3 2.2 3.2)	(2.1 3.3 1.3)
(3.1 2.2 1.3)	(3.2 1.1 2.3)	(1.3 2.2 3.1)	(2.1 3.3 1.2)
(3.1 2.3 1.3)	(3.2 1.3 2.3)	(1.3 2.1 3.1)	(2.1 3.2 1.2)
(3.2 2.2 1.2)	(3.1 1.1 2.1)	(1.2 2.2 3.2)	(2.3 3.3 1.3)
(3.2 2.2 1.3)	(3.1 1.1 2.3)	(1.2 2.2 3.1)	(2.3 3.3 1.2)
(3.2 2.3 1.3)	(3.1 1.3 2.3)	(1.2 2.1 3.1)	(2.3 3.2 1.2)
(3.3 2.3 1.3)	(3.3 1.3 2.3)	(1.1 2.1 3.1)	(2.2 3.2 1.2)
(3.3 2.2 1.1)	(3.3 1.1 2.2)	(1.12.2 3.3)	(2.2 3.3 1.1)

Als letzten Schritt können wir nun, ausgehend von den nicht-symplerotischen Zeichenklassen, aus dieser Tabelle zu jedem Homomorphismus seine je drei Heteromorphismen herauslesen. Wir notieren sie hier jedoch in nicht-invertierter Form und teilen sie entsprechend den drei semiotischen Funktionen in (M ⇒ O), (O ⇒ I) und (M ⇒ I) ein:

(3.1 2.1 1.1)	(3.2 1.2 2.2)	(1.3 2.3 3.3)	(2.1 3.1 1.1)
(1.1 ⇒ 2.1)	(1.2 ⇒ 2.2)	(1.3 ⇒ 2.3)	(1.1 ⇒ 2.1)
(2.1 ⇒ 3.1)	(2.2 ⇒ 3.2)	(2.3 ⇒ 3.3)	(2.1 ⇒ 3.1)
(1.1 ⇒ 3.1)	(1.2 ⇒ 3.2)	(1.3 ⇒ 3.3)	(1.1 ⇒ 3.1)

(3.1 2.1 1.2)	(3.2 1.2 2.1)	(1.3 2.3 3.2)	(2.1 3.1 1.3)
(1.2 ⇒ 2.1)	(1.2 ⇒ 2.1)	(1.3 ⇒ 2.3)	(1.3 ⇒ 2.1)
(2.1 ⇒ 3.1)	(2.1 ⇒ 3.2)	(2.3 ⇒ 3.2)	(2.1 ⇒ 3.1)
(1.2 ⇒ 3.1)	(1.2 ⇒ 3.2)	(1.3 ⇒ 3.2)	(1.3 ⇒ 3.1)

(3.1 2.1 1.3)	(3.2 1.2 2.3)	(1.3 2.3 3.1)	(2.1 3.1 1.2)
(1.3 $\Rightarrow$ 2.1)	(1.2 $\Rightarrow$ 2.3)	(1.3 $\Rightarrow$ 2.3)	(1.2 $\Rightarrow$ 2.1)
(2.1 $\Rightarrow$ 3.1)	(2.3 $\Rightarrow$ 3.2)	(2.3 $\Rightarrow$ 3.1)	(2.1 $\Rightarrow$ 3.1)
(1.3 $\Rightarrow$ 3.1)	(1.2 $\Rightarrow$ 3.2)	(1.3 $\Rightarrow$ 3.1)	(1.2 $\Rightarrow$ 3.1)

(3.1 2.2 1.2)	(3.2 1.1 2.1)	(1.3 2.2 3.2)	(2.1 3.3 1.3)
(1.2 $\Rightarrow$ 2.2)	(1.1 $\Rightarrow$ 2.1)	(1.3 $\Rightarrow$ 2.2)	(1.3 $\Rightarrow$ 2.1)
(2.2 $\Rightarrow$ 3.1)	(2.1 $\Rightarrow$ 3.2)	(2.2 $\Rightarrow$ 3.2)	(2.1 $\Rightarrow$ 3.3)
(1.2 $\Rightarrow$ 3.1)	(1.1 $\Rightarrow$ 3.2)	(1.3 $\Rightarrow$ 3.2)	(1.3 $\Rightarrow$ 3.3)

(3.1 2.2 1.3)	(3.2 1.1 2.3)	(1.3 2.2 3.1)	(2.1 3.3 1.2)
(1.3 $\Rightarrow$ 2.2)	(1.1 $\Rightarrow$ 2.3)	(1.3 $\Rightarrow$ 2.2)	(1.2 $\Rightarrow$ 2.1)
(2.2 $\Rightarrow$ 3.1)	(2.3 $\Rightarrow$ 3.2)	(2.2 $\Rightarrow$ 3.1)	(2.1 $\Rightarrow$ 3.3)
(1.3 $\Rightarrow$ 3.1)	(1.1 $\Rightarrow$ 3.2)	(1.3 $\Rightarrow$ 3.1)	(1.2 $\Rightarrow$ 3.3)

(3.1 2.3 1.3)	(3.2 1.3 2.3)	(1.3 2.1 3.1)	(2.1 3.2 1.2)
(1.3 $\Rightarrow$ 2.3)	(1.3 $\Rightarrow$ 2.3)	(1.3 $\Rightarrow$ 2.1)	(1.2 $\Rightarrow$ 2.1)
(2.3 $\Rightarrow$ 3.1)	(2.3 $\Rightarrow$ 3.2)	(2.1 $\Rightarrow$ 3.1)	(2.1 $\Rightarrow$ 3.2)
(1.3 $\Rightarrow$ 3.1)	(1.3 $\Rightarrow$ 3.2)	(1.3 $\Rightarrow$ 3.1)	(1.2 $\Rightarrow$ 3.2)

(3.2 2.2 1.2)	(3.1 1.1 2.1)	(1.2 2.2 3.2)	(2.3 3.3 1.3)
(1.2 $\Rightarrow$ 2.2)	(1.1 $\Rightarrow$ 2.1)	(1.2 $\Rightarrow$ 2.2)	(1.3 $\Rightarrow$ 2.3)
(2.2 $\Rightarrow$ 3.2)	(2.1 $\Rightarrow$ 3.1)	(2.2 $\Rightarrow$ 3.2)	(2.3 $\Rightarrow$ 3.3)
(1.2 $\Rightarrow$ 3.2)	(1.1 $\Rightarrow$ 3.1)	(1.2 $\Rightarrow$ 3.2)	(1.3 $\Rightarrow$ 3.3)

(3.2 2.2 1.3)	(3.1 1.1 2.3)	(1.2 2.2 3.1)	(2.3 3.3 1.2)
(1.3 $\Rightarrow$ 2.2)	(1.1 $\Rightarrow$ 2.3)	(1.2 $\Rightarrow$ 2.2)	(1.2 $\Rightarrow$ 2.3)
(2.2 $\Rightarrow$ 3.2)	(2.3 $\Rightarrow$ 3.1)	(2.2 $\Rightarrow$ 3.1)	(2.3 $\Rightarrow$ 3.3)
(1.3 $\Rightarrow$ 3.2)	(1.1 $\Rightarrow$ 3.2)	(1.2 $\Rightarrow$ 3.1)	(1.2 $\Rightarrow$ 3.3)

(3.2 2.3 1.3)	(3.1 1.3 2.3)	(1.2 2.1 3.1)	(2.3 3.2 1.2)
(1.3 ⇒ 2.3)	(1.3 ⇒ 2.3)	(1.2 ⇒ 2.1)	(1.2 ⇒ 2.3)
(2.3 ⇒ 3.2)	(2.3 ⇒ 3.1)	(2.1 ⇒ 3.1)	(2.3 ⇒ 3.2)
(1.3 ⇒ 3.2)	(1.3 ⇒ 3.1)	(1.2 ⇒ 3.1)	(1.2 ⇒ (3.2)

(3.3 2.3 1.3)	(3.3 1.3 2.3)	(1.1 2.1 3.1)	(2.2 3.2 1.2)
(1.3 ⇒ 2.3)	(1.3 ⇒ 2.3)	(1.1 ⇒ 2.1)	(1.2 ⇒ 2.2)
(2.3 ⇒ 3.3)	(2.3 ⇒ 3.3)	(2.1 ⇒ 3.1)	(2.2 ⇒ 3.2)
(1.3 ⇒ 3.3)	(1.3 ⇒ 3.3)	(1.1 ⇒ 3.1)	(1.2 ⇒ 3.2)

(3.3 2.2 1.1)	(3.3 1.1 2.2)	(1.1 2.2 3.3)	(2.2 3.3 1.1)
(1.1 ⇒ 2.2)	(1.1 ⇒ 2.2)	(1.1 ⇒ 2.2)	(1.1 ⇒ 2.2)
(2.2 ⇒ 3.3)	(2.2 ⇒ 3.3)	(2.2 ⇒ 3.3)	(2.2 ⇒ 3.3)
(1.1 ⇒ 3.3)	(1.1 ⇒ 3.3)	(1.1 ⇒ 3.3)	(1.1 ⇒ 3.3)

Damit lassen sich nun semiotische Diamanten konstruieren, welche der folgenden Forderung Kaehrs (2008, S. 1) nicht mehr widersprechen: "Diamonds are not triadic-trichotomic but genuinely tetradic, chiastic, antidromic and 4-fold. Hence, diamonds are not semiotical".

Bibliographie

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Günther, Gotthard, Das Bewusstsein der Maschinen. Krefeld 1963

Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. Bd. 1. Hamburg 1976

Kaehr, Rudolf, Towards Diamonds. Glasgow 2007. Digitalisat: http://www.thinkartlab.com/pkl/lola/Towards_Diamonds.pdf

Kaehr, Rudolf, Diamond semiotics. 2008 http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf

Toth, Alfred, In Transit. Klagenfurt 2008 (2008a)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008b)

Toth, Alfred, Symplerose und Transjunktion. In: Electronic Journal for Mathematical Semiotics, 2008c

Echte und falsche semiotische Diamanten

1. Wenn man eine reelle Zeichenklasse der allgemeinen Form

Zkl = (3.a 2.b 1.c)

dualisiert, bekommt man eine Realitätsthematik der Form

Rth = (c.1 b.2 a.3).

Wenn man hingegen eine komplexe Zeichenklasse der Form (vgl. Toth 2009)

Zkl = (3.ia 2.ib 1.ic)

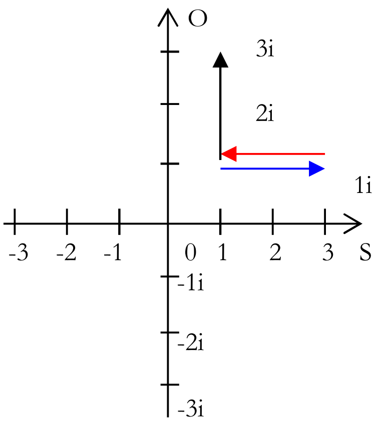
dualisiert, sieht die Realitätsthematik wie folgt aus

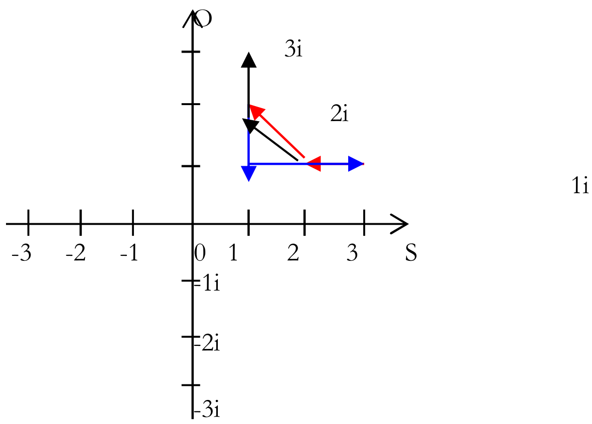
Rth = (ia.3 ib.2 ic.1),

d.h. während bei reellen Realitätsthematiken für alle (x, y) x ∈ Abszisse und y ∈ Ordinate gilt, ist es für komplexe Realitätsthematiken gerade umgekehrt.

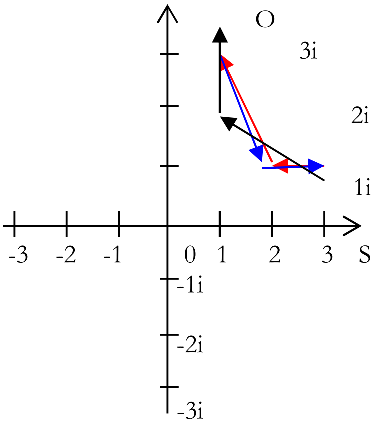
2. In den folgenden 10 Graphen sind für alle Peirceschen Zeichenklassen (rot) einerseits die reellen (blau), anderseits die komplexen Realitätsthematiken (schwarz) eingetragen:

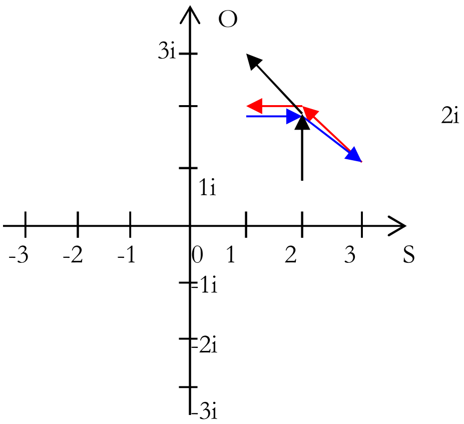
1. <<3.i1>, <2.i1>, <1.i1>> × <<i1.1>, <i1.2>, <i1.3>>
2. <<3.i1>, <2.i1>, <1.i2>> × <<i2.1>, <i1.2>, <i1.3>>



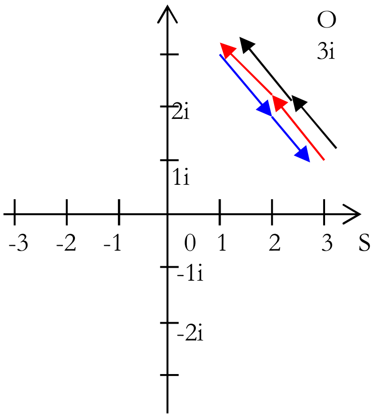


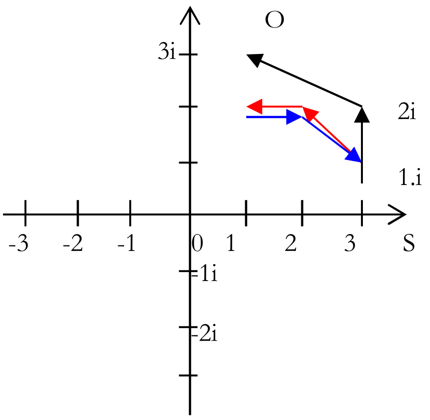
3. <<3.i1>, <2.i1>, <1.i3>> × <<i3.1>, <i1.2>, <i1.3>>
4. <<3.i1>, <2.i2>, <1.i2>> × <<i2.1>, <i2.2>, <i1.3>>



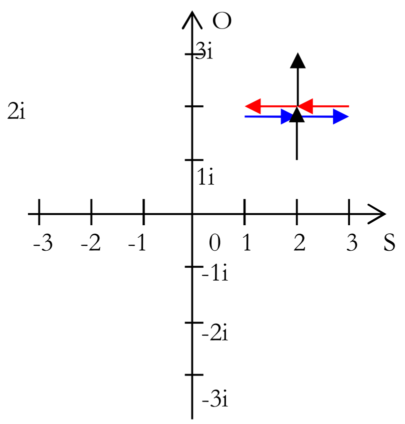


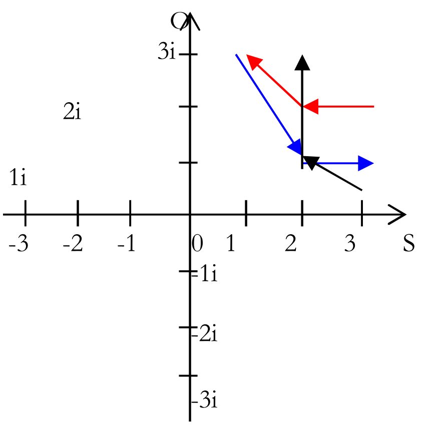
5. <<3.i1>, <2.i2>, <1.i3>> × <<i3.1>, <i2.2>, <i1.3>>
6. <<3.i1>, <2.i3>, <1.i3>> × <<i3.1>, <i3.2>, <i1.3>>



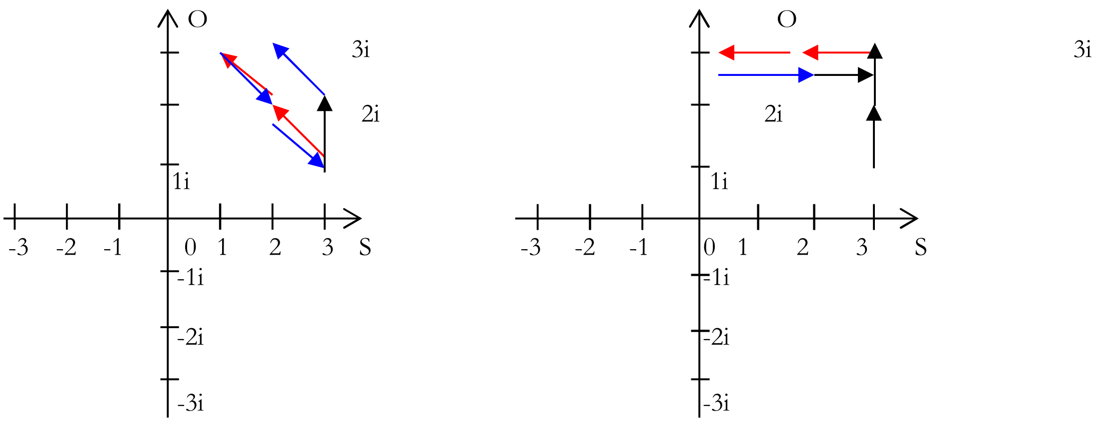


7. <<3.i2>, <2.i2>, <1.i2>> × <<i2.1>, <i2.2>, <i2.3>>
8. <<3.i2>, <2.i2>, <1.i3>> ×<<i3.1>, <i2.2>, <i2.3>>





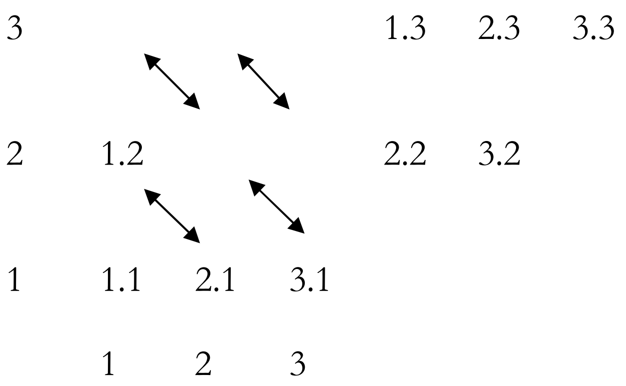
9. <<3.i2>, <2.i3>, <1.i3>> × <<i3.1>, <i3.2>, <i2.3>>
10. <<3.i3>, <2.i3>, <1.i3>> × <<i3.1>, <i3.2>, <i3.3>>



3. Schreiben wir für komplexe Realitätsthematik Rthc und für reele Rthr, dann haben wir also

Rthr(3.a 2.b 1.c) = (c.1 b.2 a.3),

wobei Δ(ZKl, Rth) = 0 nur im alle der eigenrealen, dualidentischen Zeichenklasse, sonst gilt immer Δ(Zkl, Rth) > 0, und zwar deshalb, weil jedes Paar konverser Subzeichen (a.b) und (a.b)° = (b.a) sowohl in einer anderen Triade als auch in einer anderen Trichotomie, d.h. sowohl in einer anderen Zeile als auch in einer anderen Spalte der Gausschen Zahlenebene liegen:



Dagegen gilt

Rthc(3.ai 2.bi 1.ci) = (ci.1 bi.2 ai.3),

d.h. wir haben

Rthc(1.ci → 2.bi → 3.ai) = (ai.3 → bi.2 → ci.1), d.h. eine komplexe Zeichenklasse und ihre Rthc verhalten sich genau so wie Morphismus und Heteromorphismus (vgl. Kaehr 2009, S. 28 ff.). Das bedeutet also fernerhin, dass sich komplexe Zeichenklassen als echte (dreistellige) semiotische Diamanten darstellen lassen; z.B.

(3.1i 2.1i 1.3i) × (3i.1 1i.2 1i.3) ≡

(3i.1 ← 1i.3)

(3.1i → 2.1i) ○ (2.1i → 1.3i)

(3.1i → 1.3i)

Umgekehrt lassen sich jedoch reelle Zeichenklassen (und Realitätsthematiken) nicht als echte semiotische Diamanten darstellen, da, wie Kaehr betont hatte, einfache Retrosemiosen der Gestalt (3.1 1.2) → (2.1 1.3) keine Heteromorphismen sind. Man könnte hier also höchstens von „falschen semiotischen Diamanten" sprechen.

Bibliographie

Kaehr, Rudolf, The Book of Diamonds. Neuausgabe. Glasgow 2009, Digitalisat: http://www.thinkartlab.com/pkl/lola/Diamond-Theory-Collection.pdf

Toth, Alfred, Komplexe semiotische Analyse. In: Electronic Journal of Mathematical Semiotics, 2009

New elements of theoretical semiotics (NETS), based on the work of Rudolf Kaehr) (= NETS, 1)

1. Recently, Professor Rudolf Kaehr has published four papers (Kaehr 2008, 2009a, b, c) in which he applies some elements of polycontextural theory to selected fundamentals of mathematical semiotics introduced by me. I have to point out that Kaehr's work on semiotics surpasses in never seen dimensions almost everything that has been elaborated in the long history of semiotics. Therefore, I have no doubt that Kaehr's studies mark the beginning of a wholly new era of formal semiotics compared to which most of the writings of the last decades will look rather poor and provisory. In the present article, I will discuss some of the new theoretical fundamentals introduced into semiotics by Kaehr.

2. As Kaehr correctly sees, the so-called "Genuine Category Class"

(3.3 2.2 1.1)

is the only sign-relation that appears in Bense's "semiotic matrix" without being a defined sign class, since sign classes (SCl) must be built upon the relational form

SCl = (3.a 2.b 1.c) with a, b, c $\in$ {.1, .2, .3}

obeying the inclusive trichotomic order

$(a \leq b \leq c)$,

but since (3.3 2.2 1.1) has the trichotomic order $(a > b > c)$, is is not considered a sign class and therefore does not figure in the list of the 10 Peircean sign classes.

Nevertheless, the Genuine Category Class has given rise to speculations about its theoretical status as well as about its applications throughout the history of theoretical semiotics. F.ex., Bense (1975, p. 93) wrote:

> "Alle diese für die (dreistufige Hauptsemiose der (neunstufigen) semiotischen Matrix charakteristischen erkenntnistheoretischen und kommunikationstheoretischen, ersichtlich auf Zeichenrelationen und Semiosen zurückführbaren Züge machen die Hauptsemiose (1.1, 2.2, 3.3) zu einer genuinen, die alle anderen möglichen Semiosen, die mit ihren stabilen Momenten in der semiotischen Matrix erkannt bzw. formuliert werden könen, **generiert** und **repräsentiert**. Sie kann daher in ihrer semiotischen Funktion, naheliegend und bei hinreichender Verallgemeinerung jenes Prinzips der Zustandsentwicklung, das Maxwell und Boltzmann für ihre Zwecke einführten, im Anschluss an die späteren Formulierungen von Planck, Takács, Lange, Chintschin u.a. als **ergodische Semiose** bezeichnet werden, um auszudrücken, dass ein bestimmter Abstraktionsfluss mit bestimmten relativ stabilen

> Abstraktionsmomenten existiert, der (relativ zur semiotischen Matrix der Gesamtheit der Semiosen und ihrer Subzeichen) als ergodischer Prozess zu beschreiben ist".

However, while there is no doubt that was Bense wrote, is true from a semantic standpoint, the formal side of generative and representative connections between the Genuine Category Class and the 10 regular sign classes is highly unclear. The Genuine Category Class is only connected to the following 6 sign classes:

(3.1 2.1 1.1), (3.1 2.2 1.2), (3.1 2.2 1.3), (3.2 2.2 1.2), (3.2 2.2 1.3), (3.3 2.3 1.3),

so that, unlike the eigenreal sign class (3.1 2.2 1.3), which is connected to all 10 sign classes and therefore induces a "determinant-theoretic duality system" (Walther 1982), the Genuine Category Class does not induce a discriminant-theoretic duality system.

However, in a new publication (Kaehr 2009c), Kaehr has shown that it is not sufficient to introduce the three fundamental categories of triadic semiotics as single objects or morphisms, but that they must be introduced as doublets, therein containing their "hetero-morphism" or "(inner) environment":

Firstness:	Peirce:	A
	Kaehr:	A | a
Secondness:	Peirce:	$A \to B$
	Kaehr	$A \to B \mid c$
Thirdness:	Peirce:	$A \to C$
	Kaehr:	$A \to C \mid b_1 \leftarrow b_2$

An informal approach to apply this so-called diamond-concept of defining the three semiotic fundamental categories not as single morphisms, but as doublets consisting of morphisms and their hetero-morphisms, can the derived from the correspondence between the fundamental categories and the so-called semiotic functions (cf. Walther 1979, pp. 113 ss.; Toth 1997, p. 33). Although Firstness is what stands for itself, it is also the domain of Thirdness in the semiotic "application function"

$(M \Rightarrow I)$ or $((.1.) \Rightarrow (.3.))$,

meaning that Firstness is what connects the whole (triadic) relation with itself (the monadic) relation, so that we can characterize Firstness with (1,3).[4]

[4] If we define a sign relation as SR = (M, (M⇒O),(O⇒I)) in Peirce's sense (followed by Walther 1979, pp. 113 ss.), consisting of a monadic, a dyadic and a triadic (part-)relation, then we omit the **fourth** part-relation (I⇒M) or (M⇒I), resp.! Therefore, the graph of SR would not be closed.

On the other hand, Secondness is what connects Firstness with Thirdness in correspondence with the semiotic "designation function" (1,2)

(M ⇒ O) or ((.1.) ⇒ (.2.)),

and Thirdness is what connects Secondness with the whole (triadic) relation, thus with itself (2,3) in correspondence with the semiotic "denomination function"

(O ⇒ I) or ((.2.) ⇒ (.3.)).

Therefore, we obain that the monocontextural set of prime-signs

PS = {.1., .2., .3.}

corresponds to the following polycontextural set of prime-signs

$PS^* = \{(.1.)_{1,3}, (.2.)_{1,2}, (.3.)_{2,3}\}$.

When we now have a look at Kaehr's "polycontextural semiotic 3-matrix" (Kaehr 2009c)

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

we recognize immediately that the genuine (identitive) sub-signs

$(3.3_{2,3})$, $(2,.2_{1,2})$, $(1.1_{1,3})$

are the only sub-signs whose "indices" are identical with the "indices" of the prime-signs. Thus, the polycontextural Genuine Category Class

$(3.3_{2,3}\ 2,.2_{1,2}\ 1.1_{1,3})$

is the **generating sign relation for all the sub-signs of the semiotic matrix and therefore for all the 10 (regular) Peircean sign classes**. This astonishing and extremely important result could not be achieved before the introduction of semiotic environments based on the doublet-definition of the semiotic fundamental categories ascribing to each semiotic morphism its hetero-morphism by Kaehr (2009c).

This generating function of the polycontextural Genuine Category Class can also be shown in the polycontextural 3-matrix itself:

$$\left(\begin{array}{ccc} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ \downarrow & \uparrow & \uparrow \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ \downarrow & \downarrow & \uparrow \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{array}\right)$$

This means that the "index" (1,3) of (1.1) generates (downwards) both the "index" 1 of (2.1) and the "index" 3 of (3.1). The "index" (1,2) generates (upwards) the "index" 1 of (1.2) and (downwards) the "index" of (3.2). And the "index" (2.3) generates (upwards) both the "index" 3 of (1.3) and the "index" (2,3) of (3.3).

A more "impressionistic" characterization of the sub-signs is:

(1.2), or Secondness of Firstness, is what both connects itself and the whole and Firstness with itself, i.e. (1,3) □ (1,2) = 1.

(1.3), or Thirdness of Firstness, is what both connects itself and the whole and Secondness with itself, i.e. (1,3) □ (2,3) = 3.

(2.3), or Thirdness of Secondness, is what both connects Firstness with itself and Secondness with itself, i.e. (1,2) □ (2,3) = 2.

(2.1), (3.1), and (3.2) have the same "indices", since they are dual to the three above defined sub-signs. As already shown, the indices of the genuine or identitive (self-dual) sub-signs are identical with those of the prime-signs.

3. Polycontexturality is based on the abolition of the four basic Laws of Thinking: The Law of Identity, the Law of the Excluded Middle, The Law of Non-Contradiction and the Law of Double Negation. However, when the Law of Identity is abolished, for semiotics, it is to expect that one of its central theories, the theory of eigenreality (Bense 1992), disappears, too. Already Kaehr (2009c) has shown that the monocontextural eigenreal dual system

$\times(3.1\ 2.2\ 1.3) = (3.1\ 2.2\ 1.3);\ (3.1\ 2.2\ 1.3) = (3.1\ 2.2\ 1.3)$

does not hold anymore in the polycontextural semiotic framework based on the above polycontextural semiotic 3-matrix:

$\times(3.1_{3}\ 2.2_{1,2}\ 1.3_{3}) = (3.1_{3}\ 2.2_{2,1}\ 1.3_{3});\ (3.1_{3}\ 2.2_{1,2}\ 1.3_{3}) \neq (3.1_{3}\ 2.2_{2,1}\ 1.3_{3})$, since through dualization, not only the sub-signs, but their "indices" are inverted as well. It follows that the 10 Peircean sign classes do not form anymore Walthers (monocontextural) "determinant-symmetric duality system" which says that each of the 10 sign classes/reality thematics is connected with every other sign class/reality thematics by at least 1 sub-sign. Since the theory of semiotic connections is based fundamentally on the concept of eigenreality, it has to be redefined, too.

However, the loss of eigenreality due to introduction of environment-contextuated sub-signs is not so unexpected as it might seem to be. Even without knowledge of the different contextures involved in the index (2.2), it is clear that in

×(3.1 2.2 1.3) = (3.1 2.2 1.3),

the rhema (3.1) of the second sign class is not identical with the rhema (3.1) of the first sign class, but is identical with the dualized legi-sign of the first sign class (1.3). The same holds for the legi-sign of the second sign class which is the dualized rhema of the first sign class and not its legi-sign. This means: The "identity" between (3.1) and x(1.3) and (1.3) and x(3.1) is a pure formal one. However, this purely formal identity stands in contradiction with the assertion of semiotics that the two rhemata

×(3.1 x y) = (3.1 x y)

are in fact rhemata and the two legi-signs

×(x y 1.3) = (x y 1.3)

are in fact legi-signs and thus semantically identical, which is, as we have just shown, not true. If this would be true, than sign-sign (1.2) and icon (2.1) and symbol (2.3) and dicent (3.2) would be identical, too.

For the set of the semiotic dual-systems, the abolishment of eigenreality implicates that there is no longer a partition into the eigenreal dual-system and the one side and the other 9 dual-systems on the other side. As it is show, dualization inverts all 10 sign classes or reality thematics in exactly the same way, i.e. through inversion of not only their sub-signs but also of their environmental contextures. Thus, all 10 sign classes and reality thematics need **two dualizations** in order to regain their original structure:

$(3.1_3\ 2.1_1\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$	×	$(3.1_3\ 2.1_1\ 1.1_{1,3})$
$(3.1_3\ 2.1_1\ 1.2_1)$	×	$(2.1_1\ 1.2_1\ 1.3_3)$	×	$(3.1_3\ 2.1_1\ 1.2_1)$
$(3.1_3\ 2.1_1\ 1.3_3)$	×	$(3.1_3\ 1.2_1\ 1.3_3)$	×	$(3.1_3\ 2.1_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$	×	$(3.1_3\ 2.2_{1,2}\ 1.2_1)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	×	$(3.1_3\ 2.2_{1,2}\ 1.3_3)$

$(3.1_3\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 1.3_3)$	×	$(3.1_3\ 2.3_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$	×	$(3.2_2\ 2.2_{1,2}\ 1.2_1)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$	×	$(3.2_2\ 2.2_{1,2}\ 1.3_3)$
$(3.2_2\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 2.3_2)$	×	$(3.2_2\ 2.3_2\ 1.3_3)$
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 3.3_{3,2})$	×	$(3.3_{2,3}\ 2.3_2\ 1.3_3)$

The same holds for the polycontextural Genuine Category Class:

$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3})$ × $(3.1_{3,1}\ 3.2_{2,1}\ 3.3_{3,2})$ × $(3.3_{2,3}\ 2.3_{1,2}\ 1.3_{1,3})$

So, in the third row, every sub-sign and every environment is not only formally, but also semantically identical with the respective sub-sign and environment in the first row.

4. In chapter 2., I had already mentioned that regular sign classes are restricted through obeying the inclusive semiotic order

$(3.a\ 2.b\ 1.c)$ with $a \leq b \leq c$.

Thus, every other order of the trichotomic values a, b, c leads to irregular sign classes. However, this restriction is one of those not so rare semiotic restrictions, which have no theoretical basis at all. Moreover, the special restriction in discussion here has not even a semantic motivation, since there is no reason, why a sign relation like, e.g.,

(3.2 2.1 1.3)

is not to be considered a (regular) sign class. An example for (2.1 1.3) is a literary metaphor, which as a metaphor is iconic (2.1) and by use of letters, i.e. conventional media, is a legi-sign (1.3). So, why should our metaphor (2.1 1.3) not be able to figure as part of a dicentic sentence, i.e. a sentence, which can be judged concerning its truth or falseness? The arbitrarily chosen German sentence

Der Zahn der Zeit hat an diesem Gebäude genagt

can surely be stated as true or false when uttered about a specific building. Generally, is does not need much fantasy to find counter-evidence against the “forbidden” (irregular) sign classes which are constructed just by the rule

$(3.a\ 2.b\ 1.c)$ with $a, b, c \in \{.1, .2, .3\}$

If we construct them, we get $3 \cdot 3 \cdot 3 = 27$ sign classes. We will note them as polycontextural sign classes, i.e. together with their contextural “indices”

$(3.1_3\ 2.1_1\ 1.1_{1,3})$	$(3.2_2\ 2.1_1\ 1.1_{1,3})$	$(3.3_{2,3}\ 2.1_1\ 1.1_{1,3})$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.2_2\ 2.1_1\ 1.2_1)$	$(3.3_{2,3}\ 2.1_1\ 1.2_1)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.2_2\ 2.1_1\ 1.3_3)$	$(3.3_{2,3}\ 2.1_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.1_{1,3})$	$(3.2_2\ 2.2_{1,2}\ 1.1_{1,3})$	~~$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3})$~~
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	**$(3.2_2\ 2.2_{1,2}\ 1.2_1)$**	$(3.3_{2,3}\ 2.2_{1,2}\ 1.2_1)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	**$(3.2_2\ 2.2_{1,2}\ 1.3_3)$**	$(3.3_{2,3}\ 2.2_{1,2}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.1_{1,3})$	$(3.2_2\ 2.3_2\ 1.1_{1,3})$	$(3.3_{2,3}\ 2.3_2\ 1.1_{1,3})$
$(3.1_3\ 2.3_2\ 1.2_1)$	$(3.2_2\ 2.3_2\ 1.2_1)$	$(3.3_{2,3}\ 2.3_2\ 1.2_1)$
$(3.1_3\ 2.3_2\ 1.3_3)$	**$(3.2_2\ 2.3_2\ 1.3_3)$**	**$(3.3_{2,3}\ 2.3_2\ 1.3_3)$**

In bold are the "regular" sign classes. **Simply by looking at the positions of the regular 10 sign classes, we recognize that they build only a sub-set or perhaps better: a fragment of the set of the 27 sign classes.** If we look at the system of the contextural "indices", this gets even clearer:

3-1-(1,3) **3-1-1** **3-1-3**	2-1-(1,3) 2-1-1 2-1-3	(2,3)-1-(1,3) (2,3)-1-1 (2,3)-1-3
3-(1,2)-(1.3) **3-(1,2)-1** **3-(1,2)-3**	2-(1,2)-(1,3) **2-(1,2)-1** **2-(1,2)-3**	~~(2,3)-(1,2)-(1,3)~~ (2,3)-(1,2)-1 (2,3)-(1,2)-3
3-2-(1,3) 3-2-1 **3-2-3**	2-2-(1,3) 2-2-1 **2-2-3**	(2,3)-2-(1,3) (2,3)-2-1 **(2,3)-2-3,**

since we recognize that each of the 3 horizontal squares has the following double structure:

3-x-y	2-x-y	(2,3)-x-y

with

$$x = \left\{ \begin{matrix} 1 \\ (1,2) \\ 2 \end{matrix} \right\} \downarrow \qquad y = \overrightarrow{\left\{ \begin{matrix} (1,3) \\ 1 \\ 3 \end{matrix} \right\}}$$

whereby, in x, (1,2) mediates between 1 and 2, and in y, (1,3) is unfolded into 1 and 3.

5. Semiotics belongs to the oldest scientific branches, although it never became so popular like, e.g., logic. However, while logic has been thoroughly formalized in the last two millennia, in semiotics, hardly anything more has been done than to produce endless and senseless discussions about the reality status of the sign (physei or thesei). Then, since the 60ies, Bense introduced formal concepts into semiotics, but he mainly saw in semiotics a branch of metamathematics rather than mathematics. The "mathematical turn" of semiotics was left for me to achieve. Although I have started in the early 80ies to try to elevate semiotics on the formal level of at least elementary mathematics, the bigger part of this work I could only publish in the last years, due to other scientific obligations. Included in these studies was the adaptation of some basic notions of Günther's polycontextural theory, which I had studied only in the 90ies. However, most semioticians - me included - have long time overseen that Günthers work has been expanded into a whole new scientific branch by his student Rudolf Kaehr. Since Kaehr's work surpasses Günther's work both in formal accuracy and in metaphysical depth, an approximation between semiotics and polycontextural theory can only be achieved from Kaehr's and not directly from Günther's work. I am convinced that the future of semiotics lies in big parts in this common semiotic-polycontextural basis. The very few examples given in this study may be sufficient to show the enormous power that emerges from this common basis. The present author has titled this article "New elements of theoretical semiotics" and even invented the acronym "NETS" in the hope that this study will not remain alone but continued in many sequels.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, Diamond semiotics. In: http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (Kaehr 2008)

Kaehr, Rudolf, Toth's semiotic diamonds. In: http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf (2009a)

Kaehr, Rudolf, XANADU's textemes. In: http://www.thinkartlab.com/pkl/lola/Xanadu-textemes/Xanadu-textemes.pdf (2009b)

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009c)

Toth, Alfred, Entwurf einer Semiotisch-Relationalen Grammatik. Tübingen 1997

Walther, Elisabeth, Nachtrag zu Trichotomischen Triaden. In: Semiosis 27, 1982, S. 15-20

Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

A poly-contextural view on triadic semiotics. (= NETS, 2)

1. The Peircean semiotic fundamental categories can, as Kaehr (2009) has shown, be redefined by using their inner semiotic environments or hetero-morphisms:

Firstness:	Peirce:	A
	Kaehr:	$A \mid a$
Secondness:	Peirce:	$A \rightarrow B$
	Kaehr	$A \rightarrow B \mid c$
Thirdness:	Peirce:	$A \rightarrow C$
	Kaehr:	$A \rightarrow C \mid b_1 \leftarrow b_2$

If we assume that M, O, I form three semiotic contextures, we get

$M\ (.1.) = R(1,3)$
$O(.2.) = R(1,2)$
$I(.3.) = R(2,3)$,

which correspond to the definition of semiotic functions (cf. Walther 1979, pp. 113 ss.):

$R(1,3) \leftrightarrow R(M, I) = (M \Rightarrow I)$
$R(1,2) \leftrightarrow R(M, O) = (M \Rightarrow O)$
$R(2,3) \leftrightarrow R(O, I) = (O \Rightarrow I)$

Therefore, the Peircean "mono-contextural" set of prime-signs

$PS = \{.1., .2., .3.\}$

can be redefined, too, as a "poly-contextural" set of prime-signs

$PS^* = \{(.1.)_{1,3}, (.2.)_{1,2}, (.3.)_{2,3}\}$.

On this basis we get, instead of the mono-contextural semiotic matrix, the following poly-contextural semiotic matrix" (Kaehr 2009):

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

2. However, as Kaehr (2009) has suggested and as it had been pointed out in Toth (2003, pp. 54 ss.), triadic semiotics has not necessarily to be built on 3 semiotic contextures, but can be constructed as fragments of 4 or more semiotic contextures. 4-contextural semiotics had been introduced extensively as pre-semiotics, embedding the Peircean triadic semiotics into tetradic semiotics containing the category (or contexture) Zeroness, already suggested in Bense (1975, pp. 45, 65 s.), in Toth (2008b). If the above triadic-3-contextural semiotic matrix is considered a fragment of a tetradic-4-contextural matrix, we get:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

In Toth (2008b), the category of Zeroness was identified with the "ontological space" introduced in Bense (1975, pp. 45 s., 65 ss.) and further developed in Stiebing (1981, 1984). Therefore, we have

(.0.) ‖ (.1., .2., .3.),

where the sign ‖ stands for the contextural border between the (categorial) object (.0.) and the sign (.1., .2., .3.).

However, as it was shown in Toth (2008c), the contextural border between the sign and its designated object is not the only transcendence involved in the sign relation. As a matter of fact, each of the three fundamental categories substitute, in a sign relation, an entity of the ontological space which is transcendent to its respective fundamental category. We thus have

(.0.) ‖ (.1., .2., .3.)

(.1.) ‖ (.0., .2., .3.)

(.2.) ‖ (.0., .1., .3.)

(.3.) ‖ (.0., .1., .2.)

Therefore, at least from a semantic standpoint, the upper border for a sign class is a 6-adic semiotics with 6 contextures as its minimum (cf. Toth 2007, pp. 186 ss.).

On the other side, if we take the above triadic 4-contextural matrix, we get the the following semiotic relations

(.0.) ‖ (.1., .2., .3.)

(.1.) ‖ (.0., .2., .3.)

(.2.) ‖ (.0., .1., .3.)

(.3.) ‖ (.0., .1., .2.)

Semantically, this means, that, if we construct the Peircean sign relation (.1., .2., .3.), we omit the categorial object. Thus, this is the normal sign relation which substitutes its object that is, therefore, transcendent to it. However, if we construct (.0., .2., .3.), (.0., .1., .3.) or (.0., .1., .2.), we omit the medium, the object, or the interpretant relation of the sign, but we abolish the basic contextural border between the sign and its designated object. I will discuss these three "abnormal" sign relations briefly:

(.1.) ‖ (.0., .2., .3.): The sign without medium, i.e. without sign-carrier. As an example, we can take Lewis Carroll's "Forest of no name": As long as Alice and the deer are in this forest, where there are no medium relations of the signs, they walk and discuss with one another. However, as soon as they get out, the deer remembers its name and can now infer the connotation "deer = shy animal", and runs frightened away (Nöth 1980, p. 75).

(.2.) ‖ (.0., .1., .3.): The sign without object, i.e. without meaning. Here, too, we have a good example in Lewis Carroll's work, this El-Dorado of pathological sign relations: The two sign-posts which direct in different directions, but at the same time to the allegedly unique object of the house of Tweedledum and Tweedledee. Nöth remarks: "Es stellt sich allerdings die Frage, ob es das durch die Wegweiser anzeigte Objekt überhaupt gibt; denn Alice trifft Tweedledum and Tweedledee nicht in einem Haus, sondern unter einem Baum stehend an" (1980, p. 74).

(.3.) ‖ (.0., .1., .2.): The sign without interpretant. Although there are at least ten different kinds of interpretant relations in Peirce's work, the primary notion of interpretant, fitting perfectly to the intuitive notion of sign, is that something is a sign *for somebody*, and therefore for a receiver in the sense of a sign obeying the communication schema. Thus, an example of a sign without interpretant is an inscription, which cannot be deciphered. Moreover, since there is no meaning in a sign relation when the interpretant is absent, we can quote as an instant here Carroll's Poem of Humpty-Dumpty to which Nöth correctly remarked: "Zwar kennt Alice das Gedicht auswendig, aber seine Bedeutung kennt sie nicht. Sie ist nicht in der Lage, die vollständige triadische Zeichenrelation herzustellen" (1980, S. 74) – denn hierzu bedürfte sie eben des Interpretantenbezugs.

3. In a work that unfortunately has not been recognized by the Stuttgart School of Semiotics, Joseph Ditterich pointed out that it is possible to consider the dyadic Saussurean sign as a sub-matrix of the triadic Peircean sign matrix (Ditterich 1990, p. 28). If we start again with the triadic matrix as a fragment of a 4-contextural semiotic matrix:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

then we realize that we do not only have, like in the case of the 3-contextural triadic semiotic matrix, 1, but 3 dyadic sub-matrices:

(1 ↔ 2)
(2 ↔ 3)
(1 ↔ 3)

Considering 0, we get in addition again 3 dyadic sub-matrices:

(0 ↔ 1)
(0 ↔ 2)
(0 ↔ 3)

In other words: There is no longer one dyadic sign model associating signifiant and signifié, but there are now 6 sign models which are based on associations between the pairs of Zeroness, Firstness, Secondness and Thirdness. Hence, Saussurean semiotics is not only just a (semiotically incomplete) sub-matrix of the basal triadic Peircean sign matrix, but even as a sub-matrix nothing else but a special case of at least 6 different sub-matrices which are completely unrecognized in Saussures "semiology" and its further developments in French structuralism.

4. The following table shows the distribution of the 9 sub-signs of the semiotic matrix over 4 semiotic contextures:

1	2	3	4
1.1		1.1	1.1
1.2			1.2
		1.3	1.3
2.1			2.1
2.2	2.2		2.2
	2.3		2.3
		3.1	3.1
	3.2		3.2
	3.3	3.3	3.3

As has been already stated above, we can now, starting from a triadic semiotics considered a fragment of a 4-contextural semiotics, construct sign classes which obey the following 4 semiotic part-relations:

SR*(1) = (.1., .2., .3.)
SR*(2) = (.0., .1., .2.)
SR*(3) = (.0., .1., .3.)
SR*(4) = (.0., .2., .3.)

For the construction of the sign classes, we stick with the inclusive semiotic order

(a.b c.d e.f) with a, ..., f $\in$ {1, 2, 3} and (b $\leq$ d $\leq$ f),

thereby reducing the maximal amount of sign relations in which a, c, e are pairwise different, from 3^3 = 27 to 10 sign classes, although this decision is questionable; cf. Kaehr (2009) and Toth (2009). For examples, cf. above, chapter 2.

4.1. Poly-contextural sign classes over SR*(1)

These are exactly the 10 Peircean sign classes plus the contextural "indices".

$(3.1_{3,4}\ \ 2.1_{1,4}\ \ 1.1_{1,3,4})$
$(3.1_{3,4}\ \ 2.1_{1,4}\ \ 1.2_{1,4})$
$(3.1_{3,4}\ \ 2.1_{1,4}\ \ 1.3_{3,4})$
$(3.1_{3,4}\ \ 2.2_{1,2,4}\ \ 1.2_{1,4})$
$(3.1_{3,4}\ \ 2.2_{1,2,4}\ \ 1.3_{3,4})$
$(3.1_{3,4}\ \ 2.3_{2,4}\ \ 1.3_{3,4})$
$(3.2_{2,4}\ \ 2.2_{1,2,4}\ \ 1.2_{1,4})$
$(3.2_{2,4}\ \ 2.2_{1,2,4}\ \ 1.3_{3,4})$
$(3.2_{2,4}\ \ 2.3_{2,4}\ \ 1.3_{3,4})$
$(3.3_{2,3,4}\ \ 2.3_{2,4}\ \ 1.3_{3,4})$

4.2. Poly-contextural sign classes over SR*(2)

These are exactly the 10 Peircean sign classes together with the contextural "indices":

$(2.1_{1,4}\quad 1.1_{1,3,4}\quad 0.1_{1,3})$
$(2.1_{1,4}\quad 1.2_{1,4}\quad 0.2_{1,2})$
$(2.1_{1,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(2.2_{1,2,4}\quad 1.2_{1,4}\quad 0.2_{1,2})$
$(2.2_{1,2,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(2.3_{2,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(2.2_{1,2,4}\quad 1.2_{1,4}\quad 0.2_{1,2})$
$(2.2_{1,2,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(2.3_{2,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(2.3_{2,4}\quad 1.3_{3,4}\quad 0.3_{2.3})$

4.3. Poly-contextural sign classes over SR*(3)

$(3.1_{3,4}\quad 1.1_{1,3,4}\quad 0.1_{1,3})$
$(3.1_{3,4}\quad 1.2_{1,4}\quad 0.2_{1,2})$
$(3.1_{3,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(3.1_{3,4}\quad 1.2_{1,4}\quad 0.2_{1,2})$
$(3.1_{3,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(3.1_{3,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(3.2_{2,4}\quad 1.2_{1,4}\quad 0.2_{1,2})$
$(3.2_{2,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(3.2_{2,4}\quad 1.3_{3,4}\quad 0.3_{2,3})$
$(3.3_{2,3,4}\quad 1.3_{3,4}\quad 0.3_{2.3})$

4.4. Poly-contextural sign classes over SR*(4)

$(3.1_{3,4}\quad 2.1_{1,4}\quad 0.1_{1,3})$
$(3.1_{3,4}\quad 2.2_{1,2,4}\quad 0.2_{1,2})$
$(3.1_{3,4}\quad 2.3_{2,4}\quad 0.3_{2,3})$
$(3.1_{3,4}\quad 2.2_{1,2,4}\quad 0.2_{1,2})$
$(3.1_{3,4}\quad 2.3_{2,4}\quad 0.3_{2,3})$
$(3.1_{3,4}\quad 2.3_{2,4}\quad 0.3_{2,3})$
$(3.2_{2,4}\quad 2.2_{1,2,4}\quad 0.2_{1,2})$
$(3.2_{2,4}\quad 2.3_{2,4}\quad 0.3_{2,3})$
$(3.2_{2,4}\quad 2.3_{2,4}\quad 0.3_{2,3})$
$(3.3_{2,3,4}\quad 2.3_{2,4}\quad 0.3_{2.3})$

From the above 4 polycontextural-semiotic systems, we can also very well see what I have called the "inheritence" of the pre-semiotic trichotomies in the semiotic trichotomies (Toth 2008a, pp. 166 ss.); cf., e.g.

$(3.1_{3,4} \quad 2.1_{1,4} \quad 0.1_{1,3}) \qquad\qquad (3.1_{3,4} \quad 2.2_{1,2,4} \quad 0.2_{1,2})$

While straight lines show the inheritence of the pre-semiotic trichotomies in the semiotic trichotomie, the dashed lines show the "inheritance" (or simply, the connection) of the contextures of zeroness to/with the higher fundamental categories.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Ditterich, Joseph, Selbstreferentielle Modellierungen. Klagenfurt 1990

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

Nöth, Winfried, Literatursemiotische Analysen zu Lewis Carrolls Alice-Büchern. Tübigen 1980

Stiebing, Hans Michael, Die Semiose von der Natur zur Kunst. In: Semiosis 23, 1981, S. 21-31

Stiebing, Hans Michael, „Objekte" zwischen Natur und Kunst. In: Oehler, Klaus, Zeichen und Realität. Akten des 3. semiotischen Kolloquiums Hamburg. Bd. 2. Tübingen 1984, S. 671-674

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2007

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)

Toth, Alfred, Semiotics and Pre-Semiotics. Klagenfurt 2008 (2008b)

Toth, Alfred, Qualitative semiotische Zahlbereiche und Transzendenzen. In: In: Electronic Journal for Mathematical Semiotics, 2008c

Toth, Alfred, New elements of theoretical semiotics (NETS), based on the work of Rudolf Kaehr. In: Electronic Journal for Mathematical Semiotics, 2009

Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

Connections of inner semiotic environments (= NETS, 3)

1. The distinction of system and environment is crucial for cybernetics. In semiotics, this distinction has been introduced by Bense (1975, pp.97 ss., 108 ss.). However, since there is no environment for category theoretic morphisms, in classical mathematics as well as in classical semiotics, semiotic environment, up to now, always means outer semiotic environment. Therefore, outer semiotic environment means, in accordance with the Peircean principle that no sign can appear alone, the connections between signs in the form of static sub-signs or dynamic semioses.

1.1. Example of sign connection by static sub-signs

(3.1 2.1 1.3)
 | |
(3.1 2.2 1.3)

1.2. Example of sign connection by dynamic semiosis

(3.1 2.1 1.2)
 | |
(3.1 2.1 1.3)

Note: In classical semiotics, pairs of dualized sub-signs are treated as identical, f. ex.:

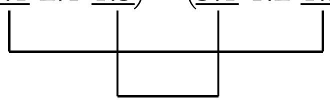
×(3.1) = (1.3)

On this strictly mono-contextural principle (cf. Kaehr 2009), the inner connections between sign classes and reality thematics are established, e.g.:

(3.1 2.1 1.3) × (3.1 1.2 1.3),

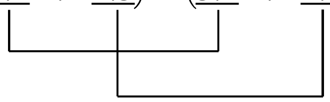
but consider

(3.1 2.1 1.3) × (3.1 1.2 1.3).



and not

(3.1 2.1 1.3) × (3.1 1.2 1.3).



because $\times(3.1) = (1.3)$ and $\times(1.3) = (3.1)$. Moreover, since, according to Kaehr (2009), we even have

$\times(\text{id}x) \neq (\text{id}x), x \in \{1, 2, 3\}$,

it follows especially that

$\times(3.1\ 2.2\ 1.3) \neq (3.1\ 2.2\ 1.3)$

in contradiction with the classical-semiotic theory of eigenreality.

The reason for the disequations is that "self-identity is able to distinguish its directionality as left (lo) and right (ro) order" (Kaehr 2009, p. 2).

From the standpoint of classical semiotics, this leads to the paradoxical situation, that, from a poly-contextural standpoint, we have on the one side

$K(a.b) = K(b.a)$,

i.e. the contexture of a sub-signs (a.b) is identical with the contexture of its dualized sub-sign. However, if not only the sub-signs, but the contexture as well is dualized

$\times(K(a.b)) \neq K(b.a.)$,

we get again a disequation.

2. In order to solve the problems caused by the above disequations, Kaehr (2009) redefined the semiotic fundamental categories:

Firstness:	Peirce:	A
	Kaehr:	$A \mid a$
Secondness:	Peirce:	$A \rightarrow B$
	Kaehr	$A \rightarrow B \mid c$
Thirdness:	Peirce:	$A \rightarrow C$
	Kaehr:	$A \rightarrow C \mid b_1 \leftarrow b_2$

In Kaehr's own words: "A composition is always accompanied by an environment of its morphisms. Therefore, an initial object or the number 1, firstness, is diamond theoretically always doubled: as itself and as its environment, i.e. (A | a). That is, as a morphism, and as a hetero-morphism. A diamond initial object is not a singular object but a doublet. Also called bi-object" (2009, p. 2).

Therefore,

PS = (.1., .2., .3.)

is the mono-contextural set of prime-signs without inner semiotic environments. Clearly, the prime-signs are not connected with one another.

However, by introducing the concept of inner semiotic environment (or hetero-morphism), we get in the case of 3-contextural PS

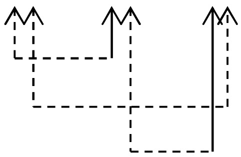
$PS_3 = ((.1.)_{1,3}, (.2.)_{1,2}, (.3.)_{2,3})$

and in the case of 4-contextural PS

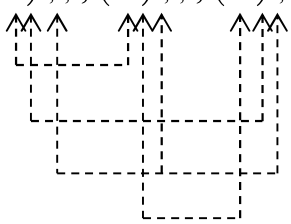
$PS_4 = ((.1.)_{1,3,4}, (.2.)_{1,2,4}, (.3.)_{2,3,4})$,

and therefore sets of prime-signs which are connected by their inner environments

$((.1.)_{1,3}, (.2.)_{1,2}, (.3.)_{2,3})$

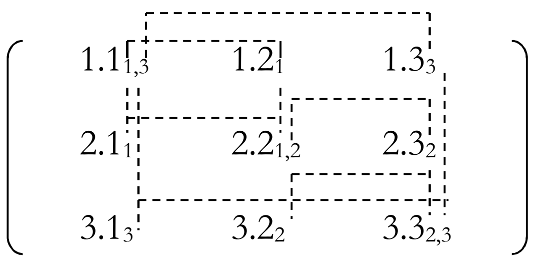


$((.1.)_{1,3,4}, (.2.)_{1,2,4}, (.3.)_{2,3,4})$,

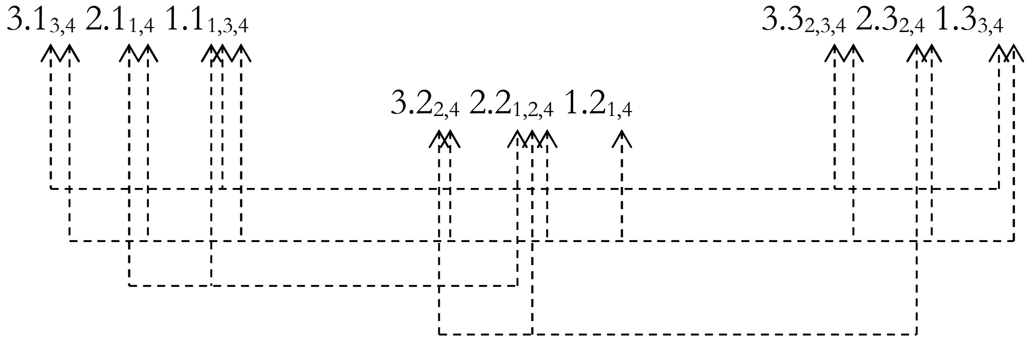


Naturally, the complexity of connections by inner semiotic environments increases with increasing number of contextures involved.

3. The sets of prime-signs are examples of connections solely by their inner semiotic environments. If we have a look at the 3-contextural triadic semiotic matrix



we recognize that in each triad and in each trichotomy the sub-signs are pairwise connected by their inner semiotic environment. It follows that there are no triadic sign relations, which are not connected by their inner semiotic environment. This is especially important for sign relation which are neither connected by static sub-signs nor by dynamic semioses, f. ex.:



The three sign classes in this example have no other than inner environmental semiotic connections.

This simple fact has tremendous consequences for the semiotic universe. Since there are pairs of sign classes which have no static nor dynamic connection, the conclusion was made in Toth (2009) that the semiotic universe is non-connected (in the topological sense). As a matter of fact, from a purely mono-contextural standpoint, the two following statements from Peirce and Karger, respectively, must appear contradictory (quotation from Toth 2009):

Walther paraphrasiert (ohne Quellenangabe des zugrunde liegendenZitats) Peirce wie folgt: “Die einzige geistige Wirkung eines Zeichens bzw. der ‘letzte logische Interpretant’, der kein Zeichen ist, aber allgemein beobachtet werden kann, ist ein ‘Wechsel der Denkgewohnheit’, wie Peirce bemerkte” (1979, S. 78). Ohne auf diese Stelle zu referieren, heisst es dann aber bei Karger: “Es ist aber so, dass eine ‘Denkgewohnheit’ ein Zeichen darstellt und der Wechsel zu einer neuen Denkgewohnheit ebenfalls. Es werden also Veränderungen am Zeichen erfahren, die wiederum zum Zeichen führen” (1986, S. 42).

However, from a poly-contextural standpoint, we can “save” the coexistence of the contradictory utterances, because even then, when an n-tuple of sign classes is topologically non-connected what concerns their sub-signs and/or semioses, it is necessarily connected by the internal semiotic invironments of their sub-signs and/or semioses. To put it in the form of

Theorem: Any n-tuple of sign-classes is connected by the heteromorphisms of their sub-signs involved, but not any n-tuple is necessarily connected by the morphisms of their sub-signs involved.

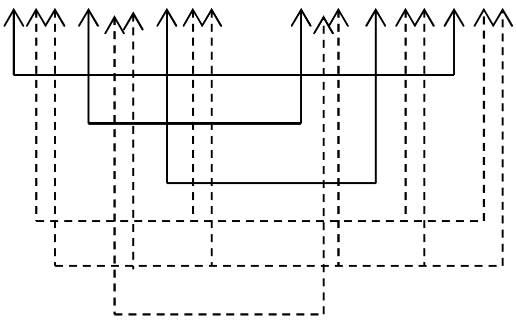
This extremely important semiotic theorem could not have found without the groundbreaking work of Rudolf Kaehr.

It therefore seems, that the change of one's habitude of mind (Wechsel der Denkgewohnheit) means indeed a loss of outer semiotic connections, but at the same time the hitherto hardly used inner semiotic connections are opening unforeseen semiotic possibilities.

4. Concluding, I want to give some examples in order to show in which semiotic areas the introduction of semiotic connections by inner environments may be helpful. In Toth (2008) I had introduced a typology of semiotic connections between sign classes and their permutations, reality thematics and their permutations, sign classes and permutations of their reality thematics, permutations of sign classes and permutations of their reality thematics.

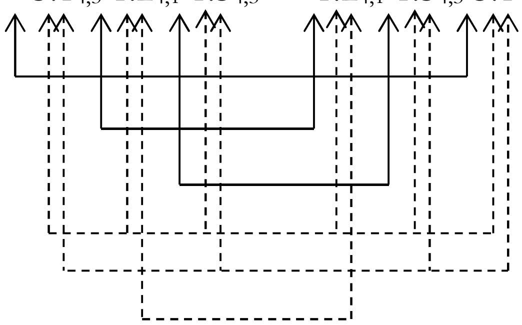
4.1. Connections of sign classes and permutations of sign classes

$3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4} \quad 2.1_{1,4}\ 1.3_{3,4}\ 3.1_{3,4}$ (SCl – Perm SCl)



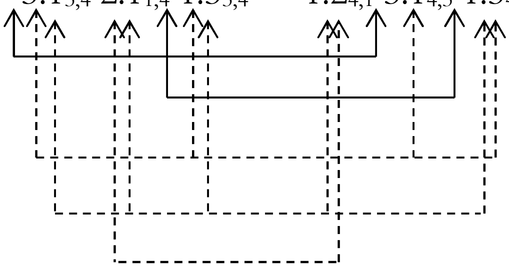
4.2. Connections of reality thematics and permutations of reality thematics

$3.1_{4,3}\ 1.2_{4,1}\ 1.3_{4,3} \quad 1.2_{4,1}\ 1.3_{4,3}\ 3.1_{4,3}$ (Rth – Perm Rth)

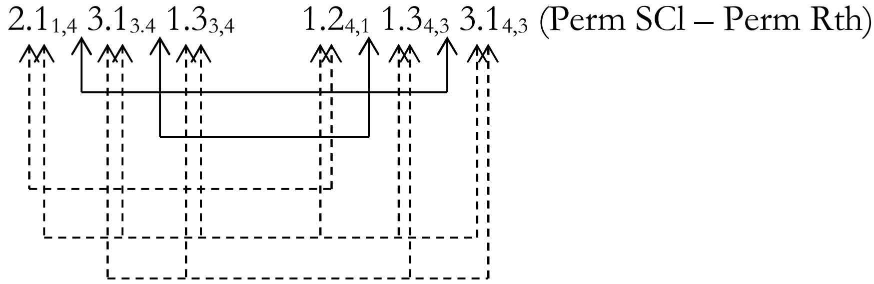


4.3. Connections of sign classes and permutations of reality thematics

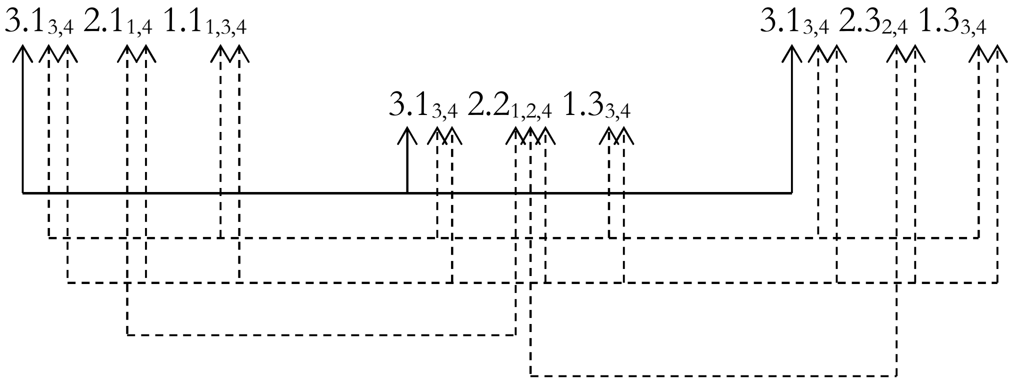
$3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4} \quad 1.2_{4,1}\ 3.1_{4,3}\ 1.3_{4,3}$ (SCl – Perm Rth)



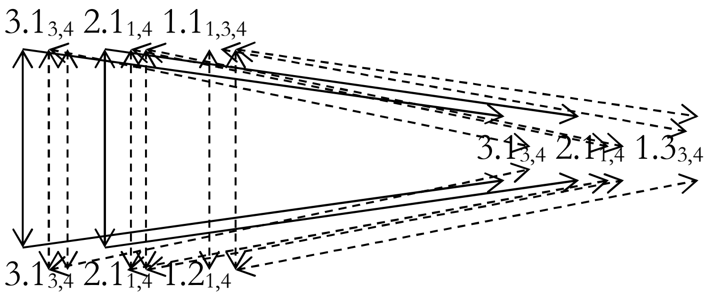
4.4. Connections of permutations of sign classes and permutations of reality thematics



4.5. Communication schemata (cf. Bense 1971, pp. 39 ss.; Toth 1993, pp. 147 ss.)



4.6. Creation schemata (cf. Bense 1976, pp. 106 ss.; Toth 1993, pp. 158 ss.)



Bibliography

Bense, Max, Zeichen und Design. Baden-Baden 1971
Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975
Bense, Max, Vermittlung der Realitäten. Baden-Baden 1975
Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)
Karger, Angelika, Zeichen und Evolution. Köln 1986
Toth, Alfred, Semiotik und Theoretische Linguistik. Tübingen 1993
Toth, Alfred, Semiotic Ghost Trains. Klagenfurt 2008

Toth, Alfred, Die semiotischen Orte des Wechsels der Denkgewohnheit. In: Electronic Journal of Mathematical Semiotics, 2009
Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

Semiotic environment systems (= NETS, 4)

1. In Bense (1975, pp. 94 ss.), we find a complex theory of semiotic environments in connection with the differentiation of virtual vs. effective triadic sign relations on the one side and the theory of pragmatic retrosemioses on the other side. Unfortunately, this theory has never even been noticed by anybody. In the present article, I will present its fundamental ideas and try to establish the connection to Kaehr's theory of "environments for transclusions in textemes" (2009b), therefore enabling to introduce both outer and inner semiotic environment systems and their interrelationships into semiotics.

2. Since contextuated sub-signs have only been introduced into semiotics by Kaehr (2009a), in semiotics, environment means always outer environment of signs. However, besides the rather trivial notion of an environment of a sign class formed by another sign class, thus meaning nothing more than sign connections, Bense (1975, pp. 97 ss.) introduced pragmatic retrosemioses of the form

$(I \Rightarrow M)$,

i.e. the so-called "application function" of the sign in the sense that, for every object O, an external interpretant I creates an M which represents this object, thereby the relation between I and M creating an outer semiotic environment of this object which is represented. Note that R(I, M) is an ordered relation to which the converse relation R(M, I) is not defined.

3. For inner semiotic environments, i.e. hetero-morphisms, we follow Kaehr (2009a, b) in assuming a triadic sign relation being a fragment of a 4-contextural sign relation. Thus,

SR(3;4) = (3.a 2.b 1.c)

operates on the following 4-contextural 3×3 polycontextural-semiotic matrix

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ \downarrow & \uparrow & \uparrow \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ \downarrow & \downarrow & \uparrow \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

Since in heteromorphisms, the arrows are inverted, but not the prime-signs constituting the sub-signs, we get the following 9 environments for the 9 sub-signs or monadic semiotic relations (left column). In opposite, in dualization, not only the arrows, but also the order of the prime-signs of the sub-signs are inverted (right column):

$E((1.1)_{1,3,4}) = (1.1)_{4,3,1}$	$D((1.1)_{1,3,4}) = (1.1)_{4,3,1}$
$E((1.2)_{1,4}) = (1.2)_{4,1}$	$D((1.2)_{1,4}) = (2.1)_{4,1}$
$E((1.3)_{3,4}) = (1.3)_{4,3}$	$D((1.3)_{3,4}) = (3.1)_{4,3}$
$E((2.1)_{1,4}) = (2.1)_{4,1}$	$D((2.1)_{1,4}) = (1.2)_{4,1}$
$E((2.2)_{1,2,4}) = (2.2)_{4,2,1}$	$D((2.2)_{1,2,4}) = (2.2)_{4,2,1}$
$E((2.3)_{2,4}) = (2.3)_{4,2}$	$D((2.3)_{2,4}) = (3.2)_{4,2}$
$E((3.1)_{3,4}) = (3.1)_{4,3}$	$D((3.1)_{3,4}) = (1.3)_{4,3}$
$E((3.2)_{2,4}) = (3.2)_{4,2}$	$D((3.2)_{2,4}) = (2.3)_{4,2}$
$E((3.3)_{2,3,4}) = (3.3)_{4,3,2}$	$D((3.3)_{2,3,4}) = (3.3)_{4,3,2}$

4. For outer semiotic environments, we follow Bense (1975, pp. 97 ss.). Therefore, every sub-sign (a.b) can be embedded into an application relation depending on the value of its trichotomy (.b). Because we stick with the semiotic inclusion order that every sign class (3.a 2.b 1.c) must obey the order ($a \leq b \leq c$), it follows, that, if (.b) = 1, we have 3 application relations, if (.b) = 2, we have 2 application relations, and, if (.b) = 3, we have 1 application relation. In the following, we show that, for every application relation, we can establish a system of 4 outer semiotic environments on the basis of Bense's pragmatic retrosemioses:

$U((1.1)_{1,3,4}) = (((3.1)_{3,4}) \Rightarrow (1.1)_{4,3,1}))$
$U((1.1)_{1,3,4}) = (((3.1)_{4,3}) \Rightarrow (1.1)_{4,3,1}))$

$U((1.1)_{4,3,1}) = (((3.1)_{3,4}) \Rightarrow (1.1)_{1,3,4}))$
$U((1.1)_{4,3,1}) = (((3.1)_{4,3}) \Rightarrow (1.1)_{1,3,4}))$

5. For the dual reality thematics of each sign class, we therefore get the following system of 4 outer semiotic environments:

$UD((1.1)_{1,3,4}) = (((1.3)_{4,3}) \Rightarrow (1.1)_{4,3,1}))$
$UD((1.1)_{1,3,4}) = (((1.3)_{3,4}) \Rightarrow (1.1)_{4,3,1}))$

$UD((1.1)_{4,3,1}) = (((1.3)_{4,3}) \Rightarrow (1.1)_{1,3,4}))$
$UD((1.1)_{4,3,1}) = (((1.3)_{3,4}) \Rightarrow (1.1)_{1,3,4}))$

6. We can finally ask if it makes sense to introduce, besides UD, the notion of the outer semiotic environment of an inner semiotic environment, UE. In doing so, we get

$UE((1.1)_{1,3,4}) = (((3.1)_{3,4}) \Rightarrow (1.1)_{4,3,1}))$
$UE((1.1)_{1,3,4}) = (((3.1)_{4,3}) \Rightarrow (1.1)_{4,3,1}))$

$UE((1.1)_{4,3,1}) = (((3.1)_{3,4}) \Rightarrow (1.1)_{1,3,4}))$
$UE((1.1)_{4,3,1}) = (((3.1)_{4,3}) \Rightarrow (1.1)_{1,3,4}))$.

As we reciognize easily, it is

$UE((a.b)_{i,jk/\varnothing}) = U((a.b)_{i,jk/\varnothing})$ (i, j, k $\in$ {1, 2, 3, 4})

This is quite an astonishing result, which we will formulate in the following semiotic theorem:

Theorem: The inner semiotic environment is already produced by the outer semiotic environment.

7. So far, we have seen that the contextural "index" of a sub-sign (a.b) in 4 contextures

$(a.b.c)_{i,jk/\varnothing}$ (i, j, k $\in$ {1, 2, 3, 4})

is either

i, j, k ("morphismic form")

or
k, j, i ("heteromorphismic form")

The heteromorphismic form appears, when a sub-sign is operated by operators E and D.

Obviously, for binary "indices" (i, k), (k, i), E and D as semiotic operators are sufficient. However, what is the semiotic meaning of the 6 possible permutations of the ternary "indices" (i, k, k):

1. (i, j, k)
2. (i, k, j)
3. (j, i, k)
4. (j, k, i)
5. (k, i, j)
6. (k, j, i)

Besides (i, j, k) and (k, j, i) we have

2. (i, k, j) which corresponds to the semiotic order of the prime-signs (M, I, O). This order corresponds to the semiotic creation schema introduced by Peirce (cf. Peirce 1976) an formalized by Bense (1976, pp. 110 ss.).

3. (j, i, k) which corresponds to the semiotic order of the prime-signs (O, M, I). This order corresponds to the semiotic communication schema introduced by Bense (1971, pp. 38 ss.) which O corresponding to the sender, M to the channel and I to the receiver of an elementary communication schema.

4. (j, k, i) which corresponds to the semiotic order of the prime-signs (O, I, M). This is the reality thematics of the semiotic creation schema (i, k, j).

5. (k, i, j) which corresponds to the semiotic order of the prime-signs (I, M, O). This is the reality thematics of the semiotic communication schema (j, i, k).

Therefore, all 6 order of the polycontextural-semiotic "indices" have a clear pragmatic definition. Thus, we can state that while

SR(M, O, I) = <[1,3,4], [1,2,4], [2,3,4])>

is the generativ-semiosic order of the sign relation (M, O, I) and

SR(M, O, I)° = <[4,3,2], [4,2,1], [4,3,1]>
ist the respective order of the dual reality thematics (I, O, M),

semiotic communication schemata can be assigned to the following two ordered sets of polycontextural-semiotic "indices"

SR(O, M, I) = <[1,2,4], [1,3,4], [2,3,4]>

SR(O, M, I)° = <[4,3,2], [4,3,1], [4,2,1]>,

and semiotic creation schemata can be assigned to

SR(M, I, O) = <[1,3,4], [2,3,4], [1,2,4]>

SR(M, I, O)° = <[4,2,1], [4,3,2], [4,3,1]>

Therefore, taking the notion of semiotic environment in its biggest possible sense, we can state that communication and creation are just special forms of environment structures of the sign model rather than practical application of cybernetic systems onto semiotics.

Bibliography

Bense, Max, Zeichen und Design. Baden-Baden 1971
Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009a)

Kaehr, Rudolf, Xanadu's textemes. http://www.thinkartlab.com/CCR/2009/02/xanadus-textems.html (2009b)

Peirce, Charles Sanders, Analysis of creation. In: Semiosis 2, 1976, pp. 5-9

Permutations of sign classes and of inner semiotic environments (= NETS, 5)

1. In Toth (2008a, pp. 177 ss.), I introduced permutations of sign classes into semiotics. In classical, semiotics, a sign class always appears in the following order of its triads:

SCl = (3.a 2.b 1.c), i.e. I, O, M,

while its dual reality thematics appears in the converse order

Rth = SCl° = (c.1 b.2 a.3), i.e. M, O, I.

However, in Bense (1971, pp. 38 ss.) semiotic communication schemata obeying the order

CoSch = (2.b 1.c 3.a), i.e. OMI

and in Bense (1976, pp. 110 ss.) semiotic creation schemata obeying the order

CrSch = (3.a 1.c 2.b), i.e. IMO

have been introduced. Thus, together with the converse relation of CoSch and CrSch,

CoSch° = (a.3 c.1 b.2), i.e. IMO
CrSch° = (b.2 c.1 a.3), i.e. OMI,

we have all 6 order types of triadic semiotic relations:

(3.a 2.b 1.c)	×	(c.1 b.2 a.3)	IOM × MOI (1)
(3.a 1.c 2.b)	×	(b.2 c.1 a.3)	IMO × OMI (2)
(2.b 1.c 3.a)	×	(a.3 c.1 b.2)	OMI × IMO (2°)
(2.b 3.a 1.c)	×	(c.1 a.3 b.2)	OIM × MIO (3)
(1.c 3.a 2.b)	×	(b.2 a.3 c.1)	MIO × OIM (3°)

and thus all possible permutations of a sign class and its reality thematic.

2. According to Kaehr (2009, p. 8), the main diagonal of the 3-contextural semiotic matrix is

$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3})$

and the main diagonal of the 3-contextural semiotic matrix as a fragment of a 4-contextural matrix is

$(3.3_{2,3,4}\ 2.2_{1,2,4}\ 1.1_{1,3,4})$.

Therefore, we have to redefine a sign class with inner semiotic environments as

$SCl+ = (3.a_{a,b,c}\ 2.b_{d,e,f}\ 1.c_{g,h,i})$, with a, ..., i ∈ {∅, 1, 2, 3, 4}

However, in addition to the 6 permutations of a sign class, we get now 6 permutations of each sub-sign of each sign class:

$(x.y)_{a,b,c}$
$(x.y)_{a,c,b}$
$(x.y)_{b,a,c}$
$(x.y)_{b,c,a}$
$(x.y)_{c,a,b}$
$(x.y)_{c,ba,}$

with x, y ∈ {1, 2, 3} and a, b, c ∈ {∅, 1, 2, 3, 4}.

Since each of the 3 sub-signs of a sign class can appear in 6 permutations, we get, purely theoretically, $6^3 = 216$ permutations of inner semiotic environments per sign class. However, only the genuine sub-signs (identitive morphisms) (1.1), (2.2), (3.3) have 3 indices unequal to, and they appear only in the following 6 sign classes:

(3.1 2.1 1.1)	(3.1 2.2 1.2)	(3.3 2.2 1.1)
	(3.1 2.2 1.3)	
	(3.2 2.2 1.2)	
	(3.2 2.2 1.3),	

so that we have a total of 6 times 216 = 1'296 sign classes. Further, the remaining 4 sign classes have 2 indices, so they can appear in only 6 times $2^3 = 48$ combination, which yields 4 times 48 = 192 sign classes. Thus, the total is 1'296 + 192 = 1'488 combinations of permutations of sign classes plus inner semiotic environments. If we further add the dual reality thematics, we have at the end 2'976 combinations of semiotic dual systems, which of course go far beyond the representative power of the system of the classical 10 Peircean sign classes.

3. Let us take, for the sake of simplicity, as an example the 4-contextural Peircean sign class

$(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4})$.

The 6 permutations with constant semiotic environments are

$(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4})$	$(2.1_{1,4}\ 1.3_{3,4}\ 3.1_{3,4})$
$(3.1_{3,4}\ 1.3_{3,4}\ 2.1_{1,4})$	$(1.3_{3,4}\ 3.1_{3,4}\ 2.1_{1,4})$
$(2.1_{1,4}\ 3.1_{3,4}\ 1.3_{3,4})$	$(1.3_{3,4}\ 2.1_{1,4}\ 3.1_{3,4})$

Now, each of these 6 permutations can be permutated again to $2^3 = 8$ combinations of inner semiotic environments:

$(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4})$	$(3.1_{4,3}\ 2.1_{1,4}\ 1.3_{3,4})$
$(3.1_{3,4}\ 2.1_{4,1}\ 1.3_{3,4})$	$(3.1_{4,3}\ 2.1_{4,1}\ 1.3_{3,4})$
$(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{4,3})$	$(3.1_{4,3}\ 2.1_{1,4}\ 1.3_{4,3})$
$(3.1_{3,4}\ 2.1_{4,1}\ 1.3_{4,3})$	$(3.1_{4,3}\ 2.1_{4,1}\ 1.3_{4,3})$

$(3.1_{3,4}\ 1.3_{3,4}\ 2.1_{1,4})$	$(3.1_{4,3}\ 1.3_{3,4}\ 2.1_{1,4})$
$(3.1_{3,4}\ 1.3_{3,4}\ 2.1_{4,1})$	$(3.1_{4,3}\ 1.3_{3,4}\ 2.1_{4,1})$
$(3.1_{3,4}\ 1.3_{4,3}\ 2.1_{1,4})$	$(3.1_{4,3}\ 1.3_{4,3}\ 2.1_{1,4})$
$(3.1_{3,4}\ 1.3_{4,3}\ 2.1_{4,1})$	$(3.1_{4,3}\ 1.3_{4,3}\ 2.1_{4,1})$

$(2.1_{1,4}\ 3.1_{3,4}\ 1.3_{3,4})$	$(2.1_{1,4}\ 3.1_{4,3}\ 1.3_{3,4})$
$(2.1_{4,1}\ 3.1_{3,4}\ 1.3_{3,4})$	$(2.1_{4,1}\ 3.1_{4,3}\ 1.3_{3,4})$
$(2.1_{1,4}\ 3.1_{3,4}\ 1.3_{4,3})$	$(2.1_{1,4}\ 3.1_{4,3}\ 1.3_{4,3})$
$(2.1_{4,1}\ 3.1_{3,4}\ 1.3_{4,3})$	$(2.1_{4,1}\ 3.1_{4,3}\ 1.3_{4,3})$

$(2.1_{1,4}\ 1.3_{3,4}\ 3.1_{3,4})$	$(2.1_{1,4}\ 1.3_{3,4}\ 3.1_{4,3})$
$(2.1_{4,1}\ 1.3_{3,4}\ 3.1_{3,4})$	$(2.1_{4,1}\ 1.3_{3,4}\ 3.1_{4,3})$
$(2.1_{1,4}\ 1.3_{4,3}\ 3.1_{3,4})$	$(2.1_{1,4}\ 1.3_{4,3}\ 3.1_{4,3})$
$(2.1_{4,1}\ 1.3_{4,3}\ 3.1_{3,4})$	$(2.1_{4,1}\ 1.3_{4,3}\ 3.1_{4,3})$

$(1.3_{3,4}\ 3.1_{3,4}\ 2.1_{1,4})$	$(1.3_{3,4}\ 3.1_{4,3}\ 2.1_{1,4})$
$(1.3_{3,4}\ 3.1_{3,4}\ 2.1_{4,1})$	$(1.3_{3,4}\ 3.1_{4,3}\ 2.1_{4,1})$
$(1.3_{4,3}\ 3.1_{3,4}\ 2.1_{1,4})$	$(1.3_{4,3}\ 3.1_{4,3}\ 2.1_{1,4})$
$(1.3_{4,3}\ 3.1_{3,4}\ 2.1_{4,1})$	$(1.3_{4,3}\ 3.1_{4,3}\ 2.1_{4,1})$

$(1.3_{3,4}\ 2.1_{1,4}\ 3.1_{3,4})$	$(1.3_{3,4}\ 2.1_{1,4}\ 3.1_{4,3})$
$(1.3_{3,4}\ 2.1_{4,1}\ 3.1_{3,4})$	$(1.3_{3,4}\ 2.1_{4,1}\ 3.1_{4,3})$
$(1.3_{4,3}\ 2.1_{1,4}\ 3.1_{3,4})$	$(1.3_{4,3}\ 2.1_{1,4}\ 3.1_{4,3})$
$(1.3_{4,3}\ 2.1_{4,1}\ 3.1_{3,4})$	$(1.3_{4,3}\ 2.1_{4,1}\ 3.1_{4,3})$

However, these 48 permutations of the original sign class $(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4})$ must be assigned a semiotic interpretation, since, unlike, e.g., in the case of the negation cycles in polycontextural logic, in semiotics, we deal with meaning and sense and not exclusively with the sign as a medium. In order interpret the combinations of inner semiotic environments, we can recur to Günther's logical-semiotic triadic sign model (1976, pp. 336 ss.), in which we have the following correspondences:

M ≡ (.1.) → objective subject (oS)
O ≡ (.2.) → objective object (oO)
I ≡ (.3.) → subjective subject (sS)

Additionally, in Toth (2008b, *passim*), the still lacking combination of subjective object was ascribed to the "quality" of Zeroness (for motivation cf. Kronthaler 1992):

Q ≡ (.0.) → subjective object (sO)

In Kaehr's contextuated semiotic matrix (2009, p. 8), Fourthness (.4.) stands for what we have introduced as Zeroness (.0.). Therefore, if we use the above abbreviations for the logical-semiotic functions, we can rewrite our 48 combinations of the sign class $(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4})$ as follows:

$(3.1_{sS, sO}\ 2.1_{oS,sO}\ 1.3_{sS,sO})$	$(3.1_{sO, sS}\ 2.1_{oS,sO}\ 1.3_{sS,sO})$
$(3.1_{sS,sO}\ 2.1_{sO, oS}\ 1.3_{sS,sO})$	$(3.1_{sO, sS}\ 2.1_{sO, oS}\ 1.3_{sS,sO})$
$(3.1_{sS,sO}\ 2.1_{oS,sO}\ 1.3_{sO\ sS})$	$(3.1_{sO, sS}\ 2.1_{oS,sO}\ 1.3_{sO\ sS})$
$(3.1_{sS,sO}\ 2.1_{sO, oS}\ 1.3_{sO, sS})$	$(3.1_{sO, sS}\ 2.1_{sO, oS}\ 1.3_{sO, sS})$
$(3.1_{sS,sO}\ 1.3_{sS,sO}\ 2.1_{oS,sO})$	$(3.1_{sO, sS}\ 1.3_{sS,sO}\ 2.1_{oS,sO})$
$(3.1_{sS,sO}\ 1.3_{sS,sO}\ 2.1_{sO, oS})$	$(3.1_{sO, sS}\ 1.3_{sS,sO}\ 2.1_{sO, oS})$
$(3.1_{sS,sO}\ 1.3_{sO, sS}\ 2.1_{oS,sO})$	$(3.1_{sO, sS}\ 1.3_{sO, sS}\ 2.1_{oS,sO})$
$(3.1_{sS,sO}\ 1.3_{sO, sS}\ 2.1_{sO, oS})$	$(3.1_{sO, sS}\ 1.3_{sO, sS}\ 2.1_{sO, oS})$
$(2.1_{oS,sO}\ 3.1_{sS,sO}\ 1.3_{sS,sO})$	$(2.1_{oS,sO}\ 3.1_{sO, sS}\ 1.3_{sS,sO})$
$(2.1_{sO, oS}\ 3.1_{sS,sO}\ 1.3_{sS,sO})$	$(2.1_{sO, oS}\ 3.1_{sO, sS}\ 1.3_{sS,sO})$
$(2.1_{oS,sO}\ 3.1_{sS,sO}\ 1.3_{sO\ sS})$	$(2.1_{oS,sO}\ 3.1_{sO, sS}\ 1.3_{sO, sS})$
$(2.1_{sO, oS}\ 3.1_{sS,sO}\ 1.3_{sO, sS})$	$(2.1_{sO, oS}\ 3.1_{sO, sS}\ 1.3_{sO, sS})$
$(2.1_{oS,sO}\ 1.3_{sS,sO}\ 3.1_{sS,sO})$	$(2.1_{oS,sO}\ 1.3_{sS,sO}\ 3.1_{sO, sS})$
$(2.1_{sO, oS}\ 1.3_{sS,sO}\ 3.1_{sS,sO})$	$(2.1_{sO, oS}\ 1.3_{sS,sO}\ 3.1_{sO, sS})$
$(2.1_{oS,sO}\ 1.3_{sO, sS}\ 3.1_{sS,sO})$	$(2.1_{oS,sO}\ 1.3_{sO, sS}\ 3.1_{sO, sS})$
$(2.1_{sO, oS}\ 1.3_{sO, sS}\ 3.1_{sS,sO})$	$(2.1_{sO, oS}\ 1.3_{sO, sS}\ 3.1_{sO, sS})$
$(1.3_{sS,sO}\ 3.1_{sS,sO}\ 2.1_{oS,sO})$	$(1.3_{sS,sO}\ 3.1_{sO, sS}\ 2.1_{oS,sO})$
$(1.3_{sS,sO}\ 3.1_{sS,sO}\ 2.1_{sO, oS})$	$(1.3_{sS,sO}\ 3.1_{sO, sS}\ 2.1_{sO, oS})$
$(1.3_{sO, sS}\ 3.1_{sS,sO}\ 2.1_{oS,sO})$	$(1.3_{sO, sS}\ 3.1_{sO, sS}\ 2.1_{oS,sO})$
$(1.3_{sO, sS}\ 3.1_{sS,sO}\ 2.1_{sO, oS})$	$(1.3_{sO, sS}\ 3.1_{sO, sS}\ 2.1_{sO, oS})$

$(1.3_{sS,sO}\ 2.1_{oS,sO}\ 3.1_{sS,sO})$ $(1.3_{sS,sO}\ 2.1_{oS,sO}\ 3.1_{sO,\ sS})$
$(1.3_{sS,sO}\ 2.1_{sO,\ oS}\ 3.1_{sS,sO})$ $(1.3_{sS,sO}\ 2.1_{sO,\ oS}\ 3.1_{sO,\ sS})$
$(1.3_{sO,\ sS}\ 2.1_{oS,sO}\ 3.1_{sS,sO})$ $(1.3_{sO,\ sS}\ 2.1_{oS,sO}\ 3.1_{sO,\ sS})$
$(1.3_{sO,\ sS}\ 2.1_{sO,\ oS}\ 3.1_{sS,sO})$ $(1.3_{sO,\ sS}\ 2.1_{sO,\ oS}\ 3.1_{sO,\ sS})$

Bibliography

Bense, Max, Zeichen und Design. Baden-Baden 1971
Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976
Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. Vol. 1. Hamburg 1976
Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html
Kronthaler, Engelbert, Zahl – Zeichen – Begriff. In: Semiosis 65-68,. 1992, S. 282-302
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)
Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 2008 (2008b)

A poly-contextural view on eigenreality (= NETS, 6)

1. According to Bense (1992), amongst the 10 Peircean sign classes and reality thematics, there is just one sign class, whose dual reality thematic is identical with the sign class:

$\times(3.1\ 2.2\ 1.3) = (3.1\ 2.2\ 1.3)$

For the other 9 sign classes, we have

$x(3.a\ 2.b\ 1.c) \neq (3.a\ 2.b\ 1.c)$ with $a, b, c \in \{1, 2, 3\}$,

f. ex.

$\times(3.1\ 2.1\ 1.3) = (3.1\ 1.2\ 1.3)$.

However, as has been pointed out earlier

$(3.1) \neq \times(1.3)$, $(1.3) \neq \times(3.1)$

and even

$(2.2) \neq \times(2.2)$ (Kaehr 2009, p.12),

which means that there is a semiotic difference between the rhema and the dualized legi-sign and between the legi-sign and the dualized rhema, as well as between the dualization of genuine sub-signs (identitive morphisms). This is, by the way, already a result from Bense's use of the Möbius band as a model for the eigenreal sign class: one turn, and one is at the same place, but on the opposite side of the ribbon. However, the consequences of this fact have not been taken care of in semiotics up to know.

2. Using Kaehr's polycontextural-semiotic 3-matrix, things get quickly clearer. So, the eigenreal sign class appears in the form

$\times(3.1_3\ 2.2_{1,2}\ 1.3_3) = (3.1_3\ 2.2_{2,1}\ 1.3_3)$,
i.e.

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \neq (3.1_3\ 2.2_{2,1}\ 1.3_3)$,

although in 3 contextures, the differences between $\times(3.1)$ and (1.3), and $\times(1.3)$ and (3.1), respectively, do not come out yet. However, if we take 4 contextures (and thus triadic semiotics as a fragment of a 4-contextural semiotics, cf. Toth 2003, pp. 54 ss.), we get

$\times(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) = (3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})$.

The result: In all semiotic contextures > 1, there is no eigenreality. As a matter of fact, there is not even eigenreality in the contexture 1, because of the semiotic difference between between ×(3.1) and (1.3), and ×(1.3) and (3.1), and possibly (2.2) and (2.2) – although identity still holds in a 1-contextural semiotics.

2. However, as it was pointed out already in Toth (2008), it is possible to produce eigenreality artificially. However, in the case of 4-contextural

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$,

we have two two-digit indices and one three-digit index. It follows, that we need two operators to produce eigenreality on the level of inner semiotic environments. Operators that work on two-digit indices are binary, like negation in logic, so they cause no problem. For our purpose, we can use Bense's operation of dualization:

×[3,4] = [4,3].

However, × is only capable of converting the order of a whole sequence of indices:

×[1,2,4] = [4,2,1],

but × cannot produce [1,4,2], [2,1,4], [2,4,1], and [4,1,2]. Therefore, we rename the dualization operation "$\times_1$" and define $\times_2$ as trialization, moving the last digit of a sequence of indices to the beginning of the sequence, f. ex.

$\times_2[1,2,4] = [4,1,2]$.

Then, we obtain, e.g.

$\times_1(124) = (421)$, $\times_1(421) = (124)$, i.e. $\times_1\times_1(124) = (124)$

$\times_2(124) = (241)$, $\times_2(241) = (412)$, $\times_2(412) = (124)$, i.e. $\times_2\times_2\times_2(124) = (124)$
$\times_1\times_2(124) = (142)$
$\times_2\times_1(124) = (214)$
$\times_1\times_2\times_2(124) = \times_2\times_1(124)$
$\times_2\times_2\times_1 = \times_1\times_2(124)$
$\times_1\times_2\times_1(124) = \times_1\times_2(124)$

$\times_2\times_1\times_2(124) = (421)$, and so on.

Therefore, we can now produce all 6 permutations of a three-digit sequence like [1,2,4] by aid of the dualization $\times_1$ and the trialization $\times_2$. Since we are up to artificially produce eigenreality, the question is: How can we produce [1,2,4]?

Because of $\times_1 (2.2_{1,2,4}) = (2.2_{4,2,1})$, we need odd cycles of trialization. However, for (3.1) and (1.3), dualization (with even cycles) is sufficient. What we thus have to introduce are field restrictions (cf., e.g., Menge 1991, pp. 141, 151) for the two classes of operators:

$\times_1\times_1 [3.1_{3,4}, 1.3_{3,4}], \times_1\times_2\times_2[2.2_{1,2,4}] (3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$
$\times_1\times_1 [3.1_{3,4}, 1.3_{3,4}], \times_2\times_2\times_2[2.2_{1,2,4}] (3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$

Therefore, these are the two easiest ways to produce eigenreality by aid of 1 binary and two ternary operators. Concluding, note that by aid of the method presented in Toth (2008), it is possible to turn every sign class into an eigenreal sign class – as long as inner semiotic environments are not been taken into account. However, by aid of the two operators $\times_1$ and $\times_2$, it is possible to turn all those sign classes into eigenreal sign classes which contain a genuine (identitive) sub-sign, thus six of the ten Peircean sign classes. For the other four sign classes, things are even easier, since there we have to deal solely with two-digit indices for which we do not need trialization. Thus, the first conclusion of this study (together with Toth 2008) is that every sign class, mono- or poly-contextural, can be turned into an eigenreal sign class. However, this result goes hand in hand with the second conclusion that eigenreality is an artificial and superfluous semiotic feature which has no relevance at all.

Bibliography

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html

Menge, Albert, Einführung in die formale Logik. 2nd ed. Darmstadt 1991

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Künstlich erzeugte Eigenrealität. In: Electronic Journal for Mathematical Semiotics, 2008

Semiotic contextural values (= NETS, 7)

1. Semiotics is a system, which is practically exclusively based on ordinal numbers. For example, the triadic relation is based on the concept of prime-signs in which the generative semiosic relation parallels the successor relation of Peano numbers (cf. Bense 1975, pp. 168 ss.; 1983, pp. 192 ss.). However, in 1980, Angelika Karger introduced a measure into semiotics based on cardinal numbers, the representation values. The representation value of any semiotic relation is calculated simply by adding the values of the prime-signs of which the relation is constructed, f. ex.

RV(2.1) = RV(1.2) = 3
RV(3.1 2.2 1.3) = RV(3.2 2.2 1.2) = 12
RV(3.3 2.3 1.3) = 15

Of course, the dual reality thematics of the sign classes as well as their permutations have the same representation value.

2. In this paper, I want to introduce a second semiotic measure based on cardinal numbers, the contextural values. According to Kaehr (2009), each sub-sign of the semiotic 3×3-matrix can be assigned a contextural index. The mapping of contextural indices to sub-signs is bijective; dual sub-signs get the same contextural index. However, the indices vary according to the contextures. E.g., the semiotic 3×3-matrix can be given for 3 or 4 contextures:

3-contextural 3×3-matrix:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

4-contextural 3×3-matrix

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

We now define the contextural value (CV) of a semiotic relation as the sum of the contextural indices of this relation, f. ex.

CV(1.1) = 1+3+4 = 8

CV(1.2) = CV(2.1) = 1+4 = 5

CV(3.1 2.2 1.3) = 21

3. We can now compare the representation and the contextural values for all 10 Peircean sign classes. We will assume as basis the 4-contextural 3×3-matrix:

$(3.1_{3,4}\quad 2.1_{1,4}\quad 1.2_{1,4})$ Kw = 17 Rpw = 10

$(3.1_{3,4}\quad 2.1_{1,4}\quad 1.3_{3,4})$ Kw = 19 Rpw = 11

$(3.1_{3,4}\quad 2.2_{1,2,4}\quad 1.2_{1,4})$ Kw = 19 Rpw = 11

$(3.1_{3,4}\quad 2.2_{1,2,4}\quad 1.3_{3,4})$ Kw = 19 Rpw = 12

$(3.2_{2,4}\quad 2.3_{2,4}\quad 1.3_{3,4})$ Kw = 19 Rpw = 14

$(3.2_{2,4}\quad 2.2_{1,2,4}\quad 1.2_{1,4})$ Kw = 18 Rpw = 12

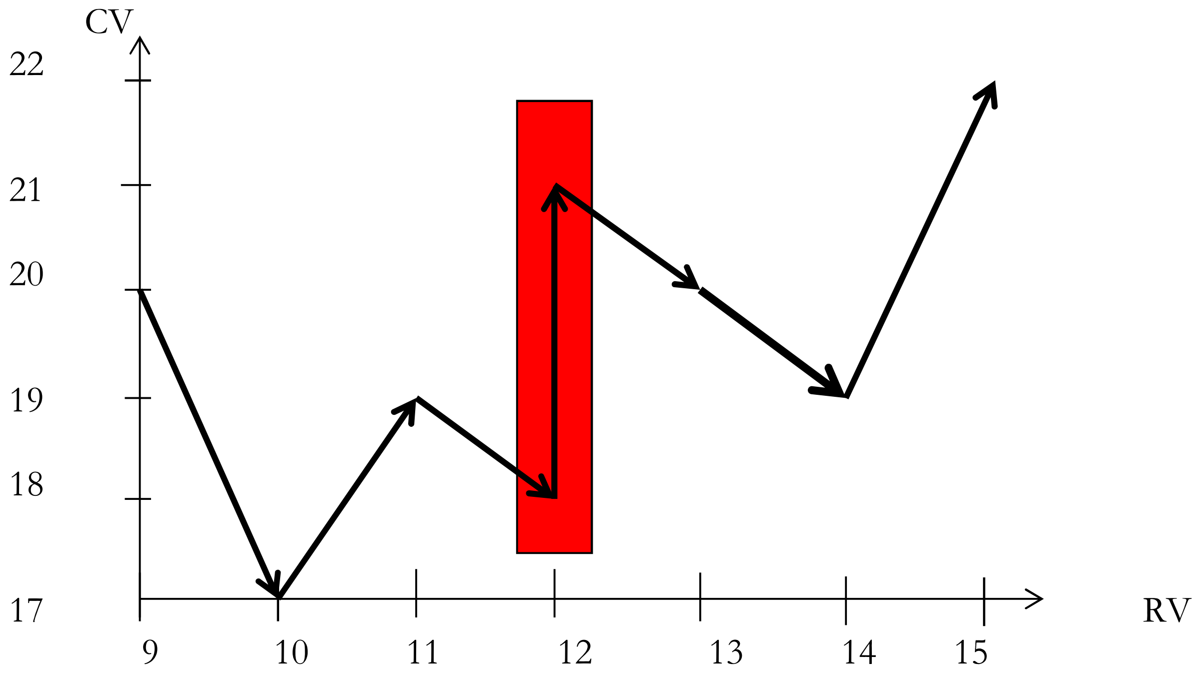
$(3.1_{3,4}\quad 2.1_{1,4}\quad 1.1_{1,3,4})$ Kw = 20 Rpw = 9

$(3.1_{3,4}\quad 2.3_{2,4}\quad 1.3_{3,4})$ Kw = 20 Rpw = 13

$(3.2_{2,4}\quad 2.2_{1,2,4}\quad 1.3_{3,4})$ Kw = 20 Rpw = 13

$(3.3_{2,3,4}\quad 2.3_{2,4}\quad 1.3_{3,4})$ Kw = 22 Rpw = 15

We can now display the interrelationship between the representation and the contextural values for the 10 sign classes in the following diagram:



Although there is no eigenreality in a poly-contextural semiotics (cf. Toth 2009) and thereby no direct connection between the “complete object” (3.2 2.2 1.2) and the "esthetic object" (3.1 2.2 1.3), as it has been pointed out in Bense (1992), there seems to be a connection between these two sign classes due to the fact that they are the only two sign classes, which have the same representation value, but lie in two different contextures.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Das Universum der Zeichen. Baden-Baden 1983

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Karger, Angelika, Über Repräsentationswerte. In: Semiosis 17/18, 1980, pp. 23-29

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

Toth, Alfred, A poly-contextural view on eigenreality. In: Electronic Journal for Mathematical Semiotics, 2009

Reference in poly-contextural semiotics (= NETS, 8)

1. Semiotic reference has already been treated thoroughly in Toth (2008a, b), but in a strictly mono-contextural semiotic frame. In this paper, I will use poly-contextural semiotics as introduced by Kaehr (2009) and in other papers.

2. The basic idea of turning mono-contextural into poly-contextural semiotics is the notion of inner semiotic environment. Every sub-sign of the semiotic matrix, an environment in the form of contextural indices is assigned. Dual sub-signs get the same indices as long as they are in the same matrix. In Toth (2009), it was shown that in a 4-contextural semiotics, the 4 contextures can be ascribed, on the basis of Günther (1976, pp. 336 ss.), to the four combinations of subject and object in a 4-contextural logic:

M ≡ (.1.) ≡	objective subject (oS):	thou/you
O ≡ (.2.) ≡	objective object (oO):	it
I ≡ (.3.) ≡	subjective subject (sS):	me/we
Q ≡ (.4.) ≡	subjective object (sO):	he, she/they

However, from the 4! = 256 possible combinations of these logical-semiotic relations, in a 4×4 4-contextural semiotic matrix, only 16 are semiotically represented:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,3} & 1.3_{1,4} & 1.4_{3,4} \\ 2.1_{1,3} & 2.2_{1,2,3} & 2.3_{1,2} & 2.4_{2,3} \\ 3.1_{1,4} & 3.2_{1,2} & 3.3_{1,2,4} & 3.4_{2,4} \\ 4.1_{3,.4} & 4.2_{2,3} & 4.3_{2,4} & 4.4_{2,3,4} \end{pmatrix}$$

Therefore, we can write the semiotic in form of the semiotically represented logical-semiotic relations:

$$\begin{pmatrix} oS/sS/sO & oS/sS & oS/sO & sS/sO \\ oS/sS & oS/oO/sS & oS/oO & oO/sS \\ oS/sO & oS/oO & oS/oO/sO & oO/sO \\ sS/sO & oO/sS & oO/sO & oO/sS/sO \end{pmatrix}$$

Therefore, the 35 possible tetradic sign classes (cf. also Toth 2007, pp. 216 ss.)

(4.1 3.1 2.1 1.1)
(4.1 3.1 2.1 1.2)
(4.1 3.1 2.1 1.3)
(4.1 3.1 2.1 1.4)

(4.1 3.1 2.2 1.2)	(4.1 3.2 2.2 1.2)
(4.1 3.1 2.2 1.3)	(4.1 3.2 2.2 1.3)
(4.1 3.1 2.2 1.4)	(4.1 3.2 2.2 1.4)

(4.1 3.1 2.3 1.3)	(4.1 3.2 2.3 1.3)	(4.1 3.3 2.3 1.3)
(4.1 3.1 2.3 1.4)	(4.1 3.2 2.3 1.4)	(4.1 3.3 2.3 1.4)

(4.1 3.1 2.4 1.4)	(4.1 3.2 2.4 1.4)	(4.1 3.3 2.4 1.4)	(4.1 3.4 2.4 1.4)

(4.2 3.2 2.2 1.2)
(4.2 3.2 2.2 1.3)
(4.2 3.2 2.2 1.4)

(4.2 3.2 2.3 1.3)	(4.2 3.3 2.3 1.3)
(4.2 3.2 2.3 1.4)	(4.2 3.3 2.3 1.4)

(4.2 3.2 2.4 1.4)	(4.2 3.3 2.4 1.4)	(4.2 3.4 2.4 1.4)

(4.3 3.3 2.3 1.3)			
(4.3 3.3 2.3 1.4)	(4.3 3.3 2.4 1.4)	(4.3 3.4 2.4 1.4)	(4.4 3.4 2.4 1.4)

can be rewritten, in a first step, as classes of semiotic indices (of inner environments)

(3,4 1,4 1,3 1,3,4)
(3,4 1,4 1,3 1,3)
(3,4 1,4 1,3 1,4)
(3,4 1,4 1,3 3,4)

(3,4 1,4 1,2,3 1,3)	(3,4 1,2 1,2,3 1,3)
(3,4 1,4 1,2,3 1,4)	(3,4 1,2 1,2,3 1,4)
(3,4 1,4 1,2,3 3,4)	(3,4 1,2 1,2,3 3,4)

(3,4 1,4 1,2 1,4)	(3,4 1,2 1,2 1,4)	(3,4 1,2,4 1,2 1,4)
(3,4 1,4 1,2 3,4)	(3,4 1,2 1,2 3,4)	(3,4 1,2,4 1,2 3,4)

(3,4 1,4 2,3 3,4)	(3,4 1,2 2,3 3,4)	(3,4 1,2,4 2,3 3,4)	(3,4 2,4 2,3 3,4)

(3,2 1,2 1,2,3 1,3)
(3,2 1,2 1,2,3 1,4)
(3,2 1,2 1,2,3 3,4)

(3,2 1,2 1,2 1,4) (3,2 1,2,4 1,2 1,4)
(3,2 1,2 1,2 3,4) (3,2 1,2,4 1,2 3,4)

(3,2 1,2 2,3 3,4) (3,2 1,2,4 2,3 3,4) (3,2 2,4 2,3 3,4)

(2,4 1,2,4 1,2 1,4)
(2,4 1,2,4 1,2 3,4) (2,4 1,2,4 2,3 3,4) (2,4 2,4 2,3 3,4) (2,3,4 2,4 2,3 3,4)

and in a second and last step as classes of logical-semiotic relations

(sS,sO oS,sO oS,sS oS,sS,sO)
(sS,sO oS,sO oS,sS oS,sS)
(sS,sO oS,sO oS,sS oS,sO)
(sS,sO oS,sO oS,sS sS,sO)

(sS,sO oS,sO oS,oO,sS oS,sS) (sS,sO oS,oO oS,oO,sS oS,sS)
(sS,sO oS,sO oS,oO,sS oS,sO) (sS,sO oS,oO oS,oO,sS oS,sO)
(sS,sO oS,sO oS,oO,sS sS,sO) (sS,sO oS,oO oS,oO,sS sS,sO)

(sS,sO oS,sO oS,oO oS,sO) (sS,sO oS,oO oS,oO oS,sO)
(sS,sO oS,oO,sO oS,oO oS,sO)

(sS,sO oS,sO oS,oO sS,sO) (sS,sO oS,oO oS,oO sS,sO)
(sS,sO oS,oO,sO oS,oO sS,sO)

(sS,sO oS,sO oO,sS sS,sO) (sS,sO oS,oO oO,sS sS,sO)
(sS,sO oS,oO,sO oO,sS sS,sO) (sS,sO oO,sO oO,sS sS,sO)

(sS,oO oS,oO oS,oO,sS oS,sS)
(sS,oO oS,oO oS,oO,sS oS,sO)
(sS,oO oS,oO oS,oO,sS sS,sO)

(sS,oO oS,oO oS,oO oS,sO) (sS,oO oS,oO,sO oS,oO oS,sO)
(sS,oO oS,oO oS,oO sS,sO) (sS,oO oS,oO,sO oS,oO sS,sO)

(sS,oO oS,oO oO,sS sS,sO) (sS,oO oS,oO,sO oO,sS sS,sO)
(sS,oO oO,sO oO,sS sS,sO)

(oO,sO oS,oO,sO oS,oO oS,sO)
(oO,sO oS,oO,sO oS,oO sS,sO) (oO,sO oS,oO,sO oO,sS sS,sO)

(oO,sO oO,sO oO,sS sS,sO)   (oO,sS,sO oO,sO oO,sS sS,sO)

These 15 sets of logical-semiotic relations thus show all possible types of reference that are poly-contextural-semiotically represented in a 4-contextural semiotic 4×4-matrix. In other words: The 15 sets contain all those types of crossings of the contextural-borders between subject and object which can be represented in a 4-contextural semiotics capable of handling the 4 types of subject-object combinations of a 4-contextural logic.

Bibliography

Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. Vol. 1. Hamburg 1976

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Reference in theoretical semiotics. In: Electronic Journal of Mathematical Semiotics, 2008a

Toth, Alfred, Substanzlose semiotische Referenz. In: Electronic Journal of Mathematical Semiotics, 2008b

Toth, Alfred, Permutations of sign classes and of inner semiotic environments. In: Electronic Journal of Mathematical Semiotics, 2009

Matching conditions for fundamental categories (= NETS, 9)

1. In Toth (2008, pp. 20 ss., pp. 51 ss.), I have given extensive lists of matching conditions of pairs of triadic sign relations. However, all these examples are monocontextural. Meanwhile, Rudolf Kaehr has added polycontextural matching conditions (Kaehr 2009). In the present article, I will suggest as a third possibility matching conditions for fundamental categories based on contextural values introduced in Toth (2009).

2. If we start with the 3-contextural 3×3 matrix

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

we recognize that we can write this matrix as a matrix of the contextural values of the respective sub-signs

$$\begin{pmatrix} 4 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 5 \end{pmatrix}$$

Therefore, we get

$M(1.2) = O^{+1}(2.3)$	$O(2.3) = M^{-1}(1.2)$
$M(1.2) = I^{+1}(3.2)$	$I(3.2) = M^{-1}(1.2)$
$M(1.2) = M^{+2}(1.3)$	$M(1.3) = M^{-2}(1.2)$
$M(1.2) = I^{+2}(3.1)$	$I(3.1) = M^{-2}(1.2)$
$M(1.2) = M^{+3}(1.1)$	$M(1.1) = M^{-3}(1.2)$
$M(1.2) = I^{+4}(3.3)$	$I(3.3) = M^{-4}(1.2)$

$O(2.3) = M^{+1}(1.3)$	$M(1.3) = O^{-1}(2.3)$
$O(2.3) = O^{+1}(2.2)$	$O(2.2) = O^{-1}(2.3)$
$O(2.3) = I^{+1}(3.1)$	$I(3.1) = O^{-1}(2.3)$
$O(2.3) = M^{+2}(1.1)$	$M(1.1) = O^{-2}(2.3)$
$O(2.3) = I^{+3}(3.3)$	$I(3.3) = O^{-3}(2.3)$

$M(1.3) = M^{+1}(1.1)$ $M(1.1) = M^{-1}(1.3)$
$M(1.3) = I^{+2}(3.3)$ $I(3.3) = M^{-2}(1.3)$

$M(1.3) = M^{+1}(1.1)$ $M(1.1) = M^{-1}(1.3)$
$M(1.3) = I^{+2}(3.3)$ $I(3.3) = M^{-2}(1.3)$

$M(1.1) = I^{+1}(3.3)$ $I(3.3) = M^{-1}(1.1)$

Moreover, we have the following identities of contextural values

$M(2.3) = I(3.2)$
$M(1.3) = O(2.2) = I(3.1)$

Thus, the main diagonal of the 3-contextural 3×3 matrix consists of three times the same contextural values – just as the respective matrix of the numerical prime-signs consists of three times the same representation values.

3. For the 4-contextural 3×3 matrix

$$\begin{pmatrix} 1.1_{1,3,4} & 1.3_{1,4} & 1.4_{3,4} \\ 3.1_{1,4} & 3.3_{1,2,4} & 3.4_{2,4} \\ 4.1_{3,.4} & 4.3_{2,4} & 4.4_{2,3,4} \end{pmatrix}$$

we get exactly the same times of matching conditions, since the contextural values differ from those of the 3-contextural 3×3 matrix just by adding +4 to each contextural value. Therefore, contextural values are not only independent of all kinds of transpositions of a semiotic matrix and thus of dualization and permutation, but they are also independent of embedding a n-contextural m×m-matrix into any n+k m×m-matrix (k ≥ 1).

Bibliography

Kaehr, Rudolf, Interactional operators in diamond semiotics. http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.pdf (2009)
Toth, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008
Toth, Alfred, Semiotic contextural values. In: Electronic Journal for Mathematical Semiotics, 2009

Some instances of qualitative preservation (= NETS, 10)

1. The German psychiatrist and writer Oskar Panizza (1853-1921) is a late representative of the radical subjectivist idealism, which has probably found its peak in Stirner's work (cf. Wiener 1978, Toth 1997). Panizza accepts the difference between outside and inside world solely as a working hypothesis. For him, thinking is hallucination, and experience is illusion (Panizza 1895, p. 21). According to him, there are no dichotomies such as Outside and Inside, Thinking and Experience, Subject and Object, etc. (1895, p. 30). However, when he is forced to explain the origin of hallucination, transcendence comes through the backdoor again in his philosophical building: "Therefore, I put the demon at the border, where I do not find anymore a *causa*, but ask for a *causa*, thus for a transcendental *causa* (...). Hence, the demon is a factor, won by necessity out of transcendence, in order to explain my thinking on This Side, which is equipped with the need of causality, and the world of appearance, connected to it. Even clearer, Panizza states later: "The demon (is) something from the Beyond" (1895, p. 27).

However, for Panizza, the demon is not only the "creative principle of the illusionist act" (1895, p. 48), but, at the same time, also "whatever comes across me in nature, after subtracting the effect of my senses" (1895, p. 49), i.e. the Thing per se. "And herewith, we have explained and constructed the 'Thing per se', however, only what concerns illusionism, experience. But here alone I encounter the question for explaining the 'Thing per se' – the question what remains after subtracting my senses from the Outer World. From the standpoint of my thinking, there is no 'Thing per se', since from here, the *entire* Outer World is illusion. But in the area of illusion, at least, I may apply my recognition, won on the standpoint of thinking, and I may call the 'per se' of my vis-à-vis, what last on him after *my* senses have been subtracted, - Demon" (1895, pp. 48 s.). In another place, Panizza calls the demon "ghost" (1894, p. 49). Thus, life appears as "haunting" (1895, p. 50), and one is remembered to the famous passage in Stirner: "Everything, which appears to you, is but the appearance of an intrinsic ghost, a ghostly appearance. For you, the world is just a world of appearance behind which the ghost itself is haunting. You see ghosts" (Stirner ap. Bauer 1984, p. 46).

On this background, Panizza formulates a semiotic paradox, which has hardly been recognized by now: "Only Death puts an end to this haunting. And end for me, since everything points out that I, my thinking, knows nothing, that my corpse – an illusionist product – lies there stinking*, a performance for the Others*. The demon retires, he stops his creative acts. And the hull, the mask, rots visibly in the illusory pleasure – of the Others, the survivors. That no rest, no rest of thinking – as far as human experience reaches – remains from me, must us, so eagerly searching for 'preservation of power', make aware that something goes down the drain, as one says, - where? Something, my thinking, goes where? And the mask rots before our eyes. It mixes into the mass of the other illusory products. It works out without rest – for our illusory view. We calculate it in nitrogen and oxygen, and the calculation works out. Inside of the world of appearance, nothing is lacking. However, the thinking, fighters for the Principle of Preservation of Power, where does the thinking go to? (1895, pp. 50 s.).

2. Semiotic preservation of quality as analogue to physical preservation of power can only work in a polycontextural semiotics which can bridge the abyss between the sign and its object (cf. Toth 1998). However, Bense tried to establish a semiotic "preservation theorem" on the basis of 1-contextural triadic semiotics. As we will see, this idea turns out to be not as bad as it seems beforehand.

For Peircean Semiotics "an absolut complete diversity of 'worlds' and 'parts of worlds', of 'being' and 'Being' (Sein und Seindes) (...) is principally not reachable for a consciousness which works over triadic sign relations" (Bense 1979, p. 59). Nevertheless, consciousness is understood as "a binary functor of being which produces the subject-object relation" (Bense 1976, p. 27), since Peirce keeps up "the difference between the object and the subject of recognition in connecting both poles through their being represented" (Walther 1989, p. 76). More exactly, "the representative connection of the sign class also indicates the epistemological subject, the realizational connection of the object thematic also indicates the epistemological object" (Gfesser 1990, p. 133). "In this way, we stipulate an intrinsic (i.e., non-transcendental) notion of recognition, whose essential process lies in de facto differentiating between (recognizable) 'world' and (recognizing) 'consciousness', but, though, in producing a real triadic relation, the 'relation of recognition'" (Bense 1976, p. 91).

Thus, "in the end, thematics of Being cannot be motivated and legitimated other than via sign thematics" (Bense 1971, p. 16). It follows, "that notions of object are only relevant with regard to a sign class and possess a semiotic reality thematic which can be discussed and judged as its connection of reality only relatively to this sign class (Bense 1976, p. 109). Therefore, sign class and reality thematic do not behave like 'platonic' and 'realistic' concepts of Being, but just like the extreme entities of the one and only identical thematic of Being" (Bense 1976, p. 85). Hence, to a sign relation and its reality thematics, there belongs also "the difference between 'onticity' and 'semioticity'" (Bense 1979, p. 19), about which a theorem of Bense orients: "With increasing semioticity, onticity of representation increases, too" (Bense 1976, p. 60). On this background, Bense formulates his "semiotic preservation theorem":

"Especially, in this connection, the dual relation of symmetry between the single sign classes and their corresponding reality thematics has to be pointed out. This relation of symmetry says that one can, in principle, represent meta-semiotically only that 'reality' or those relationships of reality, which one can represent semiotically. Therefore, the representaton values (i.e. the sums of the fundamental prime-sign numbers) of a sign class are invariant towards the dual transformation of a sign class into its reality thematic. This semiotic 'preservation theorem" can be regarded as a consequence of a theorem that had been already formulated [in Bense 1976, pp. 60, 62, v.s.] and which says that with increasing semioticity of representativity also its onticity increases in the same degree" (Bense 1981, p. 259).

3. Thus, on the first sight, Panizza's paradox cannot arise in a semiotic metaphysics built on triadic Peircean semiotics, since Bense semiotic "preservation theorem" implies that "media, object and interpretant of a sign lie in one and the same world" (Gfesser 1990, p. 139). Max Bense himself had seen already very early: "Being (das Seiende) appears as a sign, and signs survive in the purely semiotic dimension of their meanings the loss of reality" (1952, p. 80). In consequence, the concepts of Panizza and of Bense are principally different. Panizza's metaphysics is transcendental because of the notion of the demon. It is aprioric, because the demon is identified with the thing per se. Further, as an illusionist concept, it is platonic. On the other side, Peircean semiotics is a "non-transcendental, a non-aprioric and a non-platonic organon" (Gfesser 1990, p. 133).

Due to the identification of the modal categories with the prime-numbers (cf. Bense 1980) and because of the paralleling of the semiosic relation of generation with the successor relation of Peano numbers (Bense 1975, pp. 168 ss.; 1983, pp. 192 ss.), the 10 Peircean sign classes are primarily quantitative relations. Therefore, sign classes cannot preserve the qualities, which they are representing, at least not outside of the narrow representative frame of the 10 sign classes. In other words: All qualities of the ontological space, which do not fit into the Bed of Procrustes of the 10 sign classes, must be lost. On the other side, Bense's "preservation theorem" holds, but simply because reality appears thematized and thus represented in reality thematic which itself is a pure function of the corresponding sign thematic, and vice versa. Therefore, semiotic dual systems span up representation schemes in which the monocontextural subject-object dichotomy holds, but also epistemological objects can only be represented in the reality thematics as dual sign classes and therefore subject to the subjects of the interpretant relations of the sign classes. Sign classes do not reach their objects, and neither do reality thematics. The distinction between sign classes and reality thematics is just a formal doubling of the semiotic representation scheme which allows some further technical insights in the thematization structures of signs – and not more.

4. Polycontextural semiotics exists only since Kaehr (2009, and further studies). If we assume the sign being a triadic relation as a fragment of 4 contextures, we can write the 10 Peircean sign classes as follows:

$(3.1_{3,4}\ 2.1_{1,4}\ 1.2_{1,4})$ CV = 17

$(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4})$ CV = 19
$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$ CV = 19
$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$ CV = 19

$(3.2_{2,4}\ 2.3_{2,4}\ 1.3_{3,4})$ CV = 19

$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$ CV = 18

$(3.1_{3,4}\ 2.1_{1,4}\ 1.1_{1,3,4})$ CV = 20
$(3.1_{3,4}\ 2.3_{2,4}\ 1.3_{3,4})$ CV = 20
$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$ CV = 20

$(3.3_{2,3,4}\ 2.3_{2,4}\ 1.3_{3,4})$ CV = 22

Since every sub-sign lies in at least 2 contextures, qualitative conservation is possible, and since these sign classes thus represent both quantities and qualities, they are no longer purely quantitative, but quanti-qualitative or quali-quantitative sign classes.

4.1. First, we want to look if Bense's monocontextural preservation theorem also holds for polycontextural sign classes. If we take as an exampe

$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ 2.2_{4,2,1}\ 2.3_{4,2})$
Although the sub-signs of the "reality thematics" contain now hetero-morphisms, the respective contextural "indices" are preserved as the sub-signs are, and also the contextural values of both "sign class" and "reality thematic" are identical. We may therefore say, that Bense's preservation theorem, although conceived for monocontextural semiotics, holds for polycontextural semiotics, too.

4.2. As the above grouping of the ten sign classes suggests, be have two groups of sign classes that have identical contextural values:

$(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4})$ CV = 19
$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$ CV = 19
$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$ CV = 19
$(3.2_{2,4}\ 2.3_{2,4}\ 1.3_{3,4})$ CV = 19

$(3.1_{3,4}\ 2.1_{1,4}\ 1.1_{1,3,4})$ CV = 20
$(3.1_{3,4}\ 2.3_{2,4}\ 1.3_{3,4})$ CV = 20
$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$ CV = 20

We are thus allowed to say that sign classes and reality thematics which have the same contextural values, are quanti-qualitative/quali-quantitative representation preserving schemes.

4.3. However, we also have 2 sign classes which have the same representation value, but lies in 2 different contextures:

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$ CV = 19 RV = 12

$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$ CV = 18 RV = 12

These two sign classes play a crucial role in monocontextural semiotics (cf. Bense 1992), since the second is the sign class of the "complete object" and the first is the sign class of the "esthetic object" which is characterized by "augmentation of Being" (Seinsvermehrung), cf. Bense (1992, p. 16). What differentiates an object from an esthetic object, is called "Mit-Realität" by Bense (1979, p. 132). Mitreality is what causes the augmentation of Being, and it seems that the differential of eigenreality qua mitreality and (objective) reality is represented by polycontextural semiotics through the difference of the CVs: $\Delta(19, 18) = 1$.

4.4. Finally, there is another fact that requires our interest: While 9 of the 10 sign classes can be ordered by increasing CV's in steps of +1, there is not sign class whose CV = 21. In other words: The last sign class (with maximal semioticity, v.s.),

$(3.3_{2,3,4}\ 2.3_{2,4}\ 1.3_{3,4})$ CV = 22

cannot be reached from the other sign classes by one-step addition of CV's. Hence, this sign class which represents the totality of all signs in the semiotic universe, cannot be "deduced logically" from the sentences represented semiotically by the other 9 sign classes – as meta-logical sentences cannot be deduced without creating paradoxes in classical logic according to the Gödel theorems. One also should note that simply from the (monocontextural) representation values, this problem does not appear, since the 10 sign classes can be mapped to the RV's 10 to 15 without any gaps of RV's.

Bibliography

Bauer, Michael, Oskar Panizza. Ein literarisches Porträt. Munich 1984

Bense, Max, Die Theorie Kafkas. Köln 1952

Bense, Max, Zeichen und Design. Baden-Baden 1971

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Bense, Max, Das Universum der Zeichen. Baden-Baden 1983

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Gfesser, Karl: Bemerkungen zum "Zeichenband". In: Walther, Elisabeth und Bayer, Udo (ed.), *Zeichen von Zeichen für Zeichen.* Baden-Baden 1990, Baden, pp. 129-141

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

Panizza, Oskar, Der Illusionismus und Die Rettung der Persönlichkeit. Leipzig 1895

Toth, Alfred, Zu Oskar Panizzas präsemiotischem Solipsismus. In: European Journal for Semiotic Studies 9, 1997, pp. 769-779

Toth, Alfred, Ist ein qualitativer semiotischer Erhaltungssatz möglich? In: Semiosis 91/92, 1998, pp. 105-112

Walther, Elisabeth, Charles Sanders Peirce – Leben und Werk. Baden-Baden 1989

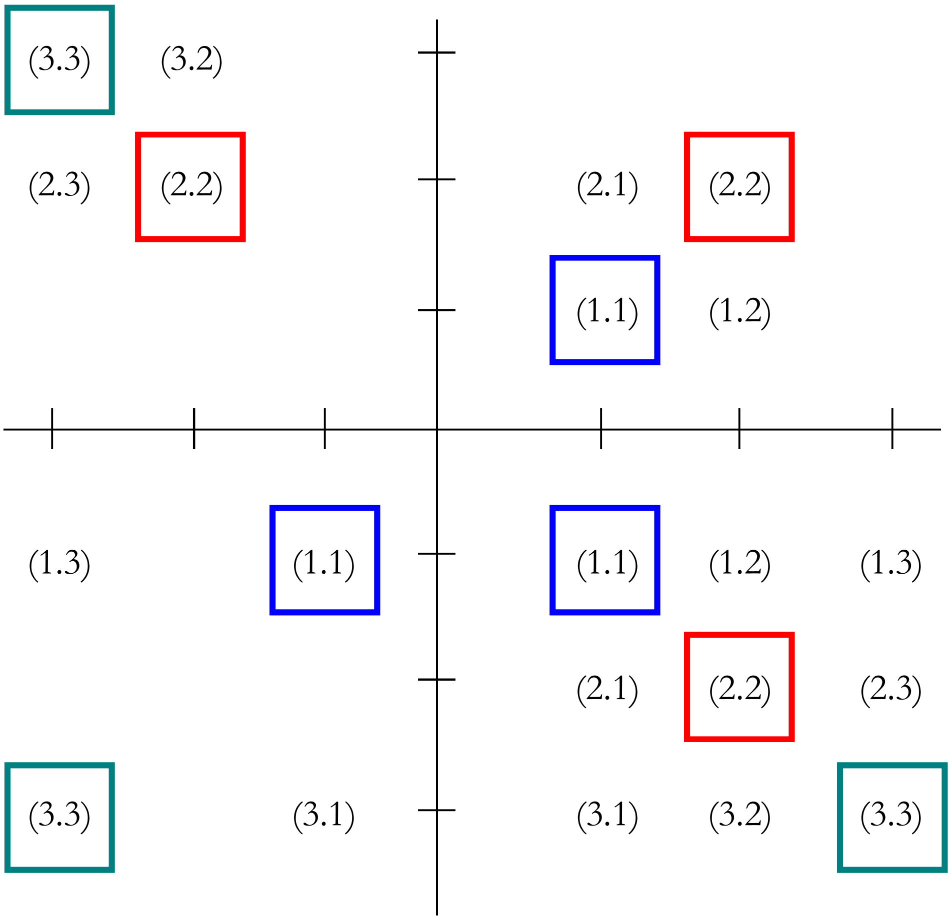
Wiener, Oswald, Über den Illusionismus. In: Panizza, Oskar, Die kriminelle Psychose, genannt Psichopatia criminalis. Munich 1978, pp. 213-237

2- and 3-dimensional display of triadic sub-signs in 4-contextural semiotics (= NETS, 11)

1. As a provisory model for semiotic contextures in 2 dimensions, the Cartesian Coordinate System had been introduced into semiotics by Toth (2001, 2008a). Instead of marking the sub-signs of the triadic semiotic matrix by algebraic signs ((a.b), (-a.b), (-a.-b), (a.-b)) for the 4 quadrants of the Gaussian number field (counterclockwise), we start with Kaehr's 4-contextural triadic matrix (Kaehr 2009a, p. 8):

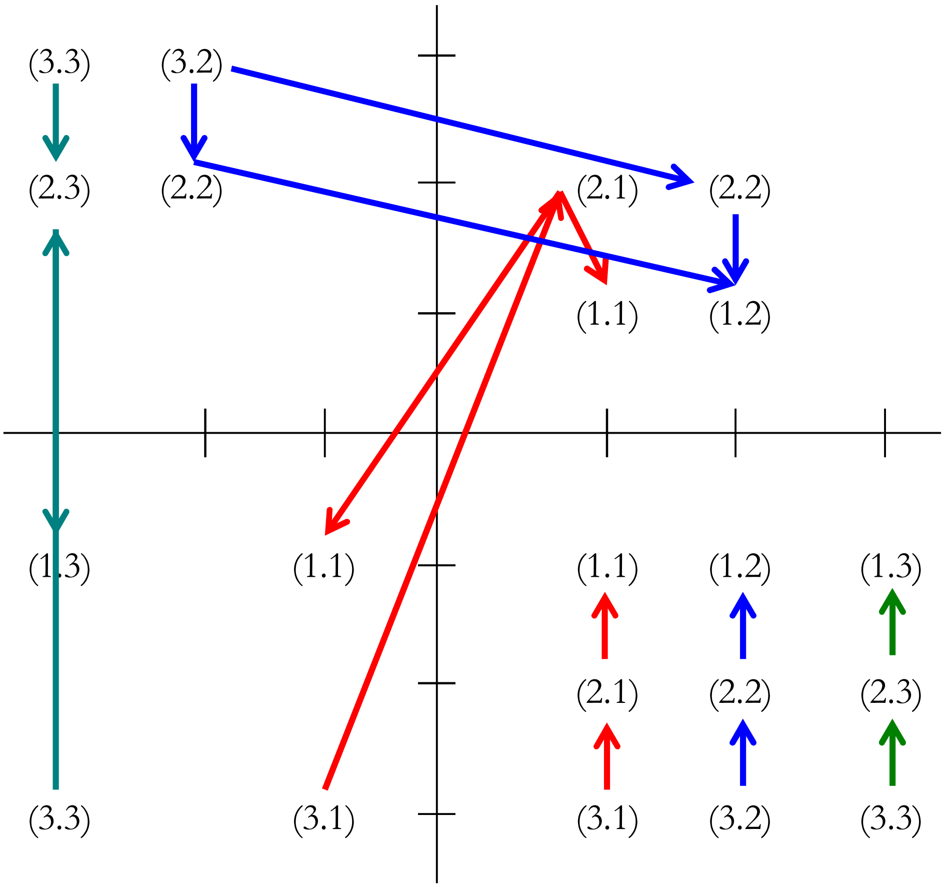
$$\begin{pmatrix} 1.1_{1,3,4} & 1.3_{1,4} & 1.4_{3,4} \\ 3.1_{1,4} & 3.3_{1,2,4} & 3.4_{2,4} \\ 4.1_{3,.4} & 4.3_{2,4} & 4.4_{2,3,4} \end{pmatrix}$$

and display the distribution of the 9 sub-signs over the 4 semiotic contextures that we assign to the 4 quadrants



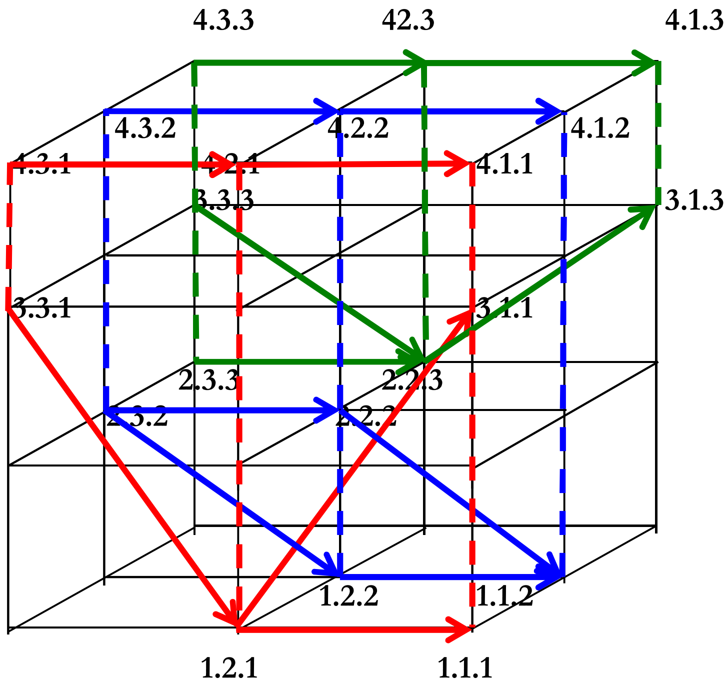
The sub-signs in frames of the same colors obey the matching conditions in connection with semiotic decomposition (cf. Kaehr 2009b).

The above coordinate system also gives a good picture of the structure of sign classes that lie in more than one contexture, extensively studied in Toth (2008a, pp. 82 ss.). In the following, we display only the three main sign classes, i.e. (3.1 2.1 1.1), (3.2 2.2 1.2), (3.3 2.3 1.3).



As one recognizes, no contextural transgressions are necessary for contexture 4.

2. Another possibility of displaying the distribution of the sub-signs over contextures is the 3-dimensional sign-cube of Stiebing (1978), which has been used in a series of papers by me (f. ex., Toth 2009). If we assign contextures to semiotic dimensions, however, we need a 3-dimensional, but 4-leveled cube. Again, we show for an example the three main sign classes:



This 3-dimensional model has the advantage that the semiotic connections between the same sub-signs in different contextures can be illustrated easily (in the graph by dashed lines).

Therefore, parametrization of sub-signs

(a.b) → (±a.±b), a, b, c ∈ {1, 2, 3}

and dimensional projection of sub-signs

(a.b) → (a.b.c), b,c ∈ {1, 2, 3}, a ∈ {1, 2, 3, ...}

can be interpreted as two ways of displaying semiotic contextures. Therefore, the models of polycontextural semiotics introduced in Toth (2008a) and (2008b) still hold after the introduction of polycontextural environments into semiotics by Kaehr (2009a, b).

Bibliography

Kaehr, Rudolf, Interactional operators in diamond semiotics. http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.pdf (2009a)

Kaehr, Rudolf, Interactional operators in diamond semiotics. http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.pdf (2009b)

Stiebing, Hans Michael, Zusammenfassungs- und Klassifikationsschemata von Wissenschaften und Theorien auf semiotischer und fundamentalkategorialer Basis. Diss. Stuttgart 1978

Toth, Alfred, Monokontexturale und polykontexturale Semiotik. In: Bernard, Jeff and Gloria Withalm (eds.), Myths, Rites, Simulacra. Proceedings of the 10th International Symposium of the Austrian Association for Semiotics, University of Applied Arts Vienna, December 2000. Vol. I: Theory and Foundations & 7th Austro-Hungarian Semio-Philosophical Colloquium. Vienna: Institute for Socio-Semiotic Studies, pp. 117-134

Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2008 (2008a)

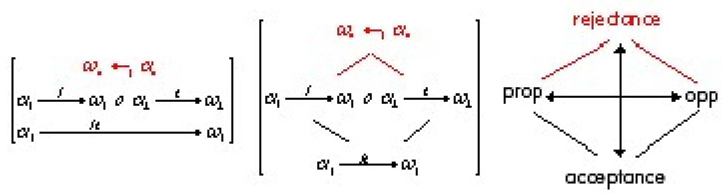
Toth, Alfred, Vorarbeiten zu einer polykontexturalen Semiotik. Klagenfurt 2008 (2008b)

Toth, Alfred, Mehrdimensionale Zeichenklassen. In: Electronic Journal for Mathematical Semiotics, 2009

Semiotic "risky bridges" vs. "spagat" in 4-contextural tetradic semiotics (NETS, 12)

1. Although – as Rudolf Kaehr has pointed out in a recent publication – the notion of "diamond" plays a crucial role in polycontextural theory since a long time, the first concise introduction into a formalized theory of diamonds goes back to Kaehr (2007). In Toth (2008), I had used the concept of diamond for semiotics, however still strictly based on 1-contextural 3-adic Peircean semiotics. Meanwhile, 3- and 4-contextural 3-adic semiotics have been applied in a new book (Toth 2009). After it has shown how incredibly big the increase of structural complexity is already in 4-contextural 3-adic semiotics, in the present article, I will go a step in the direction of 4-contextural 4-adic semiotics. In doing so, it shows that besides the elementary notions of diamond theory – morphisms and heteromorphisms – a quite new concept of semiotic connection between semiotic dyadic sub-signs shows up which has been called "risky bridge" by Kaehr (2007, p. 12).

2. In a polycontextural 3-adic diamond



the middle figure, taken from Kaehr (2007), shows the 2 basic types of semiotic mappings:

1. the morphism $\alpha_1 \rightarrow \omega_1$ and
2. the hetermorphism $\omega_4 \leftarrow \alpha_4$

If the above diamond serves as a model for a composition of a sign by its sub-signs, then the ω's must be object relations, since

$SCl = ((M \rightarrow O).(O \rightarrow I)) \rightarrow (M \rightarrow I)$,

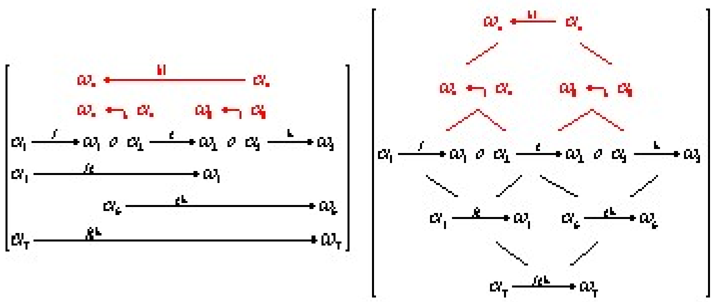
thus, the following pairs of morphisms and heteromorphisms are possible in a 4-contextural 3-adic semiotics:

$(2.1)_1 \rightarrow (2.1)_1$	$(2.1)_1 \leftarrow (2.1)_1$
$(2.1)_1 \rightarrow (2.1)_4$	$(2.1)_4 \leftarrow (2.1)_1$
$(2.1)_4 \rightarrow (2.1)_1$	$(2.1)_1 \leftarrow (2.1)_4$
$(2.1)_4 \rightarrow (2.1)_4$	$(2.1)_4 \leftarrow (2.1)_4$

$(2.2)_1 \rightarrow (2.2)_1$	$(2.2)_1 \leftarrow (2.2)_1$
$(2.2)_1 \rightarrow (2.2)_2$	$(2.2)_2 \leftarrow (2.2)_1$
$(2.2)_1 \rightarrow (2.2)_4$	$(2.2)_4 \leftarrow (2.2)_1$
$(2.2)_2 \rightarrow (2.2)_1$	$(2.2)_1 \leftarrow (2.2)_2$
$(2.2)_2 \rightarrow (2.2)_2$	$(2.2)_2 \leftarrow (2.2)_2$
$(2.2)_2 \rightarrow (2.2)_4$	$(2.2)_4 \leftarrow (2.2)_2$
$(2.2)_4 \rightarrow (2.2)_1$	$(2.2)_1 \leftarrow (2.2)_4$
$(2.2)_4 \rightarrow (2.2)_2$	$(2.2)_4 \leftarrow (2.2)_2$
$(2.2)_4 \rightarrow (2.2)_4$	$(2.2)_4 \leftarrow (2.2)_4$

$(2.3)_1 \rightarrow (2.3)_1$	$(2.3)_1 \leftarrow (2.3)_1$
$(2.3)_1 \rightarrow (2.3)_4$	$(2.3)_4 \leftarrow (2.3)_1$
$(2.3)_4 \rightarrow (2.3)_1$	$(2.3)_1 \leftarrow (2.3)_4$
$(2.3)_4 \rightarrow (2.3)_4$	$(2.3)_4 \leftarrow (2.3)_4$

3. However, if we now take as a model for sign-composition out of sub-signs the following polycontextural 4-adic diamond, taken also form Kaehr (2007)



then we have got a third type of semiotic mapping: "We can bridge the separated arrows by the arrow (kl), which is a balancing act over the gap, called *spagat*. If we want to compromise , we can build a *risky bridge* (lgk), which is involving acceptional and the rejectional arrows" (Kaehr 2007, p. 12).

Let's take as an example the 4-adic sign class

(3.2 2.2 1.2 0.2).

Its composition out of dyads is

(3.2 → 2.2) ◊ (2.2 → 1.2) ◊ (1.2 → 0.2)

In addition to 3-adic sign classes (O ≡ O), here, we must determine the pairs of morphisms and heteromorphisms also in (M ≡ M).

Therefore, spagats in 4-adic sign classes are just heteromorphisms like in 3-adic sign classes, but the new type of risky bridge appearing here is thus

$g = (2.2 \rightarrow 1.2)$
$l = (2.2 \leftarrow 3.2)$
$k = (3.2 \leftarrow 0.2)$

$lgk = (3.2 \leftarrow 0.2) \lozenge (2.2 \rightarrow 1.2) \lozenge (2.2 \leftarrow 3.2)$,

where $(3.2 \leftarrow 0.2)$ and $(2.2 \leftarrow 3.2)$ denote rejection, while $(2.2 \rightarrow 1.2)$ acception.

By introducing risky bridges vs. spagats into semiotics, it shows again, that diamond theory offers astonishing new perspectives for sign theory.

Bibliography

Kaehr, Rudolf, The book of Diamonds. Glasgow 2007. Digitalisat: http://rudys-diamond-strategies.blogspot.com/2007/06/book-of-diamonds-intro.html (2007)

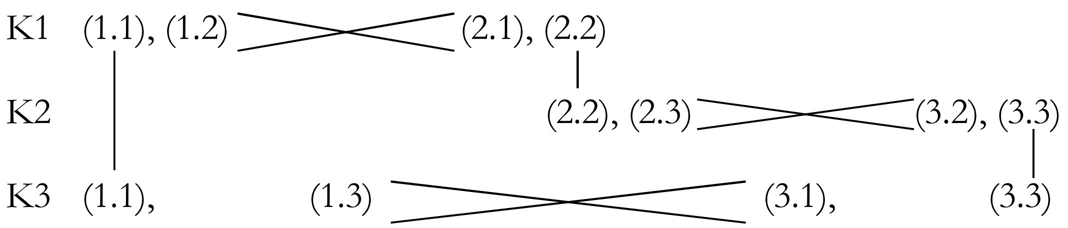
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Connections of sub-signs in contextures

For 3-adic semiotics, we have as best choices for polycontextural semiotic matrices either the 3-contextural or the 4-contextural matrix (cf. Kaehr 2009a, b). Let us start with the 3-contextural matrix. As one sees, the contextures or inner environments are scramble the order of the sub-signs in the following matrix:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

If we order horizontally only sub-signs, which lie in the same contexture, we get the following 3-level system:



There are three types of connections of the sub-signs in this scheme: First, the connections by inner environments (cf. Toth 2009):

(1.1), (1.2)
(2.1), (2.2)
(2.2), (2.3)
(3.2), (3.3)
(1.1), (1.3)
(3.1), (3.3)

Second, the connections by identical sub-signs (static via sub-signs and dynamic via their corresponding morphisms):

(1.1) (2.2) (3.3)
| | |
(1.1) (2.2) (3.3),

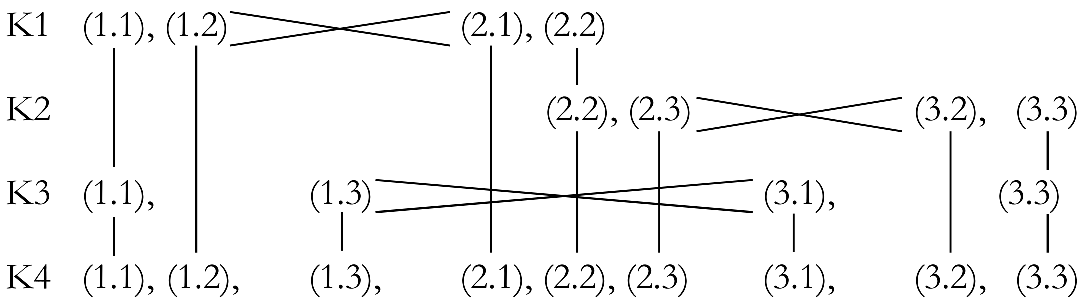
hence this kind of semiotic connection exists only between the genuine sub-signs, i.e. identitive morphisms.

Third, chiastic connections between pairs of converse sub-signs:

(1.2) × (2.1)
(2.3) × (3.2)
(1.3) × (3.1)

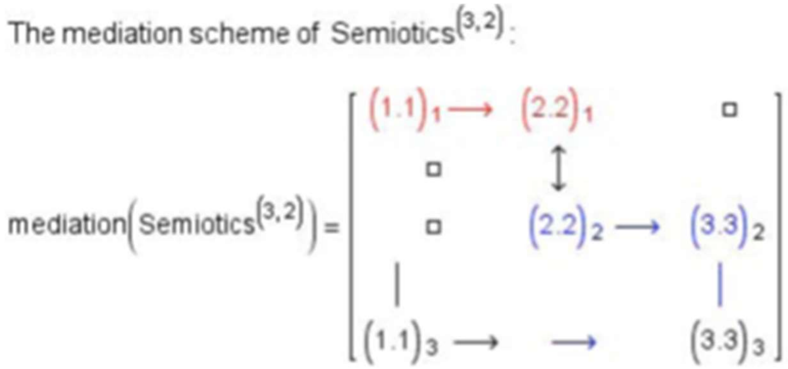
As one sees, both scheme and its types of connections are exhaustive, i.e. they are sufficient to describe the 3-contextural semiotic 3×3 matrix completely.

If we now proceed to the 4-contextural semiotic 3×3 matrix, we obtain



Of course, this scheme is exhaustive too, but with an enormous accretion of structure in K4 and mediating level between K2 and K3, compared to the scheme of 3-contextural 3×3 matrix.

3. As a marginal note, it has to be pointed out that schemes 1 and 2 have nothing to do with polycontextural schemes of mediation by decomposition; cf. the following schema for 3-contextural 3-adic semiotic by Kaehr (2009b, p. 5):



Chiastic structure

$$\textit{Order}\text{ relations} = \left\{ \begin{array}{l} (1.1)_1 \longrightarrow (2.2)_1, \\ (2.2)_2 \longrightarrow (3.3)_2, \\ (1.1)_3 \longrightarrow (3.3)_3 \end{array} \right\}.$$

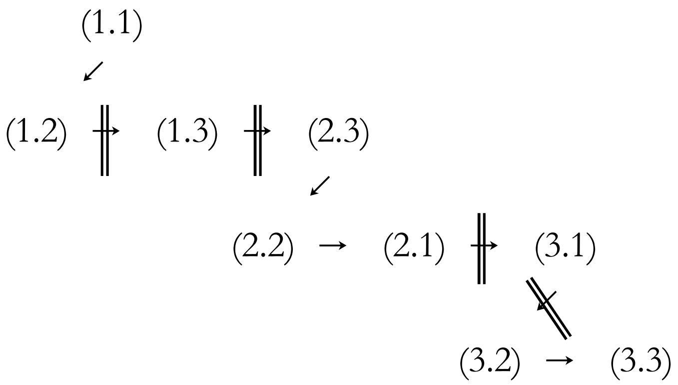
$$\textit{Exchange}\text{ relation} = \{(2.2)_1 \updownarrow (2.2)_2\}.$$

$$\textit{Coincidence}\text{ relations} = \left\{ \begin{array}{l} (1.1)_1 - (1.1)_3, \\ (3.3)_2 - (3.3)_3 \end{array} \right\}.$$

For systems, $m = 3$, $n = 2$, the matrix$^{(3,2)}$ and scheme$^{(3,2)}$ representation coincide.

In decomposition schemes like the one above, each of the (3, 2) partial sets of the (3, 3) full set does not contain the full amount of sub-signs necessary to construct not only the complete set of the 10 Peircean sign classes, but even one single sing class, provided that the semiotic law holds that every sign class must consist of 3 sub-signs which are pairwise different.

3. However, schemes like the two presented here, based on polycontextural semiotics, show some similarity to the so-called "scheme of sign-intern superization", based on monocontextural semiotics and presented by Bense (cf. Walther 1979, p. 120). Let us first have a look at the scheme from the standpoint of 3-contextural semiotics:



Provided the scheme is based on a 3-contextural semiotics, there are the following contexture borders:

$(1.2_1 \,||\, 1.3_3)$

$(1.3_3 \,||\, 2.3_2)$

$(2.1_1 \,||\, 3.1_3)$

$(3.1_3 \,||\, 3.2_2)$

However, by transgressing into a scheme with 4 contextures, they are eliminated, since then we have

$(1.2_{1,4} \nparallel 1.3_{3,4})$

$(1.3_{3,4} \nparallel 2.3_{2,4})$

$(2.1_{1,4} \nparallel 3.1_{3,4})$

$(3.1_{3,4} \nparallel 3.2_{2,4})$.

Therefore, if we use $\mathfrak{C}(x)$ for "the set of sub-signs lying in contexture x", we get for the 3-contextural 3×3 matrix:

$\mathfrak{C}(1.1) = ((1.1), (1.2), (1.3), (2.1), (2.2), (3.1), (3.3))$

$\mathfrak{C}(1.2) = ((1.1), (1.2), (2.1), (2.2))$

$\mathfrak{C}(1.3) = ((1.1), (1.3), (3.1), (3.3))$

$\mathfrak{C}(2.1) = ((1.1), (1.2), (2.1), (2.2))$

$\mathfrak{C}(2.2) = ((1.1), (1.2), (2.1), (2.2), (2.3), (3.2), (3.3))$

$\mathfrak{C}(2.3) = ((2.2), (2.3), (3.2), (3.3))$

$\mathfrak{C}(3.1) = ((1.1), (1.3), (3.1), (3.3))$

$\mathfrak{C}(3.2) = ((2.2), (2.3), (3.2), (3.3))$

$\mathfrak{C}(3.3) = ((1.1), (1.3), (2.2), (2.3), (3.1), (3.2), (3.3))$,

and we have

1. $\mathfrak{C}(a.b) = \mathfrak{C}((a.b)°)$
2. $\cap\mathfrak{C}(a.b) = \varnothing$
3. $\cup\mathfrak{C}(a.b) = \mathbf{S}$ ($\mathbf{S}$ = set of sub-signs)
4. $\max|\mathfrak{C}(1, 2, 3, ..., n)| = (n-2)$.

Bibliography

Kaehr, Rudolf, Toth's semiotic diamonds. http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf (2009a)

Kaehr, Rudolf, Interactional operators in diamond semiotics. http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.pdf (2009b)

Toth, Alfred, Connections of inner semiotic environments. In: Electronic Journal of Mathematical Semiotics, 2009

Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

Mediation between morphisms and heteromorphisms in semiotic systems

1. In his new paper, Rudolf Kaehr (2009c) has defined the Diamond relation as follows:

Diamond relation DiamRel:

$R \in Cat,\ r \in Sat$

$$(R, r)^{(m)} \Longleftrightarrow Rel^{(m)} \,\|\, rel^{(m-1)}$$

Thus each relation R belongs, qua morphism, to a category, while each relation r belongs, qua heteromorphism, to a “saltatory”. Morphism and heteromorphism are not dual, but complementary, and so are category and saltatory.

2. However, in semiotics (cf. Kaehr 2009a, b), the unmediated 2-valued opposition between morphism and heteromorphism works only with sign classes that are constructed from 2-adic sub-signs of maximal contexture 3, e.g.:

$\times(2.1)_1 = (2.1)_1 \quad \| \quad R(2.1)_1 = (1.2)_1$

$\times(2.2)_{1,2} = (2.2)_{1,2} \quad \| \quad R(2.2)_{1,2} = (2.2)_{2,1}$

“×” means here (monocontextural) dualization, “R” (polycontextural) reflection, thus dualization changes the order of the prime-signs constituting a sub-sign, while reflection also turns around the order of the contextures. So far, we have

Morphism: $(a.b)_i \rightarrow$ Heteromorphism: $(b.a)_i$

Morphism: $(a.b)_{i,j} \rightarrow$ Heteromorphisms: $(b.a)_{j,i}$

However, already here one possible mediation is lacking:

Morphism: $(a.b)_{i,j} \rightarrow$???: $(a.b)_{j,i}$

In other words: We need an operation “Y”, which turns around only the contextures of a sub-sign, but not the sub-sign itself.

3. But now let us proceed to 4-contextural 3-adic semiotics. In order to make sure what we are speaking about, I present here again the 10 Peircean sign classes as 4-contextural sign classes:

$(3.1_{3,4}\quad 2.1_{1,4}\quad 1.1_{1,3,4})$
$(3.1_{3,4}\quad 2.1_{1,4}\quad 1.2_{1,4})$
$(3.1_{3,4}\quad 2.1_{1,4}\quad 1.3_{3,4})$
$(3.1_{3,4}\quad 2.2_{1,2,4}\quad 1.2_{1,4})$
$(3.1_{3,4}\quad 2.2_{1,2,4}\quad 1.3_{3,4})$
$(3.1_{3,4}\quad 2.3_{2,4}\quad 1.3_{3,4})$
$(3.2_{2,4}\quad 2.2_{1,2,4}\quad 1.2_{1,4})$
$(3.2_{2,4}\quad 2.2_{1,2,4}\quad 1.3_{3,4})$
$(3.2_{2,4}\quad 2.3_{2,4}\quad 1.3_{3,4})$
$(3.3_{2,3}\quad 2.3_{2,4}\quad 1.3_{3})$

As one sees, the genuine sub-signs (identitive morphisms) lie in 3 contextures, so that the maximal scheme for 4-contextural 3-adic sign classes is

$SCl(4,3) = (3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$

And here now the real problems with the semiotic adaptation of Diamond theory start:

1. A sub-sign like

$(a.b)_{i,j,k}$

as a morphism has no only its heteromorphisms

$(a.b)_{k,j,i}$,

but also 4 more “mediative” morphisms

$(a.b)_{i,k,j}$, $(a.b)_{k,j,i}$, $(a.b)_{j,i,k}$ and $(a.b)_{j,k,i}$.

2. By aid of our three operations above, we also get

$\times(a.b)_{i,j,k} = (b.a)_{i,j,k}$	$R(a.b)_{i,j,k} = (b.a)_{k,j,i}$	$Y(a.b)_{i,j,k} = (a.b)_{k,j,i}$
$\times(a.b)_{k,j,i} = (b.a)_{k,i,j}$	$R(a.b)_{k,j,i} = (b.a)_{i,j,k}$	$Y(a.b)_{k,j,i} = (a.b)_{i,j,k}$
$\times(a.b)_{i,k,j} = (b.a)_{i,k,j}$	$R(a.b)_{i,k,j} = (b.a)_{j,k,i}$	$Y(a.b)_{i,k,j} = (a.b)_{j,k,i}$
$\times(a.b)_{k,j,i} = (b.a)_{k,i,j}$	$R(a.b)_{k,j,i} = (b.a)_{i,j,k}$	$Y(a.b)_{k,j,i} = (a.b)_{i,j,k}$
$\times(a.b)_{j,i,k} = (b.a)_{j,i,k}$	$R(a.b)_{j,i,k} = (b.a)_{k,i,j}$	$Y(a.b)_{j,i,k} = (a.b)_{k,i,j}$
$\times(a.b)_{j,k,i} = (b.a)_{j,k,i}$	$R(a.b)_{j,k,i} = (b.a)_{i,k,j}$	$Y(a.b)_{j,k,i} = (a.b)_{i,k,j}$

3. Now, a 3-adic sign class consists per definitionem of three sub-signs:

$SCl(4,3) = (3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$

However, this means that we can permute first the sign class as such:

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$
$(3.a_{i,j,k}\ 1.c_{,j,k}\ 2.b_{,j,k})$
$(2.b_{,j,k}\ 3.a_{,j,k}\ 1.c_{i,j,k})$
$(2.b_{,j,k}\ 1.c_{,j,k}\ 3.a_{,j,k})$
$(1.c_{i,j,k}\ 3.a_{,j,k}\ 2.b_{,j,k})$
$(1.c_{,j,k}\ 2.b_{i,j,k}\ 3.a_{,j,k})$,

and second for also the contextures, and this for all three sub-signs. Therefore, we get exactly 126 permutations of (combinations of) sign classes and contextures per sign class (cf. Toth 2009). The combined permutations look for the first permutation, i.e. the sign class in "degenerative-retrosemiosic order":

$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{ijk})$		
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{ikj})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{ikj})$	
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{jik})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{jik})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{jki})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{jki})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{kij})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{kij})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{ijk}\ 2.b_{ijk}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{ikj}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{ijk}\ 2.b_{jki}\ 1.c_{jki})$		
$(3.a_{ijk}\ 2.b_{jki}\ 1.c_{kij})$	$(3.a_{ijk}\ 2.b_{kij}\ 1.c_{kij})$	
$(3.a_{ijk}\ 2.b_{jki}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{kij}\ 1.c_{kji})$	$(3.a_{ijk}\ 2.b_{kji}\ 1.c_{kji})$
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{ijk})$		
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{ikj})$	$(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{ikj})$	
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{jik})$	$(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{jik})$	$(3.a_{ikj}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{jki})$	$(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{jki})$	$(3.a_{ikj}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{kij})$	$(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{kij})$	$(3.a_{ikj}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{ikj}\ 2.b_{ijk}\ 1.c_{kji})$	$(3.a_{ikj}\ 2.b_{ikj}\ 1.c_{kji})$	$(3.a_{ikj}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{ikj}\ 2.b_{jki}\ 1.c_{jki})$		
$(3.a_{ikj}\ 2.b_{jki}\ 1.c_{kij})$	$(3.a_{ikj}\ 2.b_{kij}\ 1.c_{kij})$	
$(3.a_{ikj}\ 2.b_{jki}\ 1.c_{kji})$	$(3.a_{ikj}\ 2.b_{kij}\ 1.c_{kji})$	$(3.a_{ikj}\ 2.b_{kji}\ 1.c_{kji})$

$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{ijk})$		
$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{ikj})$	$(3.a_{jik}\ 2.b_{ikj}\ 1.c_{ikj})$	
$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{jik})$	$(3.a_{jik}\ 2.b_{ikj}\ 1.c_{jik})$	$(3.a_{jik}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{jki})$	$(3.a_{jik}\ 2.b_{ikj}\ 1.c_{jki})$	$(3.a_{jik}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{kij})$	$(3.a_{jik}\ 2.b_{ikj}\ 1.c_{kij})$	$(3.a_{jik}\ 2.b_{jik}\ 1.c_{kij})$

$(3.a_{jik}\ 2.b_{ijk}\ 1.c_{kji})$ $(3.a_{jik}\ 2.b_{ikj}\ 1.c_{kji})$ $(3.a_{jik}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{jik}\ 2.b_{jki}\ 1.c_{jki})$
$(3.a_{jik}\ 2.b_{jki}\ 1.c_{kij})$ $(3.a_{jik}\ 2.b_{kij}\ 1.c_{kij})$
$(3.a_{jik}\ 2.b_{jki}\ 1.c_{kji})$ $(3.a_{jik}\ 2.b_{kij}\ 1.c_{kji})$ $(3.a_{jik}\ 2.b_{kji}\ 1.c_{kji})$

$(3.a_{jki}\ 2.b_{ijk}\ 1.c_{ijk})$
$(3.a_{jki}\ 2.b_{ijk}\ 1.c_{ikj})$ $(3.a_{jki}\ 2.b_{ikj}\ 1.c_{ikj})$
$(3.a_{jki}\ 2.b_{ijk}\ 1.c_{jik})$ $(3.a_{jki}\ 2.b_{ikj}\ 1.c_{jik})$ $(3.a_{jki}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{jki}\ 2.b_{ijk}\ 1.c_{jki})$ $(3.a_{jki}\ 2.b_{ikj}\ 1.c_{jki})$ $(3.a_{jki}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{jki}\ 2.b_{ijk}\ 1.c_{kij})$ $(3.a_{jki}\ 2.b_{ikj}\ 1.c_{kij})$ $(3.a_{jki}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{jki}\ 2.b_{ijk}\ 1.c_{kji})$ $(3.a_{jki}\ 2.b_{ikj}\ 1.c_{kji})$ $(3.a_{jki}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{jki}\ 2.b_{jki}\ 1.c_{jki})$
$(3.a_{jki}\ 2.b_{jki}\ 1.c_{kij})$ $(3.a_{jki}\ 2.b_{kij}\ 1.c_{kij})$
$(3.a_{jki}\ 2.b_{jki}\ 1.c_{kji})$ $(3.a_{jki}\ 2.b_{kij}\ 1.c_{kji})$ $(3.a_{jki}\ 2.b_{kji}\ 1.c_{kji})$

$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{ijk})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{ikj})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{ikj})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{jik})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{jik})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{jki})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{jki})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{kij})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{kij})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{kij}\ 2.b_{ijk}\ 1.c_{kji})$ $(3.a_{kij}\ 2.b_{ikj}\ 1.c_{kji})$ $(3.a_{kij}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{kij}\ 2.b_{jki}\ 1.c_{jki})$
$(3.a_{kij}\ 2.b_{jki}\ 1.c_{kij})$ $(3.a_{kij}\ 2.b_{kij}\ 1.c_{kij})$
$(3.a_{kij}\ 2.b_{jki}\ 1.c_{kji})$ $(3.a_{kij}\ 2.b_{kij}\ 1.c_{kji})$ $(3.a_{kij}\ 2.b_{kji}\ 1.c_{kji})$
$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{ijk})$
$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{ikj})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{ikj})$
$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{jik})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{jik})$ $(3.a_{kji}\ 2.b_{jik}\ 1.c_{jik})$
$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{jki})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{jki})$ $(3.a_{kji}\ 2.b_{jik}\ 1.c_{jki})$
$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{kij})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{kij})$ $(3.a_{kji}\ 2.b_{jik}\ 1.c_{kij})$
$(3.a_{kji}\ 2.b_{ijk}\ 1.c_{kji})$ $(3.a_{kji}\ 2.b_{ikj}\ 1.c_{kji})$ $(3.a_{kji}\ 2.b_{jik}\ 1.c_{kji})$

$(3.a_{kji}\ 2.b_{jki}\ 1.c_{jki})$
$(3.a_{kji}\ 2.b_{jki}\ 1.c_{kij})$ $(3.a_{kji}\ 2.b_{kij}\ 1.c_{kij})$
$(3.a_{kji}\ 2.b_{jki}\ 1.c_{kji})$ $(3.a_{kji}\ 2.b_{kij}\ 1.c_{kji})$ $(3.a_{kji}\ 2.b_{kji}\ 1.c_{kji})$

Thus, we get for all 6 permutations $6 \cdot 126 = 756$ sign classes, and for all 10 sign classes therefore 7'560 sign classes. However, we must not forget the structural potential that lies in our three above operators, ×, R, and Y, so that at the end we have a semiotic system of no less **than 22'680 sign classes**.

4. But that is not all. In Toth (2008), based on Stiebing (1978), I had introduced 3-dimensional sign classes into semiotics. Monocontextural 3-dimensional sign classes have the following form

3-SCl = ((a.b.c) (d.e.f) (g.h.i)),

or, if we use Peirce's "normal form"

3-SCl = ((a.3.b) (c.2.d) (e.1.f)),

whereby one sees that (a, c, e) are the so-called "dimensional numbers". Because of the triadic form of each sub-sign, the geometrical model of 3-SCl is a cube, but we can still make it higher by adding more levels. Since, for the embedded

2-SCl = $((3.b_{i,j,k})\ (2.d_{i,j,k})\ (1.f_{i,j,k}))$,

we got 22'680 sign classes, and since there are n-levels, we do not only get

$3! \cdot$ 22'680 = 136'080, but

$n! \cdot$ 22'680 different sign classes (a, c, e $\in$ {1, 2, 3, ..., n}

for sign classes constructed from 3-adic instead of 2-adic sub-signs.

5. However, the results obtained in this little contribution have enormous consequences for Diamond theory itself, because theoretically, we can surpass 3-adic sign classes and introduce 4-adic, 5-adic, 6-adic (, ..., -adic?) sign classes, the structural complexity of which grows astronomically because of the permutations, especially, if we also proceed to more than 4 contextures. Finally, we also can construct semiotic hypercubes and other nice high-dimensional polytopes that are not anymore based on cubic 3-sign classes, which are just made higher by adding more storeys, but by adding more dimensions. Since there are no formal restrictions concerning the order of dimensional numbers amongst themselves as well as in connection with prime-sign-numbers, already for a 4-dimensional (f.ex. tesseractic) sign model we have

4-SCl = ((a.b.3.c) (d.e.2.f) (g.h.1.i),
4-SCl = ((b.a.3.c) (e.d.2.f) (h.g.1.i)
(plus combinations)
4-SCl = ((3.a.b.c) (2.d.e.f) (1.g.h.i)
4-SCl = ((3.a.c.d) (2.d.f.e) (1.g.i.h)
(plus combinations)

If it is made clear, for which dimensions the variables of the dimensional numbers stand, we can also "scramble" them. Moreover, from the above constructions it results that a sign class can at the same time lie in more than 1 and maximally in 4 dimensions (if it is tesserarctic) as well as in several contextures (both qua sub-signs). That from here we have exciting connections to a quaternionic semiotics, I have already shown in a series of papers. Summa summarum, the incredibly huge amount of structural growth by introduction of contextures, permutations and dimensions in semiotics is hard to foresee.

However, to come back to Diamond theory, Kaehr has made clear that a diamond cannot deal with more than a pair of morphism and heteromorphism, the categorial/ saltatorial equivalents for logical proposition and rejection. However, as we have shown already in chapter 1 of this paper, already in a 4-contextural semiotics, we have 4 mediative morphisms between morphisms and heteromorphisms. Therefore, the diamond model has to be substituted by another polygon. (And in high-dimensional semiotics even by a polytope?) Am I right that therefore, Leinster's n-category theory could give a model for a n-category/saltatory diamond theory (at least what concerns the semiotic dimensional numbers? And then: What about the set theoretic, arithmetic, logical and also topological consequences for the mediative morphisms of the original Diamond theory? However it will be, it seems to me that for once there may be an enormous input from semiotics for the future of "graphematics" (as Kaehr says), while up to now, semiotics has only profited from polycontexturals sciences, but never contributed to them.

Bibliography

Kaehr, Rudolf, Diamond semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (Kaehr 2009a)

Kaehr, Rudolf, Toth's semiotic diamonds. In: http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf (2009b)

Kaehr, Rudolf, Diamond relations. In: http://www.thinkartlab.com/pkl/lola/Diamond%20Relations/Diamond%20Relations.pdf (2009c)

Stiebing, Hans Michael, Zusammenfassungs- und Klassifikationsschemata von Wissenschaften und Theorien auf semiotischer und fundamentalkategorialer Basis. PhD dissertation, Stuttgart 1978

Toth, Alfred, 3-dimensionale Semiotik. In: Electronic Journal for Mathematical Semiotics, 2008

Toth, Alfred, The Trip into the Light. In: Electronic Journal for Mathematical Semiotics, 2009

3-contextural 3-adic semiotic systems

1. The following 3-contextural-3-adic semiotic matrix, suggested by Kaehr (2008)

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

is not the only 3-contextural 3-adic semiotic matrix. Based on Günther (1979, pp. 231 ss.), we shall state two basic rules for the design of semiotic matrices:

1. The first element of the first row and column, A(1,1) = 1.
2. The matrix must contain at least once the three fundamental categories (1, 2, 3) making up a 3-adic matrix.

However, rule 2 allows a big number of matrices which consist mainly out of 1's or 2's or 3's and thus reduce the changes that a sub-signs appear in more than contexture, massively. Therefore, it seems to be appropriate if we restrict the above two rules by the following additional rule:

3. The 1. row of a 3-contextural 3-adic semiotic matrix contain only permutations of {1, 3};
   The 2. row of a 3-contextural 3-adic semiotic matrix contain only permutations of {1, 2};
   The 3. row of a 3-contextural 3-adic semiotic matrix contain only permutations of {2, 3}.

Since the elements A(x,x) or i = y, i.e. the elements of the main diagonal are the only elements in a 3-contextural matrix that lie in 3 contextures, we get the following possibilities for the 1. row of a 3-contextural 3-adic semiotic matrix:

A(1,1) = {(1,2), (1,3), (2,3)}
A(1,2) = A(1,3) = {1, 2, 3}
Then, we have for the 2. and 3. row:
A(2,1) = A(2,3) = {1, 2, 3}
A(2,2) = {(1,2), (1,3), (2,3)}

A(3,1) = A(3,2) = {1, 2, 3}
A(3,3) = {(1,2), (1,3), (2,3)}

2. We will now have a look at the 48 different 3-contextural 3-adic matrices.

(1,2)	1	2
1	(1.3)	3
2	3	(2,3)

1

(1,2)	1	2
1	(2.3)	3
2	3	(1,3)

2

(1,3)	1	2
1	(1.2)	3
2	3	(2,3)

3

(1,3)	1	2
1	(2.3)	3
2	3	(1,2)

4

(2,3)	1	2
1	(1.3)	3
2	3	(1,2)

5

(2,3)	1	2
1	(1,2)	3
2	3	(1,3)

6

(1,2)	2	1
2	(1,3)	3
1	3	(2,3)

7

(1,2)	2	1
2	(2,3)	3
1	3	(1,3)

8

(1,3)	2	1
2	(1,2)	3
1	3	(2,3)

9

(1,3)	2	1
2	(2,3)	3
1	3	(1,2)

10

(2,3)	2	1
2	(1,3)	3
1	3	(1,2)

11

(2,3)	2	1
2	(1,2)	3
1	3	(1,3)

12

(1,2)	1	3
1	(1,3)	2
3	2	(2,3)

13

(1,2)	1	3
1	(2,3)	2
3	2	(1,3)

14

(1,3)	1	3
1	(1,2)	2
3	2	(2,3)

15

(1,3)	1	3
1	(2,3)	2
3	2	(1,3)

16

(2,3)	1	3
1	(1,3)	2
3	2	(1,2)

17

(2,3)	1	3
1	(1,2)	2
3	2	(1,3)

18

(1,2)	3	1
3	(1,3)	2
1	2	(2,3)

19

(1,2)	3	1
3	(2,3)	2
1	2	(1,3)

20

(1,3)	3	1
3	(1,2)	2
1	2	(2,3)

21

(1,3)	3	1
3	(2,3)	2
1	2	(1,3)

22

(2,3)	3	1
3	(1,3)	2
1	2	(1,2)

23

(2,3)	3	1
3	(1,2)	2
1	2	(1,3)

24

(1,2)	1	2
1	(1,3)	3
2	3	(2,3)

25

(1,2)	1	2
1	(2,3)	3
2	3	(1,3)

26

(1,3)	1	2
1	(1,2)	3
2	3	(2,3)

27

(1,3)	1	2
1	(2,3)	3
2	3	(1,3)

28

(2,3)	1	2
1	(1,3)	3
2	3	(1,2)

29

(2,3)	1	2
1	(1,2)	3
2	3	(1,3)

30

(1,2)	2	1
2	(1,3)	3
1	3	(2,3)

31

(1,2)	2	1
2	(2,3)	3
1	3	(1,3)

32

(1,3)	2	1
2	(1,2)	3
1	3	(2,3)

33

(1,3)	2	1
2	(2,3)	3
1	3	(1,3)

34

(2,3)	2	1
2	(1,2)	3
1	3	(1,3)

35

(2,3)	2	1
2	(1,3)	3
1	3	(1,2)

36

(1,2)	1	3
1	(1,3)	2
3	2	(2,3)

37

(1,2)	1	3
1	(2,3)	2
3	2	(1,3)

38

(1,3)	1	3
1	(1,2)	2
3	2	(2,3)

39

(1,3)	1	3
1	(2,3)	2
3	2	(1,3)

40

(2,3)	1	3
1	(1,3)	2
3	2	(1,2)

41

(2,3)	1	3
1	(1,2)	2
3	2	(1,3)

42

(1,2)	3	1
3	(1,3)	2
1	2	(2,3)

43

(1,2)	3	1
3	(2,3)	2
1	2	(1,3)

44

(1,3)	3	1
3	(1,2)	2
1	2	(2,3)

45

(1,3)	3	1
3	(2,3)	2
1	2	(1,3)

46

(2,3)	3	1
3	(1,3)	2
1	2	(1,2)

47

(2,3)	3	1
3	(1,2)	2
1	2	(1,3)

48

3. Finally, we come to the actual 3-contextural 3-adic semiotic systems. They are based on the 48 3×3 3-contextural matrices, but it is not enough anymore to note them in the form of semiotic dual systems consisting of sign class plus dual reality thematic. First, polycontextural sign classes are not dual, but complementary, since not only the sub-signs, but also their indices are converted. And, generally, as has

been pointed out in earlier works, we have now to distinguish between 4 and not only 2 “standard semiotic forms” whose union we call in this article “semiotic system”:

1. $(a.b)_{i,j}$ 3. $(b.a)_{i,j}$
2. $(a.b)_{j,i}$ 4. $(b.a)_{j,i}$

Therefore, monocontextural dualization appear in two forms (nos. 3 and 4), but non-dualized forms do, too (nos. 1 and 2), and we better rename/name the 4 semiotic operations:

1. $Nm(a.b)_{i,j} = (a.b)_{i,j}$ (morphismic normal form)
2. $Nh(a.b)_{i,j} = (a.b)_{j,i}$ (heteromorphismic normal form)
3. $R(a.b)_{i,j} = (b.a)_{i,j}$ (reflection)
4. $D(a.b)_{i,j} = (b.a)_{j,i}$ (dualization)

(In n-contextural semiotic systems with n > 3, “mediative morphismic normal forms” appear; cf. Toth 2009.)

On the following pages, I will now present not all 48 3-contextural semiotic systems in their 4 standard semiotic forms.

3.1. 3-contextural semiotic system 1/48

Nm	**Nh**	**R**	**D**
$(3.1_2\ 2.1_1\ 1.1_{1,2})$	$(3.1_2\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_2)$	$(1.1_{2,1}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.2_1)$	$(3.1_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.1_2)$	$(2,.1_1\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.3_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.1_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{1,3}\ 1.2_1)$	$(3.2_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_3)$	$(2.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_2)$	$(3.2_3\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.2_3)$	$(3.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{2,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,2}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{2,3})$	$(3.1_2\ 3.2_3\ 3.3_{3,2})$

3.2. 3-contextural semiotic system 2/48

Nm	**Nh**	**R**	**D**
$(3.1_2\ 2.1_1\ 1.1_{1,2})$	$(3.1_2\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_2)$	$(1.1_{2,1}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$

$(3.1_{2}\ 2.2_{2,3}\ 1.2_{1})$	$(3.1_{2}\ 2.2_{3,2}\ 1.2_{1})$	$(1.2_{1}\ 2.2_{2,3}\ 3.1_{2})$	$(2.1_{1}\ 2.2_{3,2}\ 1.3_{2})$
$(3.1_{2}\ 2.2_{2,3}\ 1.3_{2})$	$(3.1_{2}\ 2.2_{3,2}\ 1.3_{2})$	$(1.3_{2}\ 2.2_{2,3}\ 3.1_{2})$	$(3.1_{2}\ 2.2_{3,2}\ 1.3_{2})$
$(3.1_{2}\ 2.3_{3}\ 1.3_{2})$	$(3.1_{2}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.1_{2})$	$(3.1_{2}\ 3.2_{3}\ 1.3_{2})$
$(3.2_{3}\ 2.2_{2,3}\ 1.2_{1})$	$(3.2_{3}\ 2.2_{3,2}\ 1.2_{1})$	$(1.2_{1}\ 2.2_{2,3}\ 3.2_{3})$	$(2.1_{1}\ 2.2_{3,2}\ 2.3_{3})$
$(3.2_{3}\ 2.2_{2,3}\ 1.3_{2})$	$(3.2_{3}\ 2.2_{3,2}\ 1.3_{2})$	$(1.3_{2}\ 2.2_{2,3}\ 3.2_{3})$	$(3.1_{2}\ 2.2_{3,2}\ 2.3_{3})$
$(3.2_{3}\ 2.3_{3}\ 1.3_{2})$	$(3.2_{3}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.2_{3})$	$(3.1_{2}\ 3.2_{3}\ 2.3_{3})$
$(3.3_{1,3}\ 2.3_{3}\ 1.3_{2})$	$(3.3_{3,1}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.3_{1,3})$	$(3.1_{2}\ 3.2_{3}\ 3.3_{3,1})$

3.3. 3-contextural semiotic system 3/48

Nm	**Nh**	**R**	**D**
$(3.1_{2}\ 2.1_{1}\ 1.1_{1,3})$	$(3.1_{2}\ 2.1_{1}\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_{1}\ 3.1_{2})$	$(1.1_{3,1}\ 1.2_{1}\ 1.3_{2})$
$(3.1_{2}\ 2.1_{1}\ 1.2_{1})$	$(3.1_{2}\ 2.1_{1}\ 1.2_{1})$	$(1.2_{1}\ 2.1_{1}\ 3.1_{2})$	$(2.1_{1}\ 1.2_{1}\ 1.3_{2})$
$(3.1_{2}\ 2.1_{1}\ 1.3_{2})$	$(3.1_{2}\ 2.1_{1}\ 1.3_{2})$	$(1.3_{2}\ 2.1_{1}\ 3.1_{2})$	$(3.1_{2}\ 1.2_{1}\ 1.3_{2})$
$(3.1_{2}\ 2.2_{1,2}\ 1.2_{1})$	$(3.1_{2}\ 2.2_{2,1}\ 1.2_{1})$	$(1.2_{1}\ 2.2_{1,2}\ 3.1_{2})$	$(2.1_{1}\ 2.2_{1,2}\ 1.3_{2})$
$(3.1_{2}\ 2.2_{1,2}\ 1.3_{2})$	$(3.1_{2}\ 2.2_{2,1}\ 1.3_{2})$	$(1.3_{2}\ 2.2_{1,2}\ 3.1_{2})$	$(3.1_{2}\ 2.2_{1,2}\ 1.3_{2})$
$(3.1_{2}\ 2.3_{3}\ 1.3_{2})$	$(3.1_{2}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.1_{2})$	$(3.1_{2}\ 3.2_{3}\ 1.3_{2})$
$(3.2_{3}\ 2.2_{1,2}\ 1.2_{1})$	$(3.2_{3}\ 2.2_{2,1}\ 1.2_{1})$	$(1.2_{1}\ 2.2_{1,2}\ 3.2_{3})$	$(2.1_{1}\ 2.2_{1,2}\ 2.3_{3})$
$(3.2_{3}\ 2.2_{1,2}\ 1.3_{2})$	$(3.2_{3}\ 2.2_{2,1}\ 1.3_{2})$	$(1.3_{2}\ 2.2_{1,2}\ 3.2_{3})$	$(3.1_{2}\ 2.2_{1,2}\ 2.3_{3})$
$(3.2_{3}\ 2.3_{3}\ 1.3_{2})$	$(3.2_{3}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.2_{3})$	$(3.1_{2}\ 3.2_{3}\ 2.3_{3})$
$(3.3_{2,3}\ 2.3_{3}\ 1.3_{2})$	$(3.3_{3,2}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.3_{2,3})$	$(3.1_{2}\ 3.2_{3}\ 3.3_{3,2})$

3.4. 3-contextural semiotic system 4/48

Nm	**Nh**	**R**	**D**
$(3.1_{2}\ 2.1_{1}\ 1.1_{1,3})$	$(3.1_{2}\ 2.1_{1}\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_{1}\ 3.1_{2})$	$(1.1_{3,1}\ 1.2_{1}\ 1.3_{2})$
$(3.1_{2}\ 2.1_{1}\ 1.2_{1})$	$(3.1_{2}\ 2.1_{1}\ 1.2_{1})$	$(1.2_{1}\ 2.1_{1}\ 3.1_{2})$	$(2.1_{1}\ 1.2_{1}\ 1.3_{2})$
$(3.1_{2}\ 2.1_{1}\ 1.3_{2})$	$(3.1_{2}\ 2.1_{1}\ 1.3_{2})$	$(1.3_{2}\ 2.1_{1}\ 3.1_{2})$	$(3.1_{2}\ 1.2_{1}\ 1.3_{2})$
$(3.1_{2}\ 2.2_{2,3}1.2_{1})$	$(3.1_{2}\ 2.2_{3,2}1.2_{1})$	$(1.2_{1}\ 2.2_{2,3}\ 3.1_{2})$	$(2.1_{1}\ 2.2_{3,2}\ 1.3_{2})$
$(3.1_{2}\ 2.2_{2,3}\ 1.3_{2})$	$(3.1_{2}\ 2.2_{3,2}\ 1.3_{2})$	$(1.3_{2}\ 2.2_{2,3}\ 3.1_{2})$	$(3.1_{2}\ 2.2_{3,2}\ 1.3_{2})$
$(3.1_{2}\ 2.3_{3}\ 1.3_{2})$	$(3.1_{2}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.1_{2})$	$(3.1_{2}\ 3.2_{3}\ 1.3_{2})$
$(3.2_{3}\ 2.2_{2,3}\ 1.2_{1})$	$(3.2_{3}\ 2.2_{3,2}\ 1.2_{1}))$	$(1.2_{1}\ 2.2_{2,3}\ 3.2_{3})$	$(2.1_{1}\ 2.2_{3,2}\ 2.3_{3})$
$(3.2_{3}\ 2.2_{2,3}\ 1.3_{2})$	$(3.2_{3}\ 2.2_{3,2}\ 1.3_{2})$	$(1.3_{2}\ 2.2_{2,3}\ 3.2_{3})$	$(3.1_{2}\ 2.2_{3,2}\ 2.3_{3})$
$(3.2_{3}\ 2.3_{3}\ 1.3_{2})$	$(3.2_{3}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.2_{3})$	$(3.1_{2}\ 3.2_{3}\ 2.3_{3})$
$(3.3_{1,2}\ 2.3_{3}\ 1.3_{2})$	$(3.3_{2,1}\ 2.3_{3}\ 1.3_{2})$	$(1.3_{2}\ 2.3_{3}\ 3.3_{1,2})$	$(3.1_{2}\ 3.2_{3}\ 3.3_{2,1})$

3.5. 3-contextural semiotic system 5/48

Nm	**Nh**	**R**	**D**
$(3.1_{2}\ 2.1_{1}\ 1.1_{2,3})$	$(3.1_{2}\ 2.1_{1}\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_{1}\ 3.1_{2})$	$(1.1_{3,2}\ 1.2_{1}\ 1.3_{2})$
$(3.1_{2}\ 2.1_{1}\ 1.2_{1})$	$(3.1_{2}\ 2.1_{1}\ 1.2_{1})$	$(1.2_{1}\ 2.1_{1}\ 3.1_{2})$	$(3.1_{1}\ 1.2_{1}\ 1.3_{2})$

$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.2_1)$	$(3.1_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.1_2)$	$(2.1_1\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.3_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.1_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{1,3}\ 1.2_1)$	$(3.2_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_3)$	$(2.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_2)$	$(3.2_3\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.2_3)$	$(3.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{1,2}\ 2.3_3\ 1.3_2)$	$(3.3_{2,1}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{1,2})$	$(3.1_2\ 3.2_3\ 3.3_{2,1})$

3.6. 3-contextural semiotic system 6/48

Nm	**Nh**	**R**	**D**
$(3.1_2\ 2.1_1\ 1.1_{2,3})$	$(3.1_2\ 2.1_1\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_1\ 3.1_2)$	$(1.1_{3,2}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{1,2}\ 1.2_1)$	$(3.1_2\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.1_2)$	$(2.1_1\ 2.2_{2,1}\ 1.3_2)$
$(3.1_2\ 2.2_{1,2}\ 1.3_2)$	$(3.1_2\ 2.2_{2,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,2}\ 3.1_2)$	$(3.1_2\ 2.2_{2,1}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{1,2}\ 1.2_1)$	$(3.2_3\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.2_3)$	$(2.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_2)$	$(3.2_3\ 2.2_{2,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,2}\ 3.2_3)$	$(3.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,1}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3\ 3.3_{1,3})$	$(3.1_2\ 3.2\ 3.3_{3,1})$

3.7. 3-contextural semiotic system 7/48

Nm	**Nh**	**R**	**D**
$(3.1_1\ 2.1_2\ 1.1_{1,2})$	$(3.1_1\ 2.1_2\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_2\ 3.1_1)$	$(1.1_{2,1}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_2)$	$(3.1_1\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.1_1)$	$(2.1_2\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,3}\ 1.2_2)$	$(3.2_3\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.2_3)$	$(2.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_1)$	$(3.2_3\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_3)$	$(3.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{2,3}\ 2.3_3\ 1.3_1)$	$(3.3_{3,2}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{2,3})$	$(3.1_1\ 3.2_3\ 3.3_{3,2})$

3.8. 3-contextural semiotic system 8/48

Nm	**Nh**	**R**	**D**
$(3.1_1\ 2.1_2\ 1.1_{1,2})$	$(3.1_1\ 2.1_2\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_2\ 3.1_1)$	$(1.1_{2,1}\ 1.2_2\ 1.3_1)$

$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.2_2)$	$(3.1_1\ 2.2_{3,2}\ 1.2_2)$	$(1.2_2\ 2.2_{2,3}\ 3.1_1)$	$(2.1_2\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3.2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2\ 1.3_1)$
$(3.2_3\ 2.2_{2,3}\ 1.2_2)$	$(3.2_3\ 2.2_{3,.2}\ 1.2_2)$	$(1.2_2\ 2.2_{2,3}\ 3.2_3)$	$(2.1_2\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.2_{2,3}\ 1.3_1)$	$(3.2_3\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.2_3)$	$(3.1_1\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_1)$	$(3.3_{3,1}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{1,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,1})$

3.9. 3-contextural semiotic system 9/48

Nm	**Nh**	**R**	**D**
$(3.1_1\ 2.1_2\ 1.1_{1,3})$	$(3.1_1\ 2.1_2\ 1.1_{3,1})$	$(1.1_{1,3}\ 1.2_2\ 1.3_1)$	$(1.1_{3,1}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(2.1_2\ 1.2_2\ 1.3_1)$	$(2.1\ 1.2\ 1_2.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_2)$	$(3.1_1\ 2.2_{2,1}\ 1.2_2)$	$(2.1_2\ 2.2_{1,2}\ 1.3_1)$	$(2.1_2\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$	$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,2}\ 1.2_2)$	$(3.2_3\ 2.2_{2,1}\ 1.2_2)$	$(2.1_2\ 2.2_{1,2}\ 2.3_3)$	$(2.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_1)$	$(3.2_3\ 2.2_{12,1}\ 1.3_1)$	$(3.1_1\ 2.2_{1,2}\ 2.3_3)$	$(3.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.1_1\ 3.2_3\ 2.3_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{2,3}\ 2.3_3\ 1.3_1)$	$(3.3_{3,2}\ 2.3_3\ 1.3_1)$	$(3.1_1\ 3.2_3\ 3.3_{2,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,2})$

3.10. 3-contextural semiotic system 10/48

Nm	**Nh**	**R**	**D**
$(3.1_1\ 2.1_2\ 1.1_{2,3})$	$(3.1_1\ 2.1_2\ 1.1_{3,2})$	$(1.1_{2,3}\ 1.2_2\ 1.3_1)$	$(1.1_{3,2}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(2.1_2\ 1.2_2\ 1.3_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_1\ 1.3_1)$	$(3.1_1\ 2.1_1\ 1.3_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_2)$	$(3.1_1\ 2.2_{2,1}\ 1.2_2)$	$(2.1_2\ 2.2_{1,2}\ 1.3_1)$	$(2.1_2\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$	$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,2}\ 1.2_2)$	$(3.2_3\ 2.2_{2,1}\ 1.2_2)$	$(2.1_2\ 2.2_{1,2}\ 2.3_3)$	$(2.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_1)$	$(3.2_3\ 2.2_{2,1}\ 1.3_1)$	$(3.1_1\ 2.2_{1,2}\ 2.3_3)$	$(3.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.1_1\ 3.2_3\ 2.3_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,1}\ 2.3_3\ 1.3_2)$	$(3.1_1\ 3.2_3\ 3.3_{1,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,1})$

3.11. 3-contextural semiotic system 11/48

Nm	Nh	R	D
$(3.1_1\ 2.1_2\ 1.1_{2,3})$	$(3.1_1\ 2.1_2\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_2\ 3.1_1)$	$(1.1_{3,2}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_2)$	$(3.1_1\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.1_1)$	$(2.1_2\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,3}\ 1.2_2)$	$(3.2_3\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.2_3)$	$(2.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_1)$	$(3.2_3\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_3)$	$(3.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,2}\ 2.3_3\ 1.3_1)$	$(3.3_{2,1}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{1,2})$	$(3.1_1\ 2.3_3\ 3.3_{2,1})$

3.12. 3-contextural semiotic system 12/48

Nm	Nh	R	D
$(3.1_1\ 2.1_2\ 1.1_{2,3})$	$(3.1_1\ 2.1_2\ 1.1_{2,3})$	$(1.1_{2,3}\ 2.1\ 3.1_1)$	$(1.1_{3,2}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2\ 2.1\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_2)$	$(3.1_1\ 2.2_{1,2}\ 1.2_2)$	$(1.2\ 2.2_{1,2}\ 3.1_1)$	$(2.1_2\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.1_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,2}\ 1.2_2)$	$(3.2_3\ 2.2_{1,2}\ 1.2_2)$	$(1.2\ 2.2_{1,2}\ 3.2)$	$(2.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_1)$	$(3.2_3\ 2.2_{1,2}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.2)$	$(3.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3\ 3.2)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_1)$	$(3.3_{1,3}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3\ 3.3_{1,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,1})$

3.13. 3-contextural semiotic system 13/48

Nm	Nh	R	D
$(3.1_3\ 2.1_1\ 1.1_{1,2})$	$(3.1_3\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_3)$	$(1.1_{2,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_1\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,2}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{2,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,2})$

3.14. 3-contextural semiotic system 14/48

Nm	Nh	R	D
$(3.1_3\ 2.1_1\ 1.1_{1,2})$	$(3.1_3\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_3)$	$(1.1_{2,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_1\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{2,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.2_{2,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,1})$

3.15. 3-contextural semiotic system 15/48

Nm	Nh	R	D
$(3.1_3\ 2.1_1\ 1.1_{1,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_1\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	$(3.1_3\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.1_3)$	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.1_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	$(3.2_2\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.2_2)$	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	$(3.2_2\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.2_2)$	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,2}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{2,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,2})$

3.16. 3-contextural semiotic system 16/48

Nm	Nh	R	D
$(3.1_3\ 2.1_1\ 1.1_{1,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_1\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{2,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.2_{2,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,1})$

3.17. 3-contextural semiotic system 17/48

Nm	**Nh**	**R**	**D**
$(3.1_3\ 2.1_1\ 1.1_{2,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,2}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_1\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,2}\ 2.3_2\ 1.3_3)$	$(3.3_{2,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,2})$	$(3.1_3\ 2.3_2\ 3.3_{2,1})$

3.18. 3-contextural semiotic system 18/48

Nm	**Nh**	**R**	**D**
$(3.1_3\ 2.1_1\ 1.1_{2,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,2}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_1\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	$(3.1_3\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.1_3)$	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.1_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	$(3.2_2\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.2_2)$	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	$(3.2_2\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.2_2)$	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,1})$

3.19. 3-contextural semiotic system 19/48

Nm	**Nh**	**R**	**D**
$(3.1_1\ 2.1_3\ 1.1_{1,2})$	$(3.1_1\ 2.1_3\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_3\ 3.1_1)$	$(1.1_{2,1}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_1\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.31_3)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_3)$	$(3.1_1\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.1_1)$	$(2.1_3\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{1,3}\ 1.2_3)$	$(3.2_2\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.2_2)$	$(2.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_1)$	$(3.2_2\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_2)$	$(3.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_1)$	$(3.3_{3,2}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{2,3})$	$(3.1_1\ 2.3_2\ 3.3_{3,2})$

3.20. 3-contextural semiotic system 20/48

Nm	Nh	R	D
(3.1_1 2.1_3 $1.1_{1,2}$)	(3.1_1 2.1_3 $1.1_{2,1}$)	($1.1_{1,2}$ 2.1_3 3.1_1)	($1.1_{2,1}$ 1.2_3 1.3_1)
(3.1_1 2.1_3 1.2_3)	(3.1_1 2.1_3 1.2_3)	(1.2_1 2.1_3 3.1_1)	(2.1_3 1.2_3 1.31_3)
(3.1_1 2.1_3 1.3_1)	(3.1_1 2.1_3 1.3_1)	(1.3_1 2.1_3 3.1_1)	(3.1_1 1.2_3 1.3_1)
(3.1_1 $2.2_{2,3}$ 1.2_3)	(3.1_1 $2.2_{3,2}$ 1.2_3)	(1.2_3 $2.2_{2,3}$ 3.1_1)	(2.1_3 $2.2_{3,2}$ 1.3_1)
(3.1_1 $2.2_{2,3}$ 1.3_1)	(3.1_1 $2.2_{3,2}$ 1.3_1)	(1.3_1 $2.2_{2,3}$ 3.1_1)	(3.1_1 $2.2_{3,2}$ 1.3_1)
(3.1_1 2.3_2 1.3_1)	(3.1_1 2.3_2 1.3_1)	(1.3_1 2.3_2 3.1_1)	(3.1_1 3.2_2 1.3_1)
(3.2_2 $2.2_{2,3}$ 1.2_3)	(3.2_2 $2.2_{3,2}$ 1.2_3)	(1.2_3 $2.2_{2,3}$ 3.2_2)	(2.1_3 $2.2_{3,2}$ 2.3_2)
(3.2_2 $2.2_{2,3}$ 1.3_1)	(3.2_2 $2.2_{3,2}$ 1.3_1)	(1.3_1 $2.2_{2,3}$ 3.2_2)	(3.1_1 $2.2_{3,2}$ 2.3_2)
(3.2_2 2.3_2 1.3_1)	(3.2_2 2.3_2 1.3_1)	(1.3_1 2.3_2 3.2_2)	(3.1_1 3.2_2 2.3_2)
($3.3_{1,3}$ 2.3_2 1.3_1)	($3.3_{3,1}$ 2.3_2 1.3_1)	(1.3_1 2.3_2 $3.3_{1,3}$)	(3.1_1 2.3_2 $3.3_{3,1}$)

3.21. 3-contextural semiotic system 21/48

Nm	Nh	R	D
(3.1_1 2.1_3 $1.1_{1,3}$)	(3.1_1 2.1_3 $1.1_{3,1}$)	($1.1_{1,3}$ 2.1_3 3.1_1)	($1.1_{3,1}$ 1.2_3 1.3_1)
(3.1_1 2.1_3 1.2_3)	(3.1_1 2.1_3 1.2_3)	(1.2_1 2.1_3 3.1_1)	(2.1_3 1.2_3 1.31_3)
(3.1_1 2.1_3 1.3_1)	(3.1_1 2.1_3 1.3_1)	(1.3_1 2.1_3 3.1_1)	(3.1_1 1.2_3 1.3_1)
(3.1_1 $2.2_{1,2}$ 1.2_3)	(3.1_1 $2.2_{2,1}$ 1.2_3)	(1.2_3 $2.2_{1,2}$ 3.1_1)	(2.1_3 $2.2_{2,1}$ 1.3_1)
(3.1_1 $2.2_{1,2}$ 1.3_1)	(3.1_1 $2.2_{2,1}$ 1.3_1)	(1.3_1 $2.2_{1,2}$ 3.1_1)	(3.1_1 $2.2_{2,1}$ 1.3_1)
(3.1_1 2.3_2 1.3_1)	(3.1_1 2.3_2 1.3_1)	(1.3_1 2.3_2 3.1_1)	(3.1_1 3.2_2 1.3_1)
(3.2_2 $2.2_{1,2}$ 1.2_3)	(3.2_2 $2.2_{2,1}$ 1.2_3)	(1.2_3 $2.2_{1,2}$ 3.2_2)	(2.1_3 $2.2_{2,1}$ 2.3_2)
(3.2_2 $2.2_{1,2}$ 1.3_1)	(3.2_2 $2.2_{2,1}$ 1.3_1)	(1.3_1 $2.2_{1,2}$ 3.2_2)	(3.1_1 $2.2_{2,1}$ 2.3_2)
(3.2_2 2.3_2 1.3_1)	(3.2_2 2.3_2 1.3_1)	(1.3_1 2.3_2 3.2_2)	(3.1_1 3.2_2 2.3_2)
($3.3_{2,3}$ 2.3_2 1.3_1)	($3.3_{3,2}$ 2.3_2 1.3_1)	(1.3_1 2.3_2 $3.3_{2,3}$)	(3.1_1 2.3_2 $3.3_{3,2}$)

3.22. 3-contextural semiotic system 22/48

Nm	Nh	R	D
(3.1_1 2.1_3 $1.1_{1,3}$)	(3.1_1 2.1_3 $1.1_{3,1}$)	($1.1_{1,3}$ 2.1_3 3.1_1)	($1.1_{3,1}$ 1.2_3 1.3_1)
(3.1_1 2.1_3 1.2_3)	(3.1_1 2.1_3 1.2_3)	(1.2_1 2.1_3 3.1_1)	(2.1_3 1.2_3 1.31_3)
(3.1_1 2.1_3 1.3_1)	(3.1_1 2.1_3 1.3_1)	(1.3_1 2.1_3 3.1_1)	(3.1_1 1.2_3 1.3_1)
(3.1_1 $2.2_{2,3}$ 1.2_3)	(3.1_1 $2.2_{3,2}$ 1.2_3)	(1.2_3 $2.2_{2,3}$ 3.1_1)	(2.1_3 $2.2_{3,2}$ 1.3_1)
(3.1_1 $2.2_{2,3}$ 1.3_1)	(3.1_1 $2.2_{3,2}$ 1.3_1)	(1.3_1 $2.2_{2,3}$ 3.1_1)	(3.1_1 $2.2_{3,2}$ 1.3_1)
(3.1_1 2.3_2 1.3_1)	(3.1_1 2.3_2 1.3_1)	(1.3_1 2.3_2 3.1_1)	(3.1_1 3.2_2 1.3_1)
(3.2_2 $2.2_{2,3}$ 1.2_3)	(3.2_2 $2.2_{3,2}$ 1.2_3)	(1.2_3 $2.2_{2,3}$ 3.2_2)	(2.1_3 $2.2_{3,2}$ 2.3_2)
(3.2_2 $2.2_{2,3}$ 1.3_1)	(3.2_2 $2.2_{3,2}$ 1.3_1)	(1.3_1 $2.2_{2,3}$ 3.2_2)	(3.1_1 $2.2_{3,2}$ 2.3_2)
(3.2_2 2.3_2 1.3_1)	(3.2_2 2.3_2 1.3_1)	(1.3_1 2.3_2 3.2_2)	(3.1_1 3.2_2 2.3_2)
($3.3_{1,2}$ 2.3_2 1.3_1)	($3.3_{2,1}$ 2.3_2 1.3_1)	(1.3_1 2.3_2 $3.3_{1,2}$)	(3.1_1 2.3_2 $3.3_{2,1}$)

3.23. 3-contextural semiotic system 23/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{2,3})$	$(3.1_1\ 2.1_3\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_3\ 3.1_1)$	$(1.1_{3,2}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_1\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.31_3)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_3)$	$(3.1_1\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.1_1)$	$(2.1_3\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{1,3}\ 1.2_3)$	$(3.2_2\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.2_2)$	$(2.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_1)$	$(3.2_2\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_2)$	$(3.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{1,2}\ 2.3_2\ 1.3_1)$	$(3.3_{2,1}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{1,2})$	$(3.1_1\ 2.3_2\ 3.3_{2,1})$

3.24. 3-contextural semiotic system 24/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{2,3})$	$(3.1_1\ 2.1_3\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_3\ 3.1_1)$	$(1.1_{3,2}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_1\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_3)$	$(3.1_1\ 2.2_{2,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,2}\ 3.1_1)$	$(2.1_3\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.1_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{1,2}\ 1.2_3)$	$(3.2_2\ 2.2_{2,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,2}\ 3.2_2)$	$(2.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_1)$	$(3.2_2\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.2_2)$	$(3.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_1)$	$(3.3_{3,1}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{1,3})$	$(3.1_1\ 2.3_2\ 3.3_{3,1})$

3.25. 3-contextural semiotic system 25/48

Nm	Nh	R	D
$(3.1_2\ 2.1_1\ 1.1_{1,2})$	$(3.1_2\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_2)$	$(1.1_{2,1}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.2_1)$	$(3.1_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.1_2)$	$(2.1_1\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.3_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.1_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{1,3}\ 1.2_1)$	$(3.2_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_3)$	$(2.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_2)$	$(3.2_3\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.2_3)$	$(3.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{2,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,2}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{2,3})$	$(3.1_2\ 2.3_3\ 3.3_{3,2})$

3.26. 3-contextural semiotic system 26/48

Nm	Nh	R	D
$(3.1_2\ 2.1_1\ 1.1_{1,2})$	$(3.1_2\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_2)$	$(1.1_{2,1}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{2,3}\ 1.2_1)$	$(3.1_2\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.1_2)$	$(2.1_1\ 2.2_{3,2}\ 1.3_2)$
$(3.1_2\ 2.2_{2,3}\ 1.3_2)$	$(3.1_2\ 2.2_{3,2}\ 1.3_2)$	$(1.3_2\ 2.2_{2,3}\ 3.1_2)$	$(3.1_2\ 2.2_{3,2}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{2,3}\ 1.2_1)$	$(3.2_3\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.2_3)$	$(2.1_1\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.2_{2,3}\ 1.3_2)$	$(3.2_3\ 2.2_{3,2}\ 1.3_2)$	$(1.3_2\ 2.2_{2,3}\ 3.2_3)$	$(3.1_2\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,1}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{1,3})$	$(3.1_2\ 2.3_3\ 3.3_{3,1})$

3.27. 3-contextural semiotic system 27/48

Nm	Nh	R	D
$(3.1_2\ 2.1_1\ 1.1_{1,3})$	$(3.1_2\ 2.1_1\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_1\ 3.1_2)$	$(1.1_{3,1}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{1,2}\ 1.2_1)$	$(3.1_2\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.1_2)$	$(2.1_1\ 2.2_{2,1}\ 1.3_2)$
$(3.1_2\ 2.2_{1,2}\ 1.3_2)$	$(3.1_2\ 2.2_{2,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,2}\ 3.1_2)$	$(3.1_2\ 2.2_{2,1}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{1,2}\ 1.2_1)$	$(3.2_3\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.2_3)$	$(2.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_2)$	$(3.2_3\ 2.2_{2,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,2}\ 3.2_3)$	$(3.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{2,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,2}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{2,3})$	$(3.1_2\ 2.3_3\ 3.3_{3,2})$

3.28. 3-contextural semiotic system 28/48

Nm	Nh	R	D
$(3.1_2\ 2.1_1\ 1.1_{1,3})$	$(3.1_2\ 2.1_1\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_1\ 3.1_2)$	$(1.1_{3,1}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{2,3}\ 1.2_1)$	$(3.1_2\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.1_2)$	$(2.1_1\ 2.2_{3,2}\ 1.3_2)$
$(3.1_2\ 2.2_{2,3}\ 1.3_2)$	$(3.1_2\ 2.2_{3,2}\ 1.3_2)$	$(1.3_2\ 2.2_{2,3}\ 3.1_2)$	$(3.1_2\ 2.2_{3,2}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{2,3}\ 1.2_1)$	$(3.2_3\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.2_3)$	$(2.1_1\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.2_{2,3}\ 1.3_2)$	$(3.2_3\ 2.2_{3,2}\ 1.3_2)$	$(1.3_2\ 2.2_{2,3}\ 3.2_3)$	$(3.1_2\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,1}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{1,3})$	$(3.1_2\ 2.3_3\ 3.3_{3,1})$

3.29. 3-contextural semiotic system 29/48

Nm	Nh	R	D
$(3.1_2\ 2.1_1\ 1.1_{2,3})$	$(3.1_2\ 2.1_1\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_1\ 3.1_2)$	$(1.1_{3,2}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.2_1)$	$(3.1_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.1_2)$	$(2.1_1\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.2_{1,3}\ 1.3_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.1_2)$	$(3.1_2\ 2.2_{3,1}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{1,3}\ 1.2_1)$	$(3.2_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_3)$	$(2.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_2)$	$(3.2_3\ 2.2_{3,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,3}\ 3.2_3)$	$(3.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{1,2}\ 2.3_3\ 1.3_2)$	$(3.3_{2,1}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{1,2})$	$(3.1_2\ 2.3_3\ 3.3_{2,1})$

3.30. 3-contextural semiotic system 30/48

Nm	Nh	R	D
$(3.1_2\ 2.1_1\ 1.1_{2,3})$	$(3.1_2\ 2.1_1\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_1\ 3.1_2)$	$(1.1_{3,2}\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.2_1)$	$(3.1_2\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_2)$	$(2.1_1\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.1_1\ 1.3_2)$	$(3.1_2\ 2.1_1\ 1.3_2)$	$(1.3_2\ 2.1_1\ 3.1_2)$	$(3.1_2\ 1.2_1\ 1.3_2)$
$(3.1_2\ 2.2_{1,2}\ 1.2_1)$	$(3.1_2\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.1_2)$	$(2.1_1\ 2.2_{2,1}\ 1.3_2)$
$(3.1_2\ 2.2_{1,2}\ 1.3_2)$	$(3.1_2\ 2.2_{2,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,2}\ 3.1_2)$	$(3.1_2\ 2.2_{2,1}\ 1.3_2)$
$(3.1_2\ 2.3_3\ 1.3_2)$	$(3.1_2\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.1_2)$	$(3.1_2\ 3.2_3\ 1.3_2)$
$(3.2_3\ 2.2_{1,2}\ 1.2_1)$	$(3.2_3\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.2_3)$	$(2.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_2)$	$(3.2_3\ 2.2_{2,1}\ 1.3_2)$	$(1.3_2\ 2.2_{1,2}\ 3.2_3)$	$(3.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_2)$	$(3.2_3\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.2_3)$	$(3.1_2\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_2)$	$(3.3_{3,1}\ 2.3_3\ 1.3_2)$	$(1.3_2\ 2.3_3\ 3.3_{1,3})$	$(3.1_2\ 2.3_3\ 3.3_{3,1})$

3.31. 3-contextural semiotic system 31/48

Nm	Nh	R	D
$(3.1_1\ 2.1_2\ 1.1_{1,2})$	$(3.1_1\ 2.1_2\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_2\ 3.1_1)$	$(1.1_{2,1}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_2)$	$(3.1_1\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.1_1)$	$(2.1_2\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,3}\ 1.2_2)$	$(3.2_3\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.2_3)$	$(2.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_1)$	$(3.2_3\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_3)$	$(3.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{2,3}\ 2.3_3\ 1.3_1)$	$(3.3_{3,2}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{2,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,2})$

3.32. 3-contextural semiotic system 32/48

Nm	Nh	R	D
$(3.1_1\ 2.1_2\ 1.1_{1,2})$	$(3.1_1\ 2.1_2\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_2\ 3.1_1)$	$(1.1_{2,1}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.2_2)$	$(3.1_1\ 2.2_{3,2}\ 1.2_2)$	$(1.2_2\ 2.2_{2,3}\ 3.1_1)$	$(2.1_2\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{2,3}\ 1.2_2)$	$(3.2_3\ 2.2_{3,2}\ 1.2_2)$	$(1.2_2\ 2.2_{2,3}\ 3.2_3)$	$(2.1_2\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.2_{2,3}\ 1.3_1)$	$(3.2_3\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.2_3)$	$(3.1_1\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_1)$	$(3.3_{3,1}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{1,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,1})$

3.33. 3-contextural semiotic system 33/48

Nm	Nh	R	D
$(3.1_1\ 2.1_2\ 1.1_{1,3})$	$(3.1_1\ 2.1_2\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_2\ 3.1_1)$	$(1.1_{3,1}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_2)$	$(3.1_1\ 2.2_{2,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,2}\ 3.1_1)$	$(2.1_2\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.1_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,2}\ 1.2_2)$	$(3.2_3\ 2.2_{2,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,2}\ 3.2_3)$	$(2.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_1)$	$(3.2_3\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.2_3)$	$(3.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{2,3}\ 2.3_3\ 1.3_1)$	$(3.3_{3,2}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{2,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,2})$

3.34. 3-contextural semiotic system 34/48

Nm	Nh	R	D
$(3.1_1\ 2.1_2\ 1.1_{1,3})$	$(3.1_1\ 2.1_2\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_2\ 3.1_1)$	$(1.1_{3,1}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.2_2)$	$(3.1_1\ 2.2_{3,2}\ 1.2_2)$	$(1.2_2\ 2.2_{2,3}\ 3.1_1)$	$(2.1_2\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{2,3}\ 1.2_2)$	$(3.2_3\ 2.2_{3,2}\ 1.2_2)$	$(1.2_2\ 2.2_{2,3}\ 3.2_3)$	$(2.1_2\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.2_{2,3}\ 1.3_1)$	$(3.2_3\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.2_3)$	$(3.1_1\ 2.2_{3,2}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,2}\ 2.3_3\ 1.3_1)$	$(3.3_{2,1}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{1,2})$	$(3.1_1\ 2.3_3\ 3.3_{2,1})$

3.35. 3-contextural semiotic system 35/48

Nm	**Nh**	**R**	**D**
$(3.1_1\ 2.1_2\ 1.1_{2,3})$	$(3.1_1\ 2.1_2\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_2\ 3.1_1)$	$(1.1_{3,2}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_2)$	$(3.1_1\ 2.2_{2,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,2}\ 3.1_1)$	$(2.1_2\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.1_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,2}\ 1.2_2)$	$(3.2_3\ 2.2_{2,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,2}\ 3.2_3)$	$(2.1_2\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,2}\ 1.3_1)$	$(3.2_3\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.2_3)$	$(3.1_1\ 2.2_{2,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,3}\ 2.3_3\ 1.3_1)$	$(3.3_{3,1}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{1,3})$	$(3.1_1\ 2.3_3\ 3.3_{3,1})$

3.36. 3-contextural semiotic system 36/48

Nm	**Nh**	**R**	**D**
$(3.1_1\ 2.1_2\ 1.1_{2,3})$	$(3.1_1\ 2.1_2\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_2\ 3.1_1)$	$(1.1_{3,2}\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.2_2)$	$(3.1_1\ 2.1_2\ 1.2_2)$	$(1.2_2\ 2.1_2\ 3.1_1)$	$(2.1_2\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.1_2\ 1.3_1)$	$(3.1_1\ 2.1_2\ 1.3_1)$	$(1.3_1\ 2.1_2\ 3.1_1)$	$(3.1_1\ 1.2_2\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_2)$	$(3.1_1\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.1_1)$	$(2.1_2\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_3\ 1.3_1)$	$(3.1_1\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.1_1)$	$(3.1_1\ 3.2_3\ 1.3_1)$
$(3.2_3\ 2.2_{1,3}\ 1.2_2)$	$(3.2_3\ 2.2_{3,1}\ 1.2_2)$	$(1.2_2\ 2.2_{1,3}\ 3.2_3)$	$(2.1_2\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.2_{1,3}\ 1.3_1)$	$(3.2_3\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_3)$	$(3.1_1\ 2.2_{3,1}\ 2.3_3)$
$(3.2_3\ 2.3_3\ 1.3_1)$	$(3.2_3\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.2_3)$	$(3.1_1\ 3.2_3\ 2.3_3)$
$(3.3_{1,2}\ 2.3_3\ 1.3_1)$	$(3.3_{2,1}\ 2.3_3\ 1.3_1)$	$(1.3_1\ 2.3_3\ 3.3_{1,2})$	$(3.1_1\ 2.3_3\ 3.3_{2,1})$

3.37. 3-contextural semiotic system 37/48

Nm	**Nh**	**R**	**D**
$(3.1_3\ 2.1_1\ 1.1_{1,2})$	$(3.1_3\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_3)$	$(1.1_{2,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_2\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_2\ 2.2_{1,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,2}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{2,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,2})$

3.38. 3-contextural semiotic system 38/48

Nm	**Nh**	**R**	**D**
$(3.1_3\ 2.1_1\ 1.1_{1,2})$	$(3.1_3\ 2.1_1\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_1\ 3.1_3)$	$(1.1_{2,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_2\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_2\ 2.2_{2,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{2,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.2_{2,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,1})$

3.39. 3-contextural semiotic system 39/48

Nm	**Nh**	**R**	**D**
$(3.1_3\ 2.1_1\ 1.1_{1,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_2\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	$(3.1_3\ 2.2_{2,1}\ 1.2_1)$	$(1.2_2\ 2.2_{1,2}\ 3.1_3)$	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.1_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	$(3.2_2\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.2_2)$	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	$(3.2_2\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.2_2)$	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,2}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{2,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,2})$

3.40. 3-contextural semiotic system 40/48

Nm	**Nh**	**R**	**D**
$(3.1_3\ 2.1_1\ 1.1_{1,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_2\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_2\ 2.2_{2,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.2_{2,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,2}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{2,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,2}\ 1.2_1)$	$(1.2_1\ 2.2_{2,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.2_{2,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,2}\ 1.3_3)$	$(1.3_3\ 2.2_{2,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,2}\ 2.3_2\ 1.3_3)$	$(3.3_{2,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,2})$	$(3.1_3\ 2.3_2\ 3.3_{2,1})$

3.41. 3-contextural semiotic system 41/48

Nm	Nh	R	D
$(3.1_3\ 2.1_1\ 1.1_{2,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,2}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_2\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.2_1)$	$(3.1_3\ 2.2_{3,1}\ 1.2_1)$	$(1.2_2\ 2.2_{1,3}\ 3.1_3)$	$(2.1_1\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,3}\ 1.3_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.1_3)$	$(3.1_3\ 2.2_{3,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,3}\ 1.2_1)$	$(3.2_2\ 2.2_{3,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,3}\ 3.2_2)$	$(2.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_3)$	$(3.2_2\ 2.2_{3,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,3}\ 3.2_2)$	$(3.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,2}\ 2.3_2\ 1.3_3)$	$(3.3_{2,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,2})$	$(3.1_3\ 2.3_2\ 3.3_{2,1})$

3.42. 3-contextural semiotic system 42/48

Nm	Nh	R	D
$(3.1_3\ 2.1_1\ 1.1_{2,3})$	$(3.1_3\ 2.1_1\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_1\ 3.1_3)$	$(1.1_{3,2}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1)$	$(3.1_3\ 2.1_1\ 1.2_1)$	$(1.2_1\ 2.1_1\ 3.1_3)$	$(2.1_2\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3)$	$(3.1_3\ 2.1_1\ 1.3_3)$	$(1.3_3\ 2.1_1\ 3.1_3)$	$(3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	$(3.1_3\ 2.2_{2,1}\ 1.2_1)$	$(1.2_2\ 2.2_{1,2}\ 3.1_3)$	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.1_3)$	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3)$	$(3.1_3\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.1_3)$	$(3.1_3\ 3.2_2\ 1.3_3)$
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	$(3.2_2\ 2.2_{2,1}\ 1.2_1)$	$(1.2_1\ 2.2_{1,2}\ 3.2_2)$	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	$(3.2_2\ 2.2_{2,1}\ 1.3_3)$	$(1.3_3\ 2.2_{1,2}\ 3.2_2)$	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$	$(3.2_2\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.2_2)$	$(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_3)$	$(3.3_{3,1}\ 2.3_2\ 1.3_3)$	$(1.3_3\ 2.3_2\ 3.3_{1,3})$	$(3.1_3\ 2.3_2\ 3.3_{3,1})$

3.43. 3-contextural semiotic system 43/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{1,2})$	$(3.1_1\ 2.1_3\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_3\ 3.1_1)$	$(1.1_{2,1}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_3\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_3)$	$(3.1_1\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.1_1)$	$(2.1_3\ 2.2_{3,1}\ 1.3_1)$
$(3.1_3\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{1,3}\ 1.2_3)$	$(3.2_2\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.2_2)$	$(2.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_1)$	$(3.2_2\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_2)$	$(3.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_1)$	$(3.3_{3,2}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{2,3})$	$(3.1_1\ 2.3_2\ 3.3_{3,2})$

3.44. 3-contextural semiotic system 44/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{1,2})$	$(3.1_1\ 2.1_3\ 1.1_{2,1})$	$(1.1_{1,2}\ 2.1_3\ 3.1_1)$	$(1.1_{2,1}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_3\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.2_3)$	$(3.1_1\ 2.2_{3,2}\ 1.2_3)$	$(1.2_3\ 2.2_{2,3}\ 3.1_1)$	$(2.1_3\ 2.2_{3,2}\ 1.3_1)$
$(3.1_3\ 2.2_{2,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{2,3}\ 1.2_3)$	$(3.2_2\ 2.2_{3,2}\ 1.2_3)$	$(1.2_3\ 2.2_{2,3}\ 3.2_2)$	$(2.1_3\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.2_{2,3}\ 1.3_1)$	$(3.2_2\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.2_2)$	$(3.1_1\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_1)$	$(3.3_{3,1}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{1,3})$	$(3.1_1\ 2.3_2\ 3.3_{3,1})$

3.45. 3-contextural semiotic system 45/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{1,3})$	$(3.1_1\ 2.1_3\ 1.1_{3,1})$	$(1.1_{1,3}\ 2.1_3\ 3.1_1)$	$(1.1_{3,1}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_3\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_3)$	$(3.1_1\ 2.2_{2,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,2}\ 3.1_1)$	$(2.1_3\ 2.2_{2,1}\ 1.3_1)$
$(3.1_3\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.1_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{1,2}\ 1.2_3)$	$(3.2_2\ 2.2_{2,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,2}\ 3.2_2)$	$(2.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_1)$	$(3.2_2\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.2_2)$	$(3.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_1)$	$(3.3_{3,2}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{2,3})$	$(3.1_1\ 2.3_2\ 3.3_{3,2})$

3.46. 3-contextural semiotic system 46/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{1,3})$	$(3.1_1\ 2.1_3\ 1.1_{31})$	$(1.1_{1,3}\ 2.1_3\ 3.1_1)$	$(1.1_{3,1}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_3\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{2,3}\ 1.2_3)$	$(3.1_1\ 2.2_{3,2}\ 1.2_3)$	$(1.2_3\ 2.2_{2,3}\ 3.1_1)$	$(2.1_3\ 2.2_{3,2}\ 1.3_1)$
$(3.1_3\ 2.2_{2,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,2}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{2,3}\ 1.2_3)$	$(3.2_2\ 2.2_{3,2}\ 1.2_3)$	$(1.2_3\ 2.2_{2,3}\ 3.2_2)$	$(2.1_3\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.2_{2,3}\ 1.3_1)$	$(3.2_2\ 2.2_{3,2}\ 1.3_1)$	$(1.3_1\ 2.2_{2,3}\ 3.2_2)$	$(3.1_1\ 2.2_{3,2}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{1,2}\ 2.3_2\ 1.3_1)$	$(3.3_{2,1}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{1,2})$	$(3.1_1\ 2.3_2\ 3.3_{2,1})$

3.47. 3-contextural semiotic system 47/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{2,3})$	$(3.1_1\ 2.1_3\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_3\ 3.1_1)$	$(1.1_{3,2}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_3\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{1,3}\ 1.2_3)$	$(3.1_1\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.1_1)$	$(2.1_3\ 2.2_{3,1}\ 1.3_1)$
$(3.1_3\ 2.2_{1,3}\ 1.3_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.1_1)$	$(3.1_1\ 2.2_{3,1}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{1,3}\ 1.2_3)$	$(3.2_2\ 2.2_{3,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,3}\ 3.2_2)$	$(2.1_3\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,3}\ 1.3_1)$	$(3.2_2\ 2.2_{3,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,3}\ 3.2_2)$	$(3.1_1\ 2.2_{3,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{1,2}\ 2.3_2\ 1.3_1)$	$(3.3_{2,1}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{1,2})$	$(3.1_1\ 2.3_2\ 3.3_{2,1})$

3.48. 3-contextural semiotic system 48/48

Nm	Nh	R	D
$(3.1_1\ 2.1_3\ 1.1_{2,3})$	$(3.1_1\ 2.1_3\ 1.1_{3,2})$	$(1.1_{2,3}\ 2.1_3\ 3.1_1)$	$(1.1_{3,2}\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.2_3)$	$(3.1_1\ 2.1_3\ 1.2_3)$	$(1.2_3\ 2.1_3\ 3.1_1)$	$(2.1_3\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.1_3\ 1.3_1)$	$(3.1_1\ 2.1_3\ 1.3_1)$	$(1.3_1\ 2.1_3\ 3.1_1)$	$(3.1_1\ 1.2_3\ 1.3_1)$
$(3.1_1\ 2.2_{1,2}\ 1.2_3)$	$(3.1_1\ 2.2_{2,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,2}\ 3.1_1)$	$(2.1_3\ 2.2_{2,1}\ 1.3_1)$
$(3.1_3\ 2.2_{1,2}\ 1.3_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.1_1)$	$(3.1_1\ 2.2_{2,1}\ 1.3_1)$
$(3.1_1\ 2.3_2\ 1.3_1)$	$(3.1_1\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.1_1)$	$(3.1_1\ 3.2_2\ 1.3_1)$
$(3.2_2\ 2.2_{1,2}\ 1.2_3)$	$(3.2_2\ 2.2_{2,1}\ 1.2_3)$	$(1.2_3\ 2.2_{1,2}\ 3.2_2)$	$(2.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_1)$	$(3.2_2\ 2.2_{2,1}\ 1.3_1)$	$(1.3_1\ 2.2_{1,2}\ 3.2_2)$	$(3.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_1)$	$(3.2_2\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.2_2)$	$(3.1_1\ 3.2_2\ 2.3_2)$
$(3.3_{1,3}\ 2.3_2\ 1.3_1)$	$(3.3_{3,1}\ 2.3_2\ 1.3_1)$	$(1.3_1\ 2.3_2\ 3.3_{1,3})$	$(3.1_1\ 2.3_2\ 3.3_{3,1})$

4. We have restricted us here to 4 of totally 6 possible combinations of triadic sign relations and morphisms/heteromorphisms:

$(3.1_1\ 2.2_{1,2}\ 1.2_3)$	**$(1.2_3\ 2.2_{1,2}\ 3.1_1)$**	$(2.1_3\ 2.2_{1,2}\ 1.3_1)$
$(3.1_1\ 2.2_{2,1}\ 1.2_3)$	$(1.2_3\ 2.2_{2,1}\ 3.1_1)$	**$(2.1_3\ 2.2_{2,1}\ 1.3_1)$**

While $(3.1_1\ 2.2_{1,2}\ 1.2_3)$ is the "morphismic normal form" and $(3.1_1\ 2.2_{2,1}\ 1.2_3)$ its complementary "heteromorphismic normal form", we could say that $(1.2_3\ 2.2_{1,2}\ 3.1_1)$ is the reflected morphismic and $(1.2_3\ 2.2_{2,1}\ 3.1_1)$ its complementary heteromorphismic form. Further, $(2.1_3\ 2.2_{2,1}\ 1.3_1)$ is the dual form to the morphismic normal form and $(2.1_3\ 2.2_{1,2}\ 1.3_1)$ the dual form to the heteromorphismic dual form. If we proceed like that, than we do not only get $48 \cdot 4 = 192$, but $48 \cdot 8 = 384$ semiotic systems, which are interconnected by static sub-signs or dynamic morphisms and/or by their inner semiotic environments.

Bibliography

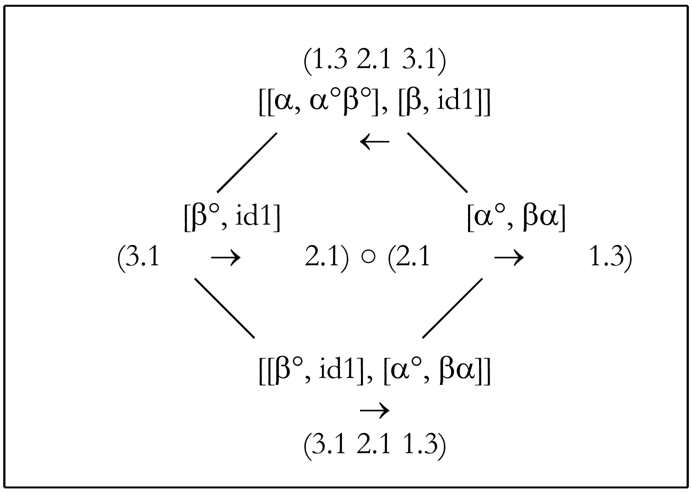
Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialek-tik. Vol. 2. Hamburg 1979

Kaehr, Rudolf, Diamond semiotics. In: http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, Connections of inner semiotic environments (NETS, 3). In: Electronic Journal for Mathematical Semiotics, 2009

3-dimensionale semiotische Diamanten

1. Das ursprünglich polykontexturale Diamanten-Modell wurde aufgrund der Arbeit von Kaehr (2007) in die Semiotik eingeführt von Toth (2008a) und (2008b, S. 177 ff.). Ein semiotischer Diamant erlaubt die gleichzeitige Darstellung einer Zeichenklasse bzw. Realitätsthematik, deren morphismische Komposition und deren Heteromorphismus, der in der Semiotik mit der Inversion der Zeichenklasse bzw. Realitätsthematik zusammenfällt. Das folgende Beispiel zeigt einen der 6 möglichen semiotischen Diamanten für die 2-dimensionale Zeichenklasse (3.1 2.1 1.3):



Die restlichen 5 sind Permutationen. Wenn wir die einzelnen Komponenten dieses Diamanten anschauen, haben wir

2-Zkl:	(3.1 2.1 1.3)
Inv(2-Zkl):	(1.3 2.1 3.1)
Comp(2-Zkl):	(3.1 → 2.1) ◊ (2.1 → 1.3)

Wie man sieht, kann also in einem 2-dimensionalen Diamanten nur entweder eine Zeichenklasse oder eine Realitätsthematik, aber nicht ein Dualsystem dargestellt werden. Ferner ist schon der 2-dimensionale Diamant insofern defektiv, als er die Darstellung inverser Kompositionen nicht erlaubt.

2. Für das allgemeine Schema der Komponenten des 2-dimensionalen semiotischen Diamanten würden wir also erwarten

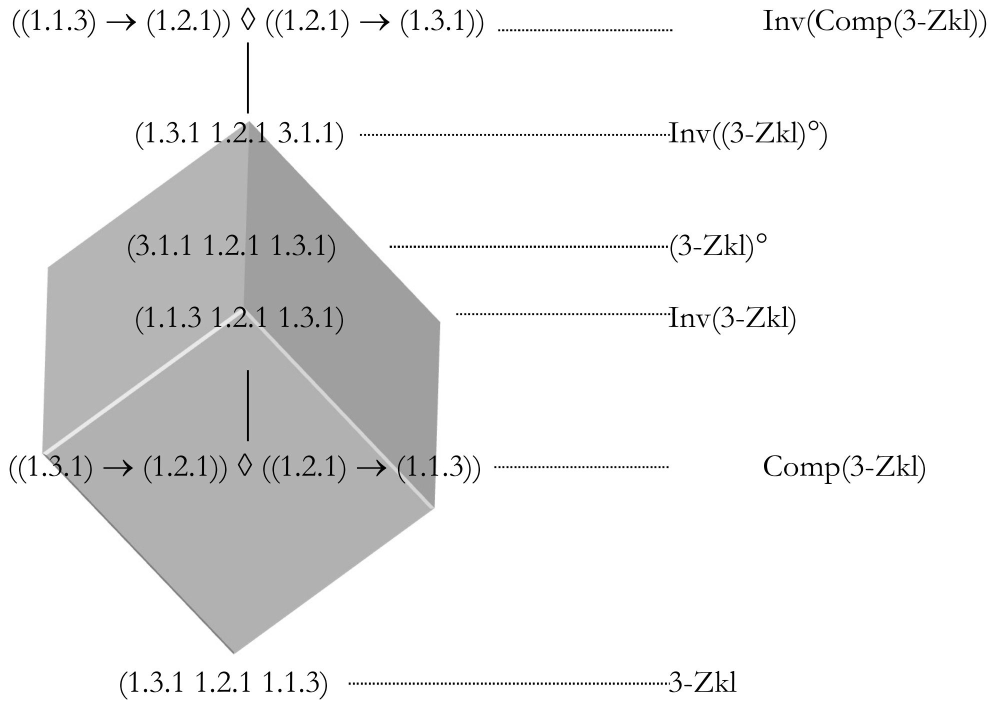
2-Zkl:	(3.a 2.b 1.c)
Inv(2-Zkl):	(1.c 2.b 3.a)
(2-Zkl)°:	(c.1 b.2 a.3)
Inv((2-Zkl)°):	(a.3 b.2 c.1)

Comp(2-Zkl):	(3.a → 2.b) ◊ (2.b → 1.c)
Inv(Comp(2-Zkl)):	(1.c → 2.b) ◊ (2.b → 3.a)

Im Falle unserer 2-Zkl (3.1 2.1 1.3) wäre das also

2-Zkl:	(3.1 2.1 1.3)
Inv(2-Zkl):	(1.3 2.1 3.1)
(2-Zkl)°:	(3.1 1.2 1.3)
Inv((2-Zkl)°):	(1.3 1.2 3.1)
Comp(2-Zkl):	(3.1 → 2.1) ◊ (2.1 → 1.3)
Inv(Comp(2-Zkl)):	(1.3 → 2.1) ◊ (2.1 → 3.1)

Wir nennen dieses Schema, bestehend aus einem Objekt (der Zeichenklasse) und den Operationen Komposition (Comp), Dualisation (°) und Inversion (Inv), ein minimales semiotisches Diamantenschema. Im folgenden zeigen wir, dass wir zu seiner Realisation einen 3-dimensionalen semiotischen Diamanten benötigen.



3. Wenn wir uns die Tabelle der durch die semiotischen Dimensionsoperatoren

η := dim(a) = W(Trd) und

ϑ := dim(a) = W(Trch)

auf das allgemeine 3-dimensionale triadische Zeichenschema

3-ZR = (a.3.b c.2.d e.1.f)

angewandten Zeichenklassen (unter Einschluss der 3-dim. Kategorienklasse) anschauen (vgl. Toth 2009a, b)

1. η(3.1 2.1 1.1) = (3.3.1 2.2.1 1.1.1)
   ϑ(3.1 2.1 1.1) = (1.3.1 1.2.1 1.1.1)

2. η(3.1 2.1 1.2) = (3.3.1 2.2.1 1.1.2)

ϑ(3.1 2.1 1.2) = (1.3.1 1.2.1 2.1.2)

3. η(3.1 2.1 1.3) = (3.3.1 2.2.1 1.1.3)
   ϑ(3.1 2.1 1.3) = (1.3.1 1.2.1 3.1.3)

4. η(3.1 2.2 1.2) = (3.3.1 2.2.2 1.1.2)
   ϑ(3.1 2.2 1.2) = (1.3.1 2.2.2 2.1.2)

5. η(3.1 2.2 1.3) = (3.3.1 2.2.2 1.1.3)
   ϑ(3.1 2.2 1.3) = (1.3.1 2.2.2 3.1.3)

6. η(3.1 2.3 1.3) = (3.3.1 2.2.3 1.1.3)
   ϑ(3.1 2.3 1.3) = (1.3.1 3.2.3 3.1.3)

7. η(3.2 2.2 1.2) = (3.3.2 2.2.2 1.1.2)
   ϑ(3.2 2.2 1.2) = (2.3.2 2.2.2 2.1.2)

8. η(3.2 2.2 1.3) = (3.3.2 2.2.2 1.1.3)
   ϑ(3.2 2.2 1.3) = (2.3.2 2.2.2 3.1.3)

9. η(3.2 2.3 1.3) = (3.3.2 2.2.3 1.1.3)
   ϑ(3.2 2.3 1.3) = (2.3.2 3.2.3 3.1.3)

10. η(3.3 2.3 1.3) = (3.3.3 2.2.3 1.1.3)
    ϑ(3.3 2.3 1.3) = (3.3.3 3.2.3 3.1.3)

11. η(3.3 2.2 1.1) = (3.3.3 2.2.2 1.1.1)
    ϑ(3.3 2.2 1.1) = (3.3.3 2.2.2 1.1.1).

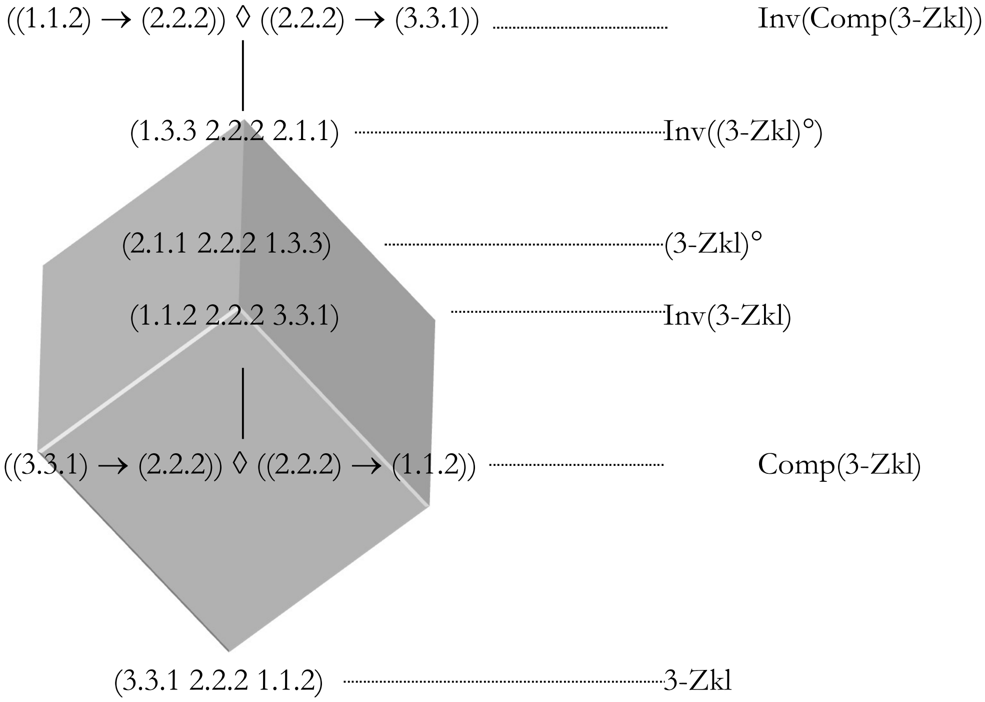
dann erkennen wir, dass 3-dimensionale semiotische Diamanten dazu benutzt werden können, um die Verteilung von inhärenten und adhärenten Dimensionszahlen bei Zeichenklassen zu bestimmen

1	3-2-1	1-1-1
2	3-2-1	1-1-2
3	3-2-1	1-1-3
4	3-2-1	1-2-2
5	3-2-1	1-2-3
6	3-2-1	1-3-3
7	3-2-1	2-2-2
8	3-2-1	2-2-3
9	3-2-1	2-3-3
10	3-2-1	3-3-3
11	3-2-1	3-2-1

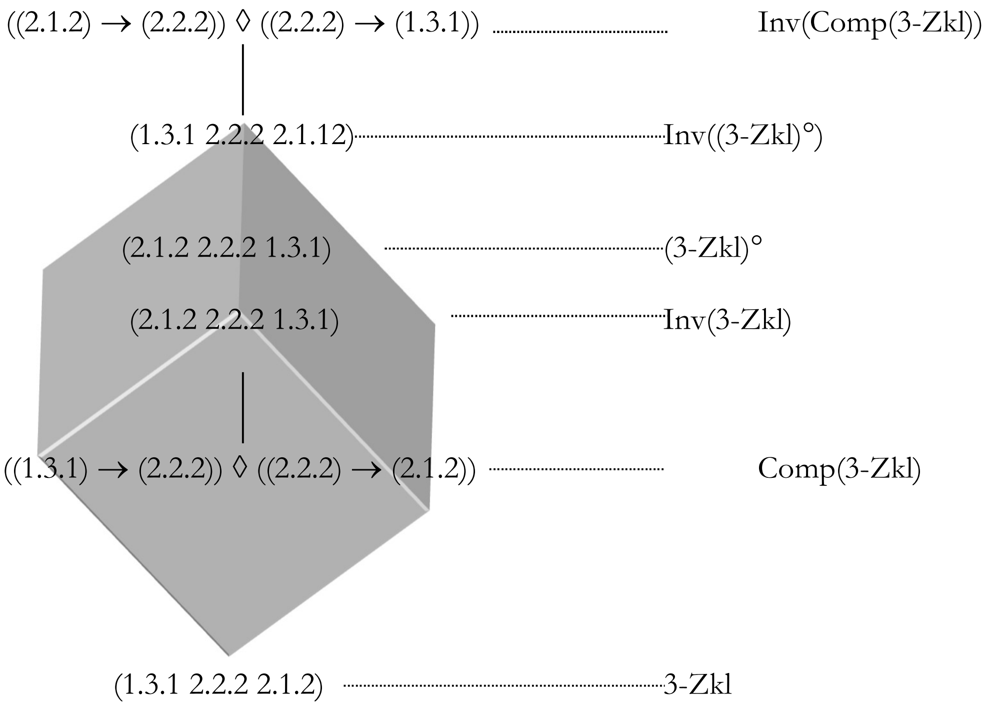
Nehmen wir als Beispiel die 4. Zeichenklasse und das Schema ihrer beiden inhärenten 3-dimensionalen Äquivalente

4. η(3.1 2.2 1.2) = (3.3.1 2.2.2 1.1.2)
ϑ(3.1 2.2 1.2) = (1.3.1 2.2.2 2.1.2)

Der 3-dimensionale semiotische Diamant von 4-η ist dann



Der 3-dimensionale semiotische Diamant von 4-ϑ ist



Die beiden semiotischen 3-Diamanten sind also bis auf die Dimensionszahlen identisch. Da 3-dimensionale semiotische Diamanten nicht nur über Zeichenklassen oder Realitätsthematiken konstruiert sind, sondern über Dualsysteme, können wir deren allgemeines Schema wie folgt notieren

3-Zkl:	(a.3.1 b.2.1 c.1.3)
Inv(3-Zkl):	(c.1.3 b.2.1 a.3.1)
(3-Zkl)°:	(3.1.c 1.2.b 1.3.a)
Inv((3-Zkl)°):	(1.3.a 1.2.b 3.1.c)

Comp(3-Zkl):	(a.3.1 → b.2.1) ◊ (b.2.1 → c.1.3)
Inv(Comp(3-Zkl)):	(c.1.3 → b.2.1) ◊ (b.2.1 → a.3.1)

Bibliographie

Kaehr, Rudolf, Towards Diamonds. Glasgow 2007. Digitalisat: http://www.thinkartlab.com/pkl/lola/Towards_Diamonds.pdf

Toth, Alfred, In Transit. Klagenfurt 2008 (2008a)

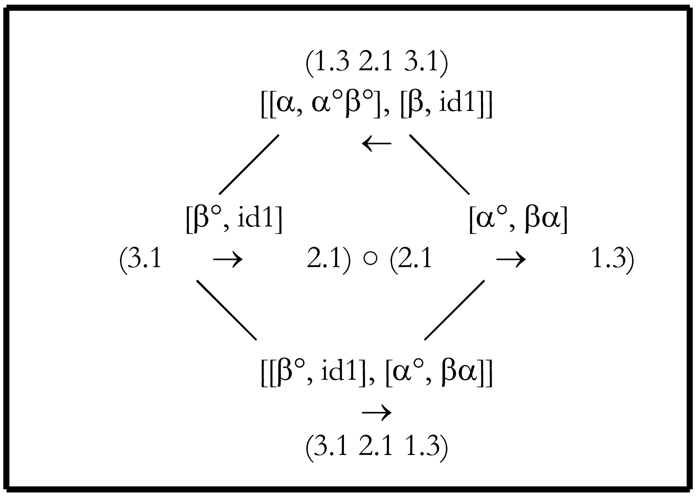
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008b)

Toth, Alfred, Inhärente und adhärente Dimensionszahlen bei Zeichenklassen. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Semiotische Differenz inhärenter Zeichenklassen. In: Electronic Journal for Mathematical Semiotics, 2009b

4-Torus und Diamant

1. Ein semiotischer Diamant wird nach Toth (2008a, S. 32 ff.) und Toth (2008b, S. 177 ff.) wie folgt schematisiert (Beispiel: 2-Zkl (3.1 2.1 1.3)):

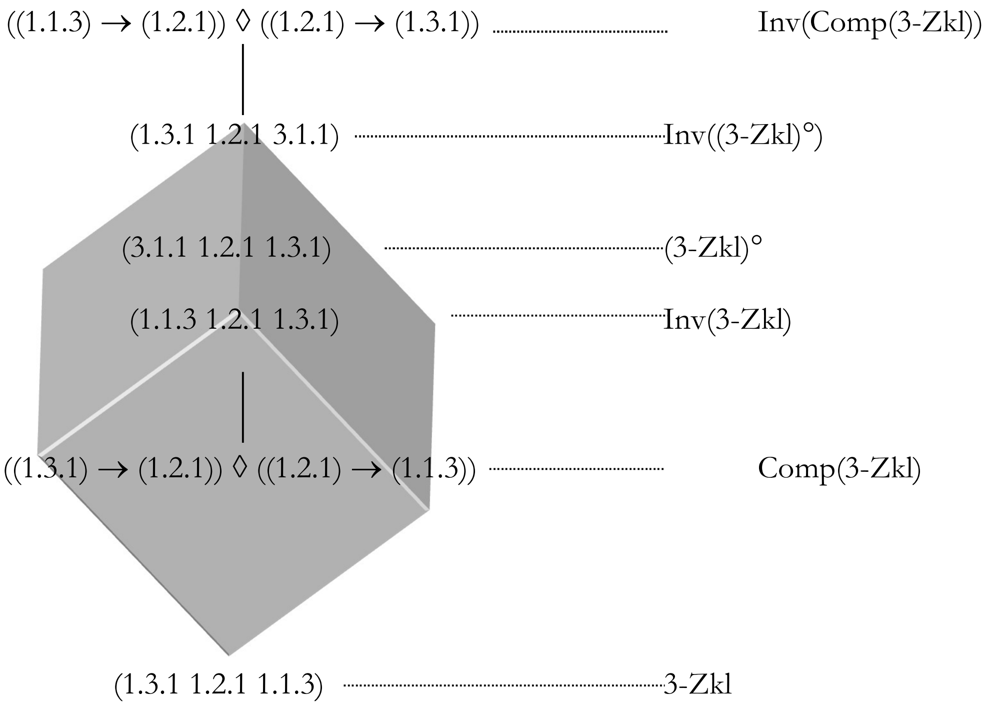


Daraus ergibt sich als allgemeines Schema der Komponenten eines 2-dimensionalen semiotischen Diamanten

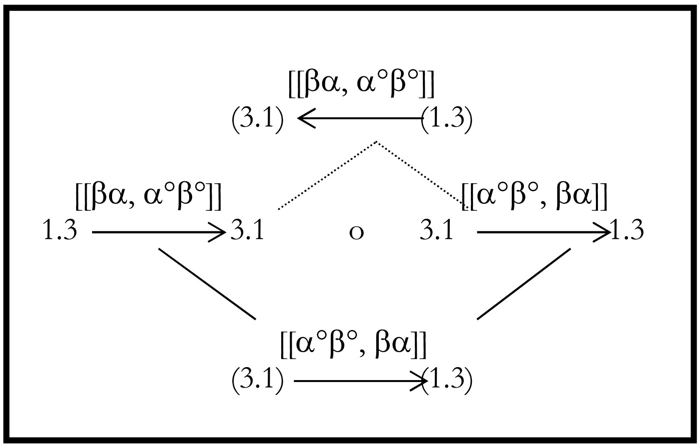
2-Zkl:	(3.a 2.b 1.c)
Inv(2-Zkl):	(1.c 2.b 3.a)
Comp(2-Zkl):	(3.a → 2.b) ◊ (2.b → 1.c)

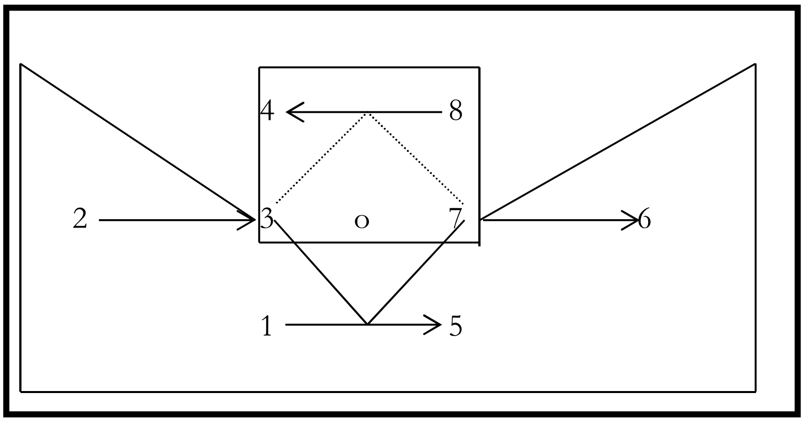
Dieser gibt also keine Auskunft über die inverse Komposition. Ferner sind 2-dimensionale Diamanten offenbar auf Zeichenklassen oder Realitätsthematiken beschränken, können also keine vollständigen Dualsysteme darstellen.

2. Um die letzteren Mängel zu beheben, wurden 3-dimensionale Diamanten eingeführt (Toth 2009a). Sie werden wie folgt schematisiert (Beispiel: 3-Zkl (3.11 1.2.1 1.3.1)):

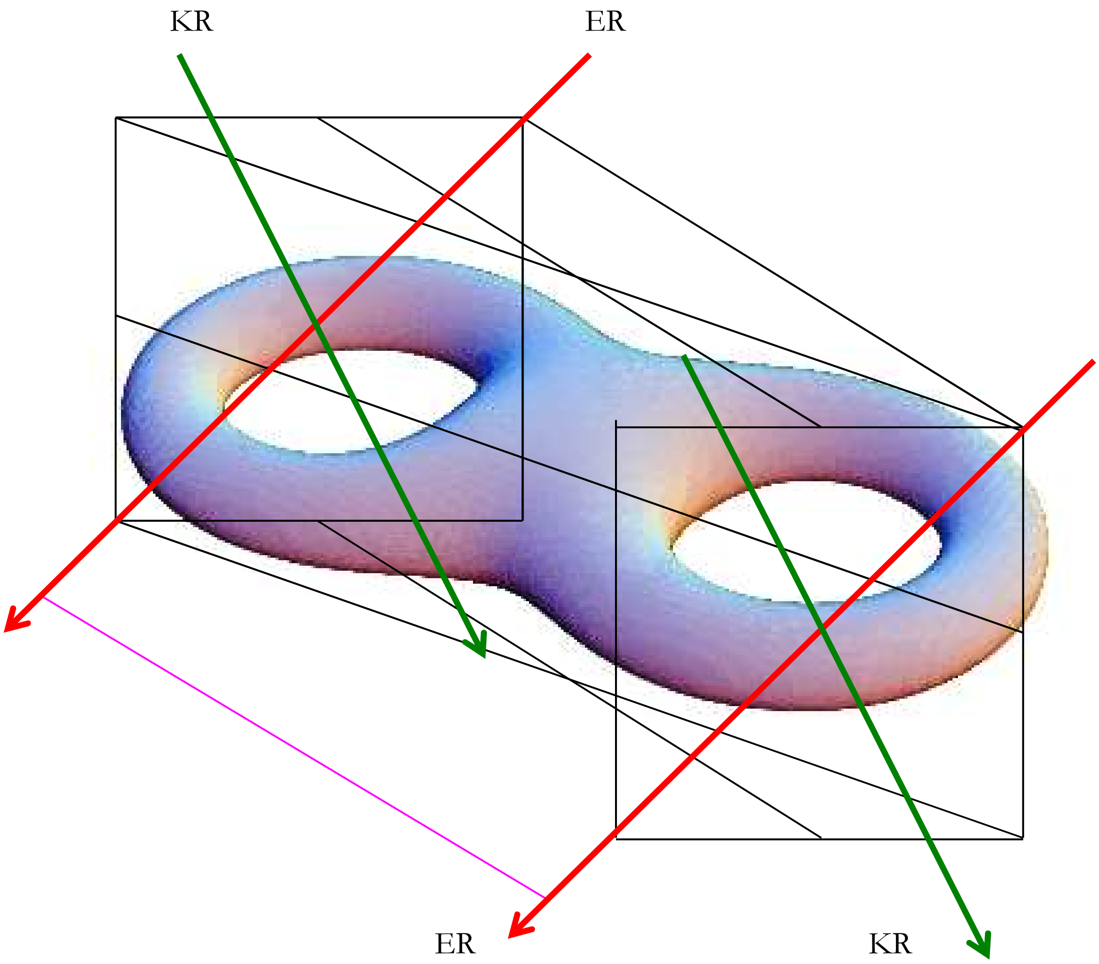


3. In Toth (2008a, S. 36) wurde gezeigt, dass 2-dimensionale semiotische Diamanten zu 2-dimensionalen semiotischen Tori isomorph sind:





Wenn man sich diesen 2-dimensionalen Torus hinten zusammengeklebt und auf 4 Dimensionen erhöht vorstellt, bekommt man einen Doppeltorus zwischen je 2 Kuben des semiotischen Tesseraktes, wo dem in der folgenden Projektion aus (Toth 2009b) nur 2 plus einige angedeutete relationale Netzstrukturen dargestellt sind:



Nach Toth (2009b) wird der einem 4-dimensionalen semiotischen Diamanten isomorphe 4-Doppeltorus durch das folgende Repräsentationsschema charakterisiert:

TK = {<a.3.3.b c.2.2.d e.1.1.f>}
mit a, c, e ∈ {1, 2, 3} und b, d, f ∈ {1, 2, 3, 4}.

Daraus ergibt sich also als allgemeines Schema der Komponenten eines 4-dimensionalen semiotischen Diamanten:

4-Zkl:	((a.3.b.c) (d.2.e.f) (g.1.h.i))
Inv(4-Zkl):	((g.1.h.i) (d.2.e.f) (a.3.b.c))
(4-Zkl)°:	((i.h.1.g) (f.e.2.d) (c.b.3.a))
Inv((4-Zkl)°):	((c.b.3.a) (f.e.2.d) (i.h.1.g))

Comp(4-Zkl):	((a.3.b.c) → (d.2.e.f)) ◊ ((d.2.e.f) → (g.1.h.i))
Inv(Comp(4-Zkl)):	((g.1.h.i) → (d.2.e.f)) ◊ ((d.2.e.f) → (a.3.b.c))

Bibliographie

Toth, Alfred, In Transit. Klagenfurt 2008 (2008a)
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008b)

Toth, Alfred, 3-dimensionale semiotische Diamanten. In: Electronic Journal for Mathematical Semiotics, 2009a
Toth, Alfred, Der Transit-Korridor. In: Electronic Journal for Mathematical Semiotics, 2009b

Categorial and saltatorial sign classes

1. Categories and saltatories are dual notions from diamond theory (cf. Kaehr 2008, 2009b). Categories are dealing with objects and morphisms, while saltatories are dealing with (co-)objects and hetero-morphisms. Together, they form bi-objects. Kaehr (2009a) has shown that amongst the bi-objects, there are bi-signs.

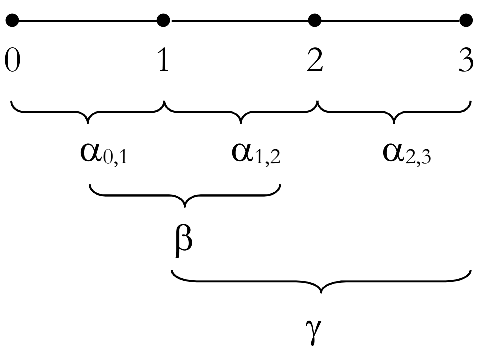
2. In this study, based on Toth (2009a, b), I introduce a "neutral" categorial notation system for sign classes and reality thematics. "Neutral" means that although contextures do no belong to the definition of these sign classes and reality thematics, they can be introduced without change in the notation system. The categorial system introduced here differs considerably from former semiotic categorial systems (cf. Toth 1997, pp. 21 ss.; Toth 2008a, pp. 159 ss.) insofar as it is based on dynamic and not on static prime-signs. This means that instead of starting with the usual static introduction of prime-signs

PS = {.1., .2., .3.),

we suggest the following dynamic introduction of prime-signs:

PS = <[[[0, 1], [[0, 2], [0, 3]]]]>

with



Therefore, we can rewrite PS as follows

PS = $[[\alpha_{0,1}], [[\alpha_{1,2}], [\alpha_{2,3}]]]$,

which allows us to generate the following categorial matrix

	$\alpha_{0,1}$	$\alpha_{1,2}$	$\alpha_{2,3}$
$\alpha_{0,1}$	$\alpha_{0,1}\alpha_{0,1}$	$\alpha_{0,1}\alpha_{1,2}$	$\alpha_{0,1}\alpha_{2,3}$
$\alpha_{1,2}$	$\alpha_{1,2}\alpha_{0,1}$	$\alpha_{1,2}\alpha_{1,2}$	$\alpha_{1,2}\alpha_{2,3}$
$\alpha_{2,3}$	$\alpha_{2,3}\alpha_{0,1}$	$\alpha_{2,3}\ \alpha_{1,2}$	$\alpha_{2,3}\alpha_{2,3}$

Now, in Toth (2009b), we have shown that Medads are trichotomically split into (0.1), (0.2) and (0.3). Further, we obtained

(0.1) = $(\alpha_0\ \alpha_{0,1})$
(0.2) = $(\alpha_0\ \alpha_{1,2})$
(0.3) = $(\alpha_0\ \alpha_{2,3})$

Therefore, we can change the above 3×3 matrix into a 4×3, i.e. a tetradic-trichotomic matrix which corresponds exactly to the "pre-semiotic matrix" introduced in Toth (2008b)

	$\alpha_{0,1}$	$\alpha_{1,2}$	$\alpha_{2,3}$
α_0	$\alpha_0\ \alpha_{0,1}$	$\alpha_0\ \alpha_{1,2}$	$\alpha_0\ \alpha_{2,3}$
$\alpha_{0,1}$	$\alpha_{0,1}\alpha_{0,1}$	$\alpha_{0,1}\alpha_{1,2}$	$\alpha_{0,1}\alpha_{2,3}$
$\alpha_{1,2}$	$\alpha_{1,2}\alpha_{0,1}$	$\alpha_{1,2}\alpha_{1,2}$	$\alpha_{1,2}\alpha_{2,3}$
$\alpha_{2,3}$	$\alpha_{2,3}\alpha_{0,1}$	$\alpha_{2,3}\ \alpha_{1,2}$	$\alpha_{2,3}\alpha_{2,3}$

Of course, this matrix is monocontextural. We can see that by comparing the sub-sign relations with their corresponding inverted relations:

(1.2) ≡ $(\alpha_{0,1}\alpha_{1,2})$ (1.2)° = (2.1) ≡ $(\alpha_{1,2}\alpha_{0,1})$
(1.3) ≡ $(\alpha_{0,1}\alpha_{2,3})$ (1.3)° = (3.1) ≡ $(\alpha_{2,3}\alpha_{0,1})$
(2.3) ≡ $(\alpha_{1,2}\alpha_{2,3})$ (2.3)° = (3.2) ≡ $(\alpha_{2,3}\ \alpha_{1,2})$

In short: Morphism stays morphism. Therefore, in order to introduce hetero-morphisms, we proceed as we did in Toth (2009c) and introduce the reflector R, which turns around not only the order of morphisms but also their indices:

(1.2) ≡ $(\alpha_{0,1}\alpha_{1,2})$ R(1.2) = $(\alpha_{2,1}\alpha_{1,0})$
(1.3) ≡ $(\alpha_{0,1}\alpha_{2,3})$ R(1.3) = $(\alpha_{3,2}\alpha_{1,0})$

$(2.3) \equiv (\alpha_{1,2}\alpha_{2,3}) \qquad R(2.3) = (\alpha_{2,1}\ \alpha_{3,2})$

Since in monocontextural matrices, dual sub-signs are identical to converted sub-signs, they are in one and the same semiotic matrix. However, since dual sub-signs are not identical to reflected sub-signs in polycontextural matrices, we need another matrix to display them (and hence n matrices for n-valued reflections or one matrix per contexture):

	$\alpha_{1,0}$	$\alpha_{2,1}$	$\alpha_{3,2}$
α_0	$\alpha_0\ \alpha_{1,0}$	$\alpha_0\ \alpha_{2,1}$	$\alpha_0\ \alpha_{3,2}$
$\alpha_{1,0}$	$\alpha_{1,0}\alpha_{1,0}$	$\alpha_{1,0}\alpha_{2,1}$	$\alpha_{1,0}\alpha_{3,2}$
$\alpha_{2,1}$	$\alpha_{2,1}\alpha_{1,0}$	$\alpha_{2,1}\alpha_{2,1}$	$\alpha_{2,1}\alpha_{3,2}$
$\alpha_{3,2}$	$\alpha_{3,2}\alpha_{1,0}$	$\alpha_{3,2}\ \alpha_{2,1}$	$\alpha_{3,2}\alpha_{3,2}$

Now, the identity law of classical logic is abolished, we have

$(2.2) \equiv (\alpha_{1,2}\alpha_{1,2}) = \times(2.2) = (\alpha_{1,2}\alpha_{1,2}) \neq$
$R(2.2) \equiv (\alpha_{2,1}\alpha_{2,1})$,

thus, the indices referring in our notation to intervals of natural numbers and not to inner environments, behave like contextures, i.e. they point out the difference between morphisms and heteromorphismus or categories and saltatories.

Therefore, we get now two semiotic systems:

1. The monocontextural semiotic system consisting of the Peircean 10 sign classes and dual(ized) reality thematics:

1. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{0,1}] \times [\alpha_{0,1}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{2,3}\alpha_{0,1}]$
2. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{1,2}] \times [\alpha_{0,1}\alpha_{1,2}, \alpha_{1,2}\alpha_{0,1}, \alpha_{2,3}\alpha_{0,1}]$
3. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{0,1}, \alpha_{2,3}\alpha_{0,1}]$
4. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}] \times [\alpha_{0,1}\alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3}\alpha_{0,1}]$
5. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3}\alpha_{0,1}]$
6. $[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{2,3}\alpha_{0,1}]$
7. $[\alpha_{2,3}\ \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}] \times [\alpha_{0,1}\alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3}\ \alpha_{1,2}]$
8. $[\alpha_{2,3}\ \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3}\ \alpha_{1,2}]$
9. $[\alpha_{2,3}\ \alpha_{1,2}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{2,3}\ \alpha_{1,2}]$
10. $[\alpha_{2,3}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}] \times [\alpha_{0,1}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{2,3}\alpha_{2,3}]$

2. The polycontextural semiotic system consisting of the 10 Peircean sign classes and reflected sign/reality thematics.

1.	$[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{0,1}]$	R	$[\alpha_{1,0}\alpha_{1,0}, \alpha_{1.0}\alpha_{2.1}, \alpha_{1.0}\alpha_{3.2}]$
2.	$[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{1,2}]$	R	$[\alpha_{2,1}\alpha_{1,0}, \alpha_{1,0}\alpha_{2,1}, \alpha_{1.0}\alpha_{3,2}]$
3.	$[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{0,1}, \alpha_{0,1}\alpha_{2,3}]$	R	$[\alpha_{3,2}\alpha_{1,0}, \alpha_{1,0}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
4.	$[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}]$	R	$[\alpha_{21}\alpha_{1,0}, \alpha_{2,1}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
5.	$[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}]$	R	$[\alpha_{3,2}\alpha_{1,0}, \alpha_{2,1}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
6.	$[\alpha_{2,3}\alpha_{0,1}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}]$	R	$[\alpha_{3,2}\alpha_{1,0}, \alpha_{3,2}\alpha_{2,1}, \alpha_{1,0}\alpha_{3,2}]$
7.	$[\alpha_{2,3}\ \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{1,2}]$	R	$[\alpha_{2,1}\alpha_{1,0}, \alpha_{2,1}\alpha_{2,1}, \alpha_{2,1}\ \alpha_{3,2}]$
8.	$[\alpha_{2,3}\ \alpha_{1,2}, \alpha_{1,2}\alpha_{1,2}, \alpha_{0,1}\alpha_{2,3}]$	R	$[\alpha_{3,2}\alpha_{1,0}, \alpha_{1,2}\alpha_{1,2}, \alpha_{2,3}\ \alpha_{1,2}]$
9.	$[\alpha_{2,3}\ \alpha_{1,2}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}]$	R	$[\alpha_{3,2}\alpha_{1,0}, \alpha_{3,2}\alpha_{2,1}, \alpha_{2,1}\ \alpha_{3,2}]$
10.	$[\alpha_{2,3}\alpha_{2,3}, \alpha_{1,2}\alpha_{2,3}, \alpha_{0,1}\alpha_{2,3}]$	R	$[\alpha_{3,2}\alpha_{1,0}, \alpha_{3,2}\alpha_{21}, \alpha_{3,2}\ \alpha_{3,2}]$

It is controversial, if an R(Scl) can be considered a reality thematics, like an ×(Scl) can; instead, we better use here the term of bi-sign, introduced into semiotics by Kaehr (2009a). However, the relation between (monocontextural) reality thematics) and (polycontextural) bi-signs or "saltatorial reality thematics" has still to be motivated.

Bibliography

Kaehr, Rudolf, Towards Diamonds. Glasgow 2008

Kaehr, Rudolf, Xanandu's textems. http://www.thinkartlab.com/pkl/lola/Xanadu-textemes/Xanadu-textemes.pdf (2009a)

Kaehr, Rudolf, Elements of diamond set theory. http://www.thinkartlab.com/pkl/lola/Elements/Elements.html (2009b)

Toth, Alfred, Entwurf einer semiotisch-relationalen Grammatik. Tübingen 1997

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)

Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 2008 (2008b)

Toth, Alfred, Medads and the triadic sign relations. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Is there a trichotomy of the Medad? In: Electronic Journal for Mathematical Semiotics, 2009b

Toth, Alfred, The Trip into the Light. Tucson, AZ 2009 (2009c)

Triads and trichotomies as adjunctions

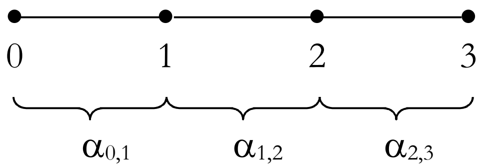
According to Toth (2009), instead of introducing prime-signs in the following static manner (cf. Bense 1980)

PS = {.1., .2., .3.),

I have suggested the following dynamic introduction of prime-signs:

PS = <[[[0, 1], [[0, 2], [0, 3]]]]>

with



If we write, as usual, the prime-signs as row and a column, we get the following categorial semiotic 3 × 3 matrix which differs considerably from the hitherto used categorial matrices in semiotics (cf. Toth 1997, pp. 21 ss.; 2008, pp. 159 ss.):

	$\alpha_{0,1}$	$\alpha_{1,2}$	$\alpha_{2,3}$
$\alpha_{0,1}$	$\alpha_{0,1}\alpha_{0,1}$	$\alpha_{0,1}\alpha_{1,2}$	$\alpha_{0,1}\alpha_{2,3}$
$\alpha_{1,2}$	$\alpha_{1,2}\alpha_{0,1}$	$\alpha_{1,2}\alpha_{1,2}$	$\alpha_{1,2}\alpha_{2,3}$
$\alpha_{2,3}$	$\alpha_{2,3}\alpha_{0,1}$	$\alpha_{2,3}\ \alpha_{1,2}$	$\alpha_{2,3}\alpha_{2,3}$

If we now define

$Z = \{(\alpha_{0,1}), (\alpha_{1,2}), (\alpha_{2,3})\}$, then we have

	$\alpha_{0,1}$	$\alpha_{1,2}$	$\alpha_{2,3}$
$\alpha_{0,1}$	$Z\alpha_{0,1}$	$Z\alpha_{1,2}$	$Z\alpha_{2,3}$
$\alpha_{1,2}$	$Z\alpha_{0,1}$	$Z\alpha_{1,2}$	$Z\alpha_{2,3}$
$\alpha_{2,3}$	$Z\alpha_{0,1}$	$Z\alpha_{1,2}$	$Z\alpha_{2,3}$

as well as

	$\alpha_{0,1}$	$\alpha_{1,2}$	$\alpha_{2,3}$
$\alpha_{0,1}$	$\alpha_{0,1}Z$	$\alpha_{0,1}Z$	$\alpha_{0,1}Z$
$\alpha_{1,2}$	$\alpha_{1,2}Z$	$\alpha_{1,2}Z$	$\alpha_{1,2}Z$
$\alpha_{2,3}$	$\alpha_{2,3}Z$	$\alpha_{2,3}Z$	$\alpha_{2,3}Z$,

Hence, if we take the structure $Z\alpha_{i,j}$, where the $[\alpha_{i,j}]$ are left-adjoint, the result is a matrix of the triads, but if we take the structure $\alpha_{0,1}Z$, where the $[\alpha_{i,j}]$ are right-adjoint, the result is a matrix of trichotomies. However, the most astonishing result is that now dual (converse) sub-signs are no longer identical inside, but between the two matrices

$Z\alpha_{1,2} \neq Z\alpha_{0,1} \wedge \alpha_{0,1}Z \neq \alpha_{1,2}Z$, but $Z\alpha_{1,2} = \alpha_{0,1}Z$
$Z\alpha_{2,3} \neq Z\alpha_{0,1} \wedge \alpha_{0,1}Z \neq \alpha_{2,3}Z$, but $Z\alpha_{2,3} = \alpha_{0,1}Z$
$Z\alpha_{2,3} \neq Z\alpha_{1,2} \wedge \alpha_{1,2}Z \neq \alpha_{2,3}Z$ but $Z\alpha_{2,3} = \alpha_{1,2}Z$.

Bibliography

Bense, Max, Die Einführung der Primzeichen. In: Ars Semeiotica 3/3, 1980, pp. 287-294

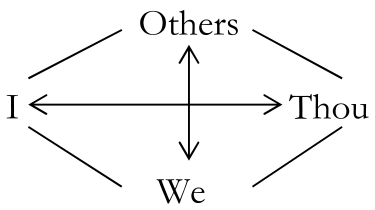
Toth, Alfred, Entwurf einer semiotisch-relationalen Grammatik. Tübingen 1997

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Toth, Alfred, Categorial and saltatorial sign classes. In: Electronic Journal for Mathematical Semiotics, 2009

Semiotic localization of logical-epistemological relations

1. We start with the following diamond model of logical-epistemological relations given by Kaehr (2007, p. 53):



In relation to Günther (1976, pp. 336 ss.), Kaehr has given the following linguistic-logical correspondences:

I = subjective Subject
Thou = objective Subject

Semiotically, according to Toth (2008, pp. 64 ss.), we can complete:

I = subjective Subject = interpretant relation
Thou = objective Subject = medium relation

Therefore, the semiotic object corresponds with the logical-epistemological (objective) object, and this lies in the diamond model there, where the double arrows cross. (By the way: This logical-semiotic correspondence, which of course holds only for a 3-valued logic and a 3-adic semiotics, is the reason why Peircean semiotics has been considered by Bense, Bayer and other as polycontexgural; cf. Toth 2007, pp. 226 ss.).

2. Therefore, we can draw an abstract semiotic diamond model for a maximally 4-contextural 3-adic semiotic as follows

$$(3.a_{k,j,i} \leftarrow 1.c_{k,j,i})$$

$$(3.a_{i,j,k} \rightarrow 2.b_{i,j,k}) \lozenge (2.b_{i,j,k} \rightarrow 1.c_{i,j,k})$$

$$(3.a_{i,j,k} \rightarrow 1.c_{i,j,k})$$

where i, j, k ∈ {∅, 1, 2, 3, 4}, and the value ∅ applies for all (a = 3), (b = 2), or (c = 1), i.e. when the values of the triad and of the trichotomy are identical.

Since in Toth (2009), it has be shown that between each three contextural indices of a morphism i,j,k and its heteromorphism k,j,i there are 4 mediative morphisms (i,k,j;

j,i,k; j,k,i; k,i,j), it has to be pointed out that the above diamondmodel is not capable of showing these mediative morphisms which lie, to speak in logical-epistemological terms, between the We ($3.a_{i,j,k} \rightarrow 1.c_{i,j,k}$) and the Others ($3.a_{k,j,i} \leftarrow 1.c_{k,j,i}$), or logically between acceptance and rejection. To put it differently: The above diamond model is only capable of dealing with such semiotic morphisms whose objects or sub-signs lie in maximally 3 contextures.

Moreover, since logical-epistemological functions are ascribed to the semiotic fundamental categories, in the diamond models, it is not possible to have other types of semiotic composition than the following one:

$(I \rightarrow O) \lozenge (O \rightarrow M)$,

since the semiotic category I stands for the subjective subject and this one stands for the linguistic relation “I”. Also, the semiotic category M stands for the objective subject and thus for the linguistic relation “thou”. Hence, from the following relations

1. $(I \rightarrow M) \lozenge (M \rightarrow O)$
2. $(O \rightarrow M) \lozenge (M \rightarrow I)$
3. $(O \rightarrow I) \lozenge (I \rightarrow M)$
4. $(M \rightarrow O) \lozenge (O \rightarrow I)$
5. $(M \rightarrow I) \lozenge (I \rightarrow O)$

the ones with O in the domain of the first morphisms or in the codomain of the second morphisms (nos. 1, 2, 3, 5) can be discarded, because the objective object cannot appear in one of the 4 subject-positions of the above diamond model. Remains no. 4, the inversion of the composition of the diamond model. Here it is to ask if this type of composition is not isomorphic to $(I \rightarrow O) \lozenge (O \rightarrow M)$, since it results from a simple rotation of the diamond model about the We-Others-axis. Finally, needless to say that the standard diamond does not work for reality thematics at all, since reality thematics are not constructed according to the principle of triadic diversity.

3. Since we have stated that the above “standard” diamond model cannot deal with semiotic objects in more than 3 contextures, it is sufficient to work with the following abstract semiotic diamond model

$(3.a_{j,i} \leftarrow 1.c_{j,i})$

$(3.a_{i,j} \rightarrow 2.b_{i,j}) \lozenge (2.b_{i,j} \rightarrow 1.c_{i,j})$

$(3.a_{i,j} \rightarrow 1.c_{i,j})$

However, this means that in all those cases where either i or j = ∅, and thus for all non-genuine sub-signs or non-identitive morphisms, respectively, morphisms and hetero-morphisms differ simply by turned around arrows. In other words: In a 3-adic semiotics that is maximally 3-contextural, heteromorphisms are nothing else than inverse morphisms. (This is, by the way, another argument that could be held for the alleged polycontexturality of Peircean semiotics asserted by Bense, Bayer and others.)

For the 10 3-contextural sign classes we then get the following diamonds:

$$(3.1_3 \leftarrow 1.1_{3,1})$$
$$(3.1_3 \rightarrow 2.1_1) \Diamond (2.1_1 \rightarrow 1.1_{1,3})$$
$$(3.1_3 \rightarrow 1.1_{1,3})$$

$$(3.1_3 \leftarrow 1.2_1)$$
$$(3.1_3 \rightarrow 2.1_1) \Diamond (2.1_1 \rightarrow 1.2_1)$$
$$(3.1_3 \rightarrow 1.2_1)$$

$$(3.1_3 \leftarrow 1.3_3)$$
$$(3.1_3 \rightarrow 2.1_1) \Diamond (2.1_1 \rightarrow 1.3_3)$$
$$(3.1_3 \rightarrow 1.3_3)$$

$$(3.1_3 \leftarrow 1.2_1)$$
$$(3.1_3 \rightarrow 2.2_{1,2}) \Diamond (2.2_{1,2} \rightarrow 1.2_1)$$
$$(3.1_3 \rightarrow 1.2_1)$$

$$(3.1_3 \leftarrow 1.3_3)$$
$$(3.1_3 \rightarrow 2.2_{1,2}) \Diamond (2.2_{1,2} \rightarrow 1.3_3)$$
$$(3.1_3 \rightarrow 1.3_3)$$

$$(3.1_3 \leftarrow 1.3_3)$$
$$(3.1_3 \rightarrow 2.3_2) \Diamond (2.3_2 \rightarrow 1.3_3)$$
$$(3.1_3 \rightarrow 1.3_3)$$

$$(3.2_2 \leftarrow 1.2_1)$$
$$(3.2_2 \rightarrow 2.2_{1,2}) \Diamond (2.2_{1,2} \rightarrow 1.2_1)$$
$$(3.2_2 \rightarrow 1.2_1)$$

$$(3.2_2 \leftarrow 1.3_3)$$
$$(3.2_2 \rightarrow 2.2_{1,2}) \Diamond (2.2_{1,2} \rightarrow 1.3_3)$$
$$(3.2_2 \rightarrow 1.3_3)$$

$$(3.2_2 \leftarrow 1.3_3)$$
$$(3.2_2 \rightarrow 2.3_2) \Diamond (2.3_2 \rightarrow 1.3_3)$$
$$(3.2_2 \rightarrow 1.3_3)$$

$$(3.3_{3,2} \leftarrow 1.3_3)$$
$$(3.3_{2,3} \rightarrow 2.3_2) \Diamond (2.3_2 \rightarrow 1.3_3)$$
$$(3.3_{2,3} \rightarrow 1.3_3)$$

Because of the above mentioned handicaps of the standard diamond modell – incapability of dealing with sub-signs in more than 3 contextures and incapability of disclosing mediative morphisms between acceptance and rejection - the use of the standard diamond model for displaying the localization of the 4 logical-epistemological relations “I”, “thou”, “we” and “others” is rather trivial insofar as it does not go over the information already contained in the 3-contextural 3×3-matrix:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix}$$

Thus, in order to localize the logical-epistemological relations in 3 semiotic contextures, we obtain

K(I) = (2, 3)
K(thou) = (1, 3)
K(we) = $(2 \rightarrow 1), (2 \rightarrow 3), (3 \rightarrow 1), (3 \rightarrow 3)$
K(others) = $(1 \rightarrow 2), (3 \rightarrow 2), (1 \rightarrow 3), (3 \rightarrow 3)$

Bibliography

Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. Vol. 1. Hamburg 1976
Kaehr, Rudolf, Towards Diamonds. Glasgow 2007
Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008
Toth, Alfred, Mediation between morphisms and heteromorphisms in semiotic systems. In: Electronic Journal for Mathematical Semiotics, 2009

Connectivity and locality in polycontextural sign classes

1. The terms connectivity and locality are borrowed from Robin Milner's theory of bigraphs (Milner 2007). Although I do not yet see how bigraphs could be fruitfully introduced into semiotics, I intend to use the two terms in order to handle connectivity in the sense of semiotic connections by sub-signs or by morphisms and the contextures, which are involved in these semiotic connections, separately.

2. First, we have a look at the 10 3-contextural 3-adic sign classes and their dual reality thematics:

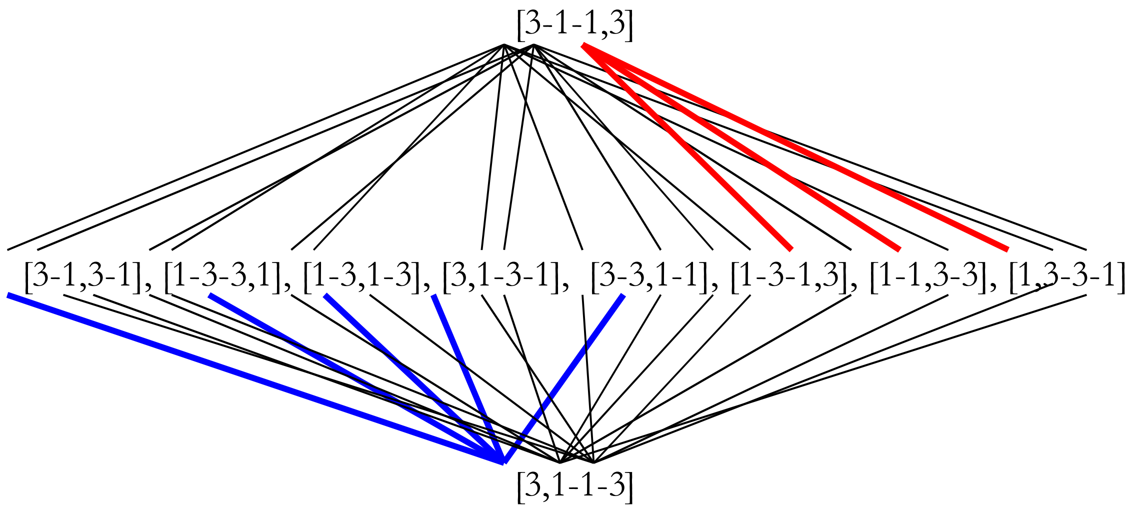
Sign classes		**Reality thematics**	**Connectivity**	**Locality**
$(3.1_3\ 2.1_1\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$	(1.1)	[3-1-1,3]/[3,1-1-3]
$(3.1_3\ 2.1_1\ 1.2_1)$	×	$(2.1_1\ 1.2_1\ 1.3_3)$	(2.1 1.2)	[3-1-1]/[1-1-3]
$(3.1_3\ 2.1_1\ 1.3_3)$	×	$(3.1_3\ 1.2_1\ 1.3_3)$	(3.1 1.3)	[3-1-3]/[3-1-3]
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$	(2.2)	[3-1,2-1]/[1-2,1-3]
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	(3.1 2.2 1.3)	[3-1,2-3]/[3-2,1-3]
$(3.1_3\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 1.3_3)$	(3.1 1.3)	[3-2-3]/[3-2-3]
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$	(2.2)	[2-1,2-1]/[1-2,1-2]
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$	(2.2)	[2-1,2-3]/[3-2,1-2]
$(3.2_2\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 2.3_2)$	(3.2 2.3)	[2-2-3]/[3-2-2]
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 3.3_{3,2})$	(3.3)	[2,3-2-3]/[3-2-3,2]

3. In concordance with Toth (2009b), we must now determine the mediating morphisms between the morphisms and heteromorphisms which "frame" the locality of the sign classes and reality thematics:

Morphisms	**Mediating Morphisms**	**Heteromorphisms**
[3-1-1,3]	[3-1,3-1], [1-3-3,1], [1-3,1-3], [3,1-3-1],	[3,1-1-3]
	[3-3,1-1], [1-3-1,3], [1-1,3-3], [1,3-3-1]	
[3-1-1]	[1-3-1]	[1-1-3]
[3-1-3]	[3-3-1], [1-3-3]	[3-1-3]* (id.)
[3-1,2-1]	[3-1-1,2], [1-1,2-3], [1-3-1,2], [1,2-3-1]	[1-2,1-3]
	[3-1-2,1], [3-2,1-1], [1-2,1-3], [1-3-2,1],	
	[2,1-3-1], [2,1-1-3]	
[3-1,2-3]	[3-3-1,2], [3-1,2-3]	[3-2,1-3]
	[3-3-2,1]	
[3-2-3]*	[3-3-2], [2-3-3]	[3-2-3]* (id.)
[2-1,2-1]	[2-1-1,2], [1,2-1-2], [1,2-2-1], [1-1,2-2],	[1-2,1-2]

	[1-2-1,2], [2-1-2,1], [2,1-1-2], [2,1-2-1], [1-2-2,1]	
[2-1,2-3]	[2-3-1,2], [1,2-2-3], [1,2-3-2], [3-1,2-2], [3-2-1,2], [2-3-2,1], [2,1-2-3], [2,1-3-2], [3-2-2,1]	[3-2,1-2]
[2-2-3]	[2-3-2]	[3-2-2]
[2,3-2-3]	[2,3-3-2], [2-2,3-3], [2-3-2,3], [3-2,3-2], [3-2-2,3], [3,2-3-2], [2-3,2-3], [2-3-3,2]	[3-2-3,2]

4. We can now visualize the contextural transgressions (internal and external) by aid of trees like for $(3.1_{3}\ 2.1_{1}\ 1.1_{1,3}) \times (1.1_{3,1}\ 1.2_{1}\ 1.3_{3})$:



Now, besides the semiotic connections by environments introduced in Toth (2009a) and the semiotic connections by static sub-signs and dynamic semiosic morphisms, introduced in Toth (2008), we have here a fourth type of semiotic connection: the connections via mediative morphisms. That this new type of semiotic connection will be specially useful for semiotics should be clear.

Bibliography

Milner, Robin, Pure bigraphs. Cambridge 2007. Digital version: http://www.cl.cam.ac.uk/~rm135/tutorial-7.pdf

Toth, Alfred, Semiotic Ghost Trains. Klagenfurt 2008

Toth, Alfred, Connections of inner semiotic environments. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Mediation between morphisms and heteromorphisms in semiotic systems. In: Electronic Journal for Mathematical Semiotics, 2009b

Semiotic paths and journeys

1. In his newest publication on polycontextural theory, Rudolf Kaehr has introduced diamond journeys, which are complementary to categorial paths. It is easiest just to copy out the formal description of the new notion of journey (Kaehr 2009b, p. 8):

3.2. Formal description of JOURN

Let denote a general bi-relation. We associate with it the *diamond* denoted by JOURN((X,x),), JOURN(X,x) or just JOURN.

Bi-objects: Bi-Elements (X,x) ☒☒ **(X, x)**.

Morphisms: Sequences (paths) of consecutive arrows,
*Hetero-morphisms:*counter-sequences of antidromic arrows.
Complementarity: Category/Saltatory

JOURN is not a product of **PATH**, i.e. JOURN != PATH x PATH but a *complementary* (and not a dual!)
interplay between PATH and co-PATH:

JOURN = compl(PATH,)

There is a *morphism* X -> Y, iff XRY☒ Cat .

There is a *hetero*-morphism x -> y, iff xry ☒ Salt.
There is a *diamond* if [Cat; Salt].

$$R^{1.2} \subseteq \left(A_0^1, A_0^2\right) \text{xx} \left(A_1^1, A_1^2\right)$$
$$\left(\text{Rr}\right) \subseteq \subseteq \left(A_0^1, a_0^2\right) \text{xx} \left(A_1^1, a_1^2\right)$$

While for categorial semiotic paths, there are extensive studies by me, f. ex. (Toth 2009a), the notion of semiotic journey has first to be introduced into semiotics.

2. If we accept that the basic sign model is the 3-adic 4-contextural sign class

SCl (3,4) = $(3.a_{i,k,j}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$

where either i, or j, or k = ∅ for all non-identitive semiotic morphisms, i.e. for all non-genuine sub-signs, since they cannot lie in 3 contextures in a 4-contextural semiotics, then we have

1. 6 different morphisms per sub-sign, i.e. a morphism, its heteromorphism, and 4 mediative morphisms (Toth 2009b) and thus for a maximal 4-contextural sub-sign:

$(a.b)_{i,j,k}$ $(a.b)_{j,k,i}$
$(a.b)_{i,k,j}$ $(a.b)_{k,i,j}$
$(a.b)_{j,i,k}$ $(a.b)_{k,j,i}$

2. If we restrict ourselves to such connections between dyads (sub-signs) that have identical fundamental categories (cf. Toth 2008, pp. 20 ss., 51 ss.), we have the following 6 types of semiotic connections:

(M → O) ◊ (O → I)	(O → I) ◊ (I → M)
(M → I) ◊ (I → O)	(I → O) ◊ (O → M)
(O → M) ◊ (M → I)	(I → M) ◊ (M → O)

3. Therefore, together with 1., we get the following 21 types

(i,j,k) ◊ (i,j,k)				
(i,j,k) ◊ (i,k,j)	(i,k,j) ◊ (i,k,j)			
(i,j,k) ◊ (j,i,k)	(i,k,j) ◊ (j,i,k)	(j,i,k) ◊ (j,i,k)		
(i,j,k) ◊ (j,k,i)	(i,k,j) ◊ (j,k,i)	(j,i,k) ◊ (j,k,i)	(j,k,i) ◊ (j,k,i)	
(i,j,k) ◊ (k,i,j)	(i,k,j) ◊ (k,i,j)	(j,i,k) ◊ (k,i,j)	(j,k,i) ◊ (k,i,j)	(k,i,j) ◊ (k,i,j)
(i,j,k) ◊ (k,j,i)	(i,k,j) ◊ (k,j,i)	(j,i,k) ◊ (k,j,i)	(j,k,i) ◊ (k,j,i)	(k,i,j) ◊ (k,j,i)

(k,j,i) ◊ (k,j,i)

for all 6 types of semiotic connections, and thus the maximal amount of 126 semiotic journeys. (Maximal, because all non-identitive 4-contextural morphisms have only two "indices", so that the effective number of combinations is massively smaller.)

3. However, in a sign class like

$(3.a_{i,j,k}\ 2.b_{k,j,i}\ 1.c_{i,k,j})$

we have

- 1 morphisms which is to await for sign classes: $(3.a_{i,j,k})$
- 1 heteromorphisms which is to await for the complementary sign class, i.e. after reflecting or dualizing the sign class: $(2.b_{k,j,i})$
- 1 mediative morphisms that does neither belong to a sign class nor to its reality thematic ("complementary sign class): $(1.c_{i,k,j})$.

Thus, the question arises which epistemological explication does a sign class have whose parts are from sign classes, from reality thematics and from something between. And what is this between, i.e. to which cognitive, epistemic, or communicative notion do the mediative morphisms belong? On the other side, only the order of the contextures, i.e. inner semiotic environments have been scrambled – the basis for a sign class, namely the Peircean sing relation (3.a 2.b 1.c) is still present. Thus, another question is for what do the contextures stand? Kaehr (2009a) has made an attempt at ascribing them to different epistemological subjects (you, thou, we, you). However, it is not clear what decides which contexture is mapped to which subject.

Bibliography

Kaehr, Rudolf, Xanadu's textemes. In: http://www.thinkartlab.com/CCR/2009/02/xanadus-textems.html (2009a)

Kaehr, Rudolf, Diamond relations. In: http://www.thinkartlab.com/pkl/lola/Diamond%20Relations/Diamond%20Relations.pdf (2009b)

Toth, Alfred, Zeichenzusammenhänge und Zeichennetze. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Mediation between morphisms and heteromorphisms in semiotic systems. In: Electronic Journal of Mathematical Semiotics, 2009b

Locality and local connectivity of polykontectural dual systems

1. Traditionally, the relationship between a sign class (SCl) and its corresponding reality thematic (RTh) is called dual, because in monocontextural semiotics, they are really dual, e.g.

(3.1 2.2 1.2)
×(3.1 2.2 1.2) = (2.1 2.2 1.3)

However, as Kaehr (2008) has shown, in all semiotic contextures K > 1, this duality not hold anymore, e.g.

$(3.1_3\ 2.2_{1,2}\ 1.2_1)$
$\times(3.1_3\ 2.2_{1,2}\ 1.2_1) = (2.1_1\ 2.2_{2,1}\ 1.3_1)$,

since (1,2 ≠ 2,1). This disequality concerns the direction between the two contextures in which the sub-sign (2.2) lies. Therefore, for sign relations, not only the locality (contexture) counts, but also the local connectivity. Thus, Kaehr replaces the term dual by the term complementary, although the operation of dualization (×) is commonly used in semiotics.

In Toth (2009), I had shown that every 3-adic 3-contextural sign class can appear in 48 combination of contextures, which construct a semiotic system. A a 3-adic 3-contextural sign class has the abstract form

$SCl(3,3) = (3.a_{i,j}\ 2.b_{k,l}\ 1.c_{m,n})$,

whereby i, ..., n ∈ {1, 2, 3} and j = l = n = ∅, unless in identitive morphisms ("genuine sub-signs").

In this supplement to Toth (2009), I introduce a very simple diagram in order to show the ontexture(s) as well as the connectivity of the contextures of each of the 3 sub-signs of the SCl(3,3). As an example, I have chosen (3.1 2.2 1.2), because here K(3.1) ≠ K(1.2), and K(2.2) lies always in two different contextures. In order to avoid arrows, the primordiality of connected contextures is pointed out by bold coloring of the primordially first element of an ordered pair of contextures.

2. Lokality and local connectivity of polykontectural dual systems

1) $(3.1_2\ 2.2_{1,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,1}\ 1.3_2)$

□	⊡	□		□	⊡	□
⊡	□	□		□	□	⊡
□	⊡	⊡		⊡	⊡	□

2) $(3.1_2\ 2.2_{2,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,2}\ 1.3_2)$

□	⊡	□		□	⊡	□
⊡	⊡	□		□	⊡	⊡
□	□	⊡		⊡	□	□

3) $(3.1_2\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_2)$

□	□	□		□	□	□
⊡	⊡	□		□	⊡	⊡
□	⊡	⊡		⊡	⊡	□

4) $(3.1_2\ 2.2_{2,3}1.2_1) \times (2.1_1\ 2.2_{3,2}\ 1.3_2)$

□	⊡	□		□	⊡	□
⊡	⊡	□		□	⊡	⊡
□	□	⊡		⊡	□	□

5) $(3.1_2\ 2.2_{1,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,1}\ 1.3_2)$

□	⊡	□		□	⊡	□
⊡	□	□		□	□	⊡
□	⊡	⊡		⊡	⊡	□

6) $(3.1_{2}\ 2.2_{1,2}\ 1.2_{1}) \times (2.1_{1}\ 2.2_{2,1}\ 1.3_{2})$

□ □ □ □ □ □
⊡ ⊡ □ □ ⊡ ⊡
□ ⊡ ⊡ ⊡ ⊡ □

7) $(3.1_{1}\ 2.2_{1,3}\ 1.2_{2}) \times (2.1_{2}\ 2.2_{3,1}\ 1.3_{1})$

□ ⊡ □ □ ⊡ □
□ □ ⊡ ⊡ □ □
⊡ ⊡ □ □ ⊡ ⊡

8) $(3.1_{1}\ 2.2_{2,3}\ 1.2_{2}) \times (2.1_{2}\ 2.2_{3,2}\ 1.3_{1})$

□ ⊡ □ □ ⊡ □
□ ⊡ ⊡ ⊡ ⊡ □
⊡ □ □ □ □ ⊡

9) $(3.1_{1}\ 2.2_{1,2}\ 1.2_{2}) \times (2.1_{2}\ 2.2_{2,1}\ 1.3_{1})$

□ □ □ □ □ □
□ ⊡ ⊡ ⊡ ⊡ □
⊡ ⊡ □ □ ⊡ ⊡

10) $(3.1_{1}\ 2.2_{1,2}\ 1.2_{2}) \times (2.1_{2}\ 2.2_{2,1}\ 1.3_{1})$

□ □ □ □ □ □
□ ⊡ ⊡ ⊡ ⊡ □
⊡ ⊡ □ □ ⊡ ⊡

11) $(3.1_{1}\ 2.2_{1,3}\ 1.2_{2}) \times (2.1_{2}\ 2.2_{3,1}\ 1.3_{1})$

□ ⊡ □ □ ⊡ □
□ □ ⊡ ⊡ □ □
⊡ ⊡ □ □ ⊡ ⊡

12) $(3.1_1\ 2.2_{1,2}\ 1.2_2) \times (2.1_2\ 2.2_{2,1}\ 1.3_1)$

□ □ □   □ □ □

□ ⊡ ⊡   ⊡ ⊡ □

⊡ ⊡ □   □ ⊡ ⊡

13) $(3.1_3\ 2.2_{1,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,1}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡

□ □ □   □ □ □

□ ⊡ ⊡   ⊡ ⊡ □

14) $(3.1_3\ 2.2_{2,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,2}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡

□ ⊡ □   □ ⊡ □

□ □ ⊡   ⊡ □ □

15) $(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3)$

⊡ □ □   □ □ ⊡

□ ⊡ □   □ ⊡ □

□ ⊡ ⊡   ⊡ ⊡ □

16) $(3.1_3\ 2.2_{2,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,2}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡

□ ⊡ □   □ ⊡ □

□ □ ⊡   ⊡ □ □

17) $(3.1_3\ 2.2_{1,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,1}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡

□ □ □   □ □ □

□ ⊡ ⊡   ⊡ ⊡ □

18) $(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3)$

⊡ □ □   □ □ ⊡

□ ⊡ □   □ ⊡ □

□ ⊡ ⊡   ⊡ ⊡ □

19) $(3.1_1\ 2.2_{1,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,1}\ 1.3_1)$

□ ⊡ ⊡   ⊡ ⊡ □

□ □ □   □ □ □

⊡ ⊡ □   □ ⊡ ⊡

20) $(3.1_1\ 2.2_{2,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,2}\ 1.3_1)$

□ ⊡ ⊡   ⊡ ⊡ □

□ ⊡ □   □ ⊡ □

⊡ □ □   □ □ ⊡

21) $(3.1_1\ 2.2_{1,2}\ 1.2_3) \times (2.1_3\ 2.2_{2,1}\ 1.3_1)$

□ □ ⊡   ⊡ □ □

□ ⊡ □   □ ⊡ □

⊡ ⊡ □   □ ⊡ ⊡

22) $(3.1_1\ 2.2_{2,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,2}\ 1.3_1)$

□ ⊡ ⊡   ⊡ ⊡ □

□ ⊡ □   □ ⊡ □

⊡ □ □   □ □ ⊡

23) $(3.1_1\ 2.2_{1,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,1}\ 1.3_1)$

□ ⊡ ⊡   ⊡ ⊡ □

□ □ □   □ □ □

⊡ ⊡ □   □ ⊡ ⊡

24) $(3.1_{1}\ 2.2_{1,2}\ 1.2_{3}) \times (2.1_{3}\ 2.2_{2,1}\ 1.3_{1})$

□ □ ⊡ ⊡ □ □

□ ⊡ □ □ ⊡ □

⊡ ⊡ □ □ ⊡ ⊡

25) $(3.1_{2}\ 2.2_{1,3}\ 1.2_{1}) \times (2.1_{1}\ 2.2_{3,1}\ 1.3_{2})$

□ ⊡ □ □ ⊡ □

⊡ □ □ □ □ ⊡

□ ⊡ ⊡ ⊡ ⊡ □

26) $(3.1_{2}\ 2.2_{2,3}\ 1.2_{1}) \times (2.1_{1}\ 2.2_{3,2}\ 1.3_{2})$

□ ⊡ □ □ ⊡ □

⊡ ⊡ □ □ ⊡ ⊡

□ □ ⊡ ⊡ □ □

27) $(3.1_{2}\ 2.2_{1,2}\ 1.2_{1}) \times (2.1_{1}\ 2.2_{2,1}\ 1.3_{2})$

□ □ □ □ □ □

⊡ ⊡ □ □ ⊡ ⊡

□ ⊡ ⊡ ⊡ ⊡ □

28) $(3.1_{2}\ 2.2_{2,3}\ 1.2_{1}) \times (2.1_{1}\ 2.2_{3,2}\ 1.3_{2})$

□ ⊡ □ □ ⊡ □

⊡ ⊡ □ □ ⊡ ⊡

□ □ ⊡ ⊡ □ □

29) $(3.1_{2}\ 2.2_{1,3}\ 1.2_{1}) \times (2.1_{1}\ 2.2_{3,1}\ 1.3_{2})$

□ ⊡ □ □ ⊡ □

⊡ □ □ □ □ ⊡

□ ⊡ ⊡ ⊡ ⊡ □

30) $(3.1_2\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_2)$

□ □ □   □ □ □

⊡ ⊡ □   □ ⊡ ⊡

□ ⊡ ⊡   ⊡ ⊡ □

31) $(3.1_1\ 2.2_{1,3}\ 1.2_2) \times (2.1_2\ 2.2_{3,1}\ 1.3_1)$

□ ⊡ □   □ ⊡ □

□ □ ⊡   ⊡ □ □

⊡ ⊡ □   □ ⊡ ⊡

32) $(3.1_1\ 2.2_{2,3}\ 1.2_2) \times (2.1_2\ 2.2_{3,2}\ 1.3_1)$

□ ⊡ □   □ ⊡ □

□ ⊡ ⊡   ⊡ ⊡ □

⊡ □ □   □ □ ⊡

33) $(3.1_1\ 2.2_{1,2}\ 1.2_2) \times (2.1_2\ 2.2_{2,1}\ 1.3_1)$

□ □ □   □ □ □

□ ⊡ ⊡   ⊡ ⊡ □

⊡ ⊡ □   □ ⊡ ⊡

34) $(3.1_1\ 2.2_{2,3}\ 1.2_2) \times (2.1_2\ 2.2_{3,2}\ 1.3_1)$

□ ⊡ □   □ ⊡ □

□ ⊡ ⊡   ⊡ ⊡ □

⊡ □ □   □ □ ⊡

35) $(3.1_1\ 2.2_{1,2}\ 1.2_2) \times (2.1_2\ 2.2_{2,1}\ 1.3_1)$

□ □ □   □ □ □

□ ⊡ ⊡   ⊡ ⊡ □

⊡ ⊡ □   □ ⊡ ⊡

36) $(3.1_1\ 2.2_{1,3}\ 1.2_2) \times (2.1_2\ 2.2_{3,1}\ 1.3_1)$

□ ⊡ □   □ ⊡ □
□ □ ⊡   ⊡ □ □
⊡ ⊡ □   □ ⊡ ⊡

37) $(3.1_3\ 2.2_{1,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,1}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡
□ □ □   □ □ □
□ ⊡ ⊡   ⊡ ⊡ □

38) $(3.1_3\ 2.2_{2,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,2}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡
□ ⊡ □   □ ⊡ □
□ □ ⊡   ⊡ □ □

39) $(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3)$

⊡ □ □   □ □ ⊡
□ ⊡ □   □ ⊡ □
□ ⊡ ⊡   ⊡ ⊡ □

40) $(3.1_3\ 2.2_{2,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,2}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡
□ ⊡ □   □ ⊡ □
□ □ ⊡   ⊡ □ □

41) $(3.1_3\ 2.2_{1,3}\ 1.2_1) \times (2.1_1\ 2.2_{3,1}\ 1.3_3)$

⊡ ⊡ □   □ ⊡ ⊡
□ □ □   □ □ □
□ ⊡ ⊡   ⊡ ⊡ □

42) $(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3)$

⊡ □ □ □ □ ⊡

□ ⊡ □ □ ⊡ □

□ ⊡ ⊡ ⊡ ⊡ □

43) $(3.1_1\ 2.2_{1,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,1}\ 1.3_1)$

□ ⊡ ⊡ ⊡ ⊡ □

□ □ □ □ □ □

⊡ ⊡ □ □ ⊡ ⊡

44) $(3.1_1\ 2.2_{2,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,2}\ 1.3_1)$

□ ⊡ ⊡ ⊡ ⊡ □

□ ⊡ □ □ ⊡ □

⊡ □ □ □ □ ⊡

45) $(3.1_1\ 2.2_{1,2}\ 1.2_3) \times (2.1_3\ 2.2_{2,1}\ 1.3_1)$

□ □ ⊡ ⊡ □ □

□ ⊡ □ □ ⊡ □

⊡ ⊡ □ □ ⊡ ⊡

46) $(3.1_1\ 2.2_{2,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,2}\ 1.3_1)$

□ ⊡ ⊡ ⊡ ⊡ □

□ ⊡ □ □ ⊡ □

⊡ □ □ □ □ ⊡

47) $(3.1_1\ 2.2_{1,3}\ 1.2_3) \times (2.1_3\ 2.2_{3,1}\ 1.3_1)$

□ ⊡ ⊡ ⊡ ⊡ □

□ □ □ □ □ □

⊡ ⊡ □ □ ⊡ ⊡

48) $(3.1_1\ 2.2_{1,2}\ 1.2_3) \times (2.1_3\ 2.2_{2,1}\ 1.3_1)$

□	□	⊡		⊡	□	□
□	⊡	□		□	⊡	□
⊡	⊡	□		□	⊡	⊡

3. Finally, we thus have

K(3.a 2.b 1.c) ≠ K(c.1 b.2 a.3),

because
K(SCl) = →, K(RTh) = ←,

and so SCl and RTh are asymmetric in K > 1.

Further, K(idx) = K(×(idx) + {1, 2}),
so
K(idx) = ↑, K(×(idx)) = ↓.

Bibliography

Kaehr, Rudolf, Diamond semiotics. In: http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, 3-contextural 3-adic semiotic systems. In: Electronic Journal of Mathematical Semiotics, 2009

Chirality in polycontextural sign relations

1. Perhaps the most exciting – or troublesome – feature that arises when inner semiotic environments are introduced in sign relations, is the disappearance one of the most central theories of semiotics: eigenreality (cf. Bense 1992). The problem is somewhat intricate:

1.1. In monocontextural semiotics, there is only 1 sign class amongst the 10 Peircean sign classes which is "identical" with it dual reality thematic[5]:

CS(3,1) = (3.1 2.2 1.3) × (3.1 2.2 1.3)

1.2. However, already in CS(3,3)-systems, reality thematic and it corresponding sign class are no longer dual-invers:

$CS(3,3) = (3.1_{3}\ 2.2_{1,2}\ 1.3_{3}) \times (3.1_{3}\ 2.2_{2,1}\ 1.3_{3})$

1.3. While in CS(3,3)-systems, at least those sub-signs which have only one contextural index seem to be unchanged or "identical", this assumptions turns out to be wrong starting with CS(3,4)-systems:

$CS(3,4) = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})$

1.4. Another very interesting observation is that dual sub-signs – as long as they appear in the same matrix – are really dual (and not complementary), i.e. they do change the order of their contextural indices; cf. the following (3,4)-matrix:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

Thus we have:

$(a.b_{i,j})° = (a.b_{i,j})$ for a, b ∈ {1, 2, 3} and i, j ∈ {1, 2, 3, 4}.

[5] Following Kaehr (2008), but also v. supra, we do not speak any longer of "dual systems" (DS), but of "complementary systems" (CS), taking care of the fact that reality thematics are only then dual to their sign classes, when they are monocontextural. (Therefore, the term CS covers both mono- and polycontextural sign relations.) From the numbers in parenthesis the first one indicates the n-adicity, the second the m-contexturality of a sign relation.

However, from this, it follows that polycontextural matrices cannot longer be considered transpositional vector spaces (cf. Toth 2007, 48 s.), since the transposed matrices do not give the sub-signs of the reality thematics anymore, which correspond to the sign classes as column-, row- or mixed column-row-vectors. In other words: Since $(a.b_{i,j})$ is not the corresponding reality thematic of $(a.b_{i,j})$, and since $(a.b_{i,j})° = (a.b_{i,j})$, we the complement-operator C, which turns $(a.b_{i,j})$ into $(a.b_{i,j})$ and thus a second matrix, hence totally two different semiotic matrices, one for sign relations and one for their corresponding reality relations:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix} \begin{pmatrix} 3.1_{4,3} & 2.1_{4,1} & 1.1_{4,3,1} \\ 3.2_{4,2} & 2.2_{4,2,1} & 1.2_{4,1} \\ 3.3_{4,3,2} & 2.3_{4,2} & 1.3_{4,3} \end{pmatrix}$$

As one can see easily, the two matrices are chiral, because their mirror pictures cannot be superimposed to one another (at least not in 3 dimensions).

2. Therefore, we are already in the center of our investigation. Thus, in order to look for chirality in polycontextural sign relations, it is necessary not only to look at the symmetry of the sub-signs, but also at the symmetry of their indices for any sign relation or reality thematic. However, the basic result from our earlier investigation (Toth 2009) is that there are no (formal or semantic) reasons to bind semiotic contextures either to specific sub-signs or to specific permutations or dualizations (complements, reflections) of sign relations. Therefore, it must be possible to put every sub-sign from a sign relation or reality thematic into any of n contextures and also in any n-tupels of contextures, whereby identitive morphisms (genuine sub-signs) alone receive the maximal number of contextural indices for a specifix contexture (the diagonals on the above matrices). In order to visualize semiotic chirality, we use double arrows (⇒, ⇐) for semiosic or retrosemiosic relations between the sub-signs of sign classes or reality thematics

(3.a 2.b 1.c)	(⇒, ⇒)	(c.1 b.2 a.3)	(⇒, ⇒)
(3.a 1.c 2.b)	(⇒, ⇐)	(b.2 c.1 a.3)	(⇒, ⇐)
(2.b 3.a 1.c)	(⇐, ⇒)	(c.1 a.3 b.2)	(⇐, ⇒)
(2.b 1.c 3.a)	(⇒, ⇐)	(a.3 c.1 b.2)	(⇒, ⇐)
(1.c 3.a 2.b)	(⇐, ⇒)	(b.2 a.3 c.1)	(⇐, ⇒)
(1.c 2.b 3.a)	(⇐,⇐)	(a.3 b.2 c.1)	(⇐,⇐)

and simple arrows (→, ←) for the order relations in the contextural indices:

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$	((→, →), (→, →), (→, →))
$(3.a_{i,k,j}\ 2.b_{i,k,j}\ 1.c_{i,k,j})$	((→, ←), (→, ←), (→, ←))

(3.$a_{j,i,k}$ 2.$b_{j,i,k}$ 1.$c_{j,i,k}$)	((←, →), (←, →), (←, →))
(3.$a_{j,k,i}$ 2.$b_{j,k,i}$ 1.$c_{j,k,i}$)	((→, ←), (→, ←), (→, ←))
(3.$a_{k,i,j}$ 2.$b_{k,i,j}$ 1.$c_{k,i,j}$)	((←, →), (←, →), (←, →))
(3.$a_{k,j,i}$ 2.$b_{k,j,i}$ 1.$c_{l,j,i}$)	((←, ←), (←, ←), (←, ←))

Although the mappings of the arrows to the sign classes and to the indices, respectively, are not bijective, we still get the main types of semiotic symmetries and asymmetries and can reconstruct the homonymic ones easily. Then, we can represent the combinations of morphismic and contextural order for all sign classes by using the following 4 groups of each 6 possibilities:

Group 1:

(⇒, ⇒)	asymmetric	
((→, →), (→, →), (→, →))	asymmetric	
(⇒, ⇒)	asymmetric	
((→, ←), (→, ←), (→, ←))	symmetric	
(⇒, ⇒)	asymmetric	
((←, →), (←, →), (←, →))	symmetric	
(⇒, ⇒)	asymmetric	
((→, ←), (→, ←), (→, ←))	symmetric	
(⇒, ⇒)	asymmetric	
((←, →), (←, →), (←, →))	symmetric	
(⇒, ⇒)	asymmetric	
((←, ←), (←, ←), (←, ←))	asymmetric	

Group 2:

(⇒, ⇐)	symmetric	
((→, →), (→, →), (→, →))	asymmetric	non-chiral
(⇒, ⇐)	symmetric	
((→, ←), (→, ←), (→, ←))	symmetric	chiral
(⇒, ⇐)	symmetric	
((←, →), (←, →), (←, →))	symmetric	chiral

(⇒, ⇐)	symmetric	
((→, ←), (→, ←), (→, ←))	symmetric	chiral
(⇒, ⇐)	symmetric	
((←, →), (←, →), (←, →))	symmetric	chiral
(⇒, ⇐)	symmetric	
((←, ←), (←, ←), (←, ←))	asymmetric	non-chiral

Group 3:

(⇐, ⇒)	symmetric	
((→, →), (→, →), (→, →))	asymmetric	non-chiral
(⇐, ⇒)	symmetric	
((→, ←), (→, ←), (→, ←))	symmetric	chiral
(⇐, ⇒)	symmetric	
((←, →), (←, →), (←, →))	symmetric	chiral
(⇐, ⇒)	symmetric	
((→, ←), (→, ←), (→, ←))	symmetric	chiral
(⇐, ⇒)	symmetric	
((←, →), (←, →), (←, →))	symmetric	chiral
(⇐, ⇒)	symmetric	
((←, ←), (←, ←), (←, ←))	asymmetric	non-chiral

Group 4:

(⇐,⇐)	asymmetric
((→, →), (→, →), (→, →))	asymmetric
(⇐,⇐)	asymmetric
((→, ←), (→, ←), (→, ←))	symmetric
(⇐,⇐)	asymmetric
((←, →), (←, →), (←, →))	symmetric

(⇐,⇐)	asymmetric
((→, ←), (→, ←), (→, ←))	symmetric
(⇐,⇐)	asymmetric
((←, →), (←, →), (←, →))	symmetric
(⇐,⇐)	asymmetric
((←, ←), (←, ←), (←, ←))	asymmetric

So, chirality obviously exists only in combinations of order of morphisms and contextures under the condition that the order of morphisms is symmetric. If it is asymmetric, there is neither chirality nor non-chirality. However, chirality need symmetry of both the order of the morphisms and the order of the contextures, since, if the order of the contextures is asymmetric, then the type is non-chiral.

As a final remark, one could state that monocontextural semiotic systems are characterized by eigenreality, while polycontextural semiotic systems are characterized by chirality. Interestingly enough, from both concepts, there are strong connections to physics.

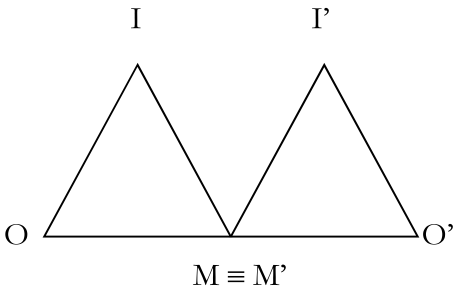
Bibliography

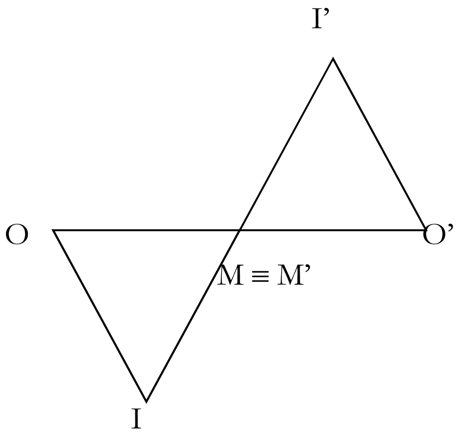
Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992
Kaehr, Rudolf, Diamond semiotics. In: http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)
Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007
Toth, Alfred, 3-contextural 3-adic semiotic systems. In: Electronic Journal for Mathematical Semiotics, 2009

Semiotic 2-, 5- and 23-categories

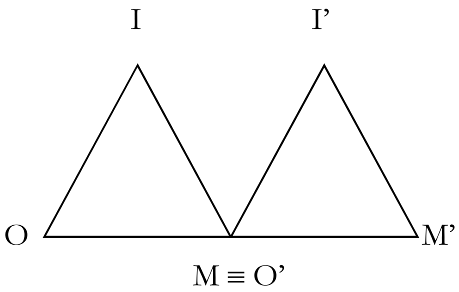
1. Sign connexes have been studied in theoretical semiotics since the beginnings (Bense 1967, 1971). Only in 2008, I have published a widely complete sign grammar showing connections of 2 and more signs in both macro-semiotic and micro-semiotic manner (Toth 2008a). One of the main results from my "General Sign Grammar" is that signs cannot only hang together in the same, but also in different fundamental categories, cf. the following examples:

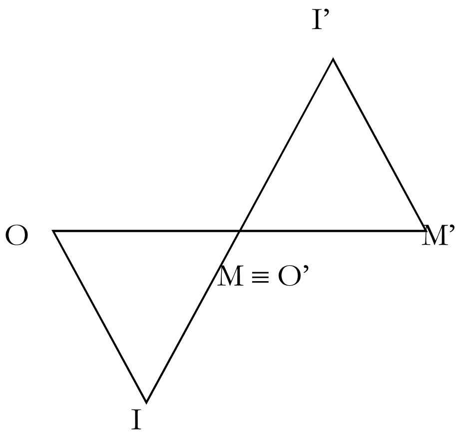
M ≡ M'



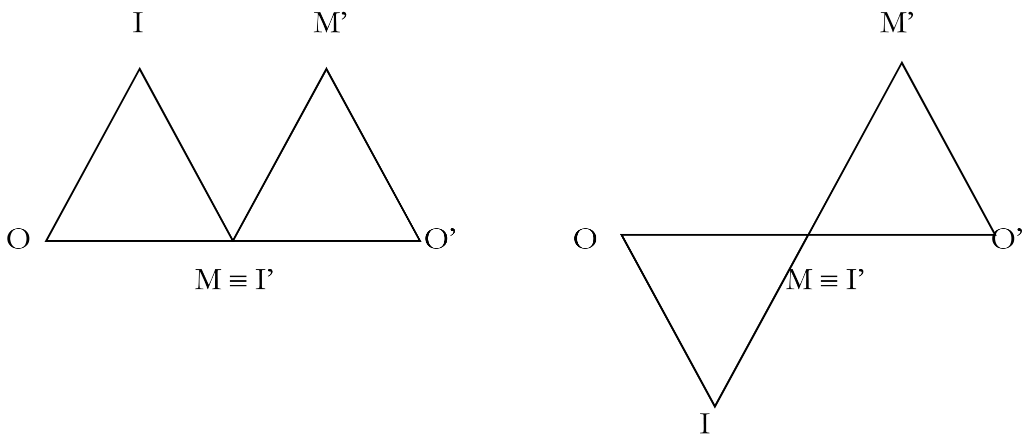


M ≡ O'

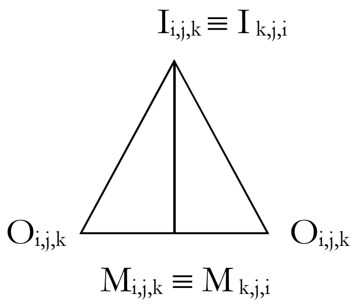




M ≡ I'



On the basis of my work and of his own studies, Kaehr (2009a) has now shown that the same two types of matching conditions also apply for "bi-signs" in "textems". For a bi-sign, we have

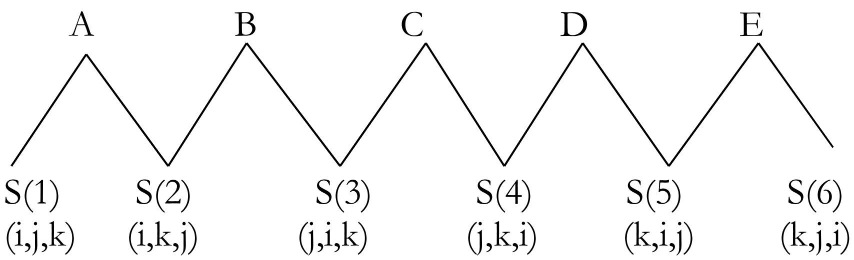


However, the matching depends here not only on the fundamental categories, but also on the contextural indices, i.e. between the morphisms (i, j, k) and the heteromorphisms (k, j, i).

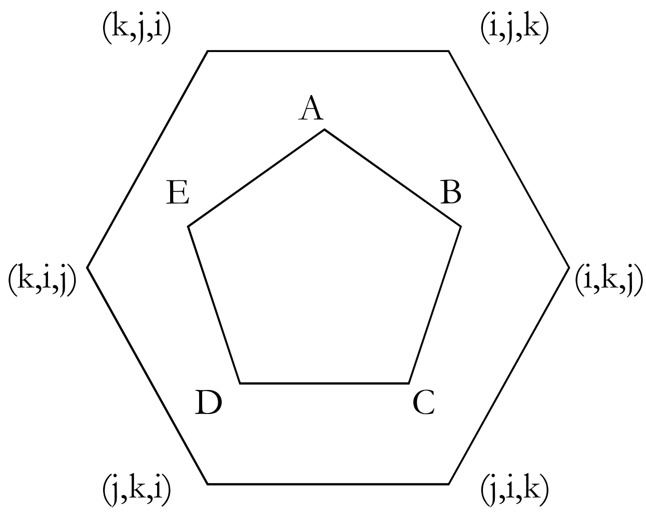
2. As I have shown in Toth (2009), besides morphisms and heteromorphisms, there are always mediative morphisms for $K > 2$. Thus, bi-signs can only exist for $K = 2$. For 3 contextures, we have the following system of morphisms, heteromorphisms and mediative morphisms:

$(i,j,k) \rightarrow (i,k,j) \rightarrow (j,i,k) \rightarrow (j,k,i) \rightarrow (k,i,j) \rightarrow (k,j,i)$,

which is a cyclic relation. Therefore, for sign relations in 3 contextures, what we need are not bi-signs, but 5-signs which could be illustrated as follows (following a 3-sign in Kaehr 2009b, p. 5):



or



3. For K = 4, we need 23-categories or 23-signs, according to the 24 permutations of the inner environments (i,j,k,l):

(ijkl), (ijlk), (ikjl), (iklj), (iljk), (ilkj),
(jikl), (jilk), (jkil), (jkli), (jlik), (jlki),
(kijl), (kilj), (kjil), (kjli), (klij), (klji),
(lijk), (likj), (ljik), (ljki), (lkij), (lkji).

For K = 5 and K = 6, we are dealing already with 119-categories and 719-categories, respectively. A motivation for these mediative morphisms can be awaited, since, after all, we are dealing with signs and thus with meaning and sense and not with "tokens" or algebraic pseudo-signs. Another question strives the need of how many fundamental categories make sense in semiotics. My last approach toward this question is Toth (2008b).

Bibliography

Bense, Max, Semiotik. Baden-Baden 1967
Bense, Max, Zeichen und Design. Baden-Baden 1971
Kaehr, Rudolf, Xanadu's textemes. http://www.thinkartlab.com/CCR/2009/02/xanadus-textems.html (2009a)

Kaehr, Rudolf, Diamond relations.
http://www.thinkartlab.com/pkl/lola/Diamond%20Relations/Diamond%20Relations.pdf (2009b)

Toth, Alfred, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008 (2008a)

Toth, Alfred, Wie viele Kontexturgrenzen hat ein Zeichen. In: In: Electronic Journal for Mathematical Semiotics, 2008b

Toth, Alfred, 3-contextural 3-adic semiotic systems. In: In: Electronic Journal for Mathematical Semiotics, 2009

The maximal system of basic homogeneous polycontextural sign relations

1. In Toth (2009), I started, based on the following abstract polycontextural semiotic matrix

$$\begin{pmatrix} 1.1_{a,b} & 1.2_{c} & 1.3_{d} \\ 2.1_{c} & 2.2_{e,f} & 2.3_{g} \\ 3.1_{d} & 3.2_{g} & 3.3_{h,i} \end{pmatrix}$$

with the fact that each of the 9 sub-signs or Cartesian products can appear in the following 4 forms

1. $(a.b)_{i,j}$ 3. $(b.a)_{i,j}$
2. $(a.b)_{j,i}$ 4. $(b.a)_{j,i}$

Therefore, on the basis of dyadic sub-signs, we get the following system of 4 basic semiotic forms

1. $Nm(a.b)_{i,j} = (a.b)_{i,j}$ (morphismic normal form)
2. $Nh(a.b)_{i,j} = (a.b)_{j,i}$ (heteromorphismic normal form)
3. $R(a.b)_{i,j} = (b.a)_{i,j}$ (reflection)
4. $D(a.b)_{i,j} = (b.a)_{j,i}$ (dualization)

2. However, if we intend to find out the basic semiotic forms for triadic sign relations (sign classes, reality thematics), we have to start with the abstract triadic sign relation, which can appear not only in 4, but in 12 outer forms:

(3.a 2.b 1.c)	(c.1 b.2 c.3)
(3.a 1.c 2.b)	(b.2 c.1 a.3)
(2.b 3.a 1.c)	(c.1 a.3 b.2)
(2.b 1.c 3.a)	(a.3 c.1 b.2)
(1.c 3.a 2.b)	(b.2 a.3 c.1)
(1.c 2.b 3.a)	(a.3 b.2 c.1)

With "outer forms", we mean here the permutations of the normal form of the sign class in the left column, and the permutations of the normal form of the dualized sign class or reality thematic in the right column.

Now, since we are up to construct maximal systems, we start not from the 3-contextural, but from the 4-contextural 3-adic semiotic matrix (Kaehr 2008, p. 8):

$$\begin{pmatrix} 1.1_{a,bj} & 1.2_{c,j} & 1.3_{d,j} \\ 2.1_{c,j} & 2.2_{e,f,j} & 2.3_{g,j} \\ 3.1_{d,j} & 3.2_{g,j} & 3.3_{h,i,j} \end{pmatrix}$$

Since the maximal number of contextural indices is 3, realized in the main diagonal, we will, for the sake of simplicity, assume that that all sub-signs of a 4.-contextural 3-adic matrix can appear in the "inner" form ($a.b_{i,j,k}$), although we actually have for non-identitive morphisms (non-genuine or "mixed" sub-signs) k = ∅.

Under this assumption, each of the three sub-signs of a sign relation can thus appear in 6 forms. Since we are here only interested in homogeneous sign relations, we have the strong restriction that each combination of contextural indices must appear as the same for all sub-signs of a specific sign relation. (For heterogeneous combinations cf. Toth 2009.) Concretely, this means that each sign relation of the form (3.a 2.b 1.c) can appear in the 6 following forms

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$
$(3.a_{i,k,j}\ 2.b_{i,k,j}\ 1.c_{i,k,j})$
$(3.a_{j,i,k}\ 2.b_{j,i,k}\ 1.c_{j,i,k})$
$(3.a_{j,k,i}\ 2.b_{j,k,i}\ 1.c_{j,k,i})$
$(3.a_{k,i,j}\ 2.b_{k,i,j}\ 1.c_{k,i,j})$
$(3.a_{k,i,j}\ 2.b_{k,i,j}\ 1.c_{k,i,j})$

Together with the permutations without consideration of indices, we therefore obtain a maximal system of 6 · 12 = 72 basic homogeneous polycontextural sign relations for each of the 10 Peircean sign classes, totally 720 sign relations.

Bibliography

Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, 3-contextural 3-adic semiotic systems. In: Electronic Journal for Mathematical Semiotics, 2009

Types of semiotic reflexivity in polycontextural semiotics

1. The two basic forms of monocontextural semiotic reflexivity, according to Bense (1992), are

1.1. The eigenreal sign class (3.1 2.2 1.3) × (3.1 2.2 1.3) whose dual reality thematic is identical to its sign class. Moreover, as Bense also pointed out, this sign class is the only one to have an in-between-symmetry: (3.1 2×2 1.3).

1.2. The class of the genuine categories (3.3 2.2 1.1) × (1.1 2.2 3.3). This sign relation is not considered a sign class because it is not built according to the semiotic inclusive order (3.a 2.b 1.c) with a ≤ b ≤ c, although it appears as main diagonal in the semiotic matrix and is thus "natural" and not constructed. Bense (1992, p. 40) speaks here about "eigenreality of weaker representation". The reason is possibly that there is an outer binnensymmetry (3.3 2.2 1.1 × 1.1 2.2 3.3) which parallels the inner binnensymmetry of (3.1 2×2 1.3).

1.3. However, as soon as inner semiotic environments are introduced (Kaehr 2008), these two types of reflexivity or eigenreality do not hold anymore, e.g.

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$; $\times(3.1_{3}\ 2.2_{1,2,4}\ 1.3_{3}) = (3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})$;
$(3.3_{2,3,4}\ 2.2_{1,2,4}\ 1.1_{1,3,4})$; $\times(3.3_{2,3,4}\ 2.2_{1,2,4}\ 1.1_{1,3,4}) = (3.3_{4,3,2}\ 2.2_{4,2,1}\ 1.1_{4,3,1})$

2. Nevertheless, Kaehr (2009) has pointed out that a pair of dyads like $(a.b_{i,j})$ and $(a.b_{j,i})$ opens a space of reflexivity for each pair, insofar as the first dyad of the pair is considered a categorial morphism $((a \rightarrow b)_{i,j})$ and the second its complementary saltatorial hetero-morphism $((a \leftarrow b)_{j,i})$. For semiotics, this means that each of the 9 sub-signs of the matrix of the dyads generating sign classes has its hetero-morphismic complement in a (complementary) matrix of the dyads generating reality thematics. In other words: The dichotomic pair of sign class/reality thematic is substituted by a pair of morphismic sign relations and hetero-morphismic sign relations between which there are mediative sign relations generated by the permutations of the contextural indices and thus mediating between the original, monocontextural concepts of dual systems consisting of sign classes and reality thematics. However, from this concept it follows that not only for the original sign classes and for the original reality thematics, but for each of the mediative sign relations there are separate semiotic matrices. Hence, a semiotic system which has more than 3 contextures requires mediative semiotic systems between their original sign classes and their original reality thematics, and only for 1 and 2 contextures, the original simple dichotomy holds, in which duality and complementarity fall together (or are not yet differentiated). So, a semiotic system with K = 3 contextures has (3! – 2) = 4 medative semiotic systems, a semiotic system with K = 4 contextures has already (4!

– 2) = 22 mediative contextures, and generally, a semiotic system with K = n contextures has of course (n! – 2) mediating semiotic systems. So, in the end we can state that eigenreality is a typical feature of monocontextural semiotics and guarantees reflexivity between the sub-signs and their semiotic processes, the morphisms. In polycontextural semiotics, eigenreality is abolished because of the possibility that a sub-sign can at the same time be located in more than one contexture and because each sub-sign has his complementary sub-sign in which not only the order of the prime-signs, but also the order of the contextures is inverted. However, the latter device is exactly how reflexivity enters polycontextural semiotics, thus, not via sub-signs and their semioses, but via contextures determining their inner semiotic environments.

3. Therefore, for each dyadic sub-sign, in polycontextural semiotics, we find

3.1. Reflexivity qua contextures alone

$(a.b_{i,j})$ vs. $(a.b_{j,i})$

3.2. Reflexivity qua sub-signs alone

$(a.b_{i,j})$ vs. $(b.a_{i,j})$

3.3. Reflexivity qua contextures and sub-signs

$(a.b_{i,j})$ vs. $(b.a_{j,i})$

However, the problem is, that these three types of reflexivity are restricted to dyads; there is no way to save monocontextural reflexivity or to introduce polycontextural reflexivity in triadic sign relations. We will show that at the hand of the above examples of monocontextural eigenreality.

3.4. Let us try to re-introduce eigenreality into the 4-contextural sign class

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$.

We start with $(1.3_{3,4}) \rightarrow (1.3_{4,3})$, i.e. through reflexivity by contextures alone:

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{4,3})$.

The only possibility to construct artificially eigenreality is not the introduction of a second index, whereby it must be hetero-morphismic:

$(3.1_{3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 1.3_{4,3})$.

In this way, we have regained the monocontextural in-between-symmetry

$(3.1_{3,4}\ 2.2_{1,2,4} \times 2.2_{4,2,1}\ 1.3_{4,3})$

as well as the eigenreality between "sign class" and "reality thematic":

$(3.1_{3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 1.3_{4,3}) \times (3.1_{3,4}\ 2.2_{1,2,4} \times 2.2_{4,2,1}\ 1.3_{4,3})$.

By doubling the object relation of the sign, we have changed a 3-adic into a 4-adic sign relation, but not adjusted the contextural indices from a 3-adic to a 4-adic sign relation. So, besides the question which epistemological status the second index has, this solution is most probably questionable or impossible.

3.5. Let us now try our luck by re-introducing "weaker eigenreality" into the 4-contextural sign class

$(3.3_{2,3,4}\ 2.2_{1,2,4}\ 1.1_{1,3,4})$.

As we quickly see, here, because of lacking binnensymmetry, we cannot apply tricks by substituting dyads by their heteromorphismic complements. But we can drop each of the three dyads and replace them by another dyad in its heteromorphismic form, until structures start to emerge:

$(3.3_{2,3,4}\ 2.2_{1,2,4}\ 3.3_{4,3,2})$.
$(1.1_{1,3,4}\ 2.2_{1,2,4}\ 1.1_{4,3,1})$

Now, we proceed like in 3.4., i.e., in both cases, we must double the object relation by inserting its heteromorphismic form:

$(3.3_{2,3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 3.3_{4,3,2})$
$(1.1_{1,3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 1.1_{4,3,1})$

Now we have even two eigenreal sign relations, which have even won binnen-symmetry by our construction. However, besides the lack of explication of the two object relations, it stays to explain why the first sign relation has no medium relation and the second no interpretant relation. In short: However one tries to save such artificial and in the end pathological sign relations, it is a fact, that in monocontextural semiotic systems eigenreality sticks to the sub-signs and their semioses, because each sign and its constituents cannot belong to more than one contexture. In polycontextural semiotic systems, however, reflexivity cannot embody reality – and thus eigenreality -, because it stays fully relational, namely bound on the order of contextures and thus depending alone of the inner semiotic environments. Reflexivity needs space to turn to itself – and there an environment. With the abolishment of the logical law of identity, eigenreality must be sacrificed, and reflexivity is moved from the static or dynamic semiotic entities to the purely relational indices of environments. Where there is no identity anymore, there can be no Eigen anymore either.

Bibliography

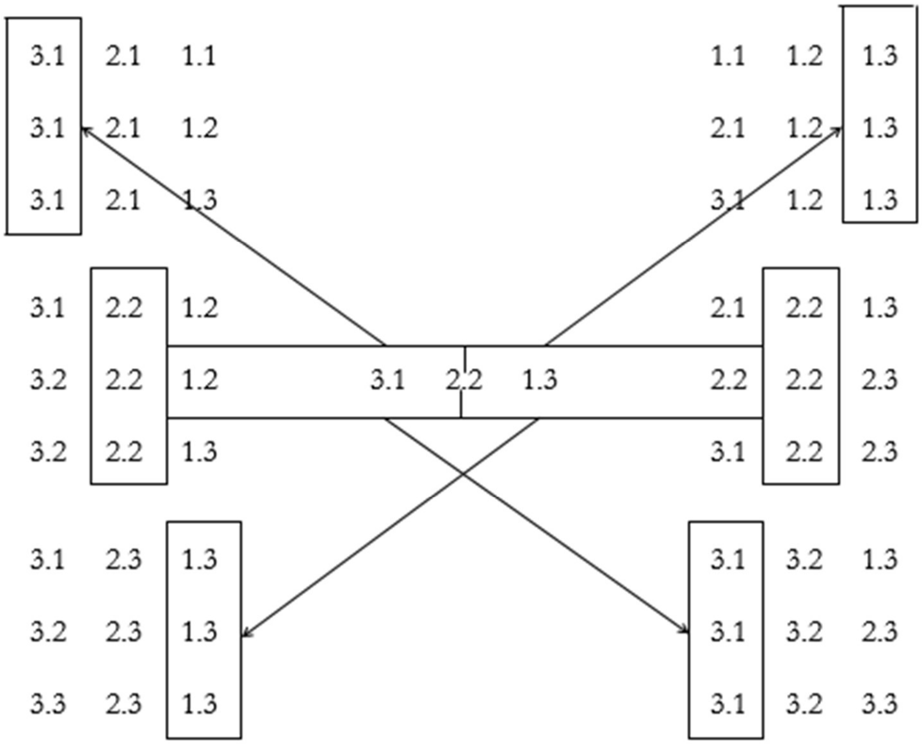
Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Kaehr, Rudolf, The category of glue. http://www.thinkartlab.com/pkl/lola/Category%20Glue/Category%20Glue.pdf (2009) (2009)

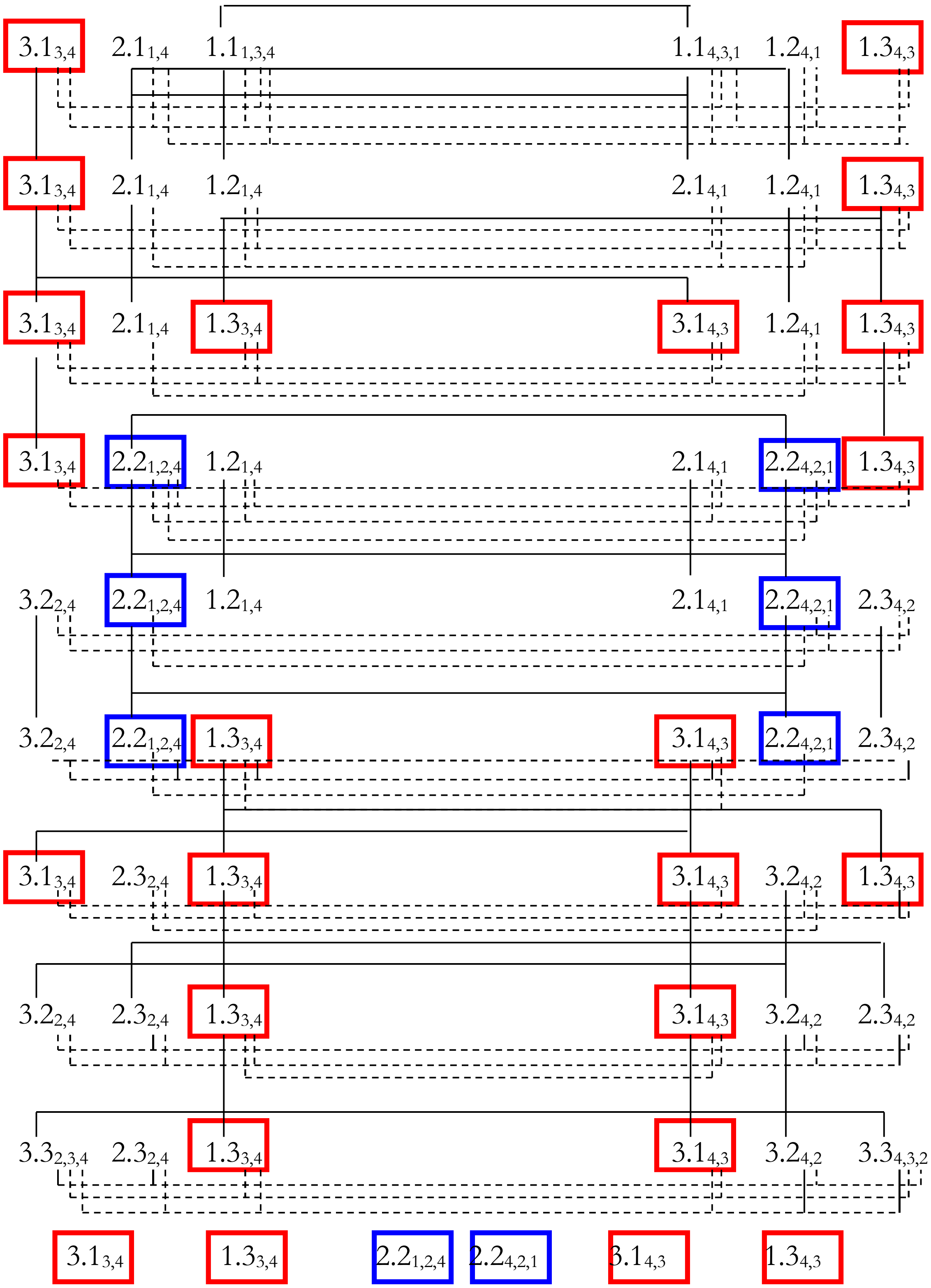
Inner and outer semiotic environments in the system of the trichotomic triads

1. Walther (1982) had shown that the monocontextural eigenreal sign class (3.1 2.2 1.3) × (3.1 2.2 1.3) first hangs together with every other sign class and reality thematic of the 10 monocontextural dual systems of Peircean semiotics. Second, the 9 sign classes and reality thematics can be ordered as "trichotomic triads" in such a way that those two times three trichotomic triads are "determined" by the eigenreal sign class:



2. As it has been shown in a series of papers by Kaehr and by me, eigenreality has to be abolished when proceding from monocontextural to polycontextural semiotic systems. The reason for this deplorable loss is that the inner environments of the sign classes involved are symmetric to those of their dual reality thematics and vice versa. Nevertheless, we will examine in the present little study if the loss of eigenreality really leads to the loss of Walther's "determinant-symmetric duality system", or not.

In the following table, the inner semiotic environments qua contural indices are dashed, and the outer semiotic environments qua shared sub-signs are straight.



Thus, we obtain a negative and positive result:

Neither the homogeneous morphismic 4-contextural sign class ($3.1_{3,4}$ $2.2_{1,2,4}$ $1.3_{3,4}$) nor the homogenous heteromorphismic 4-contextural sign class class ($3.1_{4,3}$ $2.2_{4,2,1}$ $1.3_{4,3}$) are sufficient to determine the system of the 4-contextural 9 Peircean sign classes and their symmetric “complementary” system of their reality thematics.

However, what determines the symmetric system of the trichotomic triads, which continue to exist when proceeding from mono- to polycontextural systems, is a hitherto unknown typ of "double sign class" which has the fundamental-categorial structure

(I, M, O, O, I, M)

or possibly

$(I \leftarrow M \rightarrow O \equiv O \rightarrow I \leftarrow M)$,

which has never shown up in the history of semiotics up to now.

This "double sign class" is not only binnensymmetric (in the cut of $O \equiv O$), but, as one easily sees, itself "eigenreal":

$(3.1_{3,4}\ 1.3_{3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 3.1_{4,3}\ 1.3_{4,3}) \times$
$(3.1_{3,4}\ 1.3_{3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 3.1_{4,3}\ 1.3_{4,3})$

However, what we have here, is now **polycontextural eigenreality**. Its general structure – at least what concern the "stronger" form of eigenreality (cf. Bense 1992, p. 40), is therefore

$SR(4\text{-}ER) = (3.a_{i,j}\ 1.c_{i,j}\ 2.b_{i,j,k}\ 2.b_{k,j,i}\ 3.a_{j,i}\ 1.c_{j,i})$.

Therefore, it seems that we have saved eigenreality after a series a more or less hopeless attempts. Thus, there will be a lot of work to continue in order to elaborate a theory of polycontextural eigenreality comparable to Bense's standard work (1992). Another question, that arises is: How many and which "intertwined" sign relations like SR(4-ER) do exist, and what are their epistemological interpretations?

Bibliography

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Walther, Elisabeth, Nachtrag zu Trichotomischen Triaden. In: Semiosis 27 1982, pp. 15-20

Chiastic and related sign connections in polycontextural semiotics

1. In Toth (2009) it was shown that monocontextural sign relations can appear in the 12 following basis structures

(3.a 2.b 1.c)	(c.1 b.2 c.3)
(3.a 1.c 2.b)	(b.2 c.1 a.3)
(2.b 3.a 1.c)	(c.1 a.3 b.2)
(2.b 1.c 3.a)	(a.3 c.1 b.2)
(1.c 3.a 2.b)	(b.2 a.3 c.1)
(1.c 2.b 3.a)	(a.3 b.2 c.1).

If we now assume, for a polycontextural semiotics, a maximum of three indices per sub-sign referring to contextures, and further, that the all three sub-signs are contextural homogeneous per sign relation, we get a semiotic basis system of 72 sign relations

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$	$(c.1_{k,i,j}\ b.2_{k,i,j}\ a.3_{k,i,j})$
$(3.a_{i,k,j}\ 2.b_{i,k,j}\ 1.c_{i,k,j})$	$(c.1_{j,k,i}\ b.2_{j,k,i}\ a.3_{j,k,i})$
$(3.a_{j,i,k}\ 2.b_{j,i,k}\ 1.c_{j,i,k})$	$(c.1_{k,i,j}\ b.2_{k,i,j}\ a.3_{k,i,j})$
$(3.a_{j,k,i}\ 2.b_{j,k,i}\ 1.c_{j,k,i})$	$(c.1_{i,k,j}\ b.2_{i,k,j}\ a.3_{i,k,j})$
$(3.a_{k,i,j}\ 2.b_{k,i,j}\ 1.c_{k,i,j})$	$(c.1_{j,i,k}\ b.2_{j,i,k}\ a.3_{j,i,k})$
$(3.a_{k,j,i}\ 2.b_{k,j,i}\ 1.c_{k,j,i})$	$(c.1_{i,j,k}\ b.2_{i,j,k}\ a.3_{i,j,kj})$,

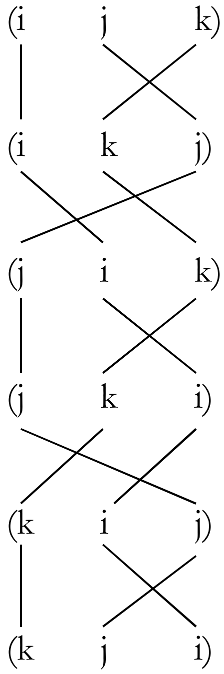
and so on for all 6 above permutations.

2. One can easily see that the connections between these 72 sign relations are quite different from the connections between the moncontextural sign relations given in Toth (2008, pp. 20 ss.). The maximal number of 72! = $6.12344584 \times 10^{103}$ connections can be split into the following groups:

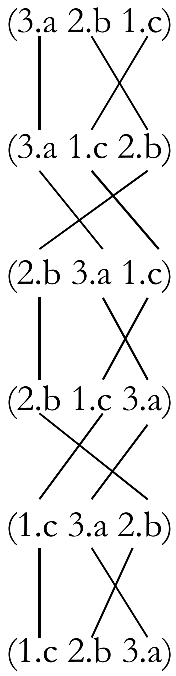
- combinations of sign classes and sign classes
- combinations of sign classes and reflections
- combinations of sign classes and dualizations
- combinations of reflections and dualizations

Other possible combinations do not furnish unexpected or otherwise “exciting” types of sign connections.

2.1. Combinations of sign classes and sign classes



2.2. Combinations of sign classes and reflections



2.3. Combinations of sign classes and dualizations

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$

$(c.1_{k,i,j}\ b.2_{k,i,j}\ a.3_{k,i,j})$

$(3.a_{i,k,j}\ 2.b_{i,k,j}\ 1.c_{i,k,j})$

$(c.1_{j,k,i}\ b.2_{j,k,i}\ a.3_{j,k,i})$

$(3.a_{j,i,k}\ 2.b_{j,i,k}\ 1.c_{j,i,k})$

$(c.1_{k,i,j}\ b.2_{k,i,j}\ a.3_{k,i,j})$

$(3.a_{j,k,i}\ 2.b_{j,k,i}\ 1.c_{j,k,i})$

$(c.1_{i,k,j}\ b.2_{i,k,j}\ a.3_{i,k,j})$

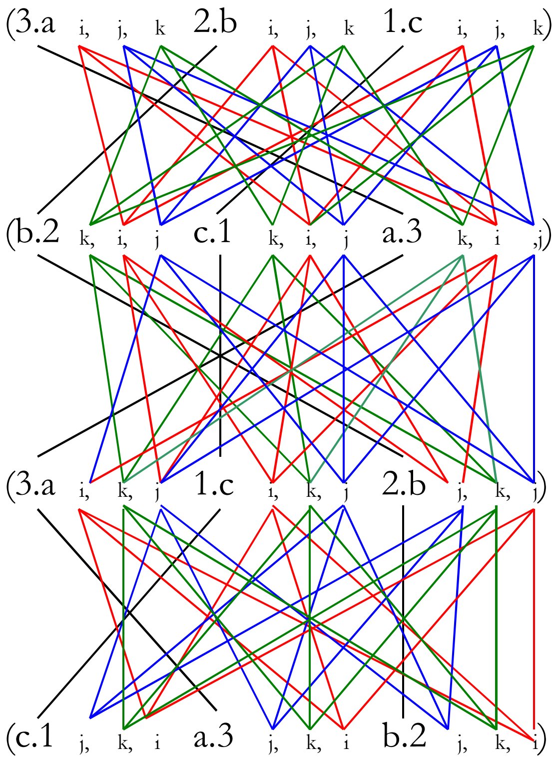
$(3.a_{k,i,j}\ 2.b_{k,i,j}\ 1.c_{k,i,j})$

$(c.1_{j,i,k}\ b.2_{j,i,k}\ a.3_{j,i,k})$

$(3.a_{k,j,i}\ 2.b_{k,j,i}\ 1.c_{k,j,i})$

$(c.1_{i,j,k}\ b.2_{i,j,k}\ a.3_{i,j,kj})$

2.4.Combinations of reflections and dualizations (excerpt)



Especially from 2.4., we can guess what an enormous complexity of connections by inner semiotic environments (contextures) result already from 3-contextural 3-adic sign relations.

Bibliography

Toth, Alfred, Semiotic Ghost Trains. Klagenfurt 2008

Toth, Alfred, The maximal system of basic homogeneous polycontextural sign relations. In: Electronic Journal for Mathematical Semiotics, 2009

The multiple reality notion in n-contextural semiotics

1. Each monocontextural sign class of the general abstract form

SCl = (3.a 2.b 1.c)

is bijectively mapped onto its dual reality thematic

×(3.a 2.b 1.c) = (c.1 b.2 a.3)

in order to form a so-called semiotic dual system:

DS = (3.a 2.b 1.c) × (c.1 b.2 a.3).

2. However, in polycontextural semiotics there is not only one, but at least two possibilities for "dualization" and thus for reality thematics. From the abstract form of the 3-contextural sign class

SCl = $(3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij})$,

we can get

$\times_1(3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) = (c.1_{i,j}\ b.2_{i,j}\ a.3_{ij})$
$\times_2(3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) = (c.1_{j,i}\ b.2_{j,i}\ a.3_{j,i})$

While the 3-contextural dual system

DS = $(3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) \times_1 (c.1_{i,j}\ b.2_{i,j}\ a.3_{ij})$

can be shown in one and the same semiotic matrix, f.ex. for

DS = $(3.1_3\ 2.2_{1,2}\ 1.2_1) \times_1 (2.1_1\ 2.2_{1,2}\ 1.3_3)$

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix}$$

the dual system

$DS = (3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) \times_2 (c.1_{j,i}\ b.2_{j,i}\ a.3_{j,i})$

needs two semiotic matrices in order to be display, f.ex. for

$DS = (3.1_3\ 2.2_{1,2}\ 1.2_1) \times_2 (2.1_1\ 2.2_{2,1}\ 1.3_3)$

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix} \quad \begin{pmatrix} 1.1_{3,1} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{2,1} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{3,2} \end{pmatrix}$$

whereby the two matrices are chiral, i.e. there is no way to superimpose the mirror pictures.

3. If have now a look at the same sign class in 4-contextures, we get

$SCl = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$

$\times_1(2.1_{1,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$
$\times_2(2.1_{4,1}\ 2.2_{4,2,1}\ 1.3_{4,3})$
$\times_3(2.1_{4,1}\ 2.2_{1,4,2}\ 1.3_{4,3})$
$\times_4(2.1_{4,1}\ 2.2_{2,1,4}\ 1.3_{4,3})$
$\times_5(2.1_{4,1}\ 2.2_{2,4,1}\ 1.3_{4,3})$
$\times_6(2.1_{4,1}\ 2.2_{4,1,2}\ 1.3_{4,3})$

and thus 6 different "reality thematics" – and these are not all, since combinations have not been looked for here.

So, while for

$1\text{-}SCl = \times_1\times_1(3.1\ 2.2\ 1.2) = (3.1\ 2.2\ 1.2)$,

we have for n-contextural sign classes with n > 1

$3\text{-}SCl = \times_2\times_2\times_2(3.1_3\ 2.2_{1,2}\ 1.2_1) = (3.1_3\ 2.2_{1,2}\ 1.2_1)$

$4\text{-}SCl = \times_3\times_3\times_3\times_3(3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4}) = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$

Regarding reality, we thus have 1 thematized reality for 1-SCl, 2 thematized realities for 3-SCl, 6 thematized realities for 4-SCl, but only as long as all sign classes are triadic! Hence generally, every n-contextural 3-adic sign class has (n-1)! thematized realities, so that n-times application of $\times_n$ closes this "semiotic Hamilton circle". It should be clear, that from these considerations, it results, that there are neither 1 nor 10 (cf. Bense 1980) nor 15 nor 35, ..., but infinite semiotic realities.

Bibliography

Bense, Max, Gotthard Günthers Universal-Metaphysik. In: Neue Zürcher Zeitung. 20./21.9.1980

Embeddings of sign relations into sign relations I

Since

$\times(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) = (3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3}) \neq (3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$

and

$\times(3.3_{2,3,4}\ 2.2_{1,2,4}\ 1.1_{1,3,4}) = (3.3_{4,3,2}\ 2.2_{4,2,1}\ 1.1_{4,3,1}) \neq (3.3_{2,3,4}\ 2.2_{1,2,4}\ 1.1_{1,3,4})$,

in Toth (2009), I have shown two possibilities of how to save eigenreality in polycontextural semiotic systems:

$$\begin{array}{lllllll} 1.\ (3.1_{3,4} & \uparrow & 2.2_{1,2,4} & \uparrow & \uparrow & & 1.3_{3,4}) \\ & 1.3_{3,4} & & 2.2_{4,2,1} & 3.1_{4,3} & & \end{array}$$

i.e., by embedding of the sign relation

$(1.3_{3,4}\ 2.2_{4,2,1}\ 3.1_{4,3})$

into the sign relation

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$.

Thus, we turn

(IOM) → (IMO≡OIM).

$$\begin{array}{lllllll} 2.\ (3.3_{2,3,4} & \uparrow & 2.2_{1,2,4} & \uparrow & 1.1_{4,3,1} & \uparrow &) \\ & 1.1_{1,3,4} & & 2.2_{4,2,1} & & 3.3_{4,3,2} & \end{array}$$

Here, the turned caused by the embedding of a sign class into a sign class is

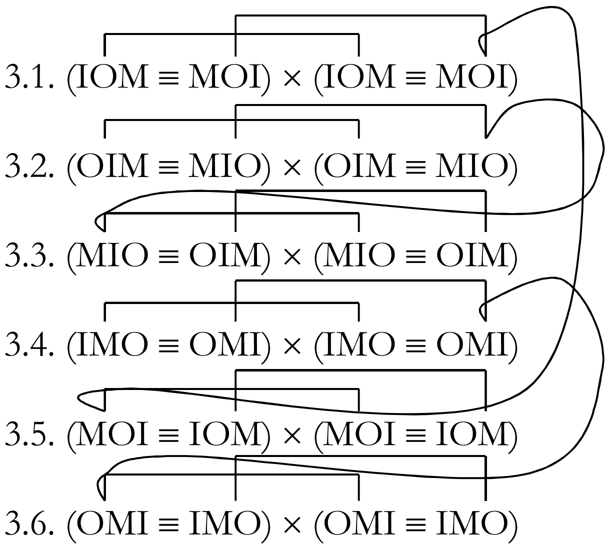
(IOM) → (IMO≡OMI).

3. Obviously, by re-establishing eigenreality, which goes lost during the transit from mono- to polycontexturality, we found a new kind of sign relation, namely the embedding of a sign relation A into the same sign relation A, but with the condition that the same fundamental categories (M, O, or I) match exactly in the middle of the new "doubled" sign relation and that the sign relation to embed consists of sub-signs whose morphisms have to be replaced by heteromorphisms, according to the

positions in the original sign relation, into which the new sign relation is embedded; cf. the above examples 1 and 2. The middle part, where X ≡ X (X ∈ (M, O, I)) match, creates binnen-symmetry. As it seems, there are exactly 6 possible types of embedding sign relations into sign relations:

3.1. (IOM ≡ MOI)
3.2. (OIM ≡ MIO)
3.3. (MIO ≡ OIM)
3.4. (IMO ≡ OMI)
3.5. (MOI ≡ IOM)
3.6. (OMI ≡ IMO)

Qua binnen-symmetry, we get, by (monocontextural) dualization, eigenreal dual systems out of these double-sign-classe and thus double-reality-thematics:



3.1. (IOM ≡ MOI) × (IOM ≡ MOI)
3.2. (OIM ≡ MIO) × (OIM ≡ MIO)
3.3. (MIO ≡ OIM) × (MIO ≡ OIM)
3.4. (IMO ≡ OMI) × (IMO ≡ OMI)
3.5. (MOI ≡ IOM) × (MOI ≡ IOM)
3.6. (OMI ≡ IMO) × (OMI ≡ IMO)

4. Although their structures (esp. the positions of the sub-signs to be embedded and their morphismic or hetero-morphismic form) have still to be scrutinized, 4-adic sign classes my be embedded into 4-adic sign classes in 4! = 24 different ways, f. ex.

(MOIQ ≡ QIOM)
(MIOQ ≡ QOIM)
(OIMQ ≡ QMIO)
(OMIQ ≡ QIMO)
(IMOQ ≡ QOMI)
(IOMQ ≡ QMOI), etc.

Another question to be investigated is which role the mediative morphisms play in the embedding of n-adic sign classes into n-adic sign classes.

Bibliography

Toth, Alfred, Types of semiotic reflexivity in polycontextural semiotics. In: Electronic Journal of Mathematical Semiotics, 2009

Semiotic Coexistence

1. In combining logic and linguistics we can look back to a long tradition, up to the theory of logical forms in generative semantics and beyond (Toth 1993, pp. 71, from a semiotic point of view). About Montague grammar, modal logic and model theoretic interpretations from a semiotic standpoint cf. Toth (2008a, pp. 47ss.). Only recently, Rudolf Kaehr has published several papers in which polycontextural logic and polycontextural semiotics are investigated together. I especially want to point to Kaehr's paper (2009a), in which the inner semiotic environments of sign relations are, for the first time, set in connection with problems of reference, therefore also bridging to the shore of linguistics. In another paper (2009b), Kaehr delivers, also first the first time, a consistent analysis of triadic semiotics and Günther's epistemological categories (cf. also Toth 2008b, pp. 64 ss.). Since we deal here with one of the most difficult problems of semiotics, this article cannot be more than a forerunner of a future theory of semiotic reference, coexistence and epistemology.

2. The beginning of the semiotic-logical theory sketched here, is, as it is so often the case in connection with phenomena at the common borders of logic, semiotics and linguistics, presented already in Gotthard Günthers work. I want to quote here the full passage, contained in the 1st foreword to Günthers "Idee und Grundriss einer nicht-aristotelischen Logik":

> "Alle bisher entwickelten Sprachen in unseren terrestrischen Hochkulturen setzen ein zweiwertiges Weltbild voraus. Ihre Reflexionsstruktur ist deshalb ebenfalls rigoros zweiwertig, und es fehlen die linguistischen Mittel, um mehrwertige Erlebnissituationen in ihnen angemessen auszudrücken. Ein Beispiel soll die Situation verdeutlichen. Der klassische Kalkül kennt einen und nur einen Begriff von 'und'. Das gleiche gilt für die deutsche, englische, französische usw. Sprache. In einer dreiwertigen Logik aber werden bereits vier (!) verschiedene und durch differente logische Funktoren identifizierte Bedeutungen von 'und' unterschieden. In unseren heutigen Umgangssprachen hat 'und' in den folgenden Konjunktionen 'ein Gegenstand *und* noch ein Gegenstand', 'Ich *und* die Gegenstände', 'Du *und* die Gegenstände', 'Wir *und* die Gegenstände' immer die gleiche Bedeutung. In anderen Worten: die klassische Logik und die an ihr spirituell orientierten Sprachen setzen voraus, dass der metaphysische Begriff der Ko-existenz so allgemein gefasst werden kann und muss, dass in ihm der Unterschied zwischen gegenständlicher Existenz und den drei möglichen Aspekten von Reflexionsexistenz irrelevant ist. Begriffe wie 'Ich', 'Du' und 'Wir' haben in der uns überlieferten Logik schlechthin keinen Sinn" (Günther 1991, p. xviii).

Before we get into the details, a remark to "den drei möglichen Aspekten von Reflexionsexistenz": In his most recent paper, Kaehr writes: "Gunther's epistemological triadism shouldn't be taken too seductively, because (t)his obsession lasted only for a short and specific time of Gunther's speculations. In the early 60ies, the dialogical concept was replaced to a much more socialist distribution of subjectivity over a mass of 'subject centers' " (Kaehr 2009b, p. 14).

Another interesting fact is that from the three basic categories of linguistic reference: animate/inanimate object, person, number, the number, too (at least singular and plural) seem to have categorical status in a polycontextural logic, when we look at Günther's example "ein Gegenstand und noch ein Gegenstand". However, the problem does not lie in the summation of two or more existential objects, but in the summation of more than one existential subject. The "We" - at least in a polycontextural logic based on "epistemological triadism" - is not considered a summation of to "I's", but – as Kaehr (2009a) had pointed out in regard to Diamond theory -, it is the area of "the Others". Being so, however, it is the opposite of the dichotomy of "I" vs. "Thou".

3. A way to overcome such problems (or pseudo-problems) is to start with a maximal system of reference as presented in theoretical linguistics and than to compare it to systems of logical and semiotic reference (Toth 2008c, vol. 2, pp. 40 ss.). Proceeding like that, we will try to find out which categories or features of a relatively complete theory of reference and coexistence is represented in polycontextural logic on the one side and in polycontextural semiotics on the other side.

3.1. In Toth (2009), it was shown that every sub-sign of a semiotic matrix can principally stand in every contexture. Concretely speaking, the following 3-contextural 3-adic semiotic matrix presented by Kaehr (2008, p. 8)

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

is one of several 3-contextural 3-adic semiotic matrices. Because of the dissemination of an n-contextural matrix into several 2-contextural matrices, what is really important in a matrix, is the diagonal whose number of indices in an n-contextural matrix is (n-1). Since a 3-contextural matrix has trivially the three contextures 1, 2, 3, the pairs of contextures as indices of the sub-signs in the main diagonal are (1,2), (1,3), (2,3), but their position is arbitrary. To put it differently: There is no law that forces (1.2) to be placed in the contexture 1 and (1.3) to be placed in the contexture 3; it can also be opposite, for example. Therefore, it follows, that also the mapping of epistemological categories onto contextures is (widely) arbitrary. For example, based on Kaehr's above 3-contextural matrix, we could suggest the following mapping:

I-Subject := 1
Thou-Subject := 2
We-Subject := 3
It-Object := 4

But already at this point, another problem arises. As I (Toth 2008a, pp. 64 ss.) and Kaehr (2009b) have shown extensively, we would rather, according to Günther (1976, pp. 336 ss.), ascribe the epistemological functions to the fundamental categories of a sign model instead of ascribing them to the inner environments of the sub-signs of a semiotic matrix. But in the latter case – however, we would be forced to deal with the problem that the dyadic sub-signs are pairs of epistemological categories, rising from "objective subject-objective subject" (1.1) via "objective object-objective object" (2.2.) to "subjective subject-subjective subject" (3.3). The question would then be which contribution the contextures would have for this system of pairs of epistemological categories. Therefore, it seems to be better to separate the semiotic fundamental categories form their "personalization" or "objectivation" in different contextures.

3.2. Languages like most Middle European languages differentiate between the following 6 subjects:

- I (ich, ego)
-thou (du, tu)
- he/she (er/sie, is/ea)
- we (wir, nos)
- you (ihr, vos)
- they (sie, ii/eae)

Grammatical difference between the gender in the 3rd (and 2nd) persons exists in some semitic languages. However, there is no trace that gender is a category relevant to logic and/or semiotics.

It is also important to see that it is not the number that produces together with the first three epistemological categories the second three epistemological categories. This results clearly from the fact that in most languages, the etymologies of I/we, thou/you, he (she)/they are not related. Number, however, is relevant for coexistence ("I and you" = "we", etc.) to be handled below.

What we therefore need for a minimal linguistic system of grammatical subjects are the 7 epistemological categories I, thou, he/she, we, you, they, plus an object. All 6 epistemological subjects can either be subjective subject, objective subject and subjective object. Hence, here it shows that epistemological categories should not be ascribed to fundamental categories, but, as we decided to do, to contextures. In Günther (1975), we read that the contextural abyss between I and Thou is as big as the contextural abyss between the Here and the Beyond. Therefore we have the following mappings between epistemological categories and semiotic contextures:

I $\rightarrow$ 1
thou $\rightarrow$ 2

he/she → 3
we → 4
you → 5
they → 6
it → 7

3.3. Finally, we can now make the step from reference to coexistence.

(I and I) → (1,1)
(I and thou) → (1,2) (thou and thou) → (2,2)
(I and he/she) → (1,3) (thou and he/she) → (2,3)
(I and we) → (1,4) (thou and we) → (2,4)
(I and you) → (1,5) (thou and you) → (2,5)
(I and they) → (1,6) (thou and they) → (2,6)
(I and it) → (1,7) (thou and it) → (2,7)

(he/she and he/she) → (3,3)
(he/she and we) → (3,4) (we and we) → (4,4)
(he/she and you) → (3,5) (we and you) → (4,5)
(he/she and they) → (3,6) (we and they) → (4,6)
(he/she an it) → (3,7) (we and it) → (4,7)

(you and you) → (5,5)
(you and they) → (5,6) (they and they) → (6,6)
(you and it) → (5,7) (they and it) → (6,7)

(it and it) → (8,8)

Thus, there are 28 combinations possible.

A first remark is that obviously, in the logical-semiotic system presented here, we have

(I + I) ≠ we; (thou + thou) ≠ (you); (he/she + he/she) ≠ (they),

thus

(1 + 1) ≠ 4; (2 + 2) ≠ 5; (3 + 3) ≠ 6.

A second remark concerns Kaehr's introduction of hetero-morphisms into Diamond theory. This truly new concept allows to model, on logical and semiotic level, the linguistic difference between

(I and thou) ≠ (thou and I), (thou and he/she) ≠ (he/she and thou),
(I and we) ≠ (we and I)

existing not in Middle European languages, but, f. ex., in Hungarian, since the order of two different subjects controls verbal agreement in such a way that the verb congruence follows in such cases the last verb. E.g.

(1) én és mi írunk "I and we are writing" (lit. I and we we-write),

but

(2) mi és én írok "We and I are writing" (lit. We and I-write).

Therefore, (1) has the contextural structure (1,4), hence morphismic, but (2) has (4,1), hence hetero-morphismic.

Although this difference is nowadays obsolete in colloquial Hungarian, it is one of the extremely seldom instances for Günther's search for polycontexturality in natural languages as cited in the passage above. In this Hungarian examples, we have

(I + we) ≠ (we + I)

and further

(I + we) ≠ (we + I) ≠ (we),

hence a second, non-classical negation in the deep structure of sentences (1) and (2). However, the mono- and the polycontextural functions are both worked out by one and the same conjunction és "and" which therefore is a logical and semiotic porte-manteau.

A third remark concern the disequations

I + thou/you (thou/you + I) ≠ we
I + he/she/they (he/she/they + I) ≠ we

which are grammaticalized in many Polynesian languages, f. ex. in Hawaiian

Pl. incl. kākou "we = you + I"

but

Pl. excl. mākou "we = he + I"

Hence, this is a second of the very rare instances of Günther's search for polycontextural structures in natural languages. The first Hawaiian expression is used in a situation where the logical subjective object (Thou) knows that he is included, e.g., to join a dinner with the subjective subject (the speaking I). However, in the second expression, the subjective object (Thou) knows that there will be, e.g., an invitation, but he is with this exlusive device nicely told that he will not be from the party. In other words: The two polycontexturally different expressions fulfil here the social function of avoiding conflict, which is typical for Polynesian.

3.4. Since we have already mapped the semiotic contextures onto Günther's epistemological categories, we can analyze all examples given in semiotic systems. On the other side, if we take Kaehr's 4-contextural 3-adic matrix whose sub-signs are more differentiated than in the corresponding 3-contextural matrix

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

we can, based on the mappings between semiotic contextures and epistemological categories, interpret this matrix as follows:

(I, he/she, we)	(I, we)	(he/she, we)
(I, we)	(I, thou, we)	(thou, we)
(he/she, we)	(thou, we)	(thou, he/she, we)

Of course, we see that this matrix is only fragment, since the epistemological categories you, and they are lacking. We may even re-interpret this matrix with the correspondences establish in the beginning of this article:

I-Subject := 1; Thou-Subject := 2; We-Subject := 3; It-Object := 4,

so that we get

(I, thou, it)	(I, it)	(we, it)
(I, it)	(I, thou, it)	(thou, we)
(we, it)	(thou, we)	(thou, we, it)

and analyze on this basis the sign classes, f. ex.

(3.1 2.2 1.2) → ((we, it), (I, thou, it), (I, it))

which is a fully new way of analysis representative systems. However, the relation between this "epistemological analysis" and the usual "model-theoretic" analysis of sign classes by Peirce (cf. Walther 1979, pp. 82 ss.) is a desideratum for the future.

Bibliography

Günther, Gotthard, Selbstbildnis im Spiegel Amerikas. In: Pongratz, Ludwig J. (Hrsg.), Philosophie in Selbstdarstellungen. Bd. 2. Hamburg 1975, pp. 1-76

Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. Bd. 1. Hamburg 1976

Günther, Gotthard, Idee und Grundriss einer nicht-aristotelischen Logik. 3rd ed. Hamburg 1991

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2008)

Kaehr, Rudolf, Diamond text theory. http://www.thinkartlab.com/pkl/media/Textems/Textems.pdf (2009a)

Kaehr, Rudolf, Triadic diamonds. http://www.thinkartlab.com/pkl/lola/Triadic%20Diamonds/Triadic%20Diamonds.pdf (2009b)

Toth, Alfred, Semiotik und Theoretische Linguistik. Tübingen 1993

Toth, Alfred, In Transit. Klagenfurt 2008 (2008a)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008b)

Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 2008 (2008c)

Toth, Alfred, Inner and outer semiotic environments in the system of the trichotomic triads. In: Electronic Journal for Mathematical Semiotics, 2009

Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

Embeddings of sign relations into sign relations II

In Toth (2009b) we had shown that there are 6 possibilities to embed a 3-adic sign class into a 3-adic sign class, according to the 6 permutations of every 3-adic sign class and the 6 matching conditions between the 3 fundamental categories:

3.1. (IOM ≡ MOI)
3.2. (OIM ≡ MIO)
3.3. (MIO ≡ OIM)
3.4. (IMO ≡ OMI)
3.5. (MOI ≡ IOM)
3.6. (OMI ≡ IMO)

Geometrically, the matching point ≡ creates a center of in-between symmetry (binnensymmetry), so that all dyads of the sign to embed to the right of the matching point change the order of their contextural indices:

1. ($3.1_{3,4}$ $2.2._{1,2.4}$ $1.3_{,3,4}$ $\uparrow$ $\uparrow$ $\uparrow$)

$1.3_{4,3}$ $2.2_{4,2,1}$ $3.1_{4,3}$) =

×($3.1_{3,4}$ $2.2._{1,2.4}$ $1.3_{,3,4}$ $1.3_{4,3}$ $2.2_{4,2,1}$ $3.1_{4,3}$) =
($3.1_{3,4}$ $2.2._{1,2.4}$ $1.3_{,3,4}$ $1.3_{4,3}$ $2.2_{4,2,1}$ $3.1_{4,3}$)

2. ($2.2_{1,2,4}$ $3.1_{3,4}$ $1.3_{3,4}$ $\uparrow$ $\uparrow$ $\uparrow$)

$1.3_{4,3}$ $3.1_{4,3}$ $2.2_{4,2,1}$)

×($2.2_{1,2,4}$ $3.1_{3,4}$ $1.3_{3,4}$ $1.3_{4,3}$ $3.1_{4,3}$ $2.2_{4,2,1}$) =
($2.2_{1,2,4}$ $3.1_{3,4}$ $1.3_{3,4}$ $1.3_{4,3}$ $3.1_{4,3}$ $2.2_{4,2,1}$

3. ($1.3_{3,4}$ $3.1_{3,4}$ $2.2_{1,2,4}$ $\uparrow$ $\uparrow$ $\uparrow$)

$2.2_{4,2,1}$ $3.1_{4,3}$ $1.3_{4,3}$

×($1.3_{3,4}$ $3.1_{3,4}$ $2.2_{1,2,4}$ $2.2_{4,2,1}$ $3.1_{4,3}$ $1.3_{4,3}$) =
($1.3_{3,4}$ $3.1_{3,4}$ $2.2_{1,2,4}$ $2.2_{4,2,1}$ $3.1_{4,3}$ $1.3_{4,3}$)

4. $(3.1_{3,4} \quad 1.3_{3,4} \quad 2.2_{1,2,4} \quad \uparrow \quad \uparrow \quad \uparrow \quad)$

$\qquad\qquad\qquad 2.2_{4,2,1} \quad 3.1_{4,3} \quad 1.3_{3,4})$

$\times(3.1_{3,4}\ 1.3_{3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 3.1_{4,3}\ 1.3_{3,4}) =$
$(3.1_{3,4}\ 1.3_{3,4}\ 2.2_{1,2,4}\ 2.2_{4,2,1}\ 3.1_{4,3}\ 1.3_{3,4})$

5. $(1.3_{3,4} \quad 2.2_{1,2,4} \quad 3.1_{3,4} \quad \uparrow \quad \uparrow \quad \uparrow \quad)$

$\qquad\qquad\qquad 3.1_{4,3} \quad 2.2_{4,2,1} \quad 1.3_{4,3})$

$\times(1.3_{3,4}\ 2.2_{1,2,4}\ 3.1_{3,4}\ 3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3}) =$
$(1.3_{3,4}\ 2.2_{1,2,4}\ 3.1_{3,4}\ 3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})$

6. $(2.2_{1,2,4} \quad 1.3_{3,4} \quad 3.1_{3,4} \quad \uparrow \quad \uparrow \quad \uparrow \quad)$

$\qquad\qquad\qquad 3.1_{4,3} \quad 1.3_{4,3} \quad 2.2_{4,2,1})$

$\times(2.2_{1,2,4}\ 1.3_{3,4}\ 3.1_{3,4}\ 3.1_{4,3}\ 1.3_{4,3}\ 2.2_{4,2,1}) =$
$(2.2_{1,2,4}\ 1.3_{3,4}\ 3.1_{3,4}\ 3.1_{4,3}\ 1.3_{4,3}\ 2.2_{4,2,1})$

As one sees, embeddings of whole sign classes into whole sign classes lead to sign graphs of six vertices. Binnensymmetry always separates sub-signs. Thus, the monocontextural case where binnensymmetry goes through a sub-sign by splitting it into its symmetric prime-signs (restricted to genuine sub-signs)

(3.1 2 × 2 1.3 × (3.1 2 × 2 1.3)

does not exist in polycontextural semiotics, which means it does not only not occur naturally, but cannot even be constructed.

Whenever two fundamental categories coincide or match, one has to make sure that one them has totally reflected contextural order:

$$\{(M\equiv M), (O\equiv O), (I\equiv I)\} \qquad \left\{\begin{array}{c} (ijk \parallel kji) \\ (ikj \parallel jki) \\ (jik \parallel kij) \end{array}\right\}$$

Since the other cases ((jki || ikj), (kij) || (jik), (kji) || (ijk)) are already included in the above 3 types, the "mediative morphisms" (Toth 2009a) ((ikj), (jik), (jki), (kij)) gather themselves together to pairs consisting of a morphism and its hetero-morphism.

Bibliography

Toth, Alfred, 3-contextural 3-adic semiotic systems. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Embeddings of sign relations into sign relations. In: Electronic Journal of Mathematical Semiotics, 2009b

On symmetry in polycontextural semiotic matrices

1. Kaehr (2008) has given the following examples for a 3- and a 4-contextural 3-adic semiotic matrix:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix} \quad \begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

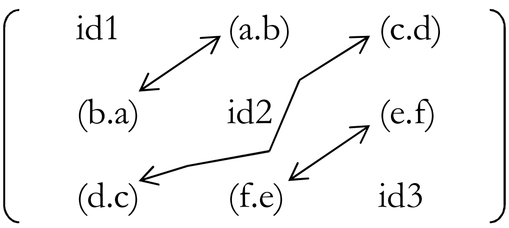
However, since there are neither formal nor semantic needs for the placing of the contextural environments to the sub-signs, in Toth (2009), I have shown many other types of both 3- and 4-contextural 3-adic matrices.

2. The maximal length of the contextural indices of an n-contextural matrix is (n-1), and this length is reserved for the contextural values of the main diagonal, the reason being the decomposability of the m×m-Matrix into (m-1)×(m-1), (m-2)×(m-2), etc. submatrices (cf. Kaehr 2009). So, all other (n^2 – n) elements of an n-contextural matrix get contextural indices of length (n-2). In the above example, the 3-contextural matrix to the left has 2-digit-length indices in the main diagonal and 1-digit-length indices otherwise.

2.1. Thus, when we start with a 3-contextural 3-adic matrix, we get the following possibilities of 1- and 2-digit-length indices:

1; 2; 3
1,2/2,1; 1,3/3,1; 2,3/3,2,

thus 6 values (1,1; 2,2; 3,3 are excluded, because this would mean that one and the same element lies two times in the same contexture). The values of the forms (a,b) and (b,a) we have taken here together, since they are just variations of one another – namely morphisms and hetero-morphisms. When we now look at the "raw scaffolding" of a 3×3 matrix:



then we see that such a matrix contains (3 · 3) – 3 = 6 different elements, i.e. elements that cannot be combined to pairs of morphisms and hetero-morphisms.

2.2. In a 4-contextural 4-adic matrix, we have the following 1-tupels:

1; 2; 3; 4,

the pairs (1,2); (1,3); (1,4); (2,3); (2,4), (3,4),

and the triples: (1,2,3); (1,2,4); (1,3,4); (2,3,4), thus 14 values.

A 4×4 Matrix has (4 · 4) – 6 = 10 different elements.

3. However, the question arises how to construct a semiotic matrix with contextuated sub-signs in the most simple and most elegant way.

3.1. In the case of the 3-contextural matrix, there are no problems, since the 6 values can be just divided over the 6 places considering that

- the trichotomy of Firstness is connected with the trichotomy of Thirdness by "the lowest interpretant (1.3)" (Bense)
- the trichotomy of Secondness builds a system of partial relations in the whole of the triadic relation because of (M ⇒ (M ⇒ O))
- the trichotomy of Thirdness is not directly connected with the partial relations of the trichotomy of Firstness, but with the trichotomy of Secondness

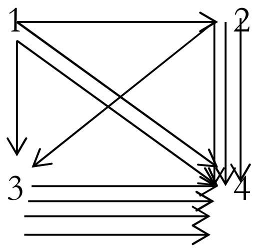
3.2. The problems start with 4-contextural matrices, since here we stand before the question of how to distribute the 14 values over 10 places. If we consider that a 4-contextural matrix can also appear as a 3-adic matrix with even less places (6), so that a 3-contextural 3-adic matrix is a fragment of a 4-contextural 4-adic matrix, we may refuse 1-tuples as contextural values. Hence we have exactly 10 values and 10 places. And as long as we are dealing with an n-contextural n-adic matrix, our more semantic argumentation may still apply here as it did above in 3.1.

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} & 1.4_{1,2} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} & 2.4_{2,4} \\ 3.1_{3,4} & 3.2_{3,1} & 3.3_{2,3,4} & 3.4_{2,3} \\ 4.1_{1,2} & 4.2_{2,4} & 4.3_{2,3} & 4.4_{1,2,3} \end{pmatrix}$$

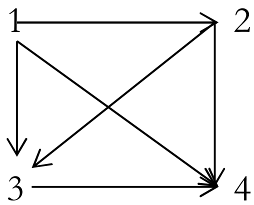
3.3. However, if we have a 4-contextural 3-adic matrix and hence 6 instead of 10 free places, we must say good-bye to 4 pairs – the question is only: to which pairs? Kaehr (2008) has solved the problem in striking simplicity, by just adding 4 as another contextural value to the contextural values of the 3-contextural 3-adic matrix:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

As one sees, the pairs (1,2); (1,3); (2,3) have disappeared, and so has the triple (1,2,3). However, if we draw the 4-contextural 3-adic matrix as a graph with the vertices 1, 2, 3, 4:



then this graphs looks highly redundant, since the following graph, too, contains all morphisms necessary for the 4-contextural 3-adic matrix:



This graph contains the pairs (1,2); (1,3); (1,4); (2,3); (2,4); (3,4); and the triples (1,2,4); (1,2,3), (1,3,4), and (2,3,4), thus, exactly the values of the 4-contextural 4-adic matrix. Therefore, the solution just to add one contextural value to the 1-tuples (→ pairs) and pairs (→ tripels) seems not be the ideal solution in order to point out that a 4-contextural 3-adic matrix is a fragment of a 4-contextural 4-adic matrix. For such cases, Kaehr's other solution, the decomposition of a matrix in its sub-matrices, seems to be a more appropriate way. Therefore we start with

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} & 1.4_{1,2} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} & 2.4_{2,4} \\ 3.1_{3,4} & 3.2_{3,1} & 3.3_{2,3,4} & 3.4_{2,3} \\ 4.1_{1,2} & 4.2_{2,4} & 4.3_{2,3} & 4.4_{1,2,3} \end{pmatrix}$$

and do not list all the possible 3×3-fragments of this 4×4-matrix, but just compare the red and the blue sub-matrices:

The red matrix is:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} \\ 3.1_{3,4} & 3.2_{1,3} & 3.3_{2,3,4} \end{pmatrix} \neq \begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

The blue matrix is:

$$\begin{pmatrix} 1.2_{1,4} & 1.3_{3,4} & 1.4_{1,2} \\ 2.2_{1,2,4} & 2.3_{1,3} & 2.4_{2,4} \\ 3.2_{3,1} & 3.3_{2,3,4} & 3.4_{2,3} \end{pmatrix}$$

However, as it stands here, the matrix is unusable. We thus have to transport the third triad to the left, and then to apply a "normal form-operator" (cf. Toth 2004). Then, the result is:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{1,3} \\ 3.1_{3,4} & 3.2_{1,3} & 3.3_{2,3,4} \end{pmatrix} \neq \begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

but the blue and the red matrix are now the same. Therefore, in both cases we get a matrix which is not the same as Kaehr's 4-contextural 3-adic matrix.

Bibliography

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2008)

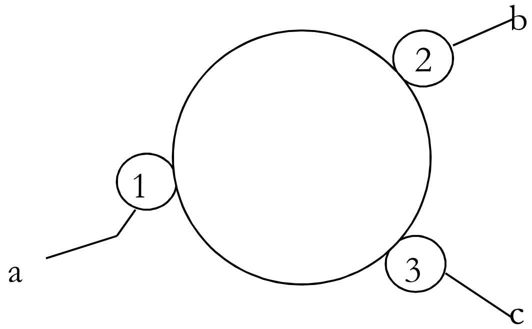
Kaehr, Rudolf, Diamond text theory. http://www.thinkartlab.com/pkl/media/Textems/Textems.pdf (2009)

Toth, Alfred, Strukturen thematisierter Realitäten in der polykontexturalen Semiotik. In: Grundlagenstudien aus Kybernetik und Geisteswissenschaft 44/4, 2004, pp. 193-198

Toth, Alfred, (668) 2009 Inner and outer semiotic environments in the system of the trichotomic triads. In: Electronic Journal for Matheamtical Semiotics, 2009

A new geometric model for polycontextural triads

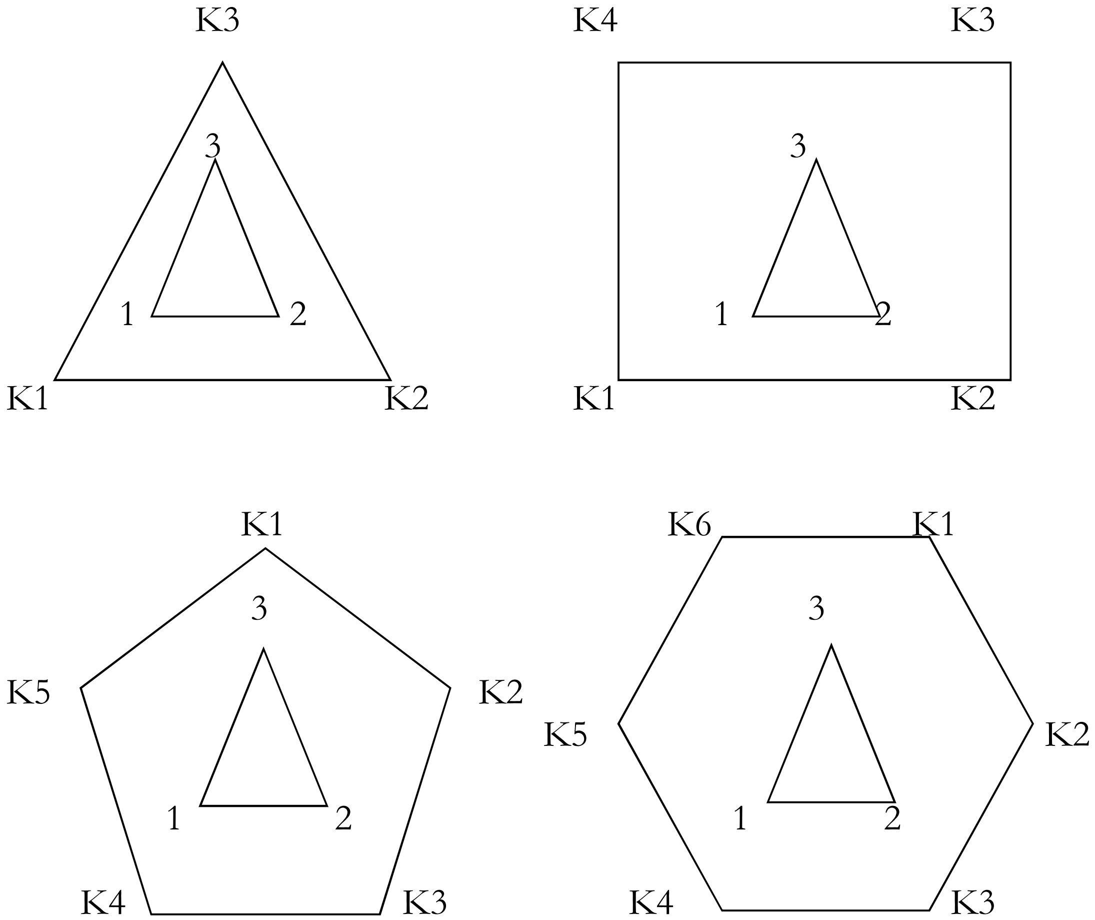
1. In 1972, in the first 3 numbers of vol. 2 of the "Journal of Cybernetics", the linguist Christopher R. Longyear presented a calculus of "triadas". Through their circle form (which is, by the way, not motivated in the three papers) they are capable of giving astonishing insights not only in the outer relationships between triads, dyads and monads as well as more complex n-ads, but also in the inner structure of triadic relations, which has escaped the linear logical models for ^3R. The abstract model of a triada presents itself like that:



According to Longyear, two triadas are equal to one another, if

1. Both triadas have three external elements.
2. Both triadas have the same three external elements.
3. Both triadas have the same order of their three elements.
4. Both triadas have the same internal structure.
5. Both triadas have the same meaning.
6. Both sides of a "shift" have arms of the same order.
7. Both sides of an internal connection have arms of the same order. (1972, p.4)

2. Longyear's triada-model can be taken as a geometric model for the Peircean triadic sign relation. However, it is not sufficient if the monocontextural sign relations are contextuated (Kaehr 2008). In this case, the sign model must be part of another model, which must be capable of representing the contextures. Since a sign can be, theoretically, in n contextures, we will prefer a polygon to a circle. Moreover, since it had been shown in Toth (2008a) that the triangle model is more appropriate to display the finesses of rotation (cf. also Toth 2008b), we will propose here a complex geometric model of a triangle embedded in an polygon, minimally triangle. Hence polycontextural signs can be displayed as follows:



As a rotation operator we may use ρ which shall work stepwise: ρ(3.1 2.1 1.3) = (1.3 3.1 2.1), ρρ(3.1 2.1 1.3) = (2.1 1.3 3.1). Thus, with ρ, all permutations of a sign relation can be generated. As for the other operators introduced in Longyear (1972), for the correspondence between (monocontextural) logic and semiotics cf. Toth (2007, pp. 143 ss.). For the application of polycontextural operators (intra- and trans-operators) cf. Toth (2003, pp. 36 ss.).

Bibliography

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2008)

Longyear, Christopher R., Further towards a triadic calculus. In: Journal of Cybernetics 2/1-3 (1972). Digital version: www.vordenker.de.

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008 (2008a)

Toth, Alfred, The semiotic wind rose. In: Electronic Journal for Mathematical Semiotics, 2008b

Polycontextural matrices

1. Kaehr (2008, p. 8) has proposed the following 3-contextural 3-adic matrix:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

From the standpoint of logic and mathematics, the existence of a matrix is per se enough; nobody has ever tried to interpret, e.g. the Sylvester-Matrix in using sense and meaning. And this is good so, since traditional logic and mathematics handle signs as tokens. However, in semiotics, we use a mathematical sign which carries sense and meaning, and therefore we must try to give the motivation of every mathematical concept that is introduced in semiotics.

2. The above semiotic matrix is interesting first, because the contextural indices hang on sub-signs which are dyads, and these dyads consist of monads or what Bense (1980) called "prime-sign" in analogy to the prime-numbers. That the monads and not the dyads are basic in semiotics, we see, e.g., then, when we dualize a dyad

$\times(a.b) = (b.a)$

and realize, that its constituents, the prime-signs, are turned around. Therefore, it is necessary to ascribe contextures not only to the sub-signs, but also to the prime-signs. And here, we are free at least from a purely formal standpoint. E.g., in a 3-contextural semiotics, we have the choice:

$a \rightarrow 1; 2; 3; (1,2); (2,3); (1.3)$ $(a \in \{.1., .2., .3.\})$

However, a Secondness (M→O) is a relation that combines a Firstness with itself, that means (1,2). And a Thirdness (O→I), consequently, is a relation that combines a Secondness with itself, that means (2,3). Now, we realize that a Firstness – quite different form Peirce's concept – is not something that stands for itself, since, for the sake of closure of the sign as a triadic relation, the Firstness is a relation, which combines itself with the whole triadic relation (1,3). Or in other words: (M→O) and (O→I) need a third mapping (M→I) for closure, so that it is impossible that Firstness as a monad stands alone, just being included in Secondness, and with Secondness in Thirdness. This has been constantly overseen in Theoretical Semiotics until Kaehr (2008) introduced the prime-signs by aid of doublets. However, since the mapping

(M→I) has been known in semiotics since decades (cf. Walther 1979, p. 73), one could have seen it.

Therefore, we can introduce the 3-contextural prime-signs as follows:

PS = {$.1._{1,3}$, $.2._{1,2}$, $.3._{2,3}$}.

Since dyads are nothing else than Cartesian products of the prime-signs onto themselves, we get

I	$.1._{1,3}$	$.2._{1,2}$	$.3._{2,3}$
$.1._{1,3}$	$1.1_{1,3}$	1.2_{1}	1.3_{3}
$.2._{1,2}$	2.1_{1}	$2.2_{1,2}$	2.3_{2}
$.3._{2,3}$	3.1_{3}	3.2_{2}	$3.3_{2,3}$

Now, we obviously have a very special law of multiplication in this matrix. The rules are:

(a,b) ♣ (a,b) = (a,b)
(a,b) ♣ (a,c) = (a,b) ♣ (c,a) = a

However, since we are free, at least from a formal standpoint, to assign any contextures to the sub-signs, it follows that the above matrix is not the only one and that we can calculate the contextural values of any semiotic matrix. Let us look at the following "alternative" matrices:

II	$.1._{1,3}$	$.2._{1,2}$	$.3._{2,3}$
$.3._{2,3}$	1.1_{3}	1.2_{2}	$1.3_{2,3}$
$.2._{1,2}$	2.1_{1}	$2.2_{1,2}$	2.3_{2}
$.1._{1,3}$	$3.1_{1,3}$	3.2_{1}	3.3_{3}

III	.1.$_{1,3}$	.2.$_{1,2}$	.3.$_{2,3}$
.2.$_{1,2}$	1.1_{1}	$1.2_{1,2}$	1.3_{2}
.1.$_{1,3}$	$2.1_{1,3}$	2.2_{1}	2.3_{3}
.3.$_{2,3}$	3.1_{3}	3.2_{2}	$3.3_{2,3}$

IV	.3.$_{2,3}$	.1.$_{1,3}$	.2.$_{1,2}$
.2.$_{1,2}$	1.1_{2}	1.2_{1}	$1.3_{1,2}$
.3.$_{2,3}$	$2.1_{2,3}$	2.2_{3}	2.3_{2}
.1.$_{1,3}$	3.1_{3}	$3.2_{1,3}$	3.3_{1}

Up to now, the following law for converse dyadic relations held:

$(a.b_{i,j}) = (b.a_{1,j})$,

as long as both sub-signs are in the same matrix. (This restriction excludes $\times(a.b_{i,j}) = (b.a_{j,i})$.) However, there is no formal reason either, why this law can not be abolished like in the 3 matrices above.

3. Up to now, sign connections have been based on shared (static) sub-signs or (dynamic) semioses, i.e. morphims between n-tuples of sign classes or reality thematics (cf. Toth 2008), e.g.

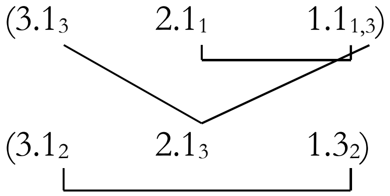
(3.1 2.1 1.1)
  | |
(3.1 2.1 1.3),
i.e. (3.1 2.1 1.1) ∩ (3.1 2.1 1.3) = (3.1 2.1).

However, what if the two sign classes do not lie in the same contextures? Cf., e.g.,

$(3.1_{3}\ 2.1_{1}\ 1.1_{1,3}) \cap (3.1_{2}\ 2.1_{3}\ 1.3_{2})$ = ??

In a monocontextural world, this intersection is as senseless as Günther's famous addition of his mother's toothache, a crocodile and the Silesian church-tower is.

We therefore have to learn to apply arithmetic operations beyond the contexture-borders. For sign connections, this means that we must give up the common sub-signs and semioses and connect only such sub-signs, which lie in the same contexture(s). Thus, we no longer connect the same sub-signs or semioses, but the same contextures:



From our three matrices above, we may guess what an enormous amount of different sign connections result from the free ascription of contextures to sub-signs.

4. If we take our above matrix I, we can distribute the sub-signs in the following manner to the contextures:

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)
K2	—	—	—	—	(2.2)	(2.3)	—	(3.2)	—
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—

However, if we take matrix II, the distribution looks like that:

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)
K2	—	(1.2)	(1.3)	—	(2.2)	(2.3)	—	—	—
K1	—	—	—	(2.1)	(2.2)	—	(1.3)	(3.2)	—

For matrix III, we get:

K3	—	—	—	(2.1)	—	(2.3)	(3.1)	—	(3.3)
K2	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.2)
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—

And for matrix IV:

K3	—	—	—	(2.1)	(2.2)	—	(3.1)	(3.2)	—
K2	(1.1)	—	(1.3)	(2.1)	—	(2.3)	—	—	—
K1	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.3)

5. Whichever matrix we use, already in 3-contextural sign relations, there are sub-signs that lie in 2 contextures, f. ex.

1. $(3.1_3\ 2.1_1\ 1.3_3)$
2. $(3.1_{1,3}\ 2.1_1\ 1.3_{2,3})$
3. $(3.1_3\ 2.1_{1,3}\ 1.3_3)$
4. $(3.1_3\ 2.1_{2,3}\ 1.3_{1,2})$

However, strictly speaking, such sign relations contain 2 sign classes, which we shall call "twin" or "multiple" sign classes:

2. $(3.1_{1,3}\ 2.1_1\ 1.3_{2,3}) \rightarrow (3.1_1\ 2.1_1\ 1.3_2) \mid (3.1_3\ 2.1_1\ 1.3_2) \mid (3.1_1\ 2.1_1\ 1.3_3) \mid (3.1_3\ 2.1_1\ 1.3_3)$
3. $(3.1_3\ 2.1_{1,3}\ 1.3_3) \rightarrow (3.1_3\ 2.1_1\ 1.3_3) \mid (3.1_3\ 2.1_3\ 1.3_3)$
4. $(3.1_3\ 2.1_{2,3}\ 1.3_{1,2}) \rightarrow (3.1_3\ 2.1_2\ 1.3_1) \mid (3.1_2\ 2.1_3\ 1.3_1) \mid (3.1_3\ 2.1_3\ 1.3_1) \mid (3.1_3\ 2.1_3\ 1.3_2)$

Another solution how to handle this "multi-ordinality", is by embedding the "ambiguous" fundamental categories into the sign relation, therefore getting to sign relations which are

tetradic: $(3.1_3\ 2.1_1\ 2.1_3\ 1.3_3)$, or

pentadic: $(3.1_3\ 2.1_2\ 2.1_3\ 1.3_1\ 1.3_2)$.

In the case of the "Genuine Category Class" we even get a

hexadic sign relation: (3.3 3.3 2.2 2.2 1.1 1.1).

If we chose this solution, we would not have to calculate with twin or multiple sign classes, but with different types of embeddings and hence besides 3-adic with 4-, 5- and 6-adic sign relations.

Bibliography

Bense, Max, Die Einführung der Primzeichen. In: Ars Semeiotica 3/3, 1980, pp. 287-294

Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008

Walther, Elisabeth, Allgemeine Zeichenlehre. 2nd ed. Stuttgart 1979

Addition of contextures of sign relations

1. According to Kronthaler (1986), "qualitative mathematics" (orig.: "Mathematik der Qualitäten") does not only allow to add objects that lie in different contextures, or to continue counting over contexture borders, but also to count contextures themselves. In this little contribution, I want to show this at the hand of the 4 systems of 3-contextures for the 9 Peircean sub-signs introduced in Toth (2009).

2. The 4 systems of 3-contextures – arbitrarily chosen and displayed in Toth (2009) – are:

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)	
K2	—	—	—	—	(2.2)	(2.3)	—	(3.2)	—	S1
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—	

K3	(1.1)	—	(1.3)	—	—	—	(3.1)	—	(3.3)	
K2	—	(1.2)	(1.3)	—	(2.2)	(2.3)	—	—	—	S2
K1	—	—	—	(2.1)	(2.2)	—	(1.3)	(3.2)	—	

K3	—	—	—	(2.1)	—	(2.3)	(3.1)	—	(3.3)	
K2	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.3)	S3
K1	(1.1)	(1.2)	—	(2.1)	(2.2)	—	—	—	—	

K3	—	—	—	(2.1)	(2.2)	—	(3.1)	(3.2)	—	
K2	(1.1)	—	(1.3)	(2.1)	—	(2.3)	—	—	—	S4
K1	—	(1.2)	(1.3)	—	—	—	—	(3.2)	(3.3)	

3. If we know look which sub-sign appears in which contextures of the 4 systems, we get

(1.1): K1 + K2 (S1) + K2 (S4)

(1.2): K1 (S1) + K2 (S2) – no representation for (1.2) in K3
(1.3): K2 + K3 (S2) + K1 (S4)
(2.1): K1 (S1) + K3 + K4 (S4)
(2.2): K1 + K2 (S1) + K4 (S4
(2.3): K2 (S2) + K3 (S3) – no representation for (2.3) in K1
(3.1): K3 (S1) – no representation for (3.1) in K1 and K2
(3.2): K2 (S1) + K1 (S2) + K3 (S4)
(3.3): K3 (S1) + K2 (S3) + K1 (S4)

In order to "fill up" the "gaps" of representation, all we would need to do is construct additional matrices, or to map the sub-signs to different contextures, respectively. Therefore, there is no need to do that but to state that unlike the mapping of natural numbers to contextures (cf. Toth 2003, pp. 56 ss.), the mapping of sub-signs to contextures is surjective. (Amongst natural numbers, e.g., the number 2 cannot be mapped to any trito-number according to Kronthaler 1986).

From the surjectivity of the mappings of sub-signs to semiotic contextures, it follows that the ideal or "balanced" representation for a dyadic sub-sign of a 3-adic sign relation is to be represented in 3 contextures. Thus, we can add the contextural systems S1 – S4 and obtain the following structure S^{1+4}:

K3	(1.1)	—	(1.3)	(2.1)	(2.2)	(2.3)	(3.1)	(3.2)	(3.3)	
K2	(1.1)	(1.2)	(1.3)	(2.1)	(2.2)	(2.3)	—	(3.2)	(3.3)	$\}$ S^{1+4}
K1	(1.2)	(1.2)	(1.3)	(2.1)	(2.3)	—	—	(3.2)	(3.3)	

When the gaps are filled, each of the ten Peircean sign classes can be represented in 3 different ways, provided that we work with 3 semiotic contextures. In the homogeneous case, we thus have

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k}) \rightarrow$

1. $(3.a_i\ 2.b_i\ 1.c_i)$
2. $(3.a_j\ 2.b_j\ 1.c_j)$
3. $(3.a_k\ 2.b_k\ 1.c_k)$,

and in the inhomogeneous cases

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k}) \rightarrow$

1. $(3.a_i\ 2.b_i\ 1.c_j)$
2. $(3.a_i\ 2.b_j\ 1.c_i)$
3. $(3.a_j\ 2.b_i\ 1.c_i)$
4. $(3.a_i\ 2.b_i\ 1.c_k)$
5. $(3.a_i\ 2.b_k\ 1.c_i)$

6. $(3.a_k\ 2.b_i\ 1.c_i)$
7. $(3.a_i\ 2.b_j\ 1.c_j)$
8. $(3.a_j\ 2.b_j\ 1.c_i)$
9. $(3.a_j\ 2.b_i\ 1.c_j)$
10. $(3.a_i\ 2.b_k\ 1.c_k)$
11. $(3.a_k\ 2.b_k\ 1.c_i)$
12. $(3.a_k\ 2.b_i\ 1.c_k)$
13. $(3.a_i\ 2.b_j\ 1.c_k)$
14. $(3.a_i\ 2.b_k\ 1.c_j)$
15. $(3.a_j\ 2.b_i\ 1.c_k)$
16. $(3.a_j\ 2.b_k\ 1.c_i)$
17. $(3.a_k\ 2.b_i\ 1.c_j)$
18. $(3.a_k\ 2.b_j\ 1.c_i)$

Bibliography

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Polycontextural matrices. In: Electronic Journal for Mathematical Semiotics, 2009

Intra- and trans- successors and predecessors in poly-contextural semiotics

1. Since the linearity of the Peano-numbers is abolished in qualitative mathematics, it is to expect that every polycontextural number has more than one predecessor and successor (Kronthaler 1986). This problem arises already on the level of the dyadic sub-signs of triadic sign relations, f. ex. in a 3-contextural semiotics:

$(3.1_3\ 2.2_{1,2}\ 1.2_1) \rightarrow (3.1_3\ 2.2_1\ 1.2_1) < (3.1_3\ 2.2_2\ 1.2_1)$

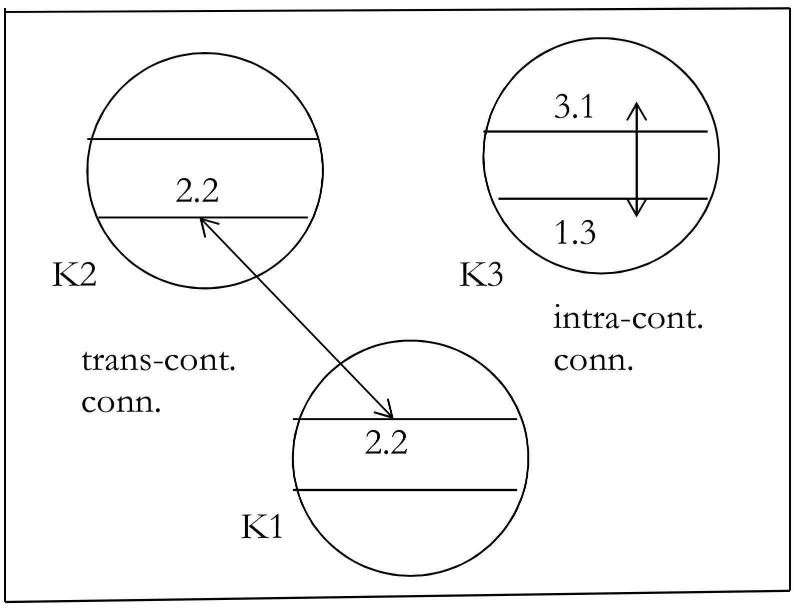
and grows with increasing number of contextures involved:

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4}) \rightarrow (3.1_3\ 2.2_1\ 1.2_1) < ... < ... (3.1_4\ 2.2_4\ 1.2_4)$

However, in polycontextural sign relations, one has to differentiate between the predecessors/successors of the sign classes and of the contextures:

$(3.1_3\ 2.2_{1,2}\ 1.2_1) \rightarrow (3.1_3\ 2.2_1\ 1.2_1) < (3.1_3\ 2.2_2\ 1.2_1)$ (contextural order)
$(2.2_1) < (2.2_2)$ (order of sub-signs)

2. Since polycontextures are understood as disseminated monocontextures, we suggest the following polycontextural sign model in order to make the problems raised in this article clear:



Thus, in $(3.1_3\ 2.2_{1,2}\ 1.3_3)$ there is trans-contextural connection

$(2.2_1) \leftrightarrow (2.2_2)$

and an intra-contextural connection.

$(3.1_3) \leftrightarrow (1.3_3)$.

An easy way to find the predecessors/successors of non-contextuated sub-signs is by aid of the semiotic matrix (cf. Toth 2008). Therefore we have

N(1.1) = {(1.1), (2.1), (1.2)}
N(1.2) = {(1.3), (2.2)}
N(1.3) = {(2.3)}
N(2.1) = {(2.2)}
N(2.2) = {(2.3), (3.2)}
N(2.3) = {(3.3)}
N(3.1) = {(3.2)}
N(3.2) = {(3.3)}

Since theoretically every sub-sign can be assigned every contexture(s), we have further

$\mathsf{N}(1.1_i) = (1.1_j)$, if $i < j$
and

$\mathsf{N}(1.2_i) = \{(1.3_j), (2.2_k)\}$, if $i < j < k$

That means that from two sub-signs (a.b), (c.d) with c > a and b > d, (c.d), N(a.b) ≠ N(c.d), but $\mathsf{N}(a.b_j) = (c.d_j)$, if j > i. Of course, this leads to a very complex system of predecessors and successors already in the sub-system of the sub-signs – and the more amongst sign classes and reality thematics.

Bibliography

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986

Toth, Alfred, Semiotic covalent bonds. In: Electronic Journal for Mathematical Semiotics, 2008

First draft of a polycontextural pre-semiotic matrix

1. Pre-Semiotics has been extensively analyzed and described in two volumes (Toth 2008). The point de départ was that the designated object as categorial object is embedded in the triadic Peircean sign relation, therefore leading to a tetradic pre-semiotic sign relation (PSR)

PSR = (3.a 2.b 1.c 0.d)

The idea of integrating the object of the sign into the sign relation itself goes back directly to Bense (1975, pp. 45 s., 65 ss.). Bense differentiated between the semiotic space of signs and the ontological space of object and assumed a transitory space between them, in which the "disposable" media mediate between the categorial object on the one side and the relational media on the other side. However, unlike the triadic relation (3.a 2.b 1.c) that consists of a monadic, a dyadic and a triadic relation, the categorial object (0.d) is a zero-relation and does behaving differently from the three other fundamental categories. According to Götz (1982, pp. 4, 28) who had picked up Bense idea, we assumed a trichotomic splitting of the categorial object into (0.1) or secandy, (0.2) or semancy, and (0.3) or selectancy. However, (0.d) as Zeroness has no triadic splitting, i.e. *(0.0), *(1.0), *(2.0), (*3.0), because these sub-signs would contradict Bense's theory of relational and categorial numbers (1975, pp. 65 s.) and would neither fit to the normal understanding, according to which a relation of a relation is meaningfull, but an object of an object is not.

Therefore, the pre-semiotic is tetradic, but trichotomic, lacking the Cartesian products marked by asterisk:

	1	2	3
0	0.1	0.2	0.3
1	1.1	1.2	1.3
2	2.1	2.2	2.3
3	3.1	3.2	3.3

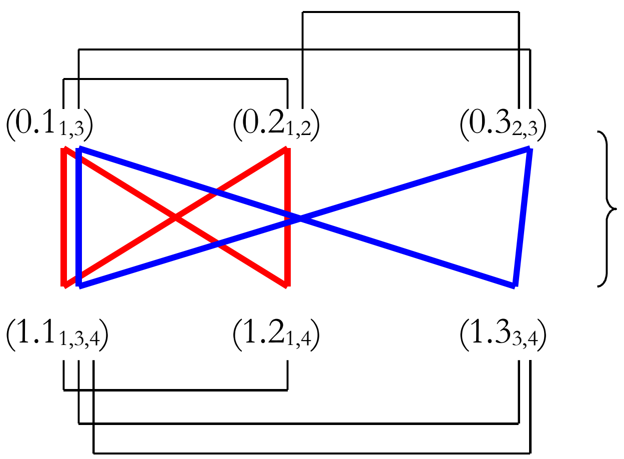
2. However, when we now go ahead and transform the monocontextural pre-semiotic matrix into a polycontextural matrix, we stand before the question if the pre-semiotic is not already a polycontextural matrix, since exactly to this behalf the categorial object had been embedded into the Peircean sign relation. This is subject that has been discussed already a couple of times. Kaehr (2008) is right when he encounters that any semiotic system in which the logical law of identity is still valid,

is monocontextural. On the other side, I am right, too, that any sign relation, in which the contextural border between sign and object is abolished, is polycontextural. However, we solve this problem quickly by following Kaehr’s way in determining for every sub-sign of the pre-semiotic matrix its inner semiotic environment. This is an n-tuple of contextures for each sub-sign. As it shows up very early, namely in sign relations, which lie in 3 contextures, sub-signs can lie in 2, 3 ... n contextures, and it is clear that by this innocent little trick the menacing law of identity is already checkmated. However, it is not quite easy to create a non-quadratic 4×3 matrix between the quadratic 3×3 and 4×4 matrices retaining the inner-matrix-symmetry of the contextural indices of pairs of converse sub-signs (e.g., $(1.2_{1,4})° = (2.1_{1,4})$, gen. $(a.b_{i,j})° = (b.a)_{1,4}$), especially because the pre-semiotic level of Zeroness (Stiebing) must be abscribed to the 1., and the semiotic levels of First-, Second- and Thirdness must be ascribed to the 2.-4. contextures. However, in this first draft, I suggest the following polycontexural pre-semiotic matrix:

	0	1	2	3
0	(0.0)	$(0.1)_{1,3}$	$(0.2)_{1,2}$	$(0.3)_{2,3}$
1	(1.0)	$(1.1)_{1,3,4}$	$(1.2)_{1,4}$	$(1.3)_{3,4}$
2	(2.0)	$(2.1)_{1,4}$	$(2.2)_{1,2,4}$	$(2.3)_{2,4}$
3	(3.0)	$(3.1)_{3,4}$	$(3.2)_{2,4}$	$(3.3)_{2,3,4}$

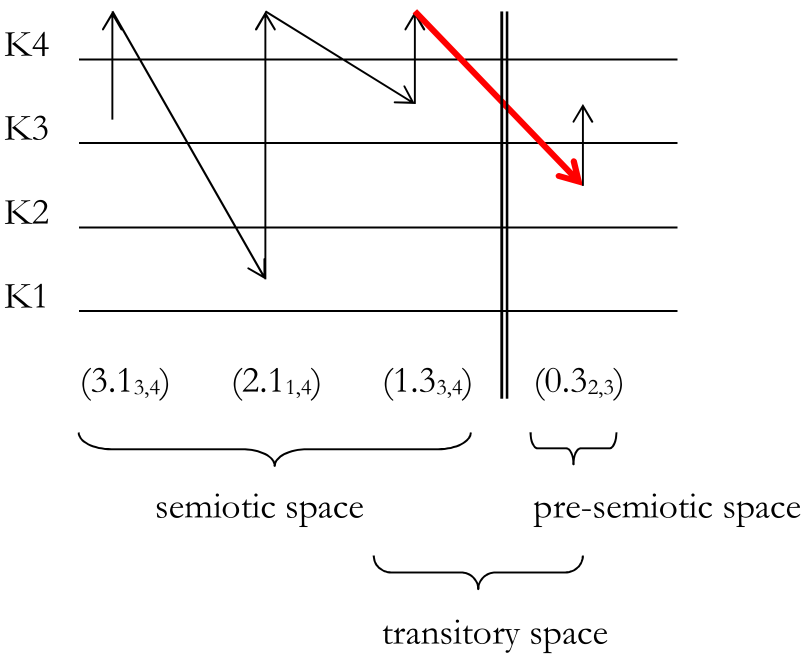
Therefore, the second occurrence of the contextural indices (1,3), (1,4), (3,4), to expect in a symmetric matrix, would have been assigned to *(1.0), *(2.0), *(3.0), and the fully excluded pseudo-relation *(0.0) would be (1,2,3).

3. Inheritance from the pre-semiotic trichotomy to the semiotic trichotomies, also extensively treated in Toth (2008), can now be formalized precisely by aid of both outer and inner semiotic connections:



Structure of mediation between “disposable” and relational media (Bense 1975)

4. Finally, what the transitory space between ontological and semiotic space concerns (Bense 1975), we can it visualize, f. ex., in the following simple schema, showing as example the pre-semiotic sign class $(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4}\ 0.3_{2,3})$:



Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 2008

Pre-semiotic-semiotic inheritance

1. In Toth (2008, pp. 166 ss.), a model for the genesis of semiosis had been introduced. It starts with Bense's (1975), Stiebings' (1981) und Götz' (1982) assumption that there is a level of Zeroness below Firstness and that therefore triadic semiotics is not the deepest level of representation. However, what is under the sign with its semiotic space, is the ontological level of the object. Now, Bense's assumption consists in positioning an in-between-level of pre-semiotics between the space of the objects and the space of the signs. On this pre-semiotic level, we have to deal with "disposable media" which developed out of "presented objects", but are not yet declared "relational media". The connection between the ontological and the semiotic space works qua a system of three invariances:

$(O°) \Rightarrow (1.1)$: Invariance of material connection;
$(O°) \Rightarrow (1.2)$: Invariance of material identification;
$(O°) \Rightarrow (1.3)$: Invariance of material existence" (Bense 1975, p. 41).

As I have pointed out (2008, p. 167a), this is a scheme of inheritence which is only then valid, if the assumption holds that already the categorial objects inhere some pre-semiotic features. It is obviously so that already at the moment when we perceive an object, we look at this object through the eyeglasses of a pre-semiotic classification scheme like "form – function – gestalt" (Wiesenfarth 1979). However, this means, on our way of classifying this object by aid of the pre-semiotic trichotomy, we have already transformed it into a "disposable" medium: f. ex., according to it form as a hammer, according to its function as a boomerang, or according to its gestalt as an icon of an animal, etc. After an object has really been declared a sign, i.e. turned from a disposable into a relational media, the pre-semiotic inheritance schema is inherited into the trichotomies of the semiotic systems.

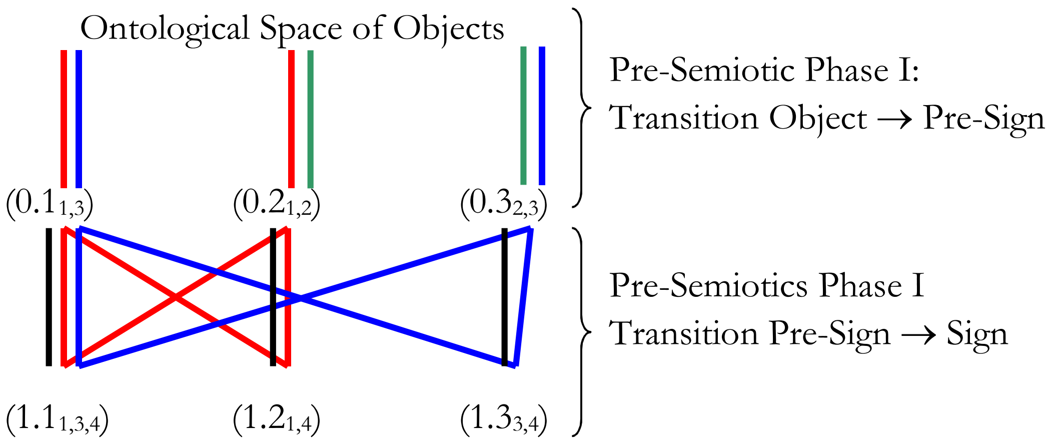
We are thus able to sketch the process of semiosis between the object and the sign as follows:

presented objects	→	disposable media	→	relational media
World of objects	→	pre-semiotics	→	semiotics

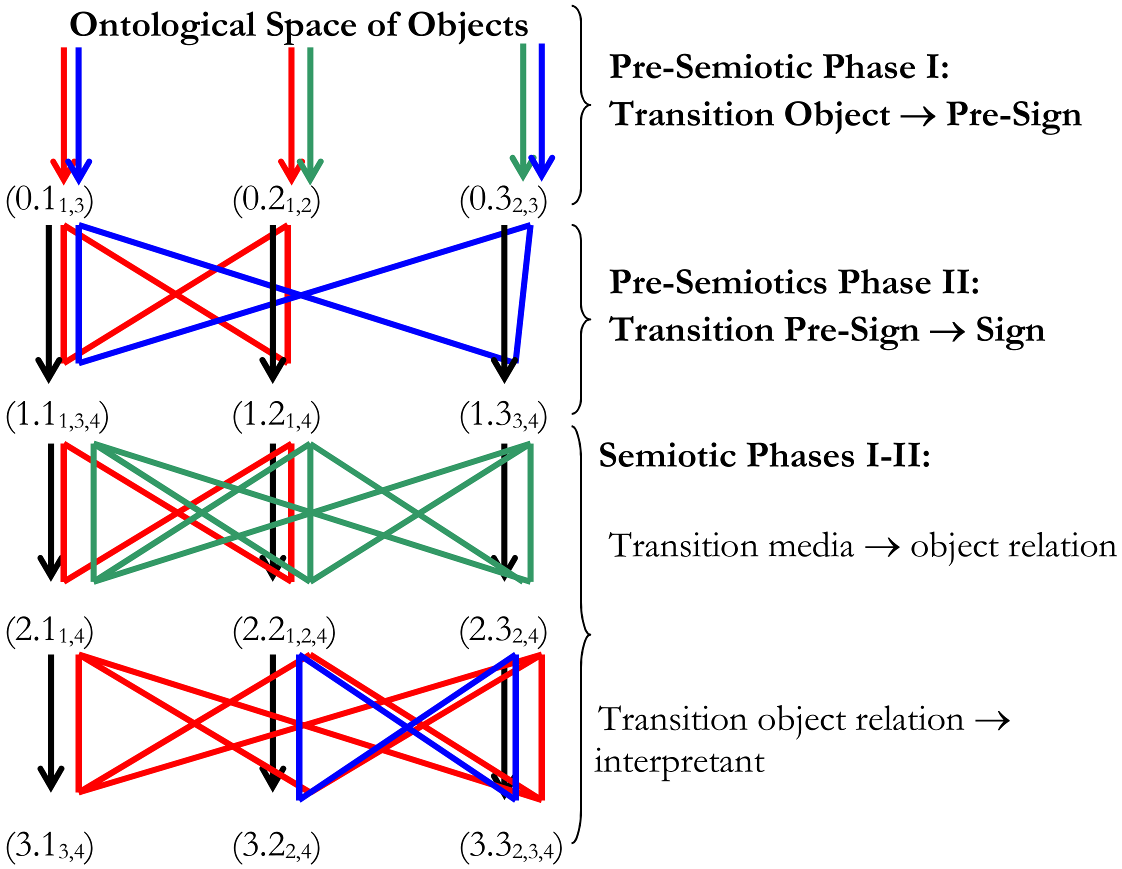
Schema of invariance: → Inheritance of invariances

1. Consistency/Secancy → 0.1
2. Identification/Semancy → 0.2
3. Existence/Selectancy → 0.3

2. But where is the level of the kenograms? If we assume, roughly speaking, that semiosis comprises the phases between object and sign, then kenogrammatics must work somewhere between ontology, pre-semiotics and semiotics. Now, one of the greatest advantages of Kaehr's introduction of polycontextural semiotics consists in just ascribing the sub-signs of a sign relation their inner semiotic envirnonments. Therefore, such polycontextural sign relations must be considered inheritance schemata, too, insofar as they are taking with them the inner environments as traces from the kenogrammatic up to the semiotic level. Therefore, the transitions between ontology and pre-semiotics on the one side and pre-semotics and semiotics on the other side form a complex semiotic system of two inheritance schemata:



The complete complex scheme of inheritances of trichotomie and inner environments in the semiosis from object to sign thus presents as follows:



The black lines denote the inheritance of the trichotomies, the colored lines the inheritance of the contextural environments.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Götz, Matthias, Schein Design. PhD dissertation, Univ. of Stuttgart, 1982

Stiebing, Hans Michael, Die Semiose von der Natur zur Kunst. In: Semiosis 23, 1981, pp. 21-31

Toth, Alfred, Semiotics and Pre-Semiotics 2 vols. Klagenfurt 2008

Wiesenfarth, Gerhard, Untersuchungen zur Kennzeichnung von Gestalt mit informationstheoretischen Methoden. PhD dissertation, Univ. of Stuttgart, 1979

The transitions between sign and object from 4 contextures

1. In Toth (2009) I have presented a new model of semiosis, based on double semiotic inheritance systems between inheritance of pre-semiotic trichotomies on the on side and inheritance of kenogrammatic contextures on the other side. An older, monocontextural approach had been presented in Toth (2008a, pp. 166 ss.).

In the present work, we start with the 4-contextural 4-adic pre-semiotic sign relation

PSR = $(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k}\ 0.d_{i,j,k})$,

introduced in Toth (2008b), which is nothing else than the 4-contextural 3-adic Peircean sign relation plus the embedded "categorial object" (Bense 1975, pp. 45 s., 65 s.).

Therefore, what interests us in this article are the different forms of transitions whose abstract schema is

$(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k}) \nparallel (0.d_{i,j,k})$,

whereby the sign $\nparallel$ stands for transition of the contextural border between sign and object.

2. However, how many polycontextural sign classes based on PSR are there? For the sake of consistency between the 10 Peircean sign classes and the 15 sign classes to be scrutinized here, we will assume that the semiotic inclusive order required for the Peircean sign classes

(3.a 2.b 1.c) with $a \leq b \leq c$ and $a, b, c \in \{1, 2, 3\}$

holds too for the 15 pre.-semiotic sign classes whose order is thus

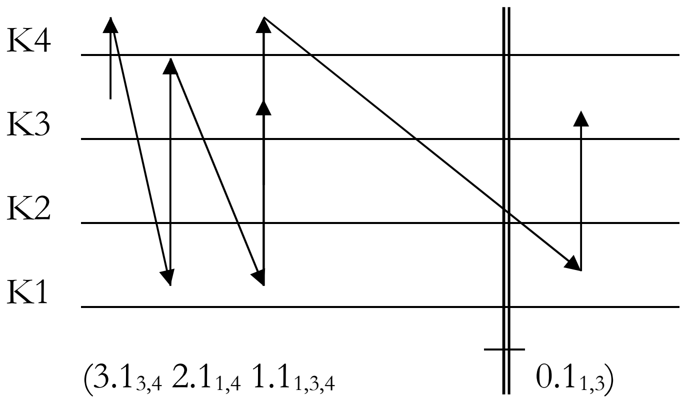
(3.a 2.b 1.c 0.d) with $a \leq b \leq c \leq d$ and $a, b, c, d \in \{1, 2, 3\}$.
Then we have the following 15 pre-semiotic 4-contextural 4-adic 3-otomic sign classes:

1	$(3.1_{3,4}\ 2.1_{1,4}\ 1.1_{1,3,4}\ 0.1_{1,3})$	×	$(1.0_{3,1}\ 1.1_{4,3,1}\ 1.2_{4,1}\ 1.3_{4,3})$
2	$(3.1_{3,4}\ 2.1_{1,4}\ 1.1_{1,3,4}\ 0.2_{1,2})$	×	$(2.0_{2,1}\ 1.1_{4,3,1}\ 1.2_{4,1}\ 1.3_{4,3})$
3	$(3.1_{3,4}\ 2.1_{1,4}\ 1.1_{1,3,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 1.1_{4,3,1}\ 1.2_{4,1}\ 1.3_{4,3})$
4	$(3.1_{3,4}\ 2.1_{1,4}\ 1.2_{1,4}\ 0.2_{1,2})$	×	$(2.0_{2,1}\ 2.1_{4,1}\ 1.2_{4,1}\ 1.3_{4,3})$
5	$(3.1_{3,4}\ 2.1_{1,4}\ 1.2_{1,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 2.1_{4,1}\ 1.2_{4,1}\ 1.3_{4,3})$

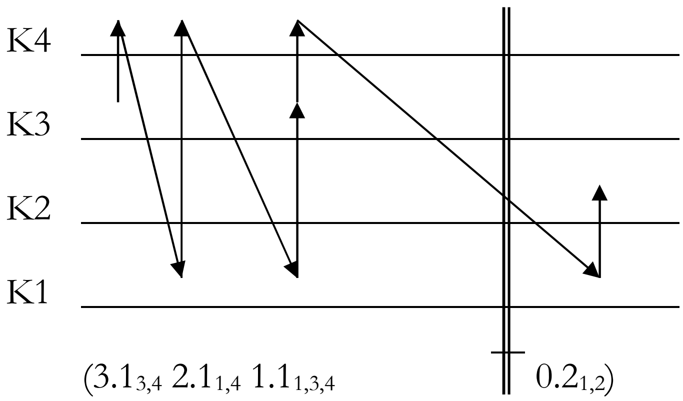
6	$(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 3.1_{4,3}\ 1.2_{4,1}\ 1.3_{4,3})$
7	$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4}\ 0.2_{1,2})$	×	$(2.0_{2,1}\ 2.1_{4,1}\ 2.2_{4,2,1}\ 1.3_{4,3})$
8	$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{\cdot 1,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 2.1_{4,1}\ 2.2_{4,2,1}\ 1.3_{4,3})$
9	$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})$
10	$(3.1_{3,4}\ 2.3_{2,4}\ 1.3_{3,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 3.1_{4,3}\ 3.2_{4,2}\ 1.3_{4,3})$
11	$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.2_{1,4}\ 0.2_{1,2})$	×	$(2.0_{2,1}\ 2.1_{4,1}\ 2.2_{4,2,1}\ 2.3_{4,2})$
12	$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.2_{1,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 2.1_{4,1}\ 2.2_{4,2,1}\ 2.3_{4,2})$
13	$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.3_{3,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 3.1_{4,3}\ 2.2_{4,2,1}\ 2.3_{4,2})$
14	$(3.2_{2,4}\ 2.3_{2,4}\ 1.3_{3,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 3.1_{4,3}\ 3.2_{4,2}\ 2.3_{4,2})$
15	$(3.3_{2,3,4}\ 2.3_{2,4}\ 1.3_{3,4}\ 0.3_{2,3})$	×	$(3.0_{3,2}\ 3.1_{4,3}\ 3.2_{4,2}\ 3.3_{4,3,2})$,

3. Now we will draw the corresponding diagrams (for the sign classes only).

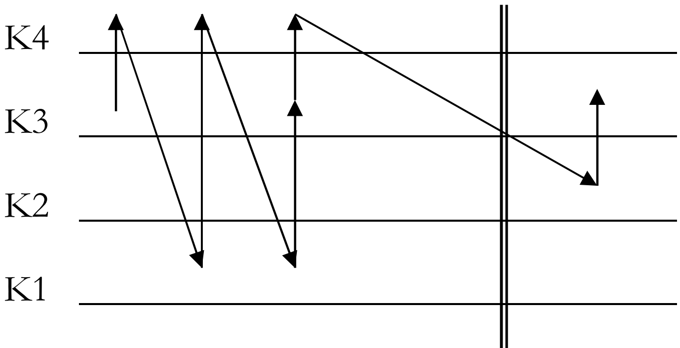
3.1. 1. PS (4-4-3)



3.2. 2. PS (4-4-3)

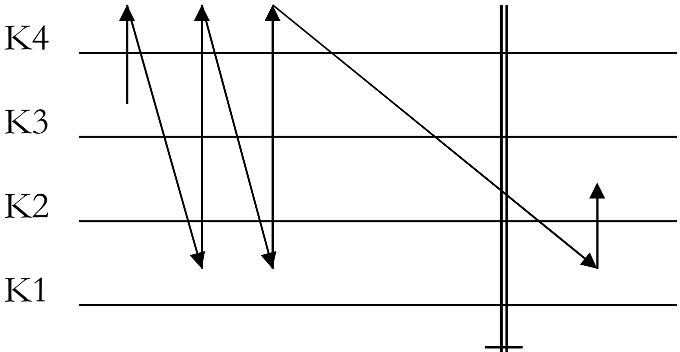


3.3. 3. PS (4-4-3)



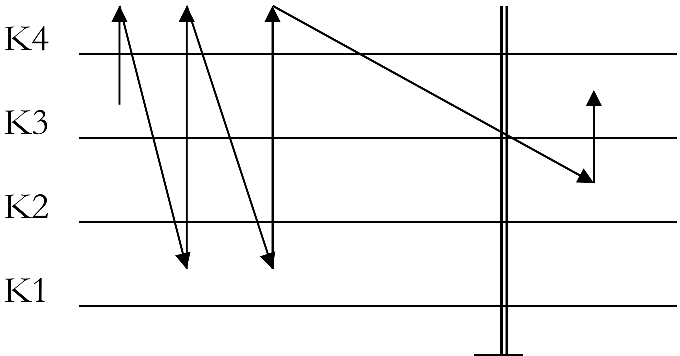
$(3.1_{3,4}\ 2.1_{1,4}\ 1.1_{1,3,4}$ ‖ $0.3_{2,3})$

3.4. 4. PS (4-4-3)



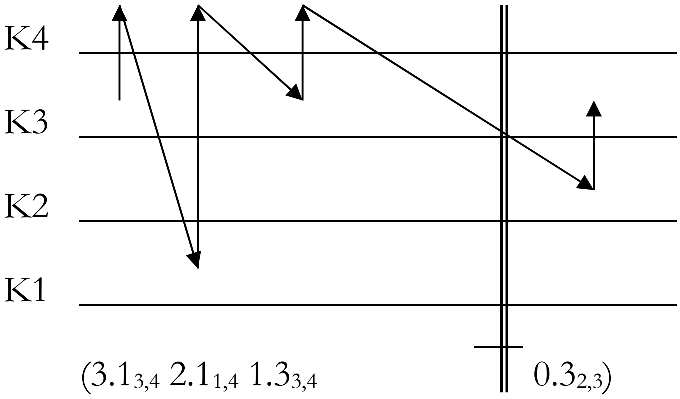
$(3.1_{3,4}\ 2.1_{1,4}\ 1.2_{1,4}$ ‖ $0.2_{1,2})$

3.5. 5. PS (4-4-3)



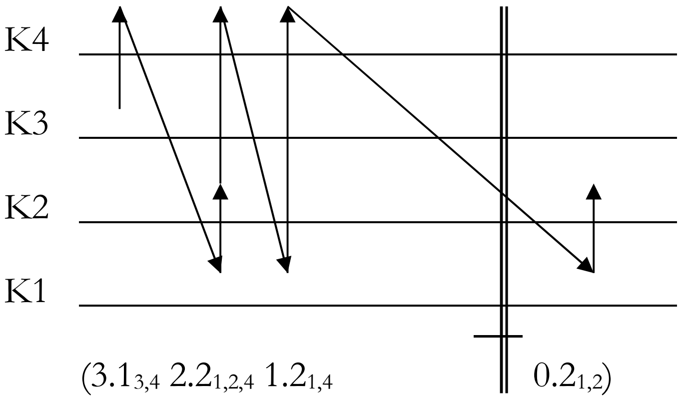
$(3.1_{3,4}\ 2.1_{1,4}\ 1.2_{1,4}$ ‖ $0.3_{2,3})$

3.6. 6. PS (4-4-3)



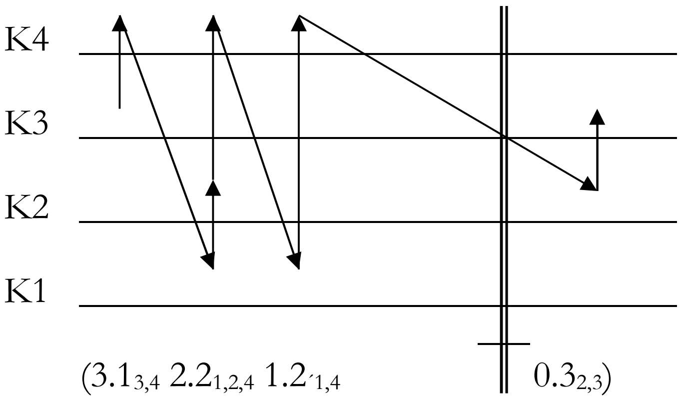
($3.1_{3,4}$ $2.1_{1,4}$ $1.3_{3,4}$ $0.3_{2,3}$)

3.7. 7. PS (4-4-3)



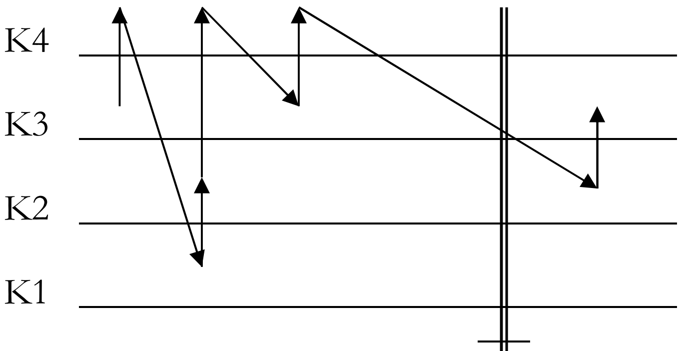
($3.1_{3,4}$ $2.2_{1,2,4}$ $1.2_{1,4}$ $0.2_{1,2}$)

3.8. 8. PS (4-4-3)



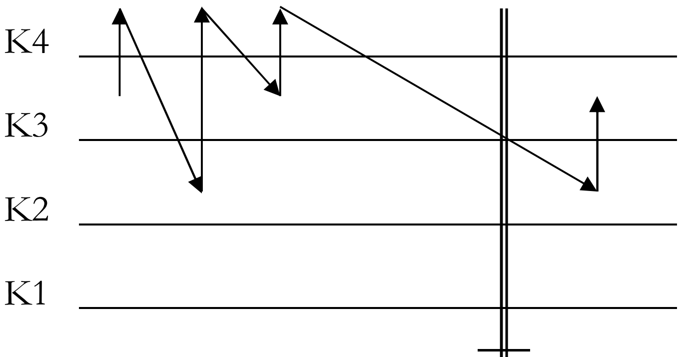
($3.1_{3,4}$ $2.2_{1,2,4}$ $1.2'_{1,4}$ $0.3_{2,3}$)

3.9. 9. PS (4-4-3)



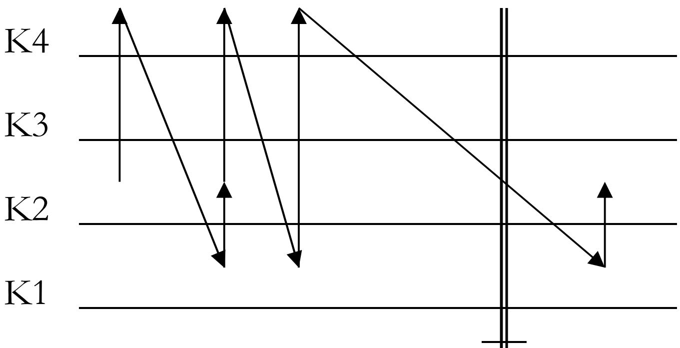
$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4} \quad 0.3_{2,3})$

3.10. 10. PS (4-4-3)



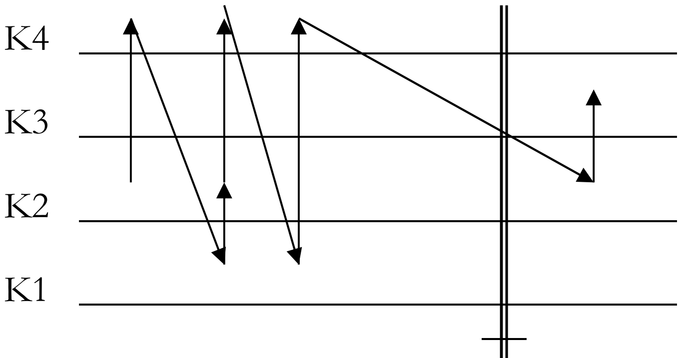
$(3.1_{3,4}\ 2.3_{2,4}\ 1.3_{3,4} \quad 0.3_{2,3})$

3.11. 11. PS (4-4-3)



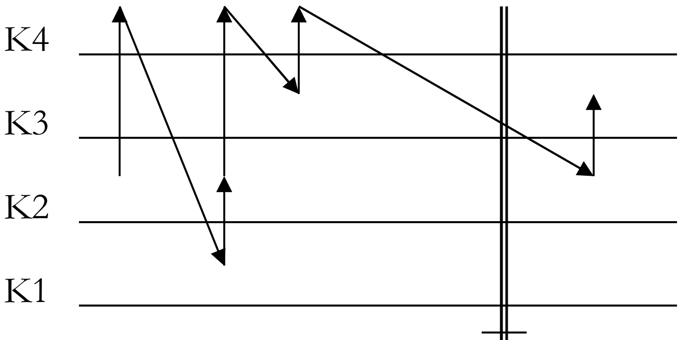
$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.2_{1,4} \quad 0.2_{1,2})$

3.12. 12. PS (4-4-3)



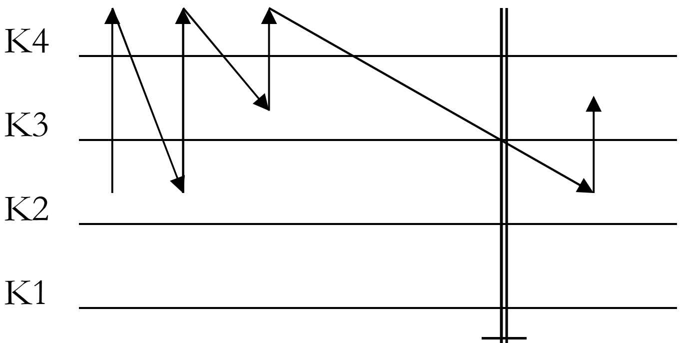
$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.2_{1,4} \quad 0.3_{2,3})$

3.13. 13. PS (4-4-3)



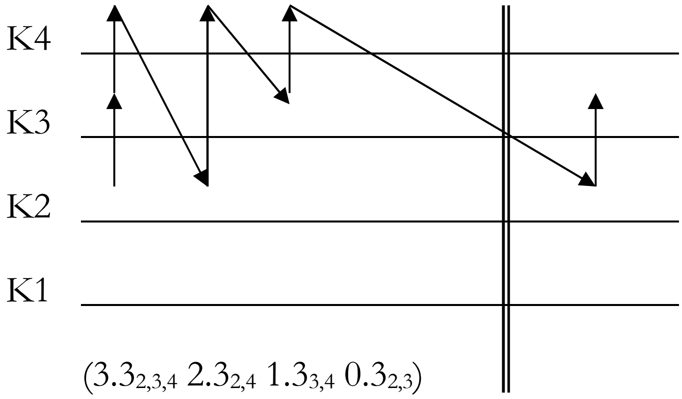
$(3.2_{2,4}\ 2.2_{1,2,4}\ 1.3_{3,4} \quad 0.3_{2,3})$

3.14. 14. PS (4-4-3)



$(3.2_{2,4}\ 2.3_{2,4}\ 1.3_{3,4} \quad 0.3_{2,3})$

3.15. 15. PS (4-4-3)



From 4 contextures, there are thus the following 6 transitions between sign and object:

1. $[1.1_{1,3,4}$ ╫ $0.1_{1,3}]$
2. $[1.1_{1,3,4}$ ╫ $0.2_{1,2}]$
3. $[1.1_{1,3,4}$ ╫ $0.3_{2,3}]$
4. $[1.2_{1,4}$ ╫ $0.2_{1,2}]$
5. $[1.2_{1,4}$ ╫$0.3_{2,3}]$
6. $[1.3_{3,4}$ ╫$0.2_{2,3}]$

Bibliography

Bense, Max, Semiotische Prozesse und Systeme.
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)
Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 200 (2008b)

Representation classes of contextural orders

1. As it is known, in changing from the monocontextural Peircean sign schema to the n-contextural 3-triadic polycontextural sign schema, both the abstract sign relation and its order type remain unchanged:

$SR(3) = (3.a_{i,j}, 2.b_{i,j}, 1.c_{i,j,}), \text{ with } a \leq b \leq c$

When we have a look at the corresponding 3-contextural matrix

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

we find the following connections between trichotomic order an contextural numbers:

	↗	(2.1)						
(3.1)	→	(2.2)	(3.2)	→	(2.2)	(3.3)		
	↘	(2.3)		↘	(2.3)		↘	(2.3)

	↗	(1)						
(3)	→	(1,2)	(2)	→	(1,2)	(2,3)		
	↘	(2)		↘	(2)		↘	(2)

	↗	(1.1)						
(2.1)	→	(1.2)	(2.2)	→	(1.2)	(2.3)		
	↘	(1.3)		↘	(1.3)		↘	(1.3)

	↗	(1,3)						
(1)	→	(1)	(1,2)	→	(1)	(2)		
	↘	(3)		↘	(1)		↘	(1)

despite (1) → (1,3), the contextural numbers get smaller with increasing trichotomical values of the interpretant and the object relations (from which we construct sign classes either by union of dyadic semioses or via matching conditions, if they are polycontextural).

2. If we now have a look at the (3^3 –10 =) 17 remaining sign classes we get, if we abolish the inclusive order restriction:

$(3.1_3\ 2.2_{1,2}\ 1.1_{1,3})$ → [3-1,2-1,3]
$(3.1_3\ 2.3_2\ 1.1_{1,3})$ → [3-2-1,3]
$(3.1_3\ 2.3_2\ 1.2_1)$ → [3-2-1]
$(3.2_2\ 2.1_1\ 1.1_{1,3})$ → [2-1-1,3]
$(3.2_2\ 2.1_1\ 1.2_1)$ → [2-1-1]
$(3.2_2\ 2.1_1\ 1.3_{1,3})$ → [2-1-1,3]
$(3.2_2\ 2.2_{1,2}\ 1.1_{1,3})$ → [2-1,2-1,3]
$(3.2_2\ 2.3_2\ 1.1_{1,3})$ → [2-2-1,3]
$(3.2_2\ 2.3_2\ 1.2_1)$ → [2-2-1]
$(3.2_2\ 2.3_2\ 1.1_{1,3})$ → [2-2-1,3]
$(3.2_2\ 2.3_2\ 1.2_1)$ → [2-2-1]
$(3.3_{2,3}\ 2.1_1\ 1.1_{1,3})$ → [2,3-1-1,3]
$(3.3_{2,3}\ 2.1_1\ 1.2_1)$ → [2,3-1-1]
$(3.3_{2,3}\ 2.1_1\ 1.3_{1,3})$ → [2,3-1-1,3]
$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3})$ → [2,3-1,2-1,3]
$(3.3_{2,3}\ 2.3_2\ 1.1_{1,3})$ → [2,3-2-1,3]
$(3.3_{2,3}\ 2.3_2\ 1.2_1)$ → [2,3-2-1],

we see that the classes of contextural orders are just complementary to those of the regular sign classes.

However, if we also recognize that in a matrix the converse sub-signs have the same contextural indices ($((a.b)_{i,j}° = (a.b)_{i,j})$, but that the order of the sub-signs in a sign class also makes it clear, which triadic value we have at certain position (i.e., e.g., a legi-sign (1.3) or an index ((1.3)° = (3.1)), we can say that the system of all possible 27 3-adic sign classes can be represented by classes of contextural orders in a non-ambiguous way. Since the same is true for sign classes and reality thematics which can be written by using environments alone, e.g.

(3-1-1,3) = (3.1 2.1 1.1)
(3-1-1) = (3.1 2.1 1.2)
(3-1-3) = (3.1 2.1 1.3), etc.

(3,1-1-3) = (1.1 1.2 1.3)

(1-1-3) = (2.1 1.2 1.3)
(3-1-3) = (3.1 1.2 1.3), etc.,

we can say that inner semiotic environments (i.e. contextural indices) are representing every semiotic relation, starting with K = 3.

Bibliography

Toth, Alfred, New elements of theoretical semiotics (NETS), based on the work of Rudolf Kaehr. In: Electronic Journal for Mathematical Semiotics, 2009

How many contextur-borders has a sign?

1. The present article refers only in its title to a former work (Toth 2008b). Our main concern here is to determine how many conextures can be assigned to a sign relation. As it is known from polycontextural theory, there is an indefinite number of two-valued, disseminated systems, according to the number of subjects to be used in the corresponding logical relations. A special problem for the sign is that its number of subjects cannot be determined. The first reason is that from a logical standpoint, the triadic sign relation consists of 3 subjects, but no object. The second reason is that the interpretant relation has a porte-manteau function for all possible subjects: the sender, the receiver, the interpreter; subjective and objective subject, and many more.

2. The only contexture border accepted up to now in semiotics, is the border between the sign and its designated object (cf. e.g., Kronthaler 1992). It says that a sign can never turn into its referring object and vice-versa. This contexture border is also the most widespread in several mystical beliefs, f. ex. in the collection of photos, hair-curls, relics, etc. which are originally supposed to enable the physical presence of the person represented by these kinds of signs.

3. If we remember that according to Bense (1975, pp. 45 s., 65 ss.) there is a pre-semiotic space between the space of the object and the space of the sign, then we are aware that an object is not directly selected as a relational media, but mediated by "disposable" media. Hence, there is another contexture border between the media of the sign relation and the material quality of the object, out of which this sign-carrier has been selected. While in the case of "artificial" signs the relationship between the sign carrier and the meaning and sense of the sign is widely arbitrary, in the case of "natural" signs, the relational media which is chosen out of an object, stands to this object in a pars-pro-toto-relation. However, also in this case, there is a contexture border between the object as thing and the object as sign carrier, caused by the interpretation of this object as a sign.

4. A third, rather trivial, but also omitted contexture border lies between the real person as a sender or receiver/intepreter and the interpretant-relation, which is per definitionem part of the sign relation. The absence of an interpretant in the Saussurean sign relation goes back to Durckheim's statement that the interpreter stays always out of the sign relation. Hence, the confusion between interpreter and interpretant and the non-recognition of the contexture border between them is the reason that the Saussurean sign is not triadic, but dyadic.

5. These three contexture-borders are borders between the inner categories of a sign relation and its outer counterparts, so they cross themselves a contextural border. However, the three fundamental categories of the sign are also separated by contexture borders inside the sign relation.

6. First (M → O), the contextural border between the media relation and the relation of the designated object. Since in the case of artificial signs, the media can be chosen arbitrarily for substituting an object, it follows, that also the relation between the media and the object relation are widely arbitrary. But even in the case where there is similarity between the media relation and the object relation, f. ex. in pictograms and related systems of "international" communication in airports, etc., media and object relation do not coincide and are thus separated by contextural border.

7. Next, we have (O → I), another important contexture border, for which we find plenty of examples, e.g., in E.T.A. Hoffmann's novel "Klein Zaches, genannt Zinnober". If O is an epistemological "Thou" and I is an epistemological (subjective) subject, then we can explain the strange effect of Zaches. Wherever Zaches appears, the speeches and the deeds of his "Thou's" are ascribed to him, i.e. to his "Ego". On the other side, all of his own speeches and deeds are ascribed to his environment. Since in reality he as an incapable knows to surround him with several capable persons, by this crossing relation between subjective and objective subject, he gets everything he wants.

8. The last contexture border of the triadic sign relation lies in (M → I). If we assume that M is a portrait, this would, e.g., mean that the painter as interpretant would get identical with his picture. If we apply this process to the end of Oscar Wilde's famous novel "The Picture of Dorian Gray", it would follow, that in that moment, when Dorian "stabs" the picture, non he (since he is object), but a the painter of the picture, Basil Hallward, dies.

9. As one can see, the opening of contextural borders is a first-rate source for fantasy, mythology, religion and horror.

10. We have abolished totally 6 inner and outer contextural borders of the sign, which we may illustrate like follows.

((3.a)	╫	(2.b)	╫	(1.c)
╫		╫		╫
Interpreter		Objekt		Mittel

The often occurring confusion between media and media relation, object and object relation, interpreter and interpretant are thus mistakes caused by non-acknowledgement of the outer contextural borders between sign and object.

11. In order to formalize our results, we have now basically two possibilities. First, we introduce 3 more categories and embed them in the triadic Peircean sign relation in order to get a 6-adic transcendental sign relation, as we had embedded the categorial object into the Peircean sign relation in order to get a 4-adic transcendental

sign relation (cf. Toth 2008a). Since all contextural borders are eliminated in such a sign relation, we will call it a complete non-transcendental sign relation (CNTSR):

CNTSR = (3.a 2.b 1.c 0.d ⦿.e ◎.f).

Now, to this hexadic sign relation, some consideration is necessary. As Bense (1975, pp. 65 ss.) had pointed out, an object can be assigned a categorial number, but no relational number. The reason is trivial: An object cannot enter a relationship with another object, unless it has been thetically introduced as a sign). An expression like "I have seen the stone of all stones" is senseless, while an expression like "I have watched the movie of all movies" is not. However, if an object has only a categorial number, but no relational number, this means semiotically, that it has only trichotomic, but no triadic values. Concretely: The categorial object (0.d) can assume the trichotomic values d $\in$ {1, 2, 3}, but the 0 is constant. Hence, there are no sub-signs like (0.0), (1.0), (2.0) and (3.0), since {0, ..., 3} are here relational number, so we would have a contradicition to Bense's theorem. And the same is valid for (⦿.e) and (◎.f), i.e. also these subsigns can only take three trichotomic values, but no triadic ones. Therefore, it follows that CNTSR is a hexadic-trichotomic sign relation.

12. A second possibility is to let the contextural indices of the regular Peircean sign class refer to the 3 transcendental categories. For that, we need at least 3 indices and thus a 4-contextural sign relation:

4-PSR = ($3.a_{i,j,k}$ $2.b_{i,j,k}$ $1.c_{i,j,k}$)

Therefore, we can define that i $\rightarrow$ transcendence of I, j $\rightarrow$ transcendence of O, and k $\rightarrow$ transcendence of M. The inner contextural borders are differentiated in this solution "automatically". Moreover, the three indices per sub-sign enable the possibility to indicate the interrelationships between the four contextures, f. ex. between the material object out of which (1.c) is selected, an the material object which is transformed by thetical introduction into a meta-object (2.b), cf. Bense (1967, p. 9). The latter possibility we do not have in the hexadic-trichotomic sign model. However, in the present solution, problems will arise then, when contextures have to be assigned to more then one function. There are the epistemological functions (subjective subject, objective subject, subjective object, objective object), there are time-contextures, the contextures of quantity and quality, etc. What we thus do, when we define 4-PSR, is basically that:

4-PSR = ($3.a_{(3.a,2.b,1.c)*}$ $2.b_{(3.a,2.b,1.c)*}$ $1.c_{(3.a,2.b,1.c)*}$),

whereby the asterisk indicates the purely categorial "relations" between transcendental objects and subjects, namely the corresponding transcendental objects and subjects of I, O, M which are separated from them, in a monocontextural logic, by a contextural abyss.

(13. The theoretically possible third solution, the combination of 11. and 12. to

CNTSR = $(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k}\ 0.d_{i,j,k}\ ⦿.e_{i,j,k}\ ◎.f_{i,j,k})$

would solve the problem of ascribing the contextures to different functions, but is over-characterized in respect to rendering transcendental categories non-transcendental, since the basic function of the contextural indices is the bridging of the abysses between the fundamental categories and their transcendental objects.)

14. Thus, we can answer the question in the title of this study: A sign has 6 contextural borders, amongst them 3 outer and 3 inner ones. However, in order to take care of bridging the contextural abysses, a 4-contextural 3-adic sign relation with transcendental categories (i.e. the regular fundamental categories) is sufficient:

4-3-PSR = $(3.a_{i,j,k}\ 2.b_{i,j,k}\ 1.c_{i,j,k})$ with $i, j, k \in \{1, 2, 3, 4\}$.

Therefore, the construction of higher n-contextural 3-adic semiotic matrices $(n > 4)$ is questionable for its semiotic use.

Bibliography

Bense, Max, Semiotik. Baden-Baden 1967

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Kronthaler, Engelbert, Zahl – Zeichen – Begriff. In: Semiosis 65-68, 1992, pp. 282-302

Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 2008 (2008a)

Toth, Alfred, Wie viele Kontexturgrenzen hat ein Zeichen? In: Electronic Journal for Mathematical Semiotics, 2008b

Polycontextural-semiotic functions

1. Monocontextural semiotic functions have been extensively studied in Toth (2008a), especially in Chapter 2 (pp. 148-178). In the present article I want to give their corresponding polycontextural functions, since they will play a crucial role after mathematical semiotics will finally have become a part of computer science. Different from a previous study (2009b), I will start here from a 4-contextural 4-adic sign model, i.e. not from a semiotic, but from a pre-semiotic sign relation, in order to maintain the framework elaborated in Toth (2008a). Therefore, we have semiotic functions with 3 or 2 variables, which can lie in 4 contextures:

Maximal variable scheme: $w = f(x_{i,j,k}, y_{i,j,k}, z_{i,j,k})$
Minimal variable scheme: $w = f(x_{i,j,k}, y_{i,j,k})$

Maximal contextural scheme: $w = f(x_{i,j,k}, y_{i,j,k}, z_{i,j,k})$
Minimal contextural scheme: $w = f(x_{i,j}, y_{i,j})$

(all schemes:) $i, j, k \in \{1, 2, 3, 4\}$

2.1. **Functions with w = $(0.1_{1,3})$**

1. $(0.1_{1,3}) = f(1.1._{1,3,4}, 2.1_{1,4})$
2. $(0.1_{1,3}) = f(1.1._{1,3,4}, 2.1_{1,4}, 3.1_{3,4})$
3. $(0.1_{1,3}) = f(1.1._{1,3,4}, 3.1_{3,4})$
4. $(0.1_{1,3}) = f(1.1._{1,3,4}, 3.1_{3,4}, 2.1_{1,4})$
5. $(0.1_{1,3}) = f(2.1_{1,4}, 1.1._{1,3,4})$
6. $(0.1_{1,3}) = f(2.1_{1,4}, 1.1._{1,3,4}, 3.1_{3,4})$
7. $(0.1_{1,3}) = f(2.1_{1,4}, 3.1_{3,4})$
8. $(0.1_{1,3}) = f(2.1_{1,4}, 3.1_{3,4}, 1.1._{1,3,4})$
9. $(0.1_{1,3}) = f(3.1_{3,4}, 1.1._{1,3,4})$
10. $(0.1_{1,3}) = f(3.1_{3,4}, 1.1._{1,3,4}, 2.1_{1,4})$
11. $(0.1_{1,3}) = f(3.1_{3,4}, 2.1_{1,4})$
12. $(0.1_{1,3}) = f(3.1_{3,4}, 2.1_{1,4}, 1.1._{1,3,4})$

2.2. **Functions with w = $(0.2_{1,2})$**

1. $(0.2_{1,2}) = f(1.1._{1,3,4}, 2.1_{1,4})$
2. $(0.2_{1,2}) = f(1.1._{1,3,4}, 2.1_{1,4}, 3.1_{3,4})$
3. $(0.2_{1,2}) = f(1.1._{1,3,4}, 3.1_{3,4})$
4. $(0.2_{1,2}) = f(1.1._{1,3,4}, 3.1_{3,4}, 2.1_{1,4})$
5. $(0.2_{1,2}) = f(1.2._{1,4}, 2.1_{1,4}, 3.1_{3,4})$
6. $(0.2_{1,2}) = f(1.2._{1,4}, 2.2_{1,2,4})$
7. $(0.2_{1,2}) = f(1.2._{1,4}, 2.2_{1,2,4}, 3.1_{3,4})$
8. $(0.2_{1,2}) = f(1.2._{1,4}, 2.2_{1,2,4}, 3.2_{2,4})$
9. $(0.2_{1,2}) = f(1.2._{1,4}, 3.1_{3,4})$

10. $(0.2_{1,2}) = f(1.2._{1,4}, 3.1_{3,4}, 2.1_{1,4})$
11. $(0.2_{1,2}) = f(1.2._{1,4}, 3.1_{3,4}, 2.2_{1,2,4})$
12. $(0.2_{1,2}) = f(1.2._{1,4}, 3.2_{2,4})$
13. $(0.2_{1,2}) = f(1.2._{1,4}, 3.2_{2,4}, 2.2_{1,2,4})$
14. $(0.2_{1,2}) = f(2.1_{1,4}, 1.1._{1,3,4})$
15. $(0.2_{1,2}) = f(2.1_{1,4}, 1.1._{1,3,4}, 3.1_{3,4})$
16. $(0.2_{1,2}) = f(2.1_{1,4}, 1.2._{1,4})$
17. $(0.2_{1,2}) = f(2.1_{1,4}, 1.2._{1,4}, 3.1_{3,4})$
18. $(0.2_{1,2}) = f(2.1_{1,4}, 3.1_{3,4})$
19. $(0.2_{1,2}) = f(2.1_{1,4}, 3.1_{3,4}, 1.1._{1,3,4})$
20. $(0.2_{1,2}) = f(2.1_{1,4}, 3.1_{3,4}, 1.2._{1,4})$
21. $(0.2_{1,2}) = f(2.2_{1,2,4}, 1.2._{1,4})$
22. $(0.2_{1,2}) = f(2.2_{1,2,4}, 1.2._{1,4}, 3.1_{3,4})$
23. $(0.2_{1,2}) = f(2.2_{1,2,4}, 1.2._{1,4}, 3.2_{2,4})$
24. $(0.2_{1,2}) = f(2.2_{1,2,4}, 3.1_{3,4})$
25. $(0.2_{1,2}) = f(2.2_{1,2,4}, 3.1_{3,4}, 1.2._{1,4})$
26. $(0.2_{1,2}) = f(2.2_{1,2,4}, 3.2_{2,4})$
27. $(0.2_{1,2}) = f(2.2_{1,2,4}, 3.2_{2,4}, 1.2._{1,4})$
28. $(0.2_{1,2}) = f(3.1_{3,4}, 1.1._{1,3,4})$
29. $(0.2_{1,2}) = f(3.1_{3,4}, 1.1._{1,3,4}, 2.1_{1,4})$
30. $(0.2_{1,2}) = f(3.1_{3,4}, 1.2._{1,4})$
31. $(0.2_{1,2}) = f(3.1_{3,4}, 1.2._{1,4}, 2.1_{1,4})$
32. $(0.2_{1,2}) = f(3.1_{3,4}, 1.2._{1,4}, 2.2_{1,2,4})$
33. $(0.2_{1,2}) = f(3.1_{3,4}, 2.1_{1,4})$
34. $(0.2_{1,2}) = f(3.1_{3,4}, 2.1_{1,4}, 1.1._{1,3,4})$
35. $(0.2_{1,2}) = f(3.1_{3,4}, 2.1_{1,4}, 1.2._{1,4})$
36. $(0.2_{1,2}) = f(3.1_{3,4}, 2.2_{1,2,4})$
37. $(0.2_{1,2}) = f(3.1_{3,4}, 2.2_{1,2,4}, 1.2._{1,4})$
38. $(0.2_{1,2}) = f(3.2_{2,4}, 1.2._{1,4})$
39. $(0.2_{1,2}) = f(3.2_{2,4}, 1.2._{1,4}, 2.2_{1,2,4})$
40. $(0.2_{1,2}) = f(3.2_{2,4}, 2.2_{1,2,4})$
41. $(0.2_{1,2}) = f(3.2_{2,4}, 2.2_{1,2,4}, 1.2._{1,4})$

2.3. **Functions with w = $(0.3_{2,3})$**

1. $(0.3_{2,3}) = f(1.1._{1,3,4}, 2.1_{1,4})$
2. $(0.3_{2,3}) = f(1.1._{1,3,4}, 2.1_{1,4}, 3.1_{3,4})$
3. $(0.3_{2,3}) = f(1.1._{1,3,4}, 3.1_{3,4})$
4. $(0.3_{2,3}) = f(1.1._{1,3,4}, 3.1_{3,4}, 2.1_{1,4})$
5. $(0.3_{2,3}) = f(1.2._{1,4}, 2.1_{1,4})$
6. $(0.3_{2,3}) = f(1.2._{1,4}, 2.1_{1,4}, 3.1_{3,4})$
7. $(0.3_{2,3}) = f(1.2._{1,4}, 2.2_{1,2,4})$
8. $(0.3_{2,3}) = f(1.2._{1,4}, 2.2_{1,2,4}, 3.1_{3,4})$
9. $(0.3_{2,3}) = f(1.2._{1,4}, 2.2_{1,2,4}, 3.2_{2,4})$

10. $(0.3_{2,3}) = f(1.2._{1,4}, 3.1_{3,4})$
11. $(0.3_{2,3}) = f(1.2._{1,4}, 3.1_{3,4}, 2.1_{1,4})$
12. $(0.3_{2,3}) = f(1.2._{1,4}, 3.1_{3,4}, 2.2_{1,2,4})$
13. $(0.3_{2,3}) = f(1.2._{1,4}, 3.2_{2,4})$
14. $(0.3_{2,3}) = f(1.2._{1,4}, 3.2_{2,4}, 2.2_{1,2,4})$
15. $(0.3_{2,3}) = f(1.3_{3,4}, 2.1_{1,4})$
16. $(0.3_{2,3}) = f(1.3_{3,4}, 2.1_{1,4}, 3.1_{3,4})$
17. $(0.3_{2,3}) = f(1.3_{3,4}, 2.2_{1,2,4})$
18. $(0.3_{2,3}) = f(1.3_{3,4}, 2.2_{1,2,4}, 3.1_{3,4})$
19. $(0.3_{2,3}) = f(1.3_{3,4}, 2.2_{1,2,4}, 3.2_{2,4})$
20. $(0.3_{2,3}) = f(1.3_{3,4}, 2.3_{2,4})$
21. $(0.3_{2,3}) = f(1.3_{3,4}, 2.3_{2,4}, 3.1_{3,4})$
22. $(0.3_{2,3}) = f(1.3_{3,4}, 2.3_{2,4}, 3.2_{2,4})$
23. $(0.3_{2,3}) = f(1.3_{3,4}, 2.3_{2,4}, 3.3_{2,3,4})$
24. $(0.3_{2,3}) = f(1.3_{3,4}, 3.1_{3,4})$
25. $(0.3_{2,3}) = f(1.3_{3,4}, 3.1_{3,4}, 2.1_{1,4})$
26. $(0.3_{2,3}) = f(1.3_{3,4}, 3.1_{3,4}, 2.2_{1,2,4})$
27. $(0.3_{2,3}) = f(1.3_{3,4}, 3.1_{3,4}, 2.3_{2,4})$
28. $(0.3_{2,3}) = f(1.3_{3,4}, 3.2_{2,4})$
29. $(0.3_{2,3}) = f(1.3_{3,4}, 3.2_{2,4}, 2.2_{1,2,4})$
30. $(0.3_{2,3}) = f(1.3_{3,4}, 3.2_{2,4}, 2.3_{2,4})$
31. $(0.3_{2,3}) = f(1.3_{3,4}, 3.3_{2,3,4})$
32. $(0.3_{2,3}) = f(1.3_{3,4}, 3.3_{2,3,4}, 2.3_{2,4})$
33. $(0.3_{2,3}) = f(2.1_{1,4}, 1.1._{1,3,4})$
34. $(0.3_{2,3}) = f(2.1_{1,4}, 1.1._{1,3,4}, 3.1_{3,4})$
35. $(0.3_{2,3}) = f(2.1_{1,4}, 1.2._{1,4}, 3.1_{3,4})$
36. $(0.3_{2,3}) = f(2.1_{1,4}, 1.3_{3,4})$
37. $(0.3_{2,3}) = f(2.1_{1,4}, 1.3_{3,4}, 3.1_{3,4})$
38. $(0.3_{2,3}) = f(2.1_{1,4}, 3.1_{3,4})$
39. $(0.3_{2,3}) = f(2.1_{1,4}, 3.1_{3,4}, 1.1_{1,3,4})$
40. $(0.3_{2,3}) = f(2.1_{1,4}, 3.1_{3,4}, 1.2._{1,4})$
41. $(0.3_{2,3}) = f(2.1_{1,4}, 3.1_{3,4}, 1.3_{3,4})$
42. $(0.3_{2,3}) = f(2.2_{1,2,4}, 1.2._{1,4})$
43. $(0.3_{2,3}) = f(2.2_{1,2,4}, 1.2._{1,4}, 3.1_{3,4})$
44. $(0.3_{2,3}) = f(2.2_{1,2,4}, 1.2._{1,4}, 3.2_{2,4})$
45. $(0.3_{2,3}) = f(2.2_{1,2,4}, 1.3_{3,4})$
46. $(0.3_{2,3}) = f(2.2_{1,2,4}, 1.3_{3,4}, 3.1_{3,4})$
47. $(0.3_{2,3}) = f(2.2_{1,2,4}, 1.3_{3,4}, 3.2_{2,4})$
48. $(0.3_{2,3}) = f(2.2_{1,2,4}, 3.1_{3,4})$
49. $(0.3_{2,3}) = f(2.2_{1,2,4}, 3.1_{3,4}, 1.2._{1,4})$
50. $(0.3_{2,3}) = f(2.2_{1,2,4}, 3.1_{3,4}, 1.3_{3,4})$
51. $(0.3_{2,3}) = f(2.2_{1,2,4}, 3.2_{2,4})$
52. $(0.3_{2,3}) = f(2.2_{1,2,4}, 3.2_{2,4}, 1.2._{1,4})$
53. $(0.3_{2,3}) = f(2.2_{1,2,4}, 3.2_{2,4}, 1.3_{3,4})$

54. $(0.3_{2,3}) = f(2.3_{2,4}, 1.3_{3,4})$
55. $(0.3_{2,3}) = f(2.3_{2,4}, 1.3_{3,4}, 3.1_{3,4})$
56. $(0.3_{2,3}) = f(2.3_{2,4}, 1.3_{3,4}, 3.2_{2,4})$
57. $(0.3_{2,3}) = f(2.3_{2,4}, 1.3_{3,4}, 3.3_{2,3,4})$
58. $(0.3_{2,3}) = f(2.3_{2,4}, 3.1_{3,4})$
59. $(0.3_{2,3}) = f(2.3_{2,4}, 3.1_{3,4}, 1.3_{3,4})$
60. $(0.3_{2,3}) = f(2.3_{2,4}, 3.2_{2,4})$
61. $(0.3_{2,3}) = f(2.3_{2,4}, 3.2_{2,4}, 1.3_{3,4})$
62. $(0.3_{2,3}) = f(2.3_{2,4}, 3.3_{2,3,4}, 1.3_{3,4})$
63. $(0.3_{2,3}) = f(3.1_{3,4}, 1.1._{1,3,4})$
64. $(0.3_{2,3}) = f(3.1_{3,4}, 1.1._{1,3,4}, 2.1_{1,4})$
65. $(0.3_{2,3}) = f(3.1_{3,4}, 1.2._{1,4})$
66. $(0.3_{2,3}) = f(3.1_{3,4}, 1.2._{1,4}, 2.1_{1,4})$
67. $(0.3_{2,3}) = f(3.1_{3,4}, 1.2._{1,4}, 2.2_{1,2,4})$
68. $(0.3_{2,3}) = f(3.1_{3,4}, 1.3_{3,4})$
69. $(0.3_{2,3}) = f(3.1_{3,4}, 1.3_{3,4}, 2.1_{1,4})$
70. $(0.3_{2,3}) = f(3.1_{3,4}, 1.3_{3,4}, 2.2_{1,2,4})$
71. $(0.3_{2,3}) = f(3.1_{3,4}, 1.3_{3,4}, 2.3_{2,4})$
72. $(0.3_{2,3}) = f(3.1_{3,4}, 2.1_{1,4})$
73. $(0.3_{2,3}) = f(3.1_{3,4}, 2.1_{1,4}, 1.1._{1,3,4})$
74. $(0.3_{2,3}) = f(3.1_{3,4}, 2.1_{1,4}, 1.2._{1,4})$
75. $(0.3_{2,3}) = f(3.1_{3,4}, 2.1_{1,4}, 1.3_{3,4})$
76. $(0.3_{2,3}) = f(3.1_{3,4}, 2.2_{1,2,4})$
77. $(0.3_{2,3}) = f(3.1_{3,4}, 2.2_{1,2,4}, 1.2._{1,4})$
78. $(0.3_{2,3}) = f(3.1_{3,4}, 2.2_{1,2,4}, 1.3_{3,4})$
79. $(0.3_{2,3}) = f(3.1_{3,4}, 2.3_{2,4})$
80. $(0.3_{2,3}) = f(3.1_{3,4}, 2.3_{2,4}, 1.3_{3,4})$
81. $(0.3_{2,3}) = f(3.2_{2,4}, 1.2._{1,4})$
82. $(0.3_{2,3}) = f(3.2_{2,4}, 1.2._{1,4}, 2.2_{1,2,4})$
83. $(0.3_{2,3}) = f(3.2_{2,4}, 1.3_{3,4})$
84. $(0.3_{2,3}) = f(3.2_{2,4}, 1.3_{3,4}, 2.2_{1,2,4})$
85. $(0.3_{2,3}) = f(3.2_{2,4}, 1.3_{3,4}, 2.3_{2,4})$
86. $(0.3_{2,3}) = f(3.2_{2,4}, 2.2_{1,2,4})$
87. $(0.3_{2,3}) = f(3.2_{2,4}, 2.2_{1,2,4}, 1.2._{1,4})$
88. $(0.3_{2,3}) = f(3.2_{2,4}, 2.2_{1,2,4}, 1.3_{3,4})$
89. $(0.3_{2,3}) = f(3.2_{2,4}, 2.3_{2,4})$
90. $(0.3_{2,3}) = f(3.2_{2,4}, 2.3_{2,4}, 1.3_{3,4})$
91. $(0.3_{2,3}) = f(3.3_{2,3,4}, 1.3_{3,4}, 2.3_{2,4})$
92. $(0.3_{2,3}) = f(3.3_{2,3,4}, 2.3_{2,4}, 1.3_{3,4})$

2.4. **Functions with w = $(1.0_{1,3})$**

1. $(1.0_{1,3}) = f(1.1._{1,3,4}, 1.2._{1,4})$
2. $(1.0_{1,3}) = f(1.1._{1,3,4}, 1.2._{1,4}, 1.3_{3,4})$

3. $(1.0_{1,3}) = f(1.1._{1,3,4}, 1.3_{3,4})$
4. $(1.0_{1,3}) = f(1.1._{1,3,4}, 1.3_{3,4}, 1.2._{1,4})$
5. $(1.0_{1,3}) = f(1.2._{1,4}, 1.1._{1,3,4})$
6. $(1.0_{1,3}) = f(1.2._{1,4}, 1.1._{1,3,4}, 1.3_{3,4})$
7. $(1.0_{1,3}) = f(1.2._{1,4}, 1.3_{3,4})$
8. $(1.0_{1,3}) = f(1.2._{1,4}, 1.3_{3,4}, 1.1._{1,3,4})$
9. $(1.0_{1,3}) = f(1.3_{3,4}, 1.1._{1,3,4})$
10. $(1.0_{1,3}) = f(1.3_{3,4}, 1.1._{1,3,4}, 1.2._{1,4})$
11. $(1.0_{1,3}) = f(1.3_{3,4}, 1.2._{1,4})$
12. $(1.0_{1,3}) = f(1.3_{3,4}, 1.2._{1,4}, 1.1._{1,3,4})$

2.5. **Functions with w = $(1.1_{1,3,4})$**

1. $(1.1._{1,3,4}) = f(0.1_{1,3}, 2.1_{1,4})$
2. $(1.1._{1,3,4}) = f(0.1_{1,3}, 2.1_{1,4}, 3.1_{3,4})$
3. $(1.1._{1,3,4}) = f(0.1_{1,3}, 3.1_{3,4})$
4. $(1.1._{1,3,4}) = f(0.1_{1,3}, 3.1_{3,4}, 2.1_{1,4})$
5. $(1.1._{1,3,4}) = f(0.2_{1,2}, 2.1_{1,4})$
6. $(1.1._{1,3,4}) = f(0.2_{1,2}, 2.1_{1,4}, 3.1_{3,4})$
7. $(1.1._{1,3,4}) = f(0.2_{1,2}, 3.1_{3,4})$
8. $(1.1._{1,3,4}) = f(0.2_{1,2}, 3.1_{3,4}, 2.1_{1,4})$
9. $(1.1._{1,3,4}) = f(0.3_{2,3}, 2.1_{1,4})$
10. $(1.1._{1,3,4}) = f(0.3_{2,3}, 2.1_{1,4}, 3.1_{3,4})$
11. $(1.1._{1,3,4}) = f(0.3_{2,3}, 3.1_{3,})$
12. $(1.1._{1,3,4}) = f(0.3_{2,3}, 3.1_{3,4}, 2.1_{1,4})$
13. $(1.1._{1,3,4}) = f(1.0_{1,3}, 1.2._{1,4})$
14. $(1.1._{1,3,4}) = f(1.0_{1,3}, 1.2._{1,4}, 1.3_{3,4})$
15. $(1.1._{1,3,4}) = f(1.0_{1,3}, 1.3_{3,4})$
16. $(1.1._{1,3,4}) = f(1.0_{1,3}, 1.3_{3,4}, 1.2._{1,4})$
17. $(1.1._{1,3,4}) = f(1.2._{1,4}, 1.0_{1,3})$
18. $(1.1._{1,3,4}) = f(1.2._{1,4}, 1.0_{1,3}, 1.3_{3,4})$
19. $(1.1._{1,3,4}) = f(1.2._{1,4}, 1.3_{3,4})$
20. $(1.1._{1,3,4}) = f(1.2._{1,4}, 1.3_{3,4}, 1.0_{1,3})$
21. $(1.1._{1,3,4}) = f(1.2._{1,4}, 1.3_{3,4}, 2.0_{1,2})$
22. $(1.1._{1,3,4}) = f(1.2._{1,4}, 1.3_{3,4}, 3.0_{2,3})$
23. $(1.1._{1,3,4}) = f(1.2._{1,4}, 2.0_{1,2})$
24. $(1.1._{1,3,4}) = f(1.2._{1,4}, 2.0_{1,2}, 1.3_{3,4})$
25. $(1.1._{1,3,4}) = f(1.2._{1,4}, 3.0_{2,3})$
26. $(1.1._{1,3,4}) = f(1.2._{1,4}, 3.0_{2,3}, 1.3_{3,4})$
27. $(1.1._{1,3,4}) = f(1.3_{3,4}, 1.0_{1,3})$
28. $(1.1._{1,3,4}) = f(1.3_{3,4}, 1.0_{1,3}, 1.2._{1,4})$
29. $(1.1._{1,3,4}) = f(1.3_{3,4}, 1.2._{1,4})$
30. $(1.1._{1,3,4}) = f(1.3_{3,4}, 1.2._{1,4}, 1.0_{1,3})$
31. $(1.1._{1,3,4}) = f(1.3_{3,4}, 1.2._{1,4}, 2.0_{1,2})$

32. $(1.1._{1,3,4}) = f(1.3_{3,4}, 1.2._{1,4}, 3.0_{2,3})$
33. $(1.1._{1,3,4}) = f(1.3_{3,4}, 2.0_{1,2})$
34. $(1.1._{1,3,4}) = f(1.3_{3,4}, 2.0_{1,2}, 1.2._{1,4})$
35. $(1.1._{1,3,4}) = f(1.3_{3,4}, 3.0_{2,3})$
36. $(1.1._{1,3,4}) = f(1.3_{3,4}, 3.0_{2,3}, 1.2._{1,4})$
37. $(1.1._{1,3,4}) = f(2.0_{1,2}, 1.2._{1,4})$
38. $(1.1._{1,3,4}) = f(2.0_{1,2}, 1.2._{1,4}, 1.3_{3,4})$
39. $(1.1._{1,3,4}) = f(2.0_{1,2}, 1.3_{3,4})$
40. $(1.1._{1,3,4}) = f(2.0_{1,2}, 1.3_{3,4}, 1.2._{1,4})$
41. $(1.1._{1,3,4}) = f(2.1_{1,4}, 0.1_{1,3})$
42. $(1.1._{1,3,4}) = f(2.1_{1,4}, 0.1_{1,3}, 3.1_{3,4})$
43. $(1.1._{1,3,4}) = f(2.1_{1,4}, 0.2_{1,2})$
44. $(1.1._{1,3,4}) = f(2.1_{1,4}, 0.2_{1,2}, 3.1_{3,4})$
45. $(1.1._{1,3,4}) = f(2.1_{1,4}, 0.3_{2,3})$
46. $(1.1._{1,3,4}) = f(2.1_{1,4}, 0.3_{2,3}, 3.1_{3,4})$
47. $(1.1._{1,3,4}) = f(2.1_{1,4}, 3.1_{3,4})$
48. $(1.1._{1,3,4}) = f(2.1_{1,4}, 3.1_{3,4}, 0.1_{1,3})$
49. $(1.1._{1,3,4}) = f(2.1_{1,4}, 3.1_{3,4}, 0.2_{1,2})$
50. $(1.1._{1,3,4}) = f(2.1_{1,4}, 3.1_{3,4}, 0.3_{2,3})$
51. $(1.1._{1,3,4}) = f(3.0_{2,3}, 1.2._{1,4})$
52. $(1.1._{1,3,4}) = f(3.0_{2,3}, 1.2._{1,4}, 1.3_{3,4})$
53. $(1.1._{1,3,4}) = f(3.0_{2,3}, 1.3_{3,4})$
54. $(1.1._{1,3,4}) = f(3.0_{2,3}, 1.3_{3,4}, 1.2._{1,4})$
55. $(1.1._{1,3,4}) = f(3.1_{3,4}, 0.1_{1,3})$
56. $(1.1._{1,3,4}) = f(3.1_{3,4}, 0.1_{1,3}, 2.1_{1,4})$
57. $(1.1._{1,3,4}) = f(3.1_{3,4}, 0.2_{1,2})$
58. $(1.1._{1,3,4}) = f(3.1_{3,4}, 0.2_{1,2}, 2.1_{1,4})$
59. $(1.1._{1,3,4}) = f(3.1_{3,4}, 0.3_{2,3})$
60. $(1.1._{1,3,4}) = f(3.1_{3,4}, 0.3_{2,3}, 2.1_{1,4})$
61. $(1.1._{1,3,4}) = f(3.1_{3,4}, 2.1_{1,4})$
62. $(1.1._{1,3,4}) = f(3.1_{3,4}, 2.1_{1,4}, 0.1_{1,3})$
63. $(1.1._{1,3,4}) = f(3.1_{3,4}, 2.1_{1,4}, 0.2_{1,2})$
64. $(1.1._{1,3,4}) = f(3.1_{3,4}, 2.1_{1,4}, 0.3_{2,3})$

2.6. **Functions with w = $(1.2._{1,4})$**

1. $(1.2._{1,4}) = f(0.2_{1,2}, 2.1_{1,4})$
2. $(1.2._{1,4}) = f(0.2_{1,2}, 2.1_{1,4}, 3.1_{3,4})$
3. $(1.2._{1,4}) = f(0.2_{1,2}, 2.2_{1,2,4})$
4. $(1.2._{1,4}) = f(0.2_{1,2}, 2.2_{1,2,4}, 3.1_{3,4})$
5. $(1.2._{1,4}) = f(0.2_{1,2}, 2.2_{1,2,4}, 3.2_{2,4})$
6. $(1.2._{1,4}) = f(0.2_{1,2}, 3.1_{3,4})$
7. $(1.2._{1,4}) = f(0.2_{1,2}, 3.1_{3,4}, 2.1_{1,4})$
8. $(1.2._{1,4}) = f(0.2_{1,2}, 3.1_{3,4}, 2.2_{1,2,4})$

9. $(1.2._{1,4}) = f(0.2_{1,2}, 3.2_{2,4})$
10. $(1.2._{1,4}) = f(0.2_{1,2}, 3.2_{2,4}, 2.2_{1,2,4})$
11. $(1.2._{1,4}) = f(0.3_{2,3}, 2.1_{1,4})$
12. $(1.2._{1,4}) = f(0.3_{2,3}, 2.1_{1,4}, 3.1_{3,4})$
13. $(1.2._{1,4}) = f(0.3_{2,3}, 2.2_{1,2,4})$
14. $(1.2._{1,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 3.1_{3,4})$
15. $(1.2._{1,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 3.2_{2,4})$
16. $(1.2._{1,4}) = f(0.3_{2,3}, 3.1_{3,4})$
17. $(1.2._{1,4}) = f(0.3_{2,3}, 3.1_{3,4}, 2.1_{1,4})$
18. $(1.2._{1,4}) = f(0.3_{2,3}, 3.1_{3,4}, 2.2_{1,2,4})$
19. $(1.2._{1,4}) = f(0.3_{2,3}, 3.2_{2,4})$
20. $(1.2._{1,4}) = f(0.3_{2,3}, 3.2_{2,4}, 2.2_{1,2,4})$
21. $(1.2._{1,4}) = f(1.0_{1,3}, 1.1._{1,3,4})$
22. $(1.2._{1,4}) = f(1.0_{1,3}, 1.1._{1,3,4}, 1.3_{3,4})$
23. $(1.2._{1,4}) = f(1.0_{1,3}, 1.3_{3,4})$
24. $(1.2._{1,4}) = f(1.0_{1,3}, 1.3_{3,4}, 1.1._{1,3,4})$
25. $(1.2._{1,4}) = f(1.1._{1,3,4}, 1.0_{1,3})$
26. $(1.2._{1,4}) = f(1.1._{1,3,4}, 1.0_{1,3}, 1.3_{3,4})$
27. $(1.2._{1,4}) = f(1.1._{1,3,4}, 1.3_{3,4})$
28. $(1.2._{1,4}) = f(1.1._{1,3,4}, 1.3_{3,4}, 1.0_{1,3})$
29. $(1.2._{1,4}) = f(1.1._{1,3,4}, 1.3_{3,4}, 2.0_{1,2})$
30. $(1.2._{1,4}) = f(1.1._{1,3,4}, 1.3_{3,4}, 3.0_{2,3})$
31. $(1.2._{1,4}) = f(1.1._{1,3,4}, 2.0_{1,2})$
32. $(1.2._{1,4}) = f(1.1._{1,3,4}, 2.0_{1,2}, 1.3_{3,4})$
33. $(1.2._{1,4}) = f(1.1._{1,3,4}, 3.0_{2,3})$
34. $(1.2._{1,4}) = f(1.1._{1,3,4}, 3.0_{2,3}, 1.3_{3,4})$
35. $(1.2._{1,4}) = f(1.3_{3,4}, 1.0_{1,3})$
36. $(1.2._{1,4}) = f(1.3_{3,4}, 1.0_{1,3}, 1.1._{1,3,4})$
37. $(1.2._{1,4}) = f(1.3_{3,4}, 1.1._{1,3,4})$
38. $(1.2._{1,4}) = f(1.3_{3,4}, 1.1._{1,3,4}, 1.0_{1,3})$
39. $(1.2._{1,4}) = f(1.3_{3,4}, 1.1._{1,3,4}, 2.0_{1,2})$
40. $(1.2._{1,4}) = f(1.3_{3,4}, 1.1._{1,3,4}, 3.0_{2,3})$
41. $(1.2._{1,4}) = f(1.3_{3,4}, 2.0_{1,2})$
42. $(1.2._{1,4}) = f(1.3_{3,4}, 2.0_{1,2}, 1.1._{1,3,4})$
43. $(1.2._{1,4}) = f(1.3_{3,4}, 2.1_{1,4})$
44. $(1.2._{1,4}) = f(1.3_{3,4}, 2.1_{1,4}, 2.0_{1,2})$
45. $(1.2._{1,4}) = f(1.3_{3,4}, 3.0_{2,3})$
46. $(1.2._{1,4}) = f(1.3_{3,4}, 3.0_{2,3}, 1.1._{1,3,4})$
47. $(1.2._{1,4}) = f(1.3_{3,4}, 3.0_{2,3}, 2.1_{1,4})$
48. $(1.2._{1,4}) = f(1.3_{3,4}, 3.0_{2,3}, 3.1_{3,4})$
49. $(1.2._{1,4}) = f(1.3_{3,4}, 3.1_{3,4})$
50. $(1.2._{1,4}) = f(1.3_{3,4}, 3.1_{3,4}, 3.0_{2,3})$
51. $(1.2._{1,4}) = f(2.0_{1,2}, 1.1._{1,3,4})$
52. $(1.2._{1,4}) = f(2.0_{1,2}, 1.3_{3,4}\ 2.1_{1,4})$

53. $(1.2._{1,4}) = f(2.0_{1,2}, 1.3_{3,4})$
54. $(1.2._{1,4}) = f(2.0_{1,2}, 1.3_{3,4}, 1.1._{1,3,4})$
55. $(1.2._{1,4}) = f(2.0_{1,2}, 2.1_{1,4})$
56. $(1.2._{1,4}) = f(2.0_{1,2}, 2.1_{1,4}, 1.3_{3,4})$
57. $(1.2._{1,4}) = f(2.1_{1,4}, 0.2_{1,2})$
58. $(1.2._{1,4}) = f(2.1_{1,4}, 0.2_{1,2}, 3.1_{3,4})$
59. $(1.2._{1,4}) = f(2.1_{1,4}, 0.3_{2,3})$
60. $(1.2._{1,4}) = f(2.1_{1,4}, 0.3_{2,3}, 3.1_{3,4})$
61. $(1.2._{1,4}) = f(2.1_{1,4}, 1.3_{3,4})$
62. $(1.2._{1,4}) = f(2.1_{1,4}, 1.3_{3,4}, 2.0_{1,2})$
63. $(1.2._{1,4}) = f(2.1_{1,4}, 1.3_{3,4}, 3.0_{2,3})$
64. $(1.2._{1,4}) = f(2.1_{1,4}, 2.0_{1,2})$
65. $(1.2._{1,4}) = f(2.1_{1,4}, 2.0_{1,2}, 1.3_{3,4})$
66. $(1.2._{1,4}) = f(2.1_{1,4}, 3.0_{2,3})$
67. $(1.2._{1,4}) = f(2.1_{1,4}, 3.0_{2,3}, 1.3_{3,4})$
68. $(1.2._{1,4}) = f(2.1_{1,4}, 3.1_{3,4})$
69. $(1.2._{1,4}) = f(2.1_{1,4}, 3.1_{3,4}, 0.2_{1,2})$
70. $(1.2._{1,4}) = f(2.1_{1,4}, 3.1_{3,4}, 0.3_{2,3})$
71. $(1.2._{1,4}) = f(2.2_{1,2,4}, 0.2_{1,2})$
72. $(1.2._{1,4}) = f(2.2_{1,2,4}, 0.2_{1,2}, 3.1_{3,4})$
73. $(1.2._{1,4}) = f(2.2_{1,2,4}, 0.2_{1,2}, 3.2_{2,4})$
74. $(1.2._{1,4}) = f(2.2_{1,2,4}, 0.3_{2,3})$
75. $(1.2._{1,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 3.1_{3,4})$
76. $(1.2._{1,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 3.2_{2,4})$
77. $(1.2._{1,4}) = f(2.2_{1,2,4}, 3.1_{3,4})$
78. $(1.2._{1,4}) = f(2.2_{1,2,4}, 3.1_{3,4}, 0.2_{1,2})$
79. $(1.2._{1,4}) = f(2.2_{1,2,4}, 3.1_{3,4}, 0.3_{2,3})$
80. $(1.2._{1,4}) = f(2.2_{1,2,4}, 3.2_{2,4})$
81. $(1.2._{1,4}) = f(2.2_{1,2,4}, 3.2_{2,4}, 0.2_{1,2})$
82. $(1.2._{1,4}) = f(2.2_{1,2,4}, 3.2_{2,4}, 0.3_{2,3})$
83. $(1.2._{1,4}) = f(3.0_{2,3}, 1.1._{1,3,4})$
84. $(1.2._{1,4}) = f(3.0_{2,3}, 1.1._{1,3,4}, 1.3_{3,4})$
85. $(1.2._{1,4}) = f(3.0_{2,3}, 1.3_{3,4})$
86. $(1.2._{1,4}) = f(3.0_{2,3}, 1.3_{3,4}, 1.1._{1,3,4})$
87. $(1.2._{1,4}) = f(3.0_{2,3}, 1.3_{3,4}, 2.1_{1,4})$
88. $(1.2._{1,4}) = f(3.0_{2,3}, 1.3_{3,4}, 3.1_{3,4})$
89. $(1.2._{1,4}) = f(3.0_{2,3}, 2.1_{1,4})$
90. $(1.2._{1,4}) = f(3.0_{2,3}, 2.1_{1,4}, 1.3_{3,4})$
91. $(1.2._{1,4}) = f(3.0_{2,3}, 3.1_{3,4})$
92. $(1.2._{1,4}) = f(3.0_{2,3}, 3.1_{3,4}, 1.3_{3,4})$
93. $(1.2._{1,4}) = f(3.1_{3,4}, 0.2_{1,2})$
94. $(1.2._{1,4}) = f(3.1_{3,4}, 0.2_{1,2}, 2.1_{1,4})$
95. $(1.2._{1,4}) = f(3.1_{3,4}, 0.2_{1,2}, 2.2_{1,2,4})$
96. $(1.2._{1,4}) = f(3.1_{3,4}, 0.3_{2,3})$

97. $(1.2._{1,4}) = f(3.1_{3,4}, 0.3_{2,3}, 2.1_{1,4})$
98. $(1.2._{1,4}) = f(3.1_{3,4}, 0.3_{2,3}, 2.2_{1,2,4})$
99. $(1.2._{1,4}) = f(3.1_{3,4}, 1.3_{3,4})$
100. $(1.2._{1,4}) = f(3.1_{3,4}, 1.3_{3,4}, 3.0_{2,3})$
101. $(1.2._{1,4}) = f(3.1_{3,4}, 2.1_{1,4})$
102. $(1.2._{1,4}) = f(3.1_{3,4}, 2.1_{1,4}, 0.2_{1,2})$
103. $(1.2._{1,4}) = f(3.1_{3,4}, 2.1_{1,4}, 0.3_{2,3})$
104. $(1.2._{1,4}) = f(3.1_{3,4}, 2.2_{1,2,4})$
105. $(1.2._{1,4}) = f(3.1_{3,4}, 2.2_{1,2,4}, 0.2_{1,2})$
106. $(1.2._{1,4}) = f(3.1_{3,4}, 2.2_{1,2,4}, 0.3_{2,3})$
107. $(1.2._{1,4}) = f(3.1_{3,4}, 3.0_{2,3})$
108. $(1.2._{1,4}) = f(3.1_{3,4}, 3.0_{2,3}, 1.3_{3,4})$
109. $(1.2._{1,4}) = f(3.2_{2,4}, 0.2_{1,2})$
110. $(1.2._{1,4}) = f(3.2_{2,4}, 0.2_{1,2}, 2.2_{1,2,4})$
111. $(1.2._{1,4}) = f(3.2_{2,4}, 0.3_{2,3})$
112. $(1.2._{1,4}) = f(3.2_{2,4}, 0.3_{2,3}, 2.2_{1,2,4})$
113. $(1.2._{1,4}) = f(3.2_{2,4}, 2.2_{1,2,4})$
114. $(1.2._{1,4}) = f(3.2_{2,4}, 2.2_{1,2,4}, 0.2_{1,2})$
115. $(1.2._{1,4}) = f(3.2_{2,4}, 2.2_{1,2,4}, 0.3_{2,3})$

2.7. **Functions with w = $(1.3_{3,4})$**

1. $(1.3_{3,4}) = f(0.3_{2,3}, 2.1_{1,4})$
2. $(1.3_{3,4}) = f(0.3_{2,3}, 2.1_{1,4}, 3.1_{3,4})$
3. $(1.3_{3,4}) = f(0.3_{2,3}, 2.2_{1,2,4})$
4. $(1.3_{3,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 3.1_{3,4})$
5. $(1.3_{3,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 3.2_{2,4})$
6. $(1.3_{3,4}) = f(0.3_{2,3}, 2.3_{2,4})$
7. $(1.3_{3,4}) = f(0.3_{2,3}, 2.3_{2,4}, 3.1_{3,4})$
8. $(1.3_{3,4}) = f(0.3_{2,3}, 2.3_{2,4}, 3.2_{2,4})$
9. $(1.3_{3,4}) = f(0.3_{2,3}, 2.3_{2,4}, 3.3_{2,3,4})$
10. $(1.3_{3,4}) = f(0.3_{2,3}, 3.1_{3,4})$
11. $(1.3_{3,4}) = f(0.3_{2,3}, 3.1_{3,4}, 2.1_{1,4})$
12. $(1.3_{3,4}) = f(0.3_{2,3}, 3.1_{3,4}, 2.2_{1,2,4})$
13. $(1.3_{3,4}) = f(0.3_{2,3}, 3.1_{3,4}, 2.3_{2,4})$
14. $(1.3_{3,4}) = f(0.3_{2,3}, 3.2_{2,4})$
15. $(1.3_{3,4}) = f(0.3_{2,3}, 3.2_{2,4}, 2.2_{1,2,4})$
16. $(1.3_{3,4}) = f(0.3_{2,3}, 3.2_{2,4}, 2.3_{2,4})$
17. $(1.3_{3,4}) = f(0.3_{2,3}, 3.3_{2,3,4})$
18. $(1.3_{3,4}) = f(0.3_{2,3}, 3.3_{2,3,4}, 2.3_{2,4})$
19. $(1.3_{3,4}) = f(1.0_{1,3}, 1.1._{1,3,4})$
20. $(1.3_{3,4}) = f(1.0_{1,3}, 1.1._{1,3,4}, 1.2._{1,4})$
21. $(1.3_{3,4}) = f(1.0_{1,3}, 1.2._{1,4})$
22. $(1.3_{3,4}) = f(1.0_{1,3}, 1.2._{1,4}, 1.1._{1,3,4})$

23. $(1.3_{3,4}) = f(1.1._{1,3,4}, 1.0_{1,3})$
24. $(1.3_{3,4}) = f(1.1._{1,3,4}, 1.0_{1,3}, 1.2._{1,4})$
25. $(1.3_{3,4}) = f(1.1._{1,3,4}, 1.2._{1,4})$
26. $(1.3_{3,4}) = f(1.1._{1,3,4}, 1.2._{1,4}, 1.0_{1,3})$
27. $(1.3_{3,4}) = f(1.1._{1,3,4}, 1.2._{1,4}, 2.0_{1,2})$
28. $(1.3_{3,4}) = f(1.1._{1,3,4}, 1.2._{1,4}, 3.0_{2,3})$
29. $(1.3_{3,4}) = f(1.1._{1,3,4}, 3.0_{2,3})$
30. $(1.3_{3,4}) = f(1.1._{1,3,4}, 3.0_{2,3}, 1.2._{1,4})$
31. $(1.3_{3,4}) = f(1.2._{1,4}, 1.0_{1,3})$
32. $(1.3_{3,4}) = f(1.2._{1,4}, 1.0_{1,3}, 1.1._{1,3,4})$
33. $(1.3_{3,4}) = f(1.2._{1,4}, 1.1._{1,3,4})$
34. $(1.3_{3,4}) = f(1.2._{1,4}, 1.1._{1,3,4}, 1.0_{1,3})$
35. $(1.3_{3,4}) = f(1.2._{1,4}, 1.1._{1,3,4}, 2.0_{1,2})$
36. $(1.3_{3,4}) = f(1.2._{1,4}, 1.1._{1,3,4}, 3.0_{2,3})$
37. $(1.3_{3,4}) = f(1.2._{1,4}, 2.0_{1,2})$
38. $(1.3_{3,4}) = f(1.2._{1,4}, 2.0_{1,2}, 1.1._{1,3,4})$
39. $(1.3_{3,4}) = f(1.2._{1,4}, 2.0_{1,2}, 2.1_{1,4})$
40. $(1.3_{3,4}) = f(1.2._{1,4}, 2.1_{1,4})$
41. $(1.3_{3,4}) = f(1.2._{1,4}, 2.1_{1,4}, 2.0_{1,2})$
42. $(1.3_{3,4}) = f(1.2._{1,4}, 2.1_{1,4}, 3.0_{2,3})$
43. $(1.3_{3,4}) = f(1.2._{1,4}, 3.0_{2,3})$
44. $(1.3_{3,4}) = f(1.2._{1,4}, 3.0_{2,3}, 1.1._{1,3,4})$
45. $(1.3_{3,4}) = f(1.2._{1,4}, 3.0_{2,3}, 2.1_{1,4})$
46. $(1.3_{3,4}) = f(1.2._{1,4}, 3.0_{2,3}, 3.1_{3,4})$
47. $(1.3_{3,4}) = f(1.2._{1,4}, 3.1_{3,4})$
48. $(1.3_{3,4}) = f(1.2._{1,4}, 3.1_{3,4}, 3.0_{2,3})$
49. $(1.3_{3,4}) = f(2.0_{1,2}, 1.1._{1,3,4})$
50. $(1.3_{3,4}) = f(2.0_{1,2}, 1.1._{1,3,4}, 1.2._{1,4})$
51. $(1.3_{3,4}) = f(2.0_{1,2}, 1.2._{1,4})$
52. $(1.3_{3,4}) = f(2.0_{1,2}, 1.2._{1,4}, 1.1._{1,3,4})$
53. $(1.3_{3,4}) = f(2.0_{1,2}, 1.2._{1,4}, 2.1_{1,4})$
54. $(1.3_{3,4}) = f(2.0_{1,2}, 2.1_{1,4})$
55. $(1.3_{3,4}) = f(2.0_{1,2}, 2.1_{1,4}, 1.2._{1,4})$
56. $(1.3_{3,4}) = f(2.0_{1,2}, 2.1_{1,4}, 2.2_{1,2,4})$
57. $(1.3_{3,4}) = f(2.0_{1,2}, 2.2_{1,2,4})$
58. $(1.3_{3,4}) = f(2.0_{1,2}, 2.2_{1,2,4}, 2.1_{1,4})$
59. $(1.3_{3,4}) = f(2.1_{1,4}, 0.3_{2,3})$
60. $(1.3_{3,4}) = f(2.1_{1,4}, 0.3_{2,3}, 3.1_{3,4})$
61. $(1.3_{3,4}) = f(2.1_{1,4}, 1.2._{1,4})$
62. $(1.3_{3,4}) = f(2.1_{1,4}, 1.2._{1,4}, 2.0_{1,2})$
63. $(1.3_{3,4}) = f(2.1_{1,4}, 1.2._{1,4}, 3.0_{2,3})$
64. $(1.3_{3,4}) = f(2.1_{1,4}, 2.0_{1,2})$
65. $(1.3_{3,4}) = f(2.1_{1,4}, 2.0_{1,2}, 1.2._{1,4})$
66. $(1.3_{3,4}) = f(2.1_{1,4}, 2.0_{1,2}, 2.2_{1,2,4})$

67. $(1.3_{3,4}) = f(2.1_{1,4}, 2.2_{1,2,4})$
68. $(1.3_{3,4}) = f(2.1_{1,4}, 2.2_{1,2,4}, 2.0_{1,2})$
69. $(1.3_{3,4}) = f(2.1_{1,4}, 2.2_{1,2,4}, 3.0_{2,3})$
70. $(1.3_{3,4}) = f(2.1_{1,4}, 3.0_{2,3})$
71. $(1.3_{3,4}) = f(2.1_{1,4}, 3.0_{2,3}, 1.2._{1,4})$
72. $(1.3_{3,4}) = f(2.1_{1,4}, 3.0_{2,3}, 2.2_{1,2,4})$
73. $(1.3_{3,4}) = f(2.1_{1,4}, 3.1_{3,4})$
74. $(1.3_{3,4}) = f(2.1_{1,4}, 3.1_{3,4}, 0.3_{2,3})$
75. $(1.3_{3,4}) = f(2.2_{1,2,4}, 0.3_{2,3})$
76. $(1.3_{3,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 3.1_{3,4})$
77. $(1.3_{3,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 3.2_{2,4})$
78. $(1.3_{3,4}) = f(2.2_{1,2,4}, 2.0_{1,2})$
79. $(1.3_{3,4}) = f(2.2_{1,2,4}, 2.0_{1,2}, 2.1_{1,4})$
80. $(1.3_{3,4}) = f(2.2_{1,2,4}, 2.1_{1,4})$
81. $(1.3_{3,4}) = f(2.2_{1,2,4}, 2.1_{1,4}, 2.0_{1,2})$
82. $(1.3_{3,4}) = f(2.2_{1,2,4}, 2.1_{1,4}, 3.0_{2,3})$
83. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.0_{2,3})$
84. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 2.1_{1,4})$
85. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 3.1_{3,4})$
86. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.1_{3,4})$
87. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.1_{3,4}, 0.3_{2,3})$
88. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.1_{3,4}, 3.0_{2,3})$
89. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.2_{2,4})$
90. $(1.3_{3,4}) = f(2.2_{1,2,4}, 3.2_{2,4}, 0.3_{2,3})$
91. $(1.3_{3,4}) = f(2.3_{2,4}, 0.3_{2,3})$
92. $(1.3_{3,4}) = f(2.3_{2,4}, 0.3_{2,3}, 3.1_{3,4})$
93. $(1.3_{3,4}) = f(2.3_{2,4}, 0.3_{2,3}, 3.2_{2,4})$
94. $(1.3_{3,4}) = f(2.3_{2,4}, 0.3_{2,3}, 3.3_{2,3,4})$
95. $(1.3_{3,4}) = f(2.3_{2,4}, 3.1_{3,4})$
96. $(1.3_{3,4}) = f(2.3_{2,4}, 3.1_{3,4}, 0.3_{2,3})$
97. $(1.3_{3,4}) = f(2.3_{2,4}, 3.2_{2,4})$
98. $(1.3_{3,4}) = f(2.3_{2,4}, 3.2_{2,4}, 0.3_{2,3})$
99. $(1.3_{3,4}) = f(2.3_{2,4}, 3.3_{2,3,4})$
100. $(1.3_{3,4}) = f(2.3_{2,4}, 3.3_{2,3,4}, 0.3_{2,3})$
101. $(1.3_{3,4}) = f(3.0_{2,3}, 1.1._{1,3,4})$
102. $(1.3_{3,4}) = f(3.0_{2,3}, 1.1._{1,3,4}, 1.2._{1,4})$
103. $(1.3_{3,4}) = f(3.0_{2,3}, 1.2._{1,4})$
104. $(1.3_{3,4}) = f(3.0_{2,3}, 1.2._{1,4}, 1.1._{1,3,4})$
105. $(1.3_{3,4}) = f(3.0_{2,3}, 1.2._{1,4}, 2.1_{1,4})$
106. $(1.3_{3,4}) = f(3.0_{2,3}, 1.2._{1,4}, 3.1_{3,4})$
107. $(1.3_{3,4}) = f(3.0_{2,3}, 2.1_{1,4})$
108. $(1.3_{3,4}) = f(3.0_{2,3}, 2.1_{1,4}, 1.2._{1,4})$
109. $(1.3_{3,4}) = f(3.0_{2,3}, 2.1_{1,4}, 2.2_{1,2,4})$
110. $(1.3_{3,4}) = f(3.0_{2,3}, 2.2_{1,2,4})$
111. $(1.3_{3,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 2.1_{1,4})$

112. $(1.3_{3,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 3.1_{3,4})$
113. $(1.3_{3,4}) = f(3.0_{2,3}, 3.1_{3,4})$
114. $(1.3_{3,4}) = f(3.0_{2,3}, 3.1_{3,4}, 1.2._{1,4})$
115. $(1.3_{3,4}) = f(3.0_{2,3}, 3.1_{3,4}, 2.2_{1,2,4})$
116. $(1.3_{3,4}) = f(3.0_{2,3}, 3.1_{3,4}, 3.2_{2,4})$
117. $(1.3_{3,4}) = f(3.0_{2,3}, 3.2_{2,4})$
118. $(1.3_{3,4}) = f(3.0_{2,3}, 3.2_{2,4}, 3.1_{3,4})$
119. $(1.3_{3,4}) = f(3.1_{3,4}, 0.3_{2,3})$
120. $(1.3_{3,4}) = f(3.1_{3,4}, 0.3_{2,3}, 2.1_{1,4})$
121. $(1.3_{3,4}) = f(3.1_{3,4}, 0.3_{2,3}, 2.2_{1,2,4})$
122. $(1.3_{3,4}) = f(3.1_{3,4}, 0.3_{2,3}, 2.3_{2,4})$
123. $(1.3_{3,4}) = f(3.1_{3,4}, 1.2._{1,4})$
124. $(1.3_{3,4}) = f(3.1_{3,4}, 1.2._{1,4}, 3.0_{2,3})$
125. $(1.3_{3,4}) = f(3.1_{3,4}, 2.1_{1,4})$
126. $(1.3_{3,4}) = f(3.1_{3,4}, 2.1_{1,4}, 0.3_{2,3})$
127. $(1.3_{3,4}) = f(3.1_{3,4}, 2.2_{1,2,4})$
128. $(1.3_{3,4}) = f(3.1_{3,4}, 2.2_{1,2,4}, 0.3_{2,3})$
129. $(1.3_{3,4}) = f(3.1_{3,4}, 2.2_{1,2,4}, 3.0_{2,3})$
130. $(1.3_{3,4}) = f(3.1_{3,4}, 2.3_{2,4})$
131. $(1.3_{3,4}) = f(3.1_{3,4}, 2.3_{2,4}, 0.3_{2,3})$
132. $(1.3_{3,4}) = f(3.1_{3,4}, 3.0_{2,3})$
133. $(1.3_{3,4}) = f(3.1_{3,4}, 3.0_{2,3}, 1.2._{1,4})$
134. $(1.3_{3,4}) = f(3.1_{3,4}, 3.0_{2,3}, 2.2_{1,2,4})$
135. $(1.3_{3,4}) = f(3.1_{3,4}, 3.0_{2,3}, 3.2_{2,4})$
136. $(1.3_{3,4}) = f(3.1_{3,4}, 3.2_{2,4})$
137. $(1.3_{3,4}) = f(3.1_{3,4}, 3.2_{2,4}, 3.0_{2,3})$
138. $(1.3_{3,4}) = f(3.2_{2,4}, 0.3_{2,3})$
139. $(1.3_{3,4}) = f(3.2_{2,4}, 0.3_{2,3}, 2.2_{1,2,4})$
140. $(1.3_{3,4}) = f(3.2_{2,4}, 0.3_{2,3}, 2.3_{2,4})$
141. $(1.3_{3,4}) = f(3.2_{2,4}, 2.2_{1,2,4})$
142. $(1.3_{3,4}) = f(3.2_{2,4}, 2.2_{1,2,4}, 0.3_{2,3})$
143. $(1.3_{3,4}) = f(3.2_{2,4}, 2.3_{2,4})$
144. $(1.3_{3,4}) = f(3.2_{2,4}, 2.3_{2,4}, 0.3_{2,3})$
145. $(1.3_{3,4}) = f(3.2_{2,4}, 3.0_{2,3})$
146. $(1.3_{3,4}) = f(3.2_{2,4}, 3.0_{2,3}, 3.1_{3,4})$
147. $(1.3_{3,4}) = f(3.2_{2,4}, 3.1_{3,4})$
148. $(1.3_{3,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
149. $(1.3_{3,4}) = f(3.3_{2,3,4}, 0.3_{2,3})$
150. $(1.3_{3,4}) = f(3.3_{2,3,4}, 0.3_{2,3}, 2.3_{2,4})$
151. $(1.3_{3,4}) = f(3.3_{2,3,4}, 2.3_{2,4})$
152. $(1.3_{3,4}) = f(3.3_{2,3,4}, 2.3_{2,4}, 0.3_{2,3})$

2.8. **Functions with w = $(2.0_{1,2})$**

1. $(2.0_{1,2}) = f(1.1._{1,3,4}, 1.2._{1,4})$
2. $(2.0_{1,2}) = f(1.1._{1,3,4}, 1.2._{1,4}, 1.3_{3,4})$
3. $(2.0_{1,2}) = f(1.1._{1,3,4}, 1.3_{3,4})$
4. $(2.0_{1,2}) = f(1.1._{1,3,4}, 1.3_{3,4}, 1.2._{1,4})$
5. $(2.0_{1,2}) = f(1.2._{1,4}, 1.1._{1,3,4})$
6. $(2.0_{1,2}) = f(1.2._{1,4}, 1.1._{1,3,4}, 1.3_{3,4})$
7. $(2.0_{1,2}) = f(1.2._{1,4}, 1.3_{3,4})$
8. $(2.0_{1,2}) = f(1.2._{1,4}, 1.3_{3,4}, 1.1._{1,3,4})$
9. $(2.0_{1,2}) = f(1.2._{1,4}, 1.3_{3,4}, 2.1_{1,4})$
10. $(2.0_{1,2}) = f(1.2._{1,4}, 2.1_{1,4}, 1.3_{3,4})$
11. $(2.0_{1,2}) = f(1.3_{3,4}, 1.1._{1,3,4})$
12. $(2.0_{1,2}) = f(1.3_{3,4}, 1.1._{1,3,4}, 1.2._{1,4})$
13. $(2.0_{1,2}) = f(1.3_{3,4}, 1.2._{1,4})$
14. $(2.0_{1,2}) = f(1.3_{3,4}, 1.2._{1,4}, 1.1._{1,3,4})$
15. $(2.0_{1,2}) = f(1.3_{3,4}, 1.2._{1,4}, 2.1_{1,4})$
16. $(2.0_{1,2}) = f(1.3_{3,4}, 2.1_{1,4})$
17. $(2.0_{1,2}) = f(1.3_{3,4}, 2.1_{1,4}, 1.2._{1,4})$
18. $(2.0_{1,2}) = f(1.3_{3,4}, 2.1_{1,4}, 2.2_{1,2,4})$
19. $(2.0_{1,2}) = f(1.3_{3,4}, 2.2_{1,2,4})$
20. $(2.0_{1,2}) = f(1.3_{3,4}, 2.2_{1,2,4}, 2.1_{1,4})$
21. $(2.0_{1,2}) = f(2.1_{1,4}, 1.2._{1,4})$
22. $(2.0_{1,2}) = f(2.1_{1,4}, 1.2._{1,4}, 1.3_{3,4})$
23. $(2.0_{1,2}) = f(2.1_{1,4}, 1.3_{3,4})$
24. $(2.0_{1,2}) = f(2.1_{1,4}, 1.3_{3,4}, 1.2._{1,4})$
25. $(2.0_{1,2}) = f(2.1_{1,4}, 1.3_{3,4}, 2.2_{1,2,4})$
26. $(2.0_{1,2}) = f(2.1_{1,4}, 2.2_{1,2,4})$
27. $(2.0_{1,2}) = f(2.1_{1,4}, 2.2_{1,2,4}, 1.3_{3,4})$
28. $(2.0_{1,2}) = f(2.1_{1,4}, 2.2_{1,2,4}, 2.3_{2,4})$
29. $(2.0_{1,2}) = f(2.1_{1,4}, 2.3_{2,4})$
30. $(2.0_{1,2}) = f(2.1_{1,4}, 2.3_{2,4}, 2.2_{1,2,4})$
31. $(2.0_{1,2}) = f(2.2_{1,2,4}, 1.3_{3,4})$
32. $(2.0_{1,2}) = f(2.2_{1,2,4}, 1.3_{3,4}, 2.1_{1,4})$
33. $(2.0_{1,2}) = f(2.2_{1,2,4}, 2.1_{1,4})$
34. $(2.0_{1,2}) = f(2.2_{1,2,4}, 2.1_{1,4}, 1.3_{3,4})$
35. $(2.0_{1,2}) = f(2.2_{1,2,4}, 2.1_{1,4}, 2.3_{2,4})$
36. $(2.0_{1,2}) = f(2.2_{1,2,4}, 2.3_{2,4})$
37. $(2.0_{1,2}) = f(2.2_{1,2,4}, 2.3_{2,4}, 2.1_{1,4})$
38. $(2.0_{1,2}) = f(2.3_{2,4}, 2.1_{1,4})$
39. $(2.0_{1,2}) = f(2.3_{2,4}, 2.1_{1,4}, 2.2_{1,2,4})$
40. $(2.0_{1,2}) = f(2.3_{2,4}, 2.2_{1,2,4})$
41. $(2.0_{1,2}) = f(2.3_{2,4}, 2.2_{1,2,4}, 2.1_{1,4})$

2.9. **Functions with w = $(2.1_{1,4})$**

1. $(2.1_{1,4}) = f(0.1_{1,3}, 1.1._{1,3,4})$
2. $(2.1_{1,4}) = f(0.1_{1,3}, 1.1._{1,3,4}, 3.1_{3,4})$
3. $(2.1_{1,4}) = f(0.2_{1,2}, 1.1._{1,3,4})$
4. $(2.1_{1,4}) = f(0.2_{1,2}, 1.1._{1,3,4}, 3.1_{3,4})$
5. $(2.1_{1,4}) = f(0.2_{1,2}, 1.2._{1,4})$
6. $(2.1_{1,4}) = f(0.2_{1,2}, 1.2._{1,4}, 3.1_{3,4})$
7. $(2.1_{1,4}) = f(0.2_{1,2}, 3.1_{3,4})$
8. $(2.1_{1,4}) = f(0.2_{1,2}, 3.1_{3,4}, 1.1._{1,3,4})$
9. $(2.1_{1,4}) = f(0.2_{1,2}, 3.1_{3,4}, 1.2._{1,4})$
10. $(2.1_{1,4}) = f(0.3_{2,3}, 1.1._{1,3,4})$
11. $(2.1_{1,4}) = f(0.3_{2,3}, 1.1._{1,3,4}, 3.1_{3,4})$
12. $(2.1_{1,4}) = f(0.3_{2,3}, 1.2._{1,4})$
13. $(2.1_{1,4}) = f(0.3_{2,3}, 1.2._{1,4}, 3.1_{3,4})$
14. $(2.1_{1,4}) = f(0.3_{2,3}, 1.3_{3,4})$
15. $(2.1_{1,4}) = f(0.3_{2,3}, 1.3_{3,4}, 3.1_{3,4})$
16. $(2.1_{1,4}) = f(0.3_{2,3}, 3.1_{3,4})$
17. $(2.1_{1,4}) = f(0.3_{2,3}, 3.1_{3,4}, 1.1._{1,3,4})$
18. $(2.1_{1,4}) = f(0.3_{2,3}, 3.1_{3,4}, 1.2._{1,4})$
19. $(2.1_{1,4}) = f(0.3_{2,3}, 3.1_{3,4}, 1.3_{3,4})$
20. $(2.1_{1,4}) = f(1.1._{1,3,4}, 0.1_{1,3})$
21. $(2.1_{1,4}) = f(1.1._{1,3,4}, 0.1_{1,3}, 3.1_{3,4})$
22. $(2.1_{1,4}) = f(1.1._{1,3,4}, 0.2_{1,2})$
23. $(2.1_{1,4}) = f(1.1._{1,3,4}, 0.2_{1,2}, 3.1_{3,4})$
24. $(2.1_{1,4}) = f(1.1._{1,3,4}, 0.3_{2,3})$
25. $(2.1_{1,4}) = f(1.1._{1,3,4}, 0.3_{2,3}, 3.1_{3,4})$
26. $(2.1_{1,4}) = f(1.1._{1,3,4}, 3.1_{3,4})$
27. $(2.1_{1,4}) = f(1.1._{1,3,4}, 3.1_{3,4}, 0.1_{1,3})$
28. $(2.1_{1,4}) = f(1.1._{1,3,4}, 3.1_{3,4}, 0.2_{1,2})$
29. $(2.1_{1,4}) = f(1.1._{1,3,4}, 3.1_{3,4}, 0.3_{2,3})$
30. $(2.1_{1,4}) = f(1.2._{1,4}, 0.2_{1,2})$
31. $(2.1_{1,4}) = f(1.2._{1,4}, 0.2_{1,2}, 3.1_{3,4})$
32. $(2.1_{1,4}) = f(1.2._{1,4}, 0.3_{2,3})$
33. $(2.1_{1,4}) = f(1.2._{1,4}, 0.3_{2,3}, 3.1_{3,4})$
34. $(2.1_{1,4}) = f(1.2._{1,4}, 1.3_{3,4}, 3.0_{2,3})$
35. $(2.1_{1,4}) = f(1.2._{1,4}, 1.3_{3,4})$
36. $(2.1_{1,4}) = f(1.2._{1,4}, 1.3_{3,4}, 2.0_{1,2})$
37. $(2.1_{1,4}) = f(1.2._{1,4}, 2.0_{1,2})$
38. $(2.1_{1,4}) = f(1.2._{1,4}, 2.0_{1,2}, 1.3_{3,4})$
39. $(2.1_{1,4}) = f(1.2._{1,4}, 3.0_{2,3})$
40. $(2.1_{1,4}) = f(1.2._{1,4}, 3.0_{2,3}, 1.3_{3,4})$
41. $(2.1_{1,4}) = f(1.2._{1,4}, 3.1_{3,4})$
42. $(2.1_{1,4}) = f(1.2._{1,4}, 3.1_{3,4}, 0.2_{1,2})$

43. $(2.1_{1,4}) = f(1.2._{1,4}, 3.1_{3,4}, 0.3_{2,3})$
44. $(2.1_{1,4}) = f(1.3_{3,4}, 0.3_{2,3})$
45. $(2.1_{1,4}) = f(1.3_{3,4}, 0.3_{2,3}, 3.1_{3,4})$
46. $(2.1_{1,4}) = f(1.3_{3,4}, 1.2._{1,4})$
47. $(2.1_{1,4}) = f(1.3_{3,4}, 1.2._{1,4}, 2.0_{1,2})$
48. $(2.1_{1,4}) = f(1.3_{3,4}, 1.2._{1,4}, 3.0_{2,3})$
49. $(2.1_{1,4}) = f(1.3_{3,4}, 2.0_{1,2})$
50. $(2.1_{1,4}) = f(1.3_{3,4}, 2.0_{1,2}, 1.2._{1,4})$
51. $(2.1_{1,4}) = f(1.3_{3,4}, 2.0_{1,2}, 2.2_{1,2,4})$
52. $(2.1_{1,4}) = f(1.3_{3,4}, 2.2_{1,2,4})$
53. $(2.1_{1,4}) = f(1.3_{3,4}, 2.2_{1,2,4}, 2.0_{1,2})$
54. $(2.1_{1,4}) = f(1.3_{3,4}, 2.2_{1,2,4}, 3.0_{2,3})$
55. $(2.1_{1,4}) = f(1.3_{3,4}, 3.0_{2,3})$
56. $(2.1_{1,4}) = f(1.3_{3,4}, 3.0_{2,3}, 1.2._{1,4})$
57. $(2.1_{1,4}) = f(1.3_{3,4}, 3.0_{2,3}, 2.2_{1,2,4})$
58. $(2.1_{1,4}) = f(1.3_{3,4}, 3.1_{3,4})$
59. $(2.1_{1,4}) = f(1.3_{3,4}, 3.1_{3,4}, 0.3_{2,3})$
60. $(2.1_{1,4}) = f(2.0_{1,2}, 1.2._{1,4})$
61. $(2.1_{1,4}) = f(2.0_{1,2}, 1.2._{1,4}, 1.3_{3,4})$
62. $(2.1_{1,4}) = f(2.0_{1,2}, 1.3_{3,4})$
63. $(2.1_{1,4}) = f(2.0_{1,2}, 1.3_{3,4}, 1.2._{1,4})$
64. $(2.1_{1,4}) = f(2.0_{1,2}, 1.3_{3,4}, 2.2_{1,2,4})$
65. $(2.1_{1,4}) = f(2.0_{1,2}, 2.2_{1,2,4})$
66. $(2.1_{1,4}) = f(2.0_{1,2}, 2.2_{1,2,4}, 1.3_{3,4})$
67. $(2.1_{1,4}) = f(2.0_{1,2}, 2.2_{1,2,4}, 2.3_{2,4})$
68. $(2.1_{1,4}) = f(2.0_{1,2}, 2.3_{2,4})$
69. $(2.1_{1,4}) = f(2.0_{1,2}, 2.3_{2,4}, 2.2_{1,2,4})$
70. $(2.1_{1,4}) = f(2.2_{1,2,4}, 1.3_{3,4})$
71. $(2.1_{1,4}) = f(2.2_{1,2,4}, 1.3_{3,4}, 2.0_{1,2})$
72. $(2.1_{1,4}) = f(2.2_{1,2,4}, 1.3_{3,4}, 3.0_{2,3})$
73. $(2.1_{1,4}) = f(2.2_{1,2,4}, 2.0_{1,2})$
74. $(2.1_{1,4}) = f(2.2_{1,2,4}, 2.0_{1,2}, 1.3_{3,4})$
75. $(2.1_{1,4}) = f(2.2_{1,2,4}, 2.0_{1,2}, 2.3_{2,4})$
76. $(2.1_{1,4}) = f(2.2_{1,2,4}, 2.3_{2,4})$
77. $(2.1_{1,4}) = f(2.2_{1,2,4}, 2.3_{2,4}, 2.0_{1,2})$
78. $(2.1_{1,4}) = f(2.2_{1,2,4}, 2.3_{2,4}, 3.0_{2,3})$
79. $(2.1_{1,4}) = f(2.2_{1,2,4}, 3.0_{2,3})$
80. $(2.1_{1,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 1.3_{3,4})$
81. $(2.1_{1,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 2.3_{2,4})$
82. $(2.1_{1,4}) = f(2.3_{2,4}, 2.0_{1,2})$
83. $(2.1_{1,4}) = f(2.3_{2,4}, 2.0_{1,2}, 2.2_{1,2,4})$
84. $(2.1_{1,4}) = f(2.3_{2,4}, 2.2_{1,2,4})$
85. $(2.1_{1,4}) = f(2.3_{2,4}, 2.2_{1,2,4}, 2.0_{1,2})$
86. $(2.1_{1,4}) = f(2.3_{2,4}, 2.2_{1,2,4}, 3.0_{2,3})$

87. $(2.1_{1,4}) = f(2.3_{2,4}, 3.0_{2,3})$
88. $(2.1_{1,4}) = f(2.3_{2,4}, 3.0_{2,3}, 2.2_{1,2,4})$
89. $(2.1_{1,4}) = f(3.0_{2,3}, 1.2._{1,4})$
90. $(2.1_{1,4}) = f(3.0_{2,3}, 1.2._{1,4}, 1.3_{3,4})$
91. $(2.1_{1,4}) = f(3.0_{2,3}, 1.3_{3,4})$
92. $(2.1_{1,4}) = f(3.0_{2,3}, 1.3_{3,4}, 1.2._{1,4})$
93. $(2.1_{1,4}) = f(3.0_{2,3}, 1.3_{3,4}, 2.2_{1,2,4})$
94. $(2.1_{1,4}) = f(3.0_{2,3}, 2.2_{1,2,4})$
95. $(2.1_{1,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 1.3_{3,4})$
96. $(2.1_{1,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 2.3_{2,4})$
97. $(2.1_{1,4}) = f(3.0_{2,3}, 2.3_{2,4})$
98. $(2.1_{1,4}) = f(3.0_{2,3}, 2.3_{2,4}, 2.2_{1,2,4})$
99. $(2.1_{1,4}) = f(3.1_{3,4}, 0.1_{1,3})$
100. $(2.1_{1,4}) = f(3.1_{3,4}, 0.1_{1,3}, 1.1._{1,3,4})$
101. $(2.1_{1,4}) = f(3.1_{3,4}, 0.2_{1,2})$
102. $(2.1_{1,4}) = f(3.1_{3,4}, 0.2_{1,2}, 1.1._{1,3,4})$
103. $(2.1_{1,4}) = f(3.1_{3,4}, 0.2_{1,2}, 1.2._{1,4})$
104. $(2.1_{1,4}) = f(3.1_{3,4}, 0.3_{2,3})$
105. $(2.1_{1,4}) = f(3.1_{3,4}, 0.3_{2,3}, 1.1._{1,3,4})$
106. $(2.1_{1,4}) = f(3.1_{3,4}, 0.3_{2,3}, 1.2._{1,4})$
107. $(2.1_{1,4}) = f(3.1_{3,4}, 0.3_{2,3}, 1.3_{3,4})$
108. $(2.1_{1,4}) = f(3.1_{3,4}, 1.1._{1,3,4})$
109. $(2.1_{1,4}) = f(3.1_{3,4}, 1.1._{1,3,4}, 0.1_{1,3})$
110. $(2.1_{1,4}) = f(3.1_{3,4}, 1.1._{1,3,4}, 0.2_{1,2})$
111. $(2.1_{1,4}) = f(3.1_{3,4}, 1.1._{1,3,4}, 0.3_{2,3})$
112. $(2.1_{1,4}) = f(3.1_{3,4}, 1.2._{1,4})$
113. $(2.1_{1,4}) = f(3.1_{3,4}, 1.2._{1,4}, 0.2_{1,2})$
114. $(2.1_{1,4}) = f(3.1_{3,4}, 1.2._{1,4}, 0.3_{2,3})$
115. $(2.1_{1,4}) = f(3.1_{3,4}, 1.3_{3,4})$
116. $(2.1_{1,4}) = f(3.1_{3,4}, 1.3_{3,4}, 0.3_{2,3})$

2.10. **Functions with w = $(2.2_{1,2,4})$**

1. $(2.2_{1,2,4}) = f(0.2_{1,2}, 1.2._{1,4})$
2. $(2.2_{1,2,4}) = f(0.2_{1,2}, 1.2._{1,4}, 3.1_{3,4})$
3. $(2.2_{1,2,4}) = f(0.2_{1,2}, 1.2._{1,4}, 3.2_{2,4})$
4. $(2.2_{1,2,4}) = f(0.2_{1,2}, 3.1_{3,4})$
5. $(2.2_{1,2,4}) = f(0.2_{1,2}, 3.1_{3,4}, 1.2._{1,4})$
6. $(2.2_{1,2,4}) = f(0.2_{1,2}, 3.2_{2,4})$
7. $(2.2_{1,2,4}) = f(0.2_{1,2}, 3.2_{2,4}, 1.2._{1,4})$
8. $(2.2_{1,2,4}) = f(0.3_{2,3}, 1.2._{1,4})$
9. $(2.2_{1,2,4}) = f(0.3_{2,3}, 1.2._{1,4}, 3.1_{3,4})$
10. $(2.2_{1,2,4}) = f(0.3_{2,3}, 1.2._{1,4}, 3.2_{2,4})$
11. $(2.2_{1,2,4}) = f(0.3_{2,3}, 1.3_{3,4})$

12. $(2.2_{1,2,4}) = f(0.3_{2,3}, 1.3_{3,4}, 3.1_{3,4})$
13. $(2.2_{1,2,4}) = f(0.3_{2,3}, 1.3_{3,4}, 3.2_{2,4})$
14. $(2.2_{1,2,4}) = f(0.3_{2,3}, 3.1_{3,4})$
15. $(2.2_{1,2,4}) = f(0.3_{2,3}, 3.1_{3,4}, 1.2._{1,4})$
16. $(2.2_{1,2,4}) = f(0.3_{2,3}, 3.1_{3,4}, 1.3_{3,4})$
17. $(2.2_{1,2,4}) = f(0.3_{2,3}, 3.2_{2,4})$
18. $(2.2_{1,2,4}) = f(0.3_{2,3}, 3.2_{2,4}, 1.2._{1,4})$
19. $(2.2_{1,2,4}) = f(0.3_{2,3}, 3.2_{2,4}, 1.3_{3,4})$
20. $(2.2_{1,2,4}) = f(1.2._{1,4}, 0.2_{1,2})$
21. $(2.2_{1,2,4}) = f(1.2._{1,4}, 0.2_{1,2}, 3.1_{3,4})$
22. $(2.2_{1,2,4}) = f(1.2._{1,4}, 0.2_{1,2}, 3.2_{2,4})$
23. $(2.2_{1,2,4}) = f(1.2._{1,4}, 0.3_{2,3})$
24. $(2.2_{1,2,4}) = f(1.2._{1,4}, 0.3_{2,3}, 3.1_{3,4})$
25. $(2.2_{1,2,4}) = f(1.2._{1,4}, 0.3_{2,3}, 3.2_{2,4})$
26. $(2.2_{1,2,4}) = f(1.2._{1,4}, 3.1_{3,4})$
27. $(2.2_{1,2,4}) = f(1.2._{1,4}, 3.1_{3,4}, 0.2_{1,2})$
28. $(2.2_{1,2,4}) = f(1.2._{1,4}, 3.1_{3,4}, 0.3_{2,3})$
29. $(2.2_{1,2,4}) = f(1.2._{1,4}, 3.2_{2,4})$
30. $(2.2_{1,2,4}) = f(1.2._{1,4}, 3.2_{2,4}, 0.2_{1,2})$
31. $(2.2_{1,2,4}) = f(1.2._{1,4}, 3.2_{2,4}, 0.3_{2,3})$
32. $(2.2_{1,2,4}) = f(1.3_{3,4}, 0.3_{2,3})$
33. $(2.2_{1,2,4}) = f(1.3_{3,4}, 0.3_{2,3}, 3.1_{3,4})$
34. $(2.2_{1,2,4}) = f(1.3_{3,4}, 0.3_{2,3}, 3.2_{2,4})$
35. $(2.2_{1,2,4}) = f(1.3_{3,4}, 2.0_{1,2})$
36. $(2.2_{1,2,4}) = f(1.3_{3,4}, 2.0_{1,2}, 2.1_{1,4})$
37. $(2.2_{1,2,4}) = f(1.3_{3,4}, 2.1_{1,4})$
38. $(2.2_{1,2,4}) = f(1.3_{3,4}, 2.1_{1,4}, 2.0_{1,2})$
39. $(2.2_{1,2,4}) = f(1.3_{3,4}, 2.1_{1,4}, 3.0_{2,3})$
40. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.0_{2,3})$
41. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.0_{2,3}, 2.1_{1,4})$
42. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.0_{2,3}, 3.1_{3,4})$
43. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.1_{3,4})$
44. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.1_{3,4}, 0.3_{2,3})$
45. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.1_{3,4}, 3.0_{2,3})$
46. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.2_{2,4})$
47. $(2.2_{1,2,4}) = f(1.3_{3,4}, 3.2_{2,4}, 0.3_{2,3})$
48. $(2.2_{1,2,4}) = f(2.0_{1,2}, 1.3_{3,4})$
49. $(2.2_{1,2,4}) = f(2.0_{1,2}, 1.3_{3,4}, 2.1_{1,4})$
50. $(2.2_{1,2,4}) = f(2.0_{1,2}, 2.1_{1,4})$
51. $(2.2_{1,2,4}) = f(2.0_{1,2}, 2.1_{1,4}, 1.3_{3,4})$
52. $(2.2_{1,2,4}) = f(2.0_{1,2}, 2.1_{1,4}, 2.3_{2,4})$
53. $(2.2_{1,2,4}) = f(2.0_{1,2}, 2.3_{2,4})$
54. $(2.2_{1,2,4}) = f(2.0_{1,2}, 2.3_{2,4}, 2.1_{1,4})$
55. $(2.2_{1,2,4}) = f(2.1_{1,4}, 1.3_{3,4})$

56. $(2.2_{1,2,4}) = f(2.1_{1,4}, 1.3_{3,4}, 2.0_{1,2})$
57. $(2.2_{1,2,4}) = f(2.1_{1,4}, 1.3_{3,4}, 3.0_{2,3})$
58. $(2.2_{1,2,4}) = f(2.1_{1,4}, 2.0_{1,2})$
59. $(2.2_{1,2,4}) = f(2.1_{1,4}, 2.0_{1,2}, 1.3_{3,4})$
60. $(2.2_{1,2,4}) = f(2.1_{1,4}, 2.0_{1,2}, 2.3_{2,4})$
61. $(2.2_{1,2,4}) = f(2.1_{1,4}, 2.3_{2,4})$
62. $(2.2_{1,2,4}) = f(2.1_{1,4}, 2.3_{2,4}, 2.0_{1,2})$
63. $(2.2_{1,2,4}) = f(2.1_{1,4}, 2.3_{2,4}, 3.0_{2,3})$
64. $(2.2_{1,2,4}) = f(2.1_{1,4}, 3.0_{2,3})$
65. $(2.2_{1,2,4}) = f(2.1_{1,4}, 3.0_{2,3}, 1.3_{3,4})$
66. $(2.2_{1,2,4}) = f(2.1_{1,4}, 3.0_{2,3}, 2.3_{2,4})$
67. $(2.2_{1,2,4}) = f(2.3_{2,4}, 2.0_{1,2})$
68. $(2.2_{1,2,4}) = f(2.3_{2,4}, 2.0_{1,2}, 2.1_{1,4})$
69. $(2.2_{1,2,4}) = f(2.3_{2,4}, 2.1_{1,4})$
70. $(2.2_{1,2,4}) = f(2.3_{2,4}, 2.1_{1,4}, 2.0_{1,2})$
71. $(2.2_{1,2,4}) = f(2.3_{2,4}, 2.1_{1,4}, 3.0_{2,3})$
72. $(2.2_{1,2,4}) = f(2.3_{2,4}, 3.0_{2,3})$
73. $(2.2_{1,2,4}) = f(2.3_{2,4}, 3.0_{2,3}, 2.1_{1,4})$
74. $(2.2_{1,2,4}) = f(2.3_{2,4}, 3.0_{2,3}, 3.1_{3,4})$
75. $(2.2_{1,2,4}) = f(2.3_{2,4}, 3.1_{3,4})$
76. $(2.2_{1,2,4}) = f(2.3_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
77. $(2.2_{1,2,4}) = f(3.0_{2,3}, 1.3_{3,4})$
78. $(2.2_{1,2,4}) = f(3.0_{2,3}, 1.3_{3,4}, 2.1_{1,4})$
79. $(2.2_{1,2,4}) = f(3.0_{2,3}, 1.3_{3,4}, 3.1_{3,4})$
80. $(2.2_{1,2,4}) = f(3.0_{2,3}, 2.1_{1,4})$
81. $(2.2_{1,2,4}) = f(3.0_{2,3}, 2.1_{1,4}, 1.3_{3,4})$
82. $(2.2_{1,2,4}) = f(3.0_{2,3}, 2.1_{1,4}, 2.3_{2,4})$
83. $(2.2_{1,2,4}) = f(3.0_{2,3}, 2.3_{2,4})$
84. $(2.2_{1,2,4}) = f(3.0_{2,3}, 2.3_{2,4}, 2.1_{1,4})$
85. $(2.2_{1,2,4}) = f(3.0_{2,3}, 2.3_{2,4}, 3.1_{3,4})$
86. $(2.2_{1,2,4}) = f(3.0_{2,3}, 3.1_{3,4})$
87. $(2.2_{1,2,4}) = f(3.0_{2,3}, 3.1_{3,4}, 1.3_{3,4})$
88. $(2.2_{1,2,4}) = f(3.0_{2,3}, 3.1_{3,4}, 2.3_{2,4})$
89. $(2.2_{1,2,4}) = f(3.1_{3,4}, 0.2_{1,2})$
90. $(2.2_{1,2,4}) = f(3.1_{3,4}, 0.2_{1,2}, 1.2._{1,4})$
91. $(2.2_{1,2,4}) = f(3.1_{3,4}, 0.3_{2,3})$
92. $(2.2_{1,2,4}) = f(3.1_{3,4}, 0.3_{2,3}, 1.2._{1,4})$
93. $(2.2_{1,2,4}) = f(3.1_{3,4}, 0.3_{2,3}, 1.3_{3,4})$
94. $(2.2_{1,2,4}) = f(3.1_{3,4}, 1.2._{1,4})$
95. $(2.2_{1,2,4}) = f(3.1_{3,4}, 1.2._{1,4}, 0.2_{1,2})$
96. $(2.2_{1,2,4}) = f(3.1_{3,4}, 1.2._{1,4}, 0.3_{2,3})$
97. $(2.2_{1,2,4}) = f(3.1_{3,4}, 1.3_{3,4})$
98. $(2.2_{1,2,4}) = f(3.1_{3,4}, 1.3_{3,4}, 0.3_{2,3})$
99. $(2.2_{1,2,4}) = f(3.1_{3,4}, 1.3_{3,4}, 3.0_{2,3})$

100. $(2.2_{1,2,4}) = f(3.1_{3,4}, 2.3_{2,4})$
101. $(2.2_{1,2,4}) = f(3.1_{3,4}, 2.3_{2,4}, 3.0_{2,3})$
102. $(2.2_{1,2,4}) = f(3.1_{3,4}, 3.0_{2,3})$
103. $(2.2_{1,2,4}) = f(3.1_{3,4}, 3.0_{2,3}, 1.3_{3,4})$
104. $(2.2_{1,2,4}) = f(3.1_{3,4}, 3.0_{2,3}, 2.3_{2,4})$
105. $(2.2_{1,2,4}) = f(3.2_{2,4}, 0.2_{1,2})$
106. $(2.2_{1,2,4}) = f(3.2_{2,4}, 0.2_{1,2}, 1.2._{1,4})$
107. $(2.2_{1,2,4}) = f(3.2_{2,4}, 0.3_{2,3})$
108. $(2.2_{1,2,4}) = f(3.2_{2,4}, 0.3_{2,3}, 1.2._{1,4})$
109. $(2.2_{1,2,4}) = f(3.2_{2,4}, 0.3_{2,3}, 1.3_{3,4})$
110. $(2.2_{1,2,4}) = f(3.2_{2,4}, 1.2._{1,4})$
111. $(2.2_{1,2,4}) = f(3.2_{2,4}, 1.2._{1,4}, 0.2_{1,2})$
112. $(2.2_{1,2,4}) = f(3.2_{2,4}, 1.2._{1,4}, 0.3_{2,3})$
113. $(2.2_{1,2,4}) = f(3.2_{2,4}, 1.3_{3,4})$
114. $(2.2_{1,2,4}) = f(3.2_{2,4}, 1.3_{3,4}, 0.3_{2,3})$

2.11. **Functions with w = $(2.3_{2,4})$**

1. $(2.3_{2,4}) = f(0.3_{2,3}, 1.3_{3,4})$
2. $(2.3_{2,4}) = f(0.3_{2,3}, 1.3_{3,4}, 3.1_{3,4})$
3. $(2.3_{2,4}) = f(0.3_{2,3}, 1.3_{3,4}, 3.2_{2,4})$
4. $(2.3_{2,4}) = f(0.3_{2,3}, 1.3_{3,4}, 3.3_{2,3,4})$
5. $(2.3_{2,4}) = f(0.3_{2,3}, 3.1_{3,4})$
6. $(2.3_{2,4}) = f(0.3_{2,3}, 3.1_{3,4}, 1.3_{3,4})$
7. $(2.3_{2,4}) = f(0.3_{2,3}, 3.2_{2,4})$
8. $(2.3_{2,4}) = f(0.3_{2,3}, 3.2_{2,4}, 1.3_{3,4})$
9. $(2.3_{2,4}) = f(0.3_{2,3}, 3.3_{2,3,4})$
10. $(2.3_{2,4}) = f(0.3_{2,3}, 3.3_{2,3,4}, 1.3_{3,4})$
11. $(2.3_{2,4}) = f(1.3_{3,4}, 0.3_{2,3})$
12. $(2.3_{2,4}) = f(1.3_{3,4}, 0.3_{2,3}, 3.1_{3,4})$
13. $(2.3_{2,4}) = f(1.3_{3,4}, 0.3_{2,3}, 3.2_{2,4})$
14. $(2.3_{2,4}) = f(1.3_{3,4}, 0.3_{2,3}, 3.3_{2,3,4})$
15. $(2.3_{2,4}) = f(1.3_{3,4}, 3.1_{3,4})$
16. $(2.3_{2,4}) = f(1.3_{3,4}, 3.1_{3,4}, 0.3_{2,3})$
17. $(2.3_{2,4}) = f(1.3_{3,4}, 3.2_{2,4})$
18. $(2.3_{2,4}) = f(1.3_{3,4}, 3.2_{2,4}, 0.3_{2,3})$
19. $(2.3_{2,4}) = f(1.3_{3,4}, 3.3_{2,3,4})$
20. $(2.3_{2,4}) = f(1.3_{3,4}, 3.3_{2,3,4}, 0.3_{2,3})$
21. $(2.3_{2,4}) = f(2.0_{1,2}, 2.1_{1,4})$
22. $(2.3_{2,4}) = f(2.0_{1,2}, 2.1_{1,4}, 2.2_{1,2,4})$
23. $(2.3_{2,4}) = f(2.0_{1,2}, 2.2_{1,2,4})$
24. $(2.3_{2,4}) = f(2.0_{1,2}, 2.2_{1,2,4}, 2.1_{1,4})$
25. $(2.3_{2,4}) = f(2.1_{1,4}, 2.0_{1,2})$
26. $(2.3_{2,4}) = f(2.1_{1,4}, 2.0_{1,2}, 2.2_{1,2,4})$

27. $(2.3_{2,4}) = f(2.1_{1,4}, 2.2_{1,2,4})$
28. $(2.3_{2,4}) = f(2.1_{1,4}, 2.2_{1,2,4}, 2.0_{1,2})$
29. $(2.3_{2,4}) = f(2.1_{1,4}, 2.2_{1,2,4}, 3.0_{2,3})$
30. $(2.3_{2,4}) = f(2.1_{1,4}, 3.0_{2,3})$
31. $(2.3_{2,4}) = f(2.1_{1,4}, 3.0_{2,3}, 2.2_{1,2,4})$
32. $(2.3_{2,4}) = f(2.2_{1,2,4}, 2.0_{1,2})$
33. $(2.3_{2,4}) = f(2.2_{1,2,4}, 2.0_{1,2}, 2.1_{1,4})$
34. $(2.3_{2,4}) = f(2.2_{1,2,4}, 2.1_{1,4})$
35. $(2.3_{2,4}) = f(2.2_{1,2,4}, 2.1_{1,4}, 2.0_{1,2})$
36. $(2.3_{2,4}) = f(2.2_{1,2,4}, 2.1_{1,4}, 3.0_{2,3})$
37. $(2.3_{2,4}) = f(2.2_{1,2,4}, 3.0_{2,3})$
38. $(2.3_{2,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 2.1_{1,4})$
39. $(2.3_{2,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 3.1_{3,4})$
40. $(2.3_{2,4}) = f(2.2_{1,2,4}, 3.1_{3,4})$
41. $(2.3_{2,4}) = f(2.2_{1,2,4}, 3.1_{3,4}, 3.0_{2,3})$
42. $(2.3_{2,4}) = f(3.0_{2,3}, 2.1_{1,4})$
43. $(2.3_{2,4}) = f(3.0_{2,3}, 2.1_{1,4}, 2.2_{1,2,4})$
44. $(2.3_{2,4}) = f(3.0_{2,3}, 2.2_{1,2,4})$
45. $(2.3_{2,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 2.1_{1,4})$
46. $(2.3_{2,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 3.1_{3,4})$
47. $(2.3_{2,4}) = f(3.0_{2,3}, 3.1_{3,4})$
48. $(2.3_{2,4}) = f(3.0_{2,3}, 3.1_{3,4}, 2.2_{1,2,4})$
49. $(2.3_{2,4}) = f(3.0_{2,3}, 3.1_{3,4}, 3.2_{2,4})$
50. $(2.3_{2,4}) = f(3.0_{2,3}, 3.2_{2,4})$
51. $(2.3_{2,4}) = f(3.0_{2,3}, 3.2_{2,4}, 3.1_{3,4})$
52. $(2.3_{2,4}) = f(3.1_{3,4}, 0.3_{2,3})$
53. $(2.3_{2,4}) = f(3.1_{3,4}, 0.3_{2,3}, 1.3_{3,4})$
54. $(2.3_{2,4}) = f(3.1_{3,4}, 1.3_{3,4})$
55. $(2.3_{2,4}) = f(3.1_{3,4}, 1.3_{3,4}, 0.3_{2,3})$
56. $(2.3_{2,4}) = f(3.1_{3,4}, 2.2_{1,2,4})$
57. $(2.3_{2,4}) = f(3.1_{3,4}, 2.2_{1,2,4}, 3.0_{2,3})$
58. $(2.3_{2,4}) = f(3.1_{3,4}, 3.0_{2,3})$
59. $(2.3_{2,4}) = f(3.1_{3,4}, 3.0_{2,3}, 2.2_{1,2,4})$
60. $(2.3_{2,4}) = f(3.1_{3,4}, 3.0_{2,3}, 3.2_{2,4})$
61. $(2.3_{2,4}) = f(3.1_{3,4}, 3.2_{2,4})$
62. $(2.3_{2,4}) = f(3.1_{3,4}, 3.2_{2,4}, 3.0_{2,3})$
63. $(2.3_{2,4}) = f(3.2_{2,4}, 0.3_{2,3})$
64. $(2.3_{2,4}) = f(3.2_{2,4}, 0.3_{2,3}, 1.3_{3,4})$
65. $(2.3_{2,4}) = f(3.2_{2,4}, 1.3_{3,4})$
66. $(2.3_{2,4}) = f(3.2_{2,4}, 1.3_{3,4}, 0.3_{2,3})$
67. $(2.3_{2,4}) = f(3.2_{2,4}, 3.0_{2,3})$
68. $(2.3_{2,4}) = f(3.2_{2,4}, 3.0_{2,3}, 3.1_{3,4})$
69. $(2.3_{2,4}) = f(3.2_{2,4}, 3.1_{3,4})$
70. $(2.3_{2,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

71. $(2.3_{2,4}) = f(3.3_{2,3,4}, 0.3_{2,3})$
72. $(2.3_{2,4}) = f(3.3_{2,3,4}, 0.3_{2,3}, 1.3_{3,4})$
73. $(2.3_{2,4}) = f(3.3_{2,3,4}, 1.3_{3,4})$
74. $(2.3_{2,4}) = f(3.3_{2,3,4}, 1.3_{3,4}, 0.3_{2,3})$

2.12. **Functions with w = $(3.0_{2,3})$**

1. $(3.0_{2,3}) = f(1.1._{1,3,4}, 1.2._{1,4})$
2. $(3.0_{2,3}) = f(1.1._{1,3,4}, 1.2._{1,4}, 1.3_{3,4})$
3. $(3.0_{2,3}) = f(1.1._{1,3,4}, 1.3_{3,4})$
4. $(3.0_{2,3}) = f(1.1._{1,3,4}, 1.3_{3,4}, 1.2._{1,4})$
5. $(3.0_{2,3}) = f(1.2._{1,4}, 1.1._{1,3,4})$
6. $(3.0_{2,3}) = f(1.2._{1,4}, 1.1._{1,3,4}, 1.3_{3,4})$
7. $(3.0_{2,3}) = f(1.2._{1,4}, 1.3_{3,4})$
8. $(3.0_{2,3}) = f(1.2._{1,4}, 1.3_{3,4}, 1.1._{1,3,4})$
9. $(3.0_{2,3}) = f(1.2._{1,4}, 1.3_{3,4}, 2.1_{1,4})$
10. $(3.0_{2,3}) = f(1.2._{1,4}, 1.3_{3,4}, 3.1_{3,4})$
11. $(3.0_{2,3}) = f(1.2._{1,4}, 2.1_{1,4})$
12. $(3.0_{2,3}) = f(1.2._{1,4}, 2.1_{1,4}, 1.3_{3,4})$
13. $(3.0_{2,3}) = f(1.2._{1,4}, 3.1_{3,4})$
14. $(3.0_{2,3}) = f(1.2._{1,4}, 3.1_{3,4}, 1.3_{3,4})$
15. $(3.0_{2,3}) = f(1.3_{3,4}, 1.1._{1,3,4})$
16. $(3.0_{2,3}) = f(1.3_{3,4}, 1.1._{1,3,4}, 1.2._{1,4})$
17. $(3.0_{2,3}) = f(1.3_{3,4}, 1.2._{1,4})$
18. $(3.0_{2,3}) = f(1.3_{3,4}, 1.2._{1,4}, 1.1._{1,3,4})$
19. $(3.0_{2,3}) = f(1.3_{3,4}, 1.2._{1,4}, 2.1_{1,4})$
20. $(3.0_{2,3}) = f(1.3_{3,4}, 1.2._{1,4}, 3.1_{3,4})$
21. $(3.0_{2,3}) = f(1.3_{3,4}, 2.1_{1,4})$
22. $(3.0_{2,3}) = f(1.3_{3,4}, 2.1_{1,4}, 1.2._{1,4})$
23. $(3.0_{2,3}) = f(1.3_{3,4}, 2.1_{1,4}, 2.2_{1,2,4})$
24. $(3.0_{2,3}) = f(1.3_{3,4}, 2.2_{1,2,4})$
25. $(3.0_{2,3}) = f(1.3_{3,4}, 2.2_{1,2,4}, 2.1_{1,4})$
26. $(3.0_{2,3}) = f(1.3_{3,4}, 2.2_{1,2,4}, 3.1_{3,4})$
27. $(3.0_{2,3}) = f(1.3_{3,4}, 3.1_{3,4})$
28. $(3.0_{2,3}) = f(1.3_{3,4}, 3.1_{3,4}, 1.2._{1,4})$
29. $(3.0_{2,3}) = f(1.3_{3,4}, 3.1_{3,4}, 2.2_{1,2,4})$
30. $(3.0_{2,3}) = f(1.3_{3,4}, 3.1_{3,4}, 3.2_{2,4})$
31. $(3.0_{2,3}) = f(1.3_{3,4}, 3.2_{2,4})$
32. $(3.0_{2,3}) = f(1.3_{3,4}, 3.2_{2,4}, 3.1_{3,4})$
33. $(3.0_{2,3}) = f(2.1_{1,4}, 1.2._{1,4})$
34. $(3.0_{2,3}) = f(2.1_{1,4}, 1.2._{1,4}, 1.3_{3,4})$
35. $(3.0_{2,3}) = f(2.1_{1,4}, 1.3_{3,4})$
36. $(3.0_{2,3}) = f(2.1_{1,4}, 1.3_{3,4}, 1.2._{1,4})$
37. $(3.0_{2,3}) = f(2.1_{1,4}, 1.3_{3,4}, 2.2_{1,2,4})$

38. $(3.0_{2,3}) = f(2.1_{1,4}, 2.2_{1,2,4})$
39. $(3.0_{2,3}) = f(2.1_{1,4}, 2.2_{1,2,4}, 1.3_{3,4})$
40. $(3.0_{2,3}) = f(2.1_{1,4}, 2.2_{1,2,4}, 2.3_{2,4})$
41. $(3.0_{2,3}) = f(2.1_{1,4}, 2.3_{2,4})$
42. $(3.0_{2,3}) = f(2.1_{1,4}, 2.3_{2,4}, 2.2_{1,2,4})$
43. $(3.0_{2,3}) = f(2.2_{1,2,4}, 1.3_{3,4})$
44. $(3.0_{2,3}) = f(2.2_{1,2,4}, 1.3_{3,4}, 2.1_{1,4})$
45. $(3.0_{2,3}) = f(2.2_{1,2,4}, 1.3_{3,4}, 3.1_{3,4})$
46. $(3.0_{2,3}) = f(2.2_{1,2,4}, 2.1_{1,4})$
47. $(3.0_{2,3}) = f(2.2_{1,2,4}, 2.1_{1,4}, 1.3_{3,4})$
48. $(3.0_{2,3}) = f(2.2_{1,2,4}, 2.1_{1,4}, 2.3_{2,4})$
49. $(3.0_{2,3}) = f(2.2_{1,2,4}, 2.3_{2,4})$
50. $(3.0_{2,3}) = f(2.2_{1,2,4}, 2.3_{2,4}, 2.1_{1,4})$
51. $(3.0_{2,3}) = f(2.2_{1,2,4}, 2.3_{2,4}, 3.1_{3,4})$
52. $(3.0_{2,3}) = f(2.2_{1,2,4}, 3.1_{3,4})$
53. $(3.0_{2,3}) = f(2.2_{1,2,4}, 3.1_{3,4}, 1.3_{3,4})$
54. $(3.0_{2,3}) = f(2.2_{1,2,4}, 3.1_{3,4}, 2.3_{2,4})$
55. $(3.0_{2,3}) = f(2.3_{2,4}, 2.1_{1,4})$
56. $(3.0_{2,3}) = f(2.3_{2,4}, 2.1_{1,4}, 2.2_{1,2,4})$
57. $(3.0_{2,3}) = f(2.3_{2,4}, 2.2_{1,2,4})$
58. $(3.0_{2,3}) = f(2.3_{2,4}, 2.2_{1,2,4}, 2.1_{1,4})$
59. $(3.0_{2,3}) = f(2.3_{2,4}, 2.2_{1,2,4}, 3.1_{3,4})$
60. $(3.0_{2,3}) = f(2.3_{2,4}, 3.1_{3,4})$
61. $(3.0_{2,3}) = f(2.3_{2,4}, 3.1_{3,4}, 2.2_{1,2,4})$
62. $(3.0_{2,3}) = f(2.3_{2,4}, 3.1_{3,4}, 3.2_{2,4})$
63. $(3.0_{2,3}) = f(2.3_{2,4}, 3.2_{2,4})$
64. $(3.0_{2,3}) = f(2.3_{2,4}, 3.2_{2,4}, 3.1_{3,4})$
65. $(3.0_{2,3}) = f(3.1_{3,4}, 1.2._{1,4})$
66. $(3.0_{2,3}) = f(3.1_{3,4}, 1.2._{1,4}, 1.3_{3,4})$
67. $(3.0_{2,3}) = f(3.1_{3,4}, 1.3_{3,4})$
68. $(3.0_{2,3}) = f(3.1_{3,4}, 1.3_{3,4}, 1.2._{1,4})$
69. $(3.0_{2,3}) = f(3.1_{3,4}, 1.3_{3,4}, 2.2_{1,2,4})$
70. $(3.0_{2,3}) = f(3.1_{3,4}, 1.3_{3,4}, 3.2_{2,4})$
71. $(3.0_{2,3}) = f(3.1_{3,4}, 2.2_{1,2,4})$
72. $(3.0_{2,3}) = f(3.1_{3,4}, 2.2_{1,2,4}, 1.3_{3,4})$
73. $(3.0_{2,3}) = f(3.1_{3,4}, 2.2_{1,2,4}, 2.3_{2,4})$
74. $(3.0_{2,3}) = f(3.1_{3,4}, 2.3_{2,4})$
75. $(3.0_{2,3}) = f(3.1_{3,4}, 2.3_{2,4}, 2.2_{1,2,4})$
76. $(3.0_{2,3}) = f(3.1_{3,4}, 2.3_{2,4}, 3.2_{2,4})$
77. $(3.0_{2,3}) = f(3.1_{3,4}, 3.2_{2,4})$
78. $(3.0_{2,3}) = f(3.1_{3,4}, 3.2_{2,4}, 1.3_{3,4})$
79. $(3.0_{2,3}) = f(3.1_{3,4}, 3.2_{2,4}, 2.3_{2,4})$
80. $(3.0_{2,3}) = f(3.1_{3,4}, 3.2_{2,4}, 3.3_{2,3,4})$
81. $(3.0_{2,3}) = f(3.2_{2,4}, 1.3_{3,4})$

82. $(3.0_{2,3}) = f(3.2_{2,4}, 1.3_{3,4}, 3.1_{3,4})$
83. $(3.0_{2,3}) = f(3.2_{2,4}, 2.3_{2,4})$
84. $(3.0_{2,3}) = f(3.2_{2,4}, 2.3_{2,4}, 3.1_{3,4})$
85. $(3.0_{2,3}) = f(3.2_{2,4}, 3.1_{3,4}, 1.3_{3,4})$
86. $(3.0_{2,3}) = f(3.2_{2,4}, 3.1_{3,4})$
87. $(3.0_{2,3}) = f(3.2_{2,4}, 3.1_{3,4}, 2.3_{2,4})$
88. $(3.0_{2,3}) = f(3.2_{2,4}, 3.1_{3,4}, 3.3_{2,3,4})$
89. $(3.0_{2,3}) = f(3.2_{2,4}, 3.3_{2,3,4}, 3.1_{3,4})$
90. $(3.0_{2,3}) = f(3.3_{2,3,4}, 3.1_{3,4})$
91. $(3.0_{2,3}) = f(3.3_{2,3,4}, 3.1_{3,4}, 3.2_{2,4})$
92. $(3.0_{2,3}) = f(3.3_{2,3,4}, 3.2_{2,4}, 3.1_{3,4})$

2.13. **Functions with w = $(3.1_{3,4})$**

1. $(3.1_{3,4}) = f(0.1_{1,3}, 1.1._{1,3,4})$
2. $(3.1_{3,4}) = f(0.1_{1,3}, 1.1._{1,3,4}, 2.1_{1,4})$
3. $(3.1_{3,4}) = f(0.1_{1,3}, 2.1_{1,4})$
4. $(3.1_{3,4}) = f(0.1_{1,3}, 2.1_{1,4}, 1.1._{1,3,4})$
5. $(3.1_{3,4}) = f(0.2_{1,2}, 1.1._{1,3,4})$
6. $(3.1_{3,4}) = f(0.2_{1,2}, 1.1._{1,3,4}, 2.1_{1,4})$
7. $(3.1_{3,4}) = f(0.2_{1,2}, 1.2._{1,4})$
8. $(3.1_{3,4}) = f(0.2_{1,2}, 1.2._{1,4}, 2.1_{1,4})$
9. $(3.1_{3,4}) = f(0.2_{1,2}, 1.2._{1,4}, 2.2_{1,2,4})$
10. $(3.1_{3,4}) = f(0.2_{1,2}, 2.1_{1,4})$
11. $(3.1_{3,4}) = f(0.2_{1,2}, 2.1_{1,4}, 1.1._{1,3,4})$
12. $(3.1_{3,4}) = f(0.2_{1,2}, 2.1_{1,4}, 1.2._{1,4})$
13. $(3.1_{3,4}) = f(0.2_{1,2}, 2.2_{1,2,4})$
14. $(3.1_{3,4}) = f(0.2_{1,2}, 2.2_{1,2,4}, 1.2._{1,4})$
15. $(3.1_{3,4}) = f(0.3_{2,3}, 1.1._{1,3,4})$
16. $(3.1_{3,4}) = f(0.3_{2,3}, 1.1._{1,3,4}, 2.1_{1,4})$
17. $(3.1_{3,4}) = f(0.3_{2,3}, 1.2._{1,4})$
18. $(3.1_{3,4}) = f(0.3_{2,3}, 1.2._{1,4}, 2.1_{1,4})$
19. $(3.1_{3,4}) = f(0.3_{2,3}, 1.2._{1,4}, 2.2_{1,2,4})$
20. $(3.1_{3,4}) = f(0.3_{2,3}, 1.3_{3,4})$
21. $(3.1_{3,4}) = f(0.3_{2,3}, 1.3_{3,4}, 2.1_{1,4})$
22. $(3.1_{3,4}) = f(0.3_{2,3}, 1.3_{3,4}, 2.2_{1,2,4})$
23. $(3.1_{3,4}) = f(0.3_{2,3}, 1.3_{3,4}, 2.3_{2,4})$
24. $(3.1_{3,4}) = f(0.3_{2,3}, 2.1_{1,4})$
25. $(3.1_{3,4}) = f(0.3_{2,3}, 2.1_{1,4}, 1.1._{1,3,4})$
26. $(3.1_{3,4}) = f(0.3_{2,3}, 2.1_{1,4}, 1.2._{1,4})$
27. $(3.1_{3,4}) = f(0.3_{2,3}, 2.1_{1,4}, 1.3_{3,4})$
28. $(3.1_{3,4}) = f(0.3_{2,3}, 2.2_{1,2,4})$
29. $(3.1_{3,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 1.2._{1,4})$
30. $(3.1_{3,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 1.3_{3,4})$

31. $(3.1_{3,4}) = f(0.3_{2,3}, 2.3_{2,4})$
32. $(3.1_{3,4}) = f(0.3_{2,3}, 2.3_{2,4}, 1.3_{3,4})$
33. $(3.1_{3,4}) = f(1.1._{1,3,4}, 0.1_{1,3})$
34. $(3.1_{3,4}) = f(1.1._{1,3,4}, 0.1_{1,3}, 2.1_{1,4})$
35. $(3.1_{3,4}) = f(1.1._{1,3,4}, 0.2_{1,2})$
36. $(3.1_{3,4}) = f(1.1._{1,3,4}, 0.2_{1,2}, 2.1_{1,4})$
37. $(3.1_{3,4}) = f(1.1._{1,3,4}, 0.3_{2,3})$
38. $(3.1_{3,4}) = f(1.1._{1,3,4}, 0.3_{2,3}, 2.1_{1,4})$
39. $(3.1_{3,4}) = f(1.1._{1,3,4}, 2.1_{1,4})$
40. $(3.1_{3,4}) = f(1.1._{1,3,4}, 2.1_{1,4}, 0.1_{1,3})$
41. $(3.1_{3,4}) = f(1.1._{1,3,4}, 2.1_{1,4}, 0.2_{1,2})$
42. $(3.1_{3,4}) = f(1.1._{1,3,4}, 2.1_{1,4}, 0.3_{2,3})$
43. $(3.1_{3,4}) = f(1.2._{1,4}, 0.2_{1,2})$
44. $(3.1_{3,4}) = f(1.2._{1,4}, 0.2_{1,2}, 2.1_{1,4})$
45. $(3.1_{3,4}) = f(1.2._{1,4}, 0.2_{1,2}, 2.2_{1,2,4})$
46. $(3.1_{3,4}) = f(1.2._{1,4}, 0.3_{2,3})$
47. $(3.1_{3,4}) = f(1.2._{1,4}, 0.3_{2,3}, 2.1_{1,4})$
48. $(3.1_{3,4}) = f(1.2._{1,4}, 0.3_{2,3}, 2.2_{1,2,4})$
49. $(3.1_{3,4}) = f(1.2._{1,4}, 1.3_{3,4})$
50. $(3.1_{3,4}) = f(1.2._{1,4}, 1.3_{3,4}, 3.0_{2,3})$
51. $(3.1_{3,4}) = f(1.2._{1,4}, 2.1_{1,4})$
52. $(3.1_{3,4}) = f(1.2._{1,4}, 2.1_{1,4}, 0.2_{1,2})$
53. $(3.1_{3,4}) = f(1.2._{1,4}, 2.1_{1,4}, 0.3_{2,3})$
54. $(3.1_{3,4}) = f(1.2._{1,4}, 2.2_{1,2,4})$
55. $(3.1_{3,4}) = f(1.2._{1,4}, 2.2_{1,2,4}, 0.2_{1,2})$
56. $(3.1_{3,4}) = f(1.2._{1,4}, 2.2_{1,2,4}, 0.3_{2,3})$
57. $(3.1_{3,4}) = f(1.2._{1,4}, 3.0_{2,3})$
58. $(3.1_{3,4}) = f(1.2._{1,4}, 3.0_{2,3}, 1.3_{3,4})$
59. $(3.1_{3,4}) = f(1.3_{3,4}, 0.3_{2,3})$
60. $(3.1_{3,4}) = f(1.3_{3,4}, 0.3_{2,3}, 2.1_{1,4})$
61. $(3.1_{3,4}) = f(1.3_{3,4}, 0.3_{2,3}, 2.2_{1,2,4})$
62. $(3.1_{3,4}) = f(1.3_{3,4}, 0.3_{2,3}, 2.3_{2,4})$
63. $(3.1_{3,4}) = f(1.3_{3,4}, 1.2._{1,4})$
64. $(3.1_{3,4}) = f(1.3_{3,4}, 1.2._{1,4}, 3.0_{2,3})$
65. $(3.1_{3,4}) = f(1.3_{3,4}, 2.1_{1,4})$
66. $(3.1_{3,4}) = f(1.3_{3,4}, 2.1_{1,4}, 0.3_{2,3})$
67. $(3.1_{3,4}) = f(1.3_{3,4}, 2.2_{1,2,4})$
68. $(3.1_{3,4}) = f(1.3_{3,4}, 2.2_{1,2,4}, 0.3_{2,3})$
69. $(3.1_{3,4}) = f(1.3_{3,4}, 2.2_{1,2,4}, 3.0_{2,3})$
70. $(3.1_{3,4}) = f(1.3_{3,4}, 2.3_{2,4})$
71. $(3.1_{3,4}) = f(1.3_{3,4}, 2.3_{2,4}, 0.3_{2,3})$
72. $(3.1_{3,4}) = f(1.3_{3,4}, 3.0_{2,3})$
73. $(3.1_{3,4}) = f(1.3_{3,4}, 3.0_{2,3}, 1.2._{1,4})$
74. $(3.1_{3,4}) = f(1.3_{3,4}, 3.0_{2,3}, 2.2_{1,2,4})$

75. $(3.1_{3,4}) = f(1.3_{3,4}, 3.0_{2,3}, 3.2_{2,4})$
76. $(3.1_{3,4}) = f(1.3_{3,4}, 3.2_{2,4})$
77. $(3.1_{3,4}) = f(1.3_{3,4}, 3.2_{2,4}, 3.0_{2,3})$
78. $(3.1_{3,4}) = f(2.1_{1,4}, 0.1_{1,3})$
79. $(3.1_{3,4}) = f(2.1_{1,4}, 0.1_{1,3}, 1.1._{1,3,4})$
80. $(3.1_{3,4}) = f(2.1_{1,4}, 0.2_{1,2})$
81. $(3.1_{3,4}) = f(2.1_{1,4}, 0.2_{1,2}, 1.1._{1,3,4})$
82. $(3.1_{3,4}) = f(2.1_{1,4}, 0.2_{1,2}, 1.2._{1,4})$
83. $(3.1_{3,4}) = f(2.1_{1,4}, 0.3_{2,3})$
84. $(3.1_{3,4}) = f(2.1_{1,4}, 0.3_{2,3}, 1.1._{1,3,4})$
85. $(3.1_{3,4}) = f(2.1_{1,4}, 0.3_{2,3}, 1.2._{1,4})$
86. $(3.1_{3,4}) = f(2.1_{1,4}, 0.3_{2,3}, 1.3_{3,4})$
87. $(3.1_{3,4}) = f(2.1_{1,4}, 1.1._{1,3,4})$
88. $(3.1_{3,4}) = f(2.1_{1,4}, 1.1._{1,3,4}, 0.1_{1,3})$
89. $(3.1_{3,4}) = f(2.1_{1,4}, 1.1._{1,3,4}, 0.2_{1,2})$
90. $(3.1_{3,4}) = f(2.1_{1,4}, 1.1._{1,3,4}, 0.3_{2,3})$
91. $(3.1_{3,4}) = f(2.1_{1,4}, 1.2._{1,4})$
92. $(3.1_{3,4}) = f(2.1_{1,4}, 1.2._{1,4}, 0.2_{1,2})$
93. $(3.1_{3,4}) = f(2.1_{1,4}, 1.2._{1,4}, 0.3_{2,3})$
94. $(3.1_{3,4}) = f(2.1_{1,4}, 1.3_{3,4})$
95. $(3.1_{3,4}) = f(2.1_{1,4}, 1.3_{3,4}, 0.3_{2,3})$
96. $(3.1_{3,4}) = f(2.2_{1,2,4}, 0.2_{1,2})$
97. $(3.1_{3,4}) = f(2.2_{1,2,4}, 0.2_{1,2}, 1.2._{1,4})$
98. $(3.1_{3,4}) = f(2.2_{1,2,4}, 0.3_{2,3})$
99. $(3.1_{3,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 1.2._{1,4})$
100. $(3.1_{3,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 1.3_{3,4})$
101. $(3.1_{3,4}) = f(2.2_{1,2,4}, 1.2._{1,4})$
102. $(3.1_{3,4}) = f(2.2_{1,2,4}, 1.2._{1,4}, 0.2_{1,2})$
103. $(3.1_{3,4}) = f(2.2_{1,2,4}, 1.2._{1,4}, 0.3_{2,3})$
104. $(3.1_{3,4}) = f(2.2_{1,2,4}, 1.3_{3,4})$
105. $(3.1_{3,4}) = f(2.2_{1,2,4}, 1.3_{3,4}, 0.3_{2,3})$
106. $(3.1_{3,4}) = f(2.2_{1,2,4}, 1.3_{3,4}, 3.0_{2,3})$
107. $(3.1_{3,4}) = f(2.2_{1,2,4}, 2.3_{2,4})$
108. $(3.1_{3,4}) = f(2.2_{1,2,4}, 2.3_{2,4}, 3.0_{2,3})$
109. $(3.1_{3,4}) = f(2.2_{1,2,4}, 3.0_{2,3})$
110. $(3.1_{3,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 1.3_{3,4})$
111. $(3.1_{3,4}) = f(2.2_{1,2,4}, 3.0_{2,3}, 2.3_{2,4})$
112. $(3.1_{3,4}) = f(2.3_{2,4}, 0.3_{2,3})$
113. $(3.1_{3,4}) = f(2.3_{2,4}, 0.3_{2,3}, 1.3_{3,4})$
114. $(3.1_{3,4}) = f(2.3_{2,4}, 1.3_{3,4})$
115. $(3.1_{3,4}) = f(2.3_{2,4}, 1.3_{3,4}, 0.3_{2,3})$
116. $(3.1_{3,4}) = f(2.3_{2,4}, 2.2_{1,2,4})$
117. $(3.1_{3,4}) = f(2.3_{2,4}, 2.2_{1,2,4}, 3.0_{2,3})$

118. $(3.1_{3,4}) = f(2.3_{2,4}, 3.0_{2,3})$
119. $(3.1_{3,4}) = f(2.3_{2,4}, 3.0_{2,3}, 2.2_{1,2,4})$
120. $(3.1_{3,4}) = f(2.3_{2,4}, 3.0_{2,3}, 3.2_{2,4})$
121. $(3.1_{3,4}) = f(2.3_{2,4}, 3.2_{2,4})$
122. $(3.1_{3,4}) = f(2.3_{2,4}, 3.2_{2,4}, 3.0_{2,3})$
123. $(3.1_{3,4}) = f(3.0_{2,3}, 1.2._{1,4})$
124. $(3.1_{3,4}) = f(3.0_{2,3}, 1.2._{1,4}, 1.3_{3,4})$
125. $(3.1_{3,4}) = f(3.0_{2,3}, 1.3_{3,4})$
126. $(3.1_{3,4}) = f(3.0_{2,3}, 1.3_{3,4}, 1.2._{1,4})$
127. $(3.1_{3,4}) = f(3.0_{2,3}, 1.3_{3,4}, 2.2_{1,2,4})$
128. $(3.1_{3,4}) = f(3.0_{2,3}, 1.3_{3,4}, 3.2_{2,4})$
129. $(3.1_{3,4}) = f(3.0_{2,3}, 2.2_{1,2,4})$
130. $(3.1_{3,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 1.3_{3,4})$
131. $(3.1_{3,4}) = f(3.0_{2,3}, 2.2_{1,2,4}, 2.3_{2,4})$
132. $(3.1_{3,4}) = f(3.0_{2,3}, 2.3_{2,4})$
133. $(3.1_{3,4}) = f(3.0_{2,3}, 2.3_{2,4}, 2.2_{1,2,4})$
134. $(3.1_{3,4}) = f(3.0_{2,3}, 2.3_{2,4}, 3.2_{2,4})$
135. $(3.1_{3,4}) = f(3.0_{2,3}, 3.2_{2,4})$
136. $(3.1_{3,4}) = f(3.0_{2,3}, 3.2_{2,4}, 1.3_{3,4})$
137. $(3.1_{3,4}) = f(3.0_{2,3}, 3.2_{2,4}, 2.3_{2,4})$
138. $(3.1_{3,4}) = f(3.0_{2,3}, 3.2_{2,4}, 3.3_{2,3,4})$
139. $(3.1_{3,4}) = f(3.0_{2,3}, 3.3_{2,3,4})$
140. $(3.1_{3,4}) = f(3.0_{2,3}, 3.3_{2,3,4}, 3.2_{2,4})$
141. $(3.1_{3,4}) = f(3.2_{2,4}, 1.3_{3,4})$
142. $(3.1_{3,4}) = f(3.2_{2,4}, 1.3_{3,4}, 3.0_{2,3})$
143. $(3.1_{3,4}) = f(3.2_{2,4}, 2.3_{2,4})$
144. $(3.1_{3,4}) = f(3.2_{2,4}, 2.3_{2,4}, 3.0_{2,3})$
145. $(3.1_{3,4}) = f(3.2_{2,4}, 3.0_{2,3})$
146. $(3.1_{3,4}) = f(3.2_{2,4}, 3.0_{2,3}, 1.3_{3,4})$
147. $(3.1_{3,4}) = f(3.2_{2,4}, 3.0_{2,3}, 2.3_{2,4})$
148. $(3.1_{3,4}) = f(3.2_{2,4}, 3.0_{2,3}, 3.3_{2,3,4})$
149. $(3.1_{3,4}) = f(3.2_{2,4}, 3.3_{2,3,4})$
150. $(3.1_{3,4}) = f(3.2_{2,4}, 3.3_{2,3,4}, 3.0_{2,3})$
151. $(3.1_{3,4}) = f(3.3_{2,3,4}, 3.0_{2,3})$
152. $(3.1_{3,4}) = f(3.3_{2,3,4}, 3.0_{2,3}, 3.2_{2,4})$
153. $(3.1_{3,4}) = f(3.3_{2,3,4}, 3.2_{2,4})$
154. $(3.1_{3,4}) = f(3.3_{2,3,4}, 3.2_{2,4}, 3.0_{2,3})$

2.14. **Functions with w = $(3.2_{2,4})$**

1. $(3.2_{2,4}) = f(0.2_{1,2}, 1.2._{1,4})$
2. $(3.2_{2,4}) = f(0.2_{1,2}, 1.2._{1,4}, 2.2_{1,2,4})$
3. $(3.2_{2,4}) = f(0.2_{1,2}, 2.2_{1,2,4})$
4. $(3.2_{2,4}) = f(0.2_{1,2}, 2.2_{1,2,4}, 1.2._{1,4})$

5. $(3.2_{2,4}) = f(0.3_{2,3}, 1.2._{1,4})$
6. $(3.2_{2,4}) = f(0.3_{2,3}, 1.2._{1,4}, 2.2_{1,2,4})$
7. $(3.2_{2,4}) = f(0.3_{2,3}, 1.3_{3,4})$
8. $(3.2_{2,4}) = f(0.3_{2,3}, 1.3_{3,4}, 2.2_{1,2,4})$
9. $(3.2_{2,4}) = f(0.3_{2,3}, 1.3_{3,4}, 2.3_{2,4})$
10. $(3.2_{2,4}) = f(0.3_{2,3}, 2.2_{1,2,4})$
11. $(3.2_{2,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 1.2._{1,4})$
12. $(3.2_{2,4}) = f(0.3_{2,3}, 2.2_{1,2,4}, 1.3_{3,4})$
13. $(3.2_{2,4}) = f(0.3_{2,3}, 2.3_{2,4})$
14. $(3.2_{2,4}) = f(0.3_{2,3}, 2.3_{2,4}, 1.3_{3,4})$
15. $(3.2_{2,4}) = f(1.2._{1,4}, 0.2_{1,2})$
16. $(3.2_{2,4}) = f(1.2._{1,4}, 0.2_{1,2}, 2.2_{1,2,4})$
17. $(3.2_{2,4}) = f(1.2._{1,4}, 0.3_{2,3})$
18. $(3.2_{2,4}) = f(1.2._{1,4}, 0.3_{2,3}, 2.2_{1,2,4})$
19. $(3.2_{2,4}) = f(1.2._{1,4}, 2.2_{1,2,4})$
20. $(3.2_{2,4}) = f(1.2._{1,4}, 2.2_{1,2,4}, 0.2_{1,2})$
21. $(3.2_{2,4}) = f(1.2._{1,4}, 2.2_{1,2,4}, 0.3_{2,3})$
22. $(3.2_{2,4}) = f(1.3_{3,4}, 0.3_{2,3})$
23. $(3.2_{2,4}) = f(1.3_{3,4}, 0.3_{2,3}, 2.2_{1,2,4})$
24. $(3.2_{2,4}) = f(1.3_{3,4}, 0.3_{2,3}, 2.3_{2,4})$
25. $(3.2_{2,4}) = f(1.3_{3,4}, 2.2_{1,2,4})$
26. $(3.2_{2,4}) = f(1.3_{3,4}, 2.2_{1,2,4}, 0.3_{2,3})$
27. $(3.2_{2,4}) = f(1.3_{3,4}, 2.3_{2,4})$
28. $(3.2_{2,4}) = f(1.3_{3,4}, 2.3_{2,4}, 0.3_{2,3})$
29. $(3.2_{2,4}) = f(1.3_{3,4}, 3.0_{2,3})$
30. $(3.2_{2,4}) = f(1.3_{3,4}, 3.0_{2,3}, 3.1_{3,4})$
31. $(3.2_{2,4}) = f(1.3_{3,4}, 3.1_{3,4})$
32. $(3.2_{2,4}) = f(1.3_{3,4}, 3.1_{3,4}, 3.0_{2,3})$
33. $(3.2_{2,4}) = f(2.2_{1,2,4}, 0.2_{1,2})$
34. $(3.2_{2,4}) = f(2.2_{1,2,4}, 0.2_{1,2}, 1.2._{1,4})$
35. $(3.2_{2,4}) = f(2.2_{1,2,4}, 0.3_{2,3})$
36. $(3.2_{2,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 1.2._{1,4})$
37. $(3.2_{2,4}) = f(2.2_{1,2,4}, 0.3_{2,3}, 1.3_{3,4})$
38. $(3.2_{2,4}) = f(2.2_{1,2,4}, 1.2._{1,4})$
39. $(3.2_{2,4}) = f(2.2_{1,2,4}, 1.2._{1,4}, 0.2_{1,2})$
40. $(3.2_{2,4}) = f(2.2_{1,2,4}, 1.2._{1,4}, 0.3_{2,3})$
41. $(3.2_{2,4}) = f(2.2_{1,2,4}, 1.3_{3,4})$
42. $(3.2_{2,4}) = f(2.2_{1,2,4}, 1.3_{3,4}, 0.3_{2,3})$
43. $(3.2_{2,4}) = f(2.3_{2,4}, 0.3_{2,3})$
44. $(3.2_{2,4}) = f(2.3_{2,4}, 0.3_{2,3}, 1.3_{3,4})$
45. $(3.2_{2,4}) = f(2.3_{2,4}, 1.3_{3,4})$
46. $(3.2_{2,4}) = f(2.3_{2,4}, 1.3_{3,4}, 0.3_{2,3})$
47. $(3.2_{2,4}) = f(2.3_{2,4}, 3.0_{2,3})$
48. $(3.2_{2,4}) = f(2.3_{2,4}, 3.0_{2,3}, 3.1_{3,4})$

49. $(3.2_{2,4}) = f(2.3_{2,4}, 3.1_{3,4})$
50. $(3.2_{2,4}) = f(2.3_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
51. $(3.2_{2,4}) = f(3.0_{2,3}, 1.3_{3,4})$
52. $(3.2_{2,4}) = f(3.0_{2,3}, 1.3_{3,4}, 3.1_{3,4})$
53. $(3.2_{2,4}) = f(3.0_{2,3}, 2.3_{2,4})$
54. $(3.2_{2,4}) = f(3.0_{2,3}, 2.3_{2,4}, 3.1_{3,4})$
55. $(3.2_{2,4}) = f(3.0_{2,3}, 3.1_{3,4})$
56. $(3.2_{2,4}) = f(3.0_{2,3}, 3.1_{3,4}, 1.3_{3,4})$
57. $(3.2_{2,4}) = f(3.0_{2,3}, 3.1_{3,4}, 2.3_{2,4})$
58. $(3.2_{2,4}) = f(3.0_{2,3}, 3.1_{3,4}, 3.3_{2,3,4})$
59. $(3.2_{2,4}) = f(3.0_{2,3}, 3.3_{2,3,4})$
60. $(3.2_{2,4}) = f(3.0_{2,3}, 3.3_{2,3,4}, 3.1_{3,4})$
61. $(3.2_{2,4}) = f(3.1_{3,4}, 1.3_{3,4})$
62. $(3.2_{2,4}) = f(3.1_{3,4}, 1.3_{3,4}, 3.0_{2,3})$
63. $(3.2_{2,4}) = f(3.1_{3,4}, 2.3_{2,4})$
64. $(3.2_{2,4}) = f(3.1_{3,4}, 2.3_{2,4}, 3.0_{2,3})$
65. $(3.2_{2,4}) = f(3.1_{3,4}, 3.0_{2,3})$
66. $(3.2_{2,4}) = f(3.1_{3,4}, 3.0_{2,3}, 1.3_{3,4})$
67. $(3.2_{2,4}) = f(3.1_{3,4}, 3.0_{2,3}, 2.3_{2,4})$
68. $(3.2_{2,4}) = f(3.1_{3,4}, 3.0_{2,3}, 3.3_{2,3,4})$
69. $(3.2_{2,4}) = f(3.1_{3,4}, 3.3_{2,3,4})$
70. $(3.2_{2,4}) = f(3.1_{3,4}, 3.3_{2,3,4}, 3.0_{2,3})$
71. $(3.2_{2,4}) = f(3.3_{2,3,4}, 3.0_{2,3})$
72. $(3.2_{2,4}) = f(3.3_{2,3,4}, 3.0_{2,3}, 3.1_{3,4})$
73. $(3.2_{2,4}) = f(3.3_{2,3,4}, 3.1_{3,4})$
74. $(3.2_{2,4}) = f(3.3_{2,3,4}, 3.1_{3,4}, 3.0_{2,3})$

2.15. **Functions with w = $(3.3_{2,3,4})$**

1. $(3.3_{2,3,4}) = f(0.3_{2,3}, 1.3_{3,4})$
2. $(3.3_{2,3,4}) = f(0.3_{2,3}, 1.3_{3,4}, 2.3_{2,4})$
3. $(3.3_{2,3,4}) = f(0.3_{2,3}, 2.3_{2,4})$
4. $(3.3_{2,3,4}) = f(0.3_{2,3}, 2.3_{2,4}, 1.3_{3,4})$
5. $(3.3_{2,3,4}) = f(1.3_{3,4}, 0.3_{2,3})$
6. $(3.3_{2,3,4}) = f(1.3_{3,4}, 0.3_{2,3}, 2.3_{2,4})$
7. $(3.3_{2,3,4}) = f(1.3_{3,4}, 2.3_{2,4})$
8. $(3.3_{2,3,4}) = f(1.3_{3,4}, 2.3_{2,4}, 0.3_{2,3})$
9. $(3.3_{2,3,4}) = f(2.3_{2,4}, 0.3_{2,3})$
10. $(3.3_{2,3,4}) = f(2.3_{2,4}, 0.3_{2,3}, 1.3_{3,4})$
11. $(3.3_{2,3,4}) = f(2.3_{2,4}, 1.3_{3,4})$
12. $(3.3_{2,3,4}) = f(2.3_{2,4}, 1.3_{3,4}, 0.3_{2,3})$
13. $(3.3_{2,3,4}) = f(3.0_{2,3}, 3.1_{3,4})$
14. $(3.3_{2,3,4}) = f(3.0_{2,3}, 3.1_{3,4}, 3.2_{2,4})$
15. $(3.3_{2,3,4}) = f(3.0_{2,3}, 3.2_{2,4})$

16. $(3.3_{2,3,4}) = f(3.0_{2,3}, 3.2_{2,4}, 3.1_{3,4})$
17. $(3.3_{2,3,4}) = f(3.1_{3,4}, 3.0_{2,3})$
18. $(3.3_{2,3,4}) = f(3.1_{3,4}, 3.0_{2,3}, 3.2_{2,4})$
19. $(3.3_{2,3,4}) = f(3.1_{3,4}, 3.2_{2,4})$
20. $(3.3_{2,3,4}) = f(3.1_{3,4}, 3.2_{2,4}, 3.0_{2,3})$
21. $(3.3_{2,3,4}) = f(3.2_{2,4}, 3.0_{2,3})$
22. $(3.3_{2,3,4}) = f(3.2_{2,4}, 3.0_{2,3}, 3.1_{3,4})$
23. $(3.3_{2,3,4}) = f(3.2_{2,4}, 3.1_{3,4})$
24. $(3.3_{2,3,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

3. The list of 1'162 polycontextural functions presented here is exhaustive for a pre-semiotic 4-contextural 4-adic sign model. However, functions, which contain values that lie in more than 1 semiotic contexture, can be "split up" into several more functions, according to combinatorics. E.g., the function

$(3.3_{2,3,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$

can be split up in

$(3.3_{2}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{3}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{2,3}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{2,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{3,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{2,3,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{2,4,3}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{3,2,4}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{3,4,2}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{4,2,3}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$
$(3.3_{4,3,2}) = f(3.2_{2,4}, 3.1_{3,4}, 3.0_{2,3})$.

For the sake of avoiding longer lists which can be produced easily, such combinatorial cases have been left away.

Another strong source for enormous increase of further semiotic functions is the permutation of the contextural indices, i.e. the "morphismic" order, its complementary "hetero-morphismic" order and the "mediative" morphimsm between them (cf. Toth 2009a).

Bibliography

Toth, Alfred, Semiotische Funktionentheorie. Klagenfurt 2008 (2008a)
Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 2008 (2008b)

Toth, Alfred, Inner and outer semiotic environments in the system of the trichotomic triads. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, How many contexture-borders has a sign? In: Electronic Journal for Mathematical Semiotics, 2009b

Illustration of contextures by Venn diagrams

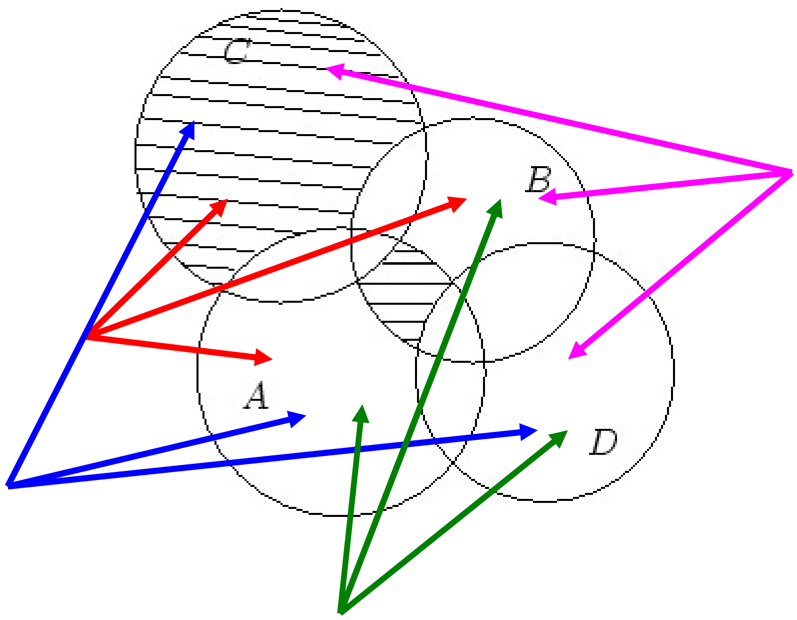
1. In the book (Toth 2009), I have constructed the following 4-adic 3-/4-otomic pre-semiotic matrix which holds in 4 contextures:

	0	1	2	3
0	$(0.0)_{3,2,1}$	$(0.1)_{1,3}$	$(0.2)_{1,2}$	$(0.3)_{2,3}$
1	$(1.0)_{3,1}$	$(1.1)_{1,3,4}$	$(1.2)_{1,4}$	$(1.3)_{3,4}$
2	$(2.0)_{2,1}$	$(2.1)_{1,4}$	$(2.2)_{1,2,4}$	$(2.3)_{2,4}$
3	$(3.0)_{3,2}$	$(3.1)_{3,4}$	$(3.2)_{2,4}$	$(3.3)_{2,3,4}$

If we have a look at the trichotomies/tetratomies, we recognize that the following contextures appear or do not appear, resp.:

0:	1, 2, 3	4
1:	1, 3, 4	2
2:	1, 2, 4	3
3:	2, 3, 4	1

The sense of this little note is fulfilled in showing that the following Venn diagram:



illustrates the mappings from the contextures to sets, when the red arrows stand of the 0-tricho-/tetratomies, the blue arrows for the 1-tricho-/tetratomies, the green arrows for the 2-tricho-/tetratomies, and the lilac arrows for the 3-tricho-/tetratomies.

Bibliography

Toth, Alfred, Elements of Theory of the Night. In: Electronic Journal for Mathematical Semiotics, 2009

Quantitative, qualitative, quanti-qualitative, and quali-quantitative sign classes

1. The 10 monocontextural Peircean sign classes are, as classes, i.e. sets, of signs quantitative sign relations, although their three dyadic sub-signs and their semioses involved are qualitatively defined. There are two possibilities to note the sign classes formally, first, as unordered sets dyads (left), second as unordered sets of the trichotomic values of the dyads:

(3.1 2.1 1.1) ≡ (1, 1, 1)
(3.1 2.1 1.2) ≡ (1, 1, 2)
(3.1 2.1 1.3) ≡ (1, 1, 3)
(3.1 2.2 1.2) ≡ (1, 2, 2)
(3.1 2.2 1.3) ≡ (1, 2, 3)
(3.1 2.3 1.3) ≡ (1, 3, 3)
(3.2 2.2 1.2) ≡ (2, 2, 2)
(3.2 2.2 1.3) ≡ (2, 2, 3)
(3.2 2.3 1.3) ≡ (2, 3, 3)
(3.3 2.3 1.3) ≡ (3, 3, 3)

An one sees, the correspondences are unambiguous.

2. As I have shown in Toth (2009), there is more than one way of mapping contextural indices to the dyadic sub-signs of a sign class. Or, in other words: As long as certain logical laws are considered, the contextures in which dyads can be placed, are almost arbitrary (Korzybski-ambiguity). Important is alone that we define in which contexture(s) we set a semiotic matrix, so that all 9 dyads get unambiguously mapped to contextures. E.g., if we start with Kaehr's (2008) proposal, we get the following 3-contextural 3-adic 3×3 semiotic matrix:

	1	2	3
1	$(1.1)_{1,3}$	$(1.2)_{1}$	$(1.3)_{3}$
2	$(2.1)_{1}$	$(2.2)_{1,2}$	$(2.3)_{2}$
3	$(3.1)_{3}$	$(3.2)_{2}$	$(3.3)_{2,3}$

Now, the mapping of contextures to sub-signs is not bijective, since we have

$(1.2)_{1} = (2.1)_{1}$
$(1.3)_{3} = (3.1)_{3}$

$(2.3)_2 = (3.2)_2$

However, nevertheless, if we set the dyads together to sign classes for which we must obey the law

SCl = (3.a 2b 1.c),

because of the fix position of the dyads, they get quasi afterwards unambiguous. From that is follows now, that, considering (SCl = (3.a 2.b 1.c)), we can substitute the sign classes by their ordered (!) sets of contextures. Needless to say, that n-tuples of contextures are also ordered (partial) sets. We thus obtain:

(3.1 2.1 1.1)	≡	(1, 1, 1)	≡	$(3.1_3\ 2.1_1\ 1.1_{1,3})$	≡	<3, 1, <1, 3>>
(3.1 2.1 1.2)	≡	(1, 1, 2)	≡	$(3.1_3\ 2.1_1\ 1.2_1)$	≡	<3, 1, 1>
(3.1 2.1 1.3)	≡	(1, 1, 3)	≡	$(3.1_3\ 2.1_1\ 1.3_3)$	≡	<3, 1, 3>
(3.1 2.2 1.2)	≡	(1, 2, 2)	≡	$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	≡	<3, <1, 2>, 1>
(3.1 2.2 1.3)	≡	(1, 2, 3)	≡	$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	≡	<3, <1, 2>, 3>
(3.1 2.3 1.3)	≡	(1, 3, 3)	≡	$3.1_3\ 2.3_2\ 1.3_3)$	≡	<3, 2, 3>
(3.2 2.2 1.2)	≡	(2, 2, 2)	≡	$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	≡	<2, <1, 2>, 1>
(3.2 2.2 1.3)	≡	(2, 2, 3)	≡	$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	≡	<2, <1, 2>, 3>
(3.2 2.3 1.3)	≡	(2, 3, 3)	≡	$(3.2_2\ 2.3_2\ 1.3_3)$	≡	<2, 2, 3>
(3.3 2.3 1.3)	≡	(3, 3, 3)	≡	$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	≡	<<2, 3>, 2, 3>

3. So, finally, we can summarize that we have

3.1. Two ways of noting quantitative sign classes: As (3.a 2.b 1.c) and as (a, b, c).

3.2. One way of noting qualitative sing classes: As $<<i, j>, <k, l>, <m, n>>$, where $j \vee l \vee n \in \{\varnothing\}$.

3.3. One way of noting quanti-qualitative sign classes: As $(3.a_{i,j}\ 2.b_{k,l}\ 1.c_{m,n})$.

3.4. One way of noting quali-quantitative sing classes: $<<i, j>_a, <k, l>_b, <m, n>_c>$, where a, b, c ∈ {.1, .2, .3}.

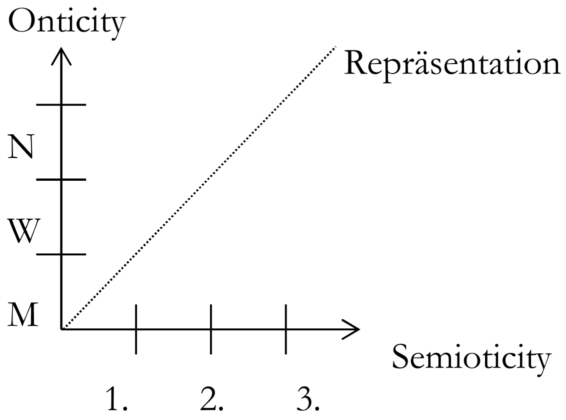
Bibliography

Kaehr, Rudolf, Diamond semiotics. Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, Polycontextural matrices. In: Electronic Journal for Mathematical Semiotics, 2009

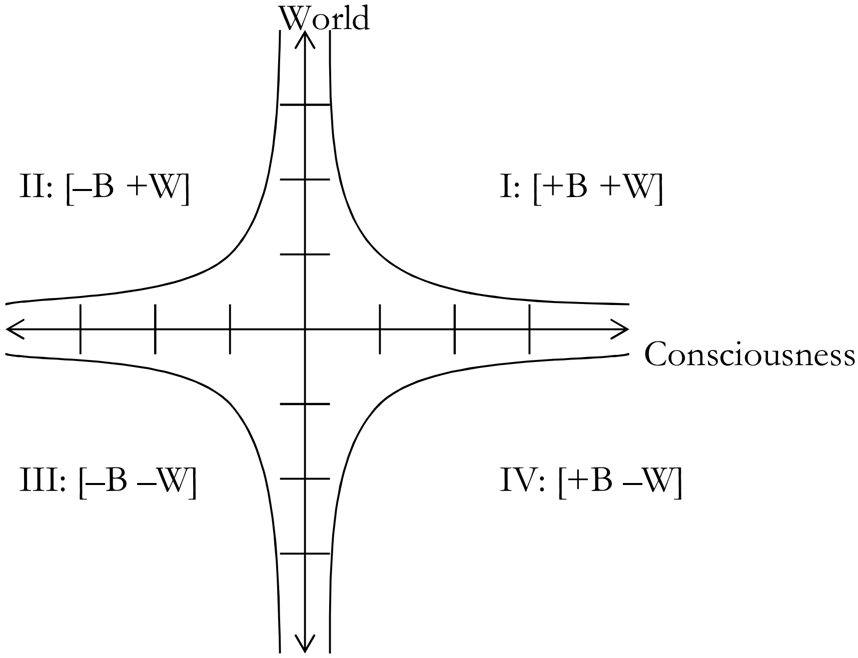
Sign relations as sets of pairs of subject and object

1. The „Theorem on Onticity and Semioticity“ (Bense 1976, p. 60) says: „With increasing semioticity, onticity of representation increases, too“:



2. However, Bense’s theorem on Onticity and Semioticity can be formulated in an even more abstract way: As I have shown (Toth 2008), the sign as a function of representaton is asymptotic both to its object and to its consciousness. We find close ideas already in Günther’s work: „Und so, wie das orthothematische Begreifen der äußeren Reflexion das absolut objektive Ansich der Dinge nie erreicht und ihm die Gegenstände ewig transzendent bleiben, so erreicht die introszendent orientierte Reflexion-in-sich niemals den inneren Grund ihrer eigenen Reflexionstätigkeit“ (1991, p. 269). „Eine Transzendenz besitzen aber heisst, einen unerreichbaren Grund haben“ (1963, p. 37). „Es ist eine ganz empirische Erfahrung, daß alle Subjektivität ‚bodenlos‘ ist. Das heisst, es liegt hinter jedem erreichten Bewußtseinszustand immer noch ein tieferer, nicht erreichter“" (1963, p. 198).

In order to display the sign function mathematically with double asymptosis, one choses easiest the hyperbolic function y = 1/x. This hyperbola is known to have two branches, which in a Cartesian Coordinate System lie in the I. and in the III. quadrant. If we also draw the negative function y = –1/x, whose branches lie in the II. and IV. quadrant, then we obtain a semiotic Cartesian Coordinate System with hyperbola-branches in all 4 quadrants:



Thus, while in Bense's Theorem the abscissa is designed for semioticity and the ordinate for onticity, in the more abstract wording of this theorem, which one could call „Theorem on World and Consciousness", the abscissa is designed for consciousness and the ordinate for onticity. Hence the question which of the two onotological notions, Being or consciousnes, has a metaphysical primordiality, is senseless: the sign as a representation function mediates between both notions.

3. Therefore, we can now write a sign class together with its reality thematic in the following form

$ZR_{sem} = [[S, O]_I, [S, O]_O, [S, O]_M] \times [[O, S]_{M°}, [O, S]_{O°}, [O, S]_{I°}]$

We put the index "sem" to refer that this is just the normal form of a sign class in quadrant I. The normal forms of the three other quadrants look as follows:

$ZR_{mat} = [[-S, O]_I, [-S, O]_O, [-S, O]_M] \times [[O, -S]_{M°}, [O, -S]_{O°} [O, -S]_{I°}]$
$ZR_{ide} = [[S, -O]_I, [S, -O]_O, [S, -O]_M] \times [[-O, S]_{M°}, [-O, S]_{O°}, [-O, S]_{I°}]$
$ZR_{meo} = [[-S, -O]_I, [-S, -O]_O, [-S, -O]_M] \times [[-O, -S]_{M°}, [-O, -S]_{O°}, [-O, -S]_{I°}]$

Now, in all four normal forms of sign classes (and reality thematics) we have

$[\pm S, \pm O]_I \in \{3, 2, <2, 3>\}$
$[\pm S, \pm O]_O \in \{1, <1, 2>, 2\}$
$[\pm S, \pm O]_M \in \{<1, 3>, 1, 3\}$

and consequently

$[\pm O, \pm S]_{I°} \in \{3, 2, <3,2>\}$
$[\pm O, \pm S]_{O°} \in \{1, <2,1>, 2\}$
$[\pm O, \pm S]_{M°} \in \{<3,1>, 1, 3\}$

Bibliography

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976
Günther, Gotthard, Das Bewusstsein der Maschinen. Krefeld 1963
Günther, Gotthard, Idee und Grundriss einer nicht-aristotelischen Logik. 3rd ed. Hamburg 1991
Toth, Alfred, The sign as a "disjunction between world and consciousness". In: Toth, Alfred, Semiotics and Pre-Semiotics. Vol.2. Klagenfurt 2008, pp. 127-144

A 3-contextural semiotic 3 × 6 matrix

1. Let us compare the following 4 combinations of sub-signs and contextures:

$(a.b)_{1,2}$, $(a.b)_{2,1}$; $(b.a)_{1,2}$, $(b.a)_{2,1}$

An unwritten rule of polycontextural semiotic matrices is that one and the same matrix must not contain both morphisms and hetero-morphisms, but only morphisms and inverse morphisms:

	1	2	3
1	$(1.1)_{1,3}$	$(1.2)_{1}$	$(1.3)_{3}$
2	$(2.1)_{1}$	$(2.2)_{1,2}$	$(2.3)_{2}$
3	$(3.1)_{3}$	$(3.2)_{2}$	$(3.3)_{2,3}$

Since each Peircean sign class possesses its dual reality thematic and since its dual dyads consist of hetero-morphisms, it follows that in order to display a full elementary semiotic system, consisting of sign- and reality thematics, one semiotic matrix is not enough like in monocontextural systems. In other words, we either use for polycontextural sign relations two or more matrices (in order also to represent the "mediative" morphisms between morphism and hetero-morphisms), or we change from 3×6 to a 3×9 (... 4×16, ...) matrix:

	1	2	3
1	$(1.1)_{1,3}$ $(1.1)_{3,1}$	$(1.2)_{1}$ $(2.1)_{1}$	$(1.3)_{3}$ $(3.1)_{3}$
2	$(2.1)_{1}$ $(1.2)_{1}$	$(2.2)_{2,1}$ $(2.2)_{1,2}$	$(2.3)_{2}$ $(3.2)_{2}$
3	$(3.1)_{3}(1.3)_{3}$	$(3.2)_{2}$ $(2.3)_{2}$	$(3.3)_{2,3}$ $(3.3)_{3,2}$

2. When we now construct the 10 Peircean sign classes, we get 10 elementary semiotic systems of hexadic-trichotomic sign classes:

(3.1 1.3 2.1 1.2 1.1 1.1) × (1.1.1.1 2.1 1.2 3.1 1.3)
(3.1 1.3 2.1 1.2 1.2 2.1) × (2.1 1.2 2.1 1.2 3.1 1.3)
(3.1 1.3 2.1 1.2 1.3 3.1) × (1.3 3.1 2.1 1.2 3.1 1.3)
(3.1 1.3 2.2 2.2 1.2 2.1) × (1.2 2.1 2.2 2.2 3.1 1.3)
(3.1 1.3 2.2 2.2 1.3 3.1) × (1.3 3.1 2.2 2.2 3.1 1.3)

(3.1 1.3 2.3 3.2 1.3 3.1) × (1.3 3.1 3.2 2.3 3.1 1.3)
(3.2 2.3 2.2 2.2 1.2 2.1) × (1.2 2.1 2.2 2.2 3.2 2.3)
(3.2 2.3 2.2 2.2 1.3 3.1) × (1.3 3.1 2.2 2.2 2.3 2.3)
(3.2 2.3 2.3 3.2 1.3 3.1) × (1.3 3.1 2.3 3.2 3.2 2.3)
(3.3 3.3 2.3 3.2 1.3 3.1) × (1.3 3.1 3.2 3.2 3.3 3.3)

All 6-adic 3-otomic sign classes are of "weaker eigenreality", like the Genuine Category Class (cf. Bense 1992, p. 40). And every pair of dyads contains the corresponding object-relation to its subject-relation and the corresponding subject-relation to its object-relation. Thus, these 10 6-adic 3-otomic sign classes are complete hybrids as far as the epistemological relations of the whole sign classes and their reality thematics as well as their constituting sub-signs concerns. In addition, they are full symmetric in their contextures.

Bibliography

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Eigenreality in mono- and polycontextural semiotic systems

1. One of the important discoveries in theoretical semiotics was that Being can only be represented (Bense 1981), but as a represented Being, it shows up in two shapes: the sign thematic and the reality thematic. According to Bense, the sign model serves here as a "doubled epistemological relation", insofar as the sign thematic indicates the subject, and the reality thematic indicates the object relation (Bense 1976, 1992). Thus, the triadic structure of the semiotic main values and the trichotomic structure of the semiotic secondary values are repeated in the epistemological schema (sign thematic), (dualization), (reality thematic). If we write down the 10 Peircean sign classes together with their dual reality thematics

(3.1 2.1 1.1) × (1.1 1.2 1.3)
(3.1 2.1 1.2) × (2.1 1.2 1.3)
(3.1 2.1 1.3) × (3.1 1.2 1.3)
(3.1 2.2 1.2) × (2.1 2.2 1.3)
(3.1 2.2 1.3) × (3.1 2.2 1.3)
(3.1 2.3 1.3) × (3.1 3.2 1.3)
(3.2 2.2 1.2) × (2.1 2.2 2.3)
(3.2 2.2 1.3) × (3.1 2.2 2.3)
(3.2 2.3 1.3) × (3.1 3.2 2.3)
(3.3 2.3 1.3) × (3.1 3.2 3.3),

we realize that only 1 sign thematic is identical with its dual reality thematic. According to Bense (1992), this is the sign class of the "sign itself", i.e. it does not refer to a reality outside of itself, but solely to its own inner reality, the semiotic reality, it is so-to-say composed of itself.

3. However, Rudolf Kaehr (2009) has shown, that eigenreality is a phenomenon that can only appear in monocontextural systems. Monocontextural systems are systems in which the logical Law of Identity (a ≡ a) is fully valid. In semiotics, this means, f. ex., that every genuine sub-sign is self-identical, i.e. that semiotic identity of identitive morphisms cannot be split into 2 or more sub-signs:

(3.1 2.×.2 1.3) × (3.1 2.×.2 1.3),

thus the Law of Identity creates, in semiotics, in-between-symmetry. Now, in polycontextural semiotics, the Law of Identity is abolished, hence, we find genuine sub-signs in more than 1 contexture already in the most simple polycontextural matrix for 3-adic 3-otomic sub-signs:

	1	2	3
1	$(1.1)_{1,3}$	$(1.2)_{1}$	$(1.3)_{3}$
2	$(2.1)_{1}$	$(2.2)_{1,2}$	$(2.3)_{2}$
3	$(3.1)_{3}$	$(3.2)_{2}$	$(3.3)_{2,3}$

i.e. we have here exchange relations between

(1.1): $1 \leftrightarrow 3$
(2.2): $1 \leftrightarrow 2$
(2.3): $2 \leftrightarrow 3$,

but the self-identity of the non-genuine sub-signs is abolished, too, since we also have

(1.2): $(1.2) \leftrightarrow (1.2)°$
(1.3): $(1.3) \leftrightarrow (1.3)°$
(2.3): $(2.3) \leftrightarrow (3.2)°$

while in dualizations, we have

(1.2): $(1.2)_{i,k} \leftrightarrow (1.2)_{k,i}$
(1.3): $(1.3)_{i,k} \leftrightarrow (1.3)_{k,i}$
(2.3): $(2.3)_{i,k} \leftrightarrow (2.3)_{k,i}$

This observation we want to write down in the following theorem:

Theorem: While pairs of converse dyadic relations of the shape ((a.b), (a.b)°) lie in the same contexture(s), pairs of dual dyadic relations of the shape ((a.b), ×(a.b)) lie in the same contextures with inverted order.

From this theorem it follows immediately:

Lemma: Eigenreality is a monocontextural structural feature because in systems with only 1 contexture, there is not differentiation between the order of contextures, and thus the pairs ((a.b), (a.b)°) and ((a.b), ×(a.b)) coincide.

4. Therefore, we can also say that binnensymmetry (in-between-symmetry) creates eigenreality in monocontextural systems. Knowing that, it will be shown now how eigenreality can be constructed in a very simple way. Lets start with the 3-contextural pseudo-eigenreal "dual" system:

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3)$

In this "complementary system" (Kaehr 2009), the sign class

$(3.1_3\ 2.2_{1,2}\ 1.3_3)$

with the inner environments of its dyadic sub-signs (<3, <1, 2>, 3>)

is assigned the reality thematic

$(3.1_3\ 2.2_{2,1}\ 1.3_3)$

with its outer environments (<3, <2, 1>, 3>). Remember that in classical semiotic dual systems the sign thematic stands for the subject and the reality thematic for the object relation. Therefore, we have

(<3, <1, 2>, 3>): Set of contextures for subject relation
(<3, <2, 1>, 3>): Set of contextures for object relation

for $(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3)$. If we now transport the genuine dyadic relation (identitive morphisms) with outer environment to its corresponding genuine dyadic relation with inner environment, we get

$(3.1_3\ 2.2_{1,2}\ 2.2_{2,1}\ 1.3_3)$.

Now, we have created binnensymmetry:

$(2.2_{1,2} \times 2.2_{2,1})$,

and thus eigenreality (theorem and lemma):

$(3.1_3\ 2.2_{1,2}\ 2.2_{2,1}\ 1.3_3) \times (3.1_3\ 2.2_{1,2}\ 2.2_{2,1}\ 1.3_3)$.

But what have we done? We have exported an object element of the representation and imported it into the subjective relation of the representation, and we have exported a subjective element of the representation and imported it into the objective relation of representation. The above complementary system is thus a hybrid semiotic system with objective share in its subjective representation and with subjective share in its objective representation.

If we not note the abstract sign relation as follows (Toth 2008)

$SR = (<\pm S\pm O>_I, <\pm S\pm O>_O, <\pm S\pm O>_M)$, considering that a sign mediates as a function between world (= object pole) and consciousness (= subject pole), then we get for our polycontextural-eigenreal representation system above

$<\pm S \pm O>_O = (2.2_{1,2})$

and

$<\pm O \pm S>_O = (2.2_{2,1})$,

so that we get

$<<\pm S \pm O><\pm O \pm S>> = (2.2_{1,2}\ 2.2_{2,1})$,

and thus again binnensymmetry and from here qua theorem and lemma eigenreality. However, we also learn something that appears only in semiotic structures with contextures K > 3, namely

$(a.b_i) \neq \times(a.b_1)$,

e.g. in the following context
$(3.1_3\ 2.1_1\ 1.3_3) \times (3.1_3\ 1.2_1\ 1.3_3) =$

$(<\pm 3 \pm 1>_3, <\pm 2 \pm 1>_1, <\pm 1 \pm 3>_3) \times (<\pm 3 \pm 1>_3, <\pm 1 \pm 2>_1, <\pm 1 \pm 3>_3)$,

that $(3.1)_3$ in the sign thematic is not $(3.1)_3$ in the reality thematic (and so on for the other two dyads), because subject- and object-position have been exchanged. To put it simply: $(3.1)_3$ (sign thematic) just looks like $(3.1)_3$ (reality thematic), because in a 3-contextural system, there are not enough contextures to let show the non-identity of dualized sub-signs, but cf. in 4 contextures: $(3.1)_{3,4} \neq ((3.1)_{4,3}$.

Bibliography

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976
Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981
Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992
Kaehr, Rudolf Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)
Toth, Alfred, The sign as a "disjunction between world and consciousness". In: Electronic Journal for Mathematical Semiotics, 2008

Speculations about the addition of contextuated sign relations

1. As Günther had pointed out in his memories (Günther 1975), by aid of polycontextural numbers one can add objects of more than one quality, i.e. not only 1 apple + 1 apple = 2 apples (preservation of quantity and quality), but also 1 apple + 1 pear = 2 ???, where 2 "fruits" just preserves the quantity, but offers a kind of compromise for the (non-) added qualities. Also, Günther pointed out that it is possible to start counting in contexture 1 and to continue in another contexture, so that the border for the Here and the Beyond(s) are getting permeable for counting processes as well as for series of numbers. And finally, again according to Günther, it is not only possible to count polycontextural numbers, but the contextures themselves, in which they lie.

2. A nice little example of how counting could look in the semiotic subsystem of the dyadic sub-signs is given by Kaehr (2009, but going back to Kaehr's notes from the late 70ies). If T denote the transjunction operator (here: mathematical "Verwerfung"!), then we have, e.g.

T((2.1), (2.2)) = (2.3)

This is an example of a verwerfung of two trichtomic values in favor of a third one offered by trichotomic semiotics (already). However, how would T operate in the following examples:

T(2.1) = (2.2)?, (2.3)?
T((2.1), (2.2), (2.3)) = ? (Unsolvable or dyad from another triad?)
T((2.1), (3.1) = ? (Unsolvable or (1.1)?)
T((1.1), (2.1), (3.3) = ? (A whole set of dyads like ((1.2), (1.3), (2.2), (2.3), (3.1), (3.2)) or unsolvable?

3. If two ore more sign classes have to be added, this was defined by Berger (1976) as the maximum of the intersection of these sign classes, e.g.

(3.1 2.2 1.2) + (3.2 2.2 1.2) = max((3.1 2.2 1.2) $\cap$ (3.2 2.2 1.2)) = (3.2 2.2 1.2),

a subtraction was defined conversely, with min- instead of maximum function. However, this addition/subtraction seems not to work of the level of the sub-sings, cf.

(3.1) + (3.2) = ?

We may also be aware that in a semiotics in which the inclusive semiotic order is eliminated, we may meet an example like follows:

(3.1 2.2 1.1) + (3.2 2.1 1.2).

Here, max((3.1 2.2 1.1) ∩ (3.2 2.1 1.2)) = ?? (3.2 ??)

4. However, if we also consider contextures, the problems start to increase quickly. Generally, we have the following problems:

4.1. Triadic values = trichotomic values = contextures, e.g.

$(3.1)_i + (3.1)_i = ?$

4.2. Triadic values = trichotomic values ≠ contextures, e.g.

$(3.1)_i + (3.1)_j = ?$

4.3. Triadic values ≠ trichotomic values ≠ contextures, e.g.

$(3.1)_i + (3.2)_j = ?$

4.4. Triadic values ≠ trichotomic values ≠ contextures, e.g.

$(2.1)_i + (3.2)_j = ?$

Of course, there are more combinations, but with these main types not only dyads, but triads (sign classes, reality thematics) can be added – provided that for question marked places there will be found a solution.

5. In order to solve the above marked problems, I shall suggest two ways:

5.1. Sub-signs with either/or – and – different triadic and trichotomic values cannot be added directly, since the prime sign-numbers (Bense 1980) are separated by contexture borders by themselves (Toth 2008, passim). If either the triadic and/or the trichotomic values coincide, they just can be added like in elementary arithmetics:

5.1.1. (3.1) + (3.1) = (3.1)

5.1.2. (3.1) + (2.1) = (3.1) + (1.2) + (2.1). This suggestion substitutes addition by composition, which is legitimated by the double nature of sub-signs as static dyads and as dynamic semioses.

5.1.3. (3.1) + (3.2) = max((3.1), (3.2)) = (3.2). Here, I agree with Berger (1976).

Now the types with contextures:

5.1.4. $(3.1)_i + (3.1)_i = (3.1)_i$

5.1.5. $(3.1)_i + (3.1)_j = (3.1)_{max((i,j)}$. Thus, I use Berger's lattice addition for contextures.

5.1.6. $(3.1)_i + (3.2)_i$ = max((3.1), (3.2)) = (3.2). (Berger)

5.1.7. $(3.1)_i + (3.2)_j$ = max((3.1), (3.2)) = (3.2); max(i, j) eat(Berger) Thus, here I use the max both for sub-signs and for their contextures.

5.1.8. $(2.1)_i + (3.2)_{jj}$ = min((2.1), (3.2)) = (2.1). Min-function is legitimated here because a triadic relation contains itself, a dyadic and monadic relation.

min(i, k). The idea is that in contextuated semiotic relations, you do not add primarily the prime-sign-numbers, but the contextures themselves.

5.1.9. $(3.1)_i + (2.1)_j$ = (3.1) + (1.2) + (2.1); min(i, j), cf. 5.1.2.

Bibliography

Bense, Max, Die Einführung der Primzeichen. In: Ars Semeiotica3/3, 1980, pp. 287-294

Berger, Wolfgang, Zur Algebra der Zeichenklassen. In: Semiosis 4, 1976, pp. 20-24

Günther, Gotthard, Selbstdarstellung im Spiegel Amerikas. In: Pongratz, L.J.: Philosophie in Selbstdarstellungen. Vol. II. Hamburg 1975, pp. 1-76

Kaehr, Rudolf, Toth's semiotic diamonds. http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf (2009)

Toth, Alfred, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008

Time as semiotic contexture

1. According to Gotthard Günther it is possible to develop a contextural notion of time: “Zeit ist, strukturtheoretisch betrachtet, nichts anderes als die Aktivierung einer Diskontexturalitätsrelation zwischen Vergangenheit und Zukunft” (1979, p. 191). As such, time can be, in concordance with the poly-contextural “Life Lines” (Günther 1979, pp. 283 ss.), linear, non-linear or multi-linear (cf. Toth 2008a, pp. 57-67). After I have given a monocontextural model for semiotic time in Toth (2008b), I will ad some more considerations here, and the basis of the contextuated sign relation introduced by Kaehr (2008).

2. Like a golden thread, the alleged timelessness of signs goes through the history of semiotics, roughly speaking from Aristoteles to Bense. Mostly, time is not even mentioned in connection with signs, although every child knows that it needs time to write something down, to make a node into the handkerchief or explain a foreigner the way to the Hofbräuhaus. In Toth (2008a, pp. 177 ss.), I have shown that every sign relation has 6 permutations:

(3.a 2.b 1.c)
(3.a 1.c 2.b)
(2.b 3.a 1.c)
(2b. 1.c 3.a)
(1.c 3.a 2.b)
(1.c 2.b 3.a)

When we insert values for the variables a, b, c ∈ {.1, .2, .3}, we get the 10 quantitative sign classes, which we can, according to Toth (2009), write as unordered sets of trichotomic values. However, if we now ascribe contextures to each dyad of every sign relation, we also get in the case of a 3-contextural sign model the 10 quantitative-qualitative sign classes. In a last step, we can use ordered sets of contextures and ordered partial sets of contextures in order to give a purely qualitative system of sign relations:

(3.1 2.1 1.1)	≡	(1, 1, 1)	≡	$(3.1_3\ 2.1_1\ 1.1_{1,3})$	≡	<3, 1, <1, 3>>
(3.1 2.1 1.2)	≡	(1, 1, 2)	≡	$(3.1_3\ 2.1_1\ 1.2_1)$	≡	<3, 1, 1>
(3.1 2.1 1.3)	≡	(1, 1, 3)	≡	$(3.1_3\ 2.1_1\ 1.3_3)$	≡	<3, 1, 3>
(3.1 2.2 1.2)	≡	(1, 2, 2)	≡	$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	≡	<3, <1, 2>, 1>
(3.1 2.2 1.3)	≡	(1, 2, 3)	≡	$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	≡	<3, <1, 2>, 3>
(3.1 2.3 1.3)	≡	(1, 3, 3)	≡	$3.1_3\ 2.3_2\ 1.3_3)$	≡	<3, 2, 3>
(3.2 2.2 1.2)	≡	(2, 2, 2)	≡	$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	≡	<2, <1, 2>, 1>
(3.2 2.2 1.3)	≡	(2, 2, 3)	≡	$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	≡	<2, <1, 2>, 3>
(3.2 2.3 1.3)	≡	(2, 3, 3)	≡	$(3.2_2\ 2.3_2\ 1.3_3)$	≡	<2, 2, 3>

(3.3 2.3 1.3) ≡ (3, 3, 3) ≡ ($3.3_{2,3}$ 2.3_{2} 1.3_{3}) ≡ <<2, 3>, 2, 3>

3. Now we come back to Günther's definition of time as "nothing else but the activation of a discontexturality relation between past and future" (1979, p. 191). Thus, we get the following system of permutations for our triadic sets of time-contextures:

$P(T_1)$ = (<3, 1, <1, 3>>, <3, <1, 3>, 1>, <<1, 3>, 3, 1>, <<1, 3>, 1, 3>, <1, 3, <1, 3>>, <1, <1, 3>, 3>>)
$P(T_2)$ = (<3, 1, 1>, <1, 3, 1>, <1, 1, 3>, <3, 1, 1>)
$P(T_3)$ = (<3, 1, 3>, <1, 3, 1>, <3, 3, 1>, <1, 3, 3>)
$P(T_4)$ = (<3, <1, 2>, 1>, <3, 1, <1, 2>>, <<1, 2>, 1, 3>, <<1, 2>, 3, 1>, <1, <1, 2>, 3>, <1, 3, <1, 2>>)
$P(T_5)$ = (<3, <1, 2>, 3>, <<1, 2>, 3, 3>, <3, 3, <1, 2>>)
$P(T_6)$ = (<3, 2, 3>, <3, 3, 2>, <2, 3, 3>)
$P(T_7)$ = (<2, <1, 2>, 1>, <2, 1, <1, 2>>, <<1, 2>, 2, 1>, <<1,2>, 1, >, <1, <1, 2>, 2>, <1, 2, <1, 2>
$P(T_8)$ = (<2, <1, 2>, 3>, <2, 3, <1, 2>>, <<1, 2>, 2, 3>, <<1.2>, 3, 2>, <3, <1, 2> 2>, <3, <1, 2>, 2>)
$P(T_9)$ = (<2, 2, 3>, <3, 2, 2>, <2, 3, 2>)
$P(T_{10})$ = (<<2, 3>, 2, 3>, <<2, 3>, 3, 2>, <3, 2, <2, 3>>, <3, <2, 3>, 2>, <2, <2, 3>, 3>, <2, 3, <2, 3>>)

These 47 combinatorial types of time-contextures are thus all that are reachable in a 3.contextural 3-adic semiotics. If we combine them again amongst themselves, we get quickly very highly complicated semiotic time-structures – in opposition, of course, to the phantasma of time-free sign notion.

Bibliography

Günther, Gotthard, Idee und Grundriss einer nicht-aristotelischen Logik. 3rd ed. Hamburg 1991

Kaehr, Rudolf, Diamond semiotics. Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a).

Toth, Alfred, Auf dem Weg zu einer polykontextural-semiotischen Theorie der Zeit. In: Electronic Journal for Mathematical Semiotics, 2008b

Toth, Alfred, Polycontextural matrices. In: Electronic Journal for Mathematical Semiotics, 2009

Primzeichen-Zahlen und semiotische Kontexturen

1. In Toth (2009) I have shown that it is possible to artificially construct eigenreality in polycontextural sign relations:

(3.1 2.2 1.3) × (3.1 2.2 1.3)

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3) \rightarrow$
$(3.1_3\ \underline{2.2_{1,2}\ 2.2_{2,1}}\ 1.3_3) \times (3.1_3\ \underline{2.2_{1,2}\ 2.2_{2,1}}\ 1.3_3)$.

Hereby we thus import a reality relation into the sign class and a sign relation into the reality thematic.

A related idea stays behind Bense's definition of prime-signs: He starts with the monocontextural eigenreal sign class

(3.1 2.2 1.3) → (3. 2. 1.) + (.1 .2 .3)

explaining it by "additive association" of the triads of the sign class and the trichotomies of the reality thematic. Bense concludes: "Diese festgestellten Zuammenhänge legitimieren m.E. ausreichend, von **Primzeichen** als den die repräsentierenden und kategorialisierenden Zeichenfunktionen zusammenfassenden Bestimmungsstücken zu sprechen" (1981, p. 23).

Already a few years before, Bense succeeded in showing the intrinsic connections between Peano-numbers and the prime-signs:

"Nunmehr ergibt die semiotische Reduktion und Explikation der Peanoschen Axiome folgende Aussagen für das semiotische Repräsentationsschema:

1. Der Präsentant ist ein Repräsentant.
2. Der Repräsentant eines Repräsentanten ist ein Repräsentant.
3. Es gibt keine zwei Präsentanten mit dem gleichen Repräsentanten.
4. Der Präsentant ist nicht Repräsentant seines Repräsentanten"
   (Bense 1975, p. 171)

In this way, Bense introduces the antecessor/successor relation into semiotics. Obviously, this ASR parallels the natural numbers in semiotics, thus creating a formal mathematical basis for semiotics. However, it also parallels relational logic, namely the n-adic calculus for n = 1, 2, 3. That the prime-signs break up having reached 3, is due to Peirce's conviction that every relation can be reduced to triad (or according to Günther 1991 due to Peirce's Christianism).

However, if semiotics has as a mathematical basis the natural rumbers, then the question arises if there cannot be negative prime-signs. At this point, it is important to underline that kenonumbers do not have negative counterparts – they are not differentiated at all into positive and negative numbers. However, in polycontextural semiotics, one counts the contextures and not some keno-sequences or morphograms corresponding with the sub-signs, the signs and the reality thematics. Thus, there is no formal obstacle to redefine a sign relation like that

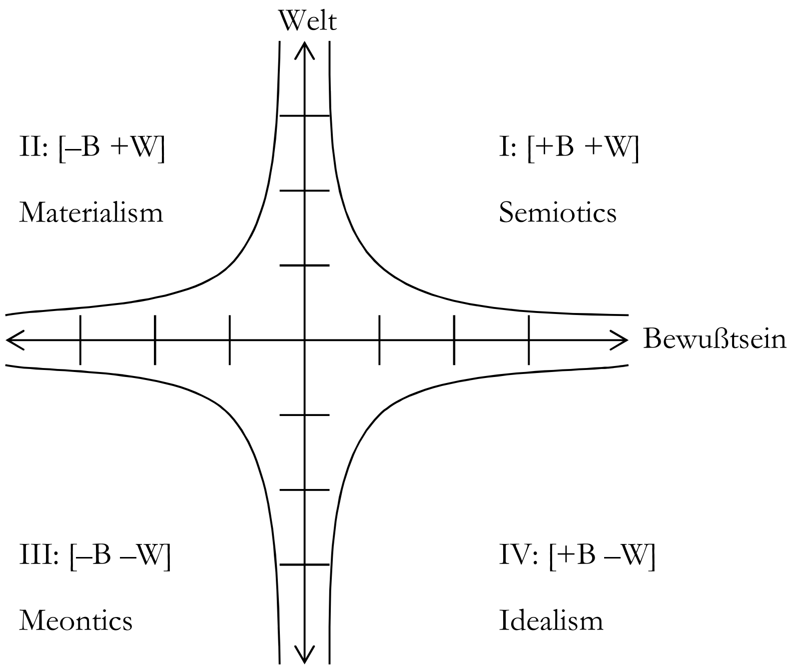
SR = (±3.±a ±2.±b ±1.±c),

so that we have on the level of the dyads the following 4 possible parametrizations:

(+a.+b)
(-a.+b)
(+a.-b)
(-a.-a).

Now these 4 types of dyads correspond with the parameters of the 4 quadrants of the Gaussian Number Field. Hence, in short, the introduction of negative prime-signs opens us up never before seen fields like complex semiotics (Toth 2007, 2008).

We get thus the following – very roughly sketched – graph as a new model for sign relation up to the number field of the complex numbers:



As one can see, too, from this graph, there is a bijection between the algebraic structure of a subsign (+a.-b, -a.+b, +a.+b, -a –b) and the epistemological structure of each dyadic sub-sign [+B -W], [-B +W] [+B +W] [-B +W]. The 4 quadrants correspond exactly both with the algebraic and the epistemological characterizations.

Moreover, there is cyclic transformation between semiotics and idealism: [+B +W] → [-B +W] → [-B -W] → [+B +W].

We can now start further semiotic inquiries by inaugurating the following 4-conxtextural 3-adic 3-otomic sign model

$$SCl = (\pm 3. \pm 1_{\underline{i,j,k}} \pm 2. \pm b_{\underline{l,m,n}} \pm 1. \pm c_{\underline{o,p,q}}) \text{ with } i, ..., q \in \{1, 2, 3, 4\}$$

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Günther, Gotthard, Grundzüge einer neuen Theorie des Denkens in Hegels Logik. 2n ed. Hamburg 1991

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2008

Toth, Alfred, Eigenreality in mono- and polycontextural semiotic systems. In: Electronic Journal for Mathematical Semiotics, 2009

Categories and multiple identity

1. The category theoretic semiotic matrix, which has been postulated by Bense and Marty looks as follows (cf. Toth 1997, pp. 21 ss.)

	1	2	3
1	id1	α	$\beta\alpha$
2	$\alpha°$	id2	β
3	$\alpha°\beta°$	$\beta°$	id3

As one recognizes, the main diagonal contains all identities which are possible in a triadic-trichotomic 3×3 matrix.

2. However, Kaehr has given the following matrices with the dramatic changes, when we step from a monocontextural to a polycontextural logic. The matrix to the left is a 3-contextural 3-adic matrix, the one to the right a 4-contextural 3-adic matrix.

	1	2	3
1	$id1_{1,3}$	α_1	α_3
2	$\alpha°_1$	$id2_{1,2}$	β_2
3	$\alpha°_1$	$\beta°_2$	$id3_{2,3}$

1	2	3
$id1_{1,3,4}$	$\alpha_{1,4}$	$\alpha_{3.4}$
$\alpha°_{1,4}$	$id2_{1,4,2}$	$\beta_{2,4}$
$\alpha\ °_{1,4}$	$\beta°_{2,4}$	$id3_{2,3}$

As the main diagonal of the 3-contextural matrix shows, every semiotic identity is split into 2, and as the 4-contextural matrix shows, there are (n-1) identities for an n-contextural matrix. One of these dramatic changes is the (alleged) vanishing of eigenreality in semiotics with contextures ≥ 3, since the reflection of the inner environments of the sub-signs prevents a dual-identical mapping of the sign class (3.1_1 $2.2_{1,2}$ 1.3_1) onto its reality thematic (3.1_1 $2.2_{2,1}$ 1.3).

3. However, in this little contribution, we want to shed a light on a formal device which I had already introduced in monocontextural semiotics in Toth (2008, pp. 159 ss.). Against every rules in mathematical category theory, I had differentiated between "static" and "dynamic" morphims. What is meant with that, I repeat here informally, since this differentiation has lead to severe misunderstandings.

3.1. The classical way of transforming a sign class into its morphisms is by exchanging the sub-signs by morphisms. E.g.

$(3.1\ 2.1\ 1.3) \equiv (\alpha°\beta°, \alpha°, \beta\alpha)$

However, this mapping is purely static, since the fact that (3.1 ...) is a triadic relation over a dyadic relation (... 2.1 ...), and (... 2.1 ....) is a dydic reation over a monadic relation (... 1.3) .is not taken into consideration. This classical device lies on the double introduction of sub-signs as being both static and being both dynamic relations.

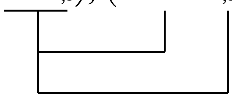
3.2. However, instead of mapping $(3.1) \rightarrow \alpha$, $(2.1) \rightarrow \beta$, $(1.3) \rightarrow \gamma$, we can proceed as follows:

$(3.1\ 2.1) \rightarrow (3.2\ 1.1), (2.1\ 1.3) \rightarrow (2.1\ 1.3)$.

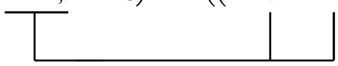
Therefore, trichotomies and triads are now linked together, and the relational dependency between the dyads is made clear. (However, there is no way how to show the differences between triadic, dyadic and monadic linking.)

In the case of polycontextural dyads, we thus get, f. ex.

$(3.1_3\ 2.1_1\ 1.3_3) \rightarrow ((3.2_2\ 1.1_{1,3}), (2.1_1\ 1.3_{,3})$



$(3.1_3\ 2.2_{1,2}\ 1.3_3) \rightarrow ((3.2_2\ 1.2_1), (2.1_1\ 1.3_{,3})$



This means: The identity-splittings of the one genuine sub-signs (identitive morphisms) is not distributed over two genuine sub-signs (identitive morphisms).

So, besides (static) categories (Eilenberg, MacLane), bi- and n-categories (Leinster) , static/dynamic saltatories (Kaehr), dynamic categories introduced here are another distinct type of category theory.

Bibliogaphy

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Additive Association

1. Additive association as a semiotic operation was mentioned in the whole semiotic writings of the Stuttgart School just one time. When Bense was showing how the eigenreal sign class (3.1 2.2 1.3) comes to appear as the side diagonal of a transposition of his semiotic 3×3 matrix

	3.	2.	1.
.1	3.1		
.2		2.2	
.3			1.3

he used the following scheme of combination of the retrosemiosic and the semiosic order of the prime-signs which he called additive association (Bense 1981, p. 204):

3. 2. 1.
.1 .2 .3

2. However, Bense did not know that every sign class can appear in 6 permutations (Toth 2008, pp. 177 ss.), so that all other types, like, e.g.,

1. 2. 3.
.1 .2 .3

1. 2. 3.
.1 .3 .2

1. .3 .2
.1 .3 .2, etc.

1. 2. 3.
.2 .3 .1

1. 2. 3.
.2 .1 .3

1. 2. 3.
.3 .2 .1

1. 2. 3.
.3 .1 .2,

totally 21 types of associative additions have remained unknown to him. As it can be shown easily, these are the "incorrect" sign classes that have never been accepted by Bense because they violate the semioitic inclusion, which sets for every sign class

SCl = (3.a 2.b 1.c)

an order

(a ≤ b ≤ c).

Now, the semiotic 3×3 matrix already contains as main diagonal the Genuine Category Class (3.3 2.2 1.1) which is built according to (a > b > c). Moreover, there are no formal obstacles against the elimination of this restriction and thus to proceed from the system of the 10 to a system of 27 (= 3^3) sign classes:

(3.1 2.1 1.1)	(3.1 2.2 1.1)	(3.1 2.3 1.1)
(3.1 2.1 1.2)	(3.1 2.2 1.2)	(3.1 2.3 1.2)
(3.1 2.1 1.3)	(3.1 2.2 1.3)	(3.1 2.3 1.3)
(3.2 2.1 1.1)	(3.2 2.2 1.1)	(3.2 2.3 1.1)
(3.2 2.1 1.2)	(3.2 2.2 1.2)	(3.2 2.3 1.2)
(3.2 2.1 1.3)	(3.2 2.2 1.3)	(3.2 2.3 1.3)
(3.3 2.1 1.1)	(3.3 2.2 1.1)	(3.3 2.3 1.1)
(3.3 2.1 1.2)	(3.3 2.2 1.2)	(3.3 2.3 1.2)
(3.3 2.1 1.3)	(3.3 2.2 1.3)	(3.3 2.3 1.3)

Thus, additive addition, that had only been used by Bense to show the construction of (3.1 2.2 1.3) and (3.3 2.2 1.1) in connection with the semiotic 3×3 matrix and their two diagonals, it can now be pointed out as The device of overcoming the senseless inclusion order and to proceed the system of the 27 sign classes whose part-set the 10 Peircean sign classes are.

Bibliography

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981
Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992
Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2007

Polycontextural-semiotic reality theory

1. In Toth (2009b), I have shown that Bense's "associative addition"

(3. 2. 1.) + (.1 .2 .3) = (3.1 2.2 1.3)

is just one special case of 21 possible mappings of an n-tuple of triadic values to an n-tuple of trichotomic values. The maximum system of sign classes we get by consistent application of this mapping is the complete system of all $3^3 = 27$ triadic-trichotomic sign classes in which the restriction that the trichotomic values of position (n+1) must not be smaller than the one on position n, is abolished:

(3.1 2.1 1.1)	(3.1 2.2 1.1)	(3.1 2.3 1.1)
(3.1 2.1 1.2)	(3.1 2.2 1.2)	(3.1 2.3 1.2)
(3.1 2.1 1.3)	(3.1 2.2 1.3)	(3.1 2.3 1.3)
(3.2 2.1 1.1)	(3.2 2.2 1.1)	(3.2 2.3 1.1)
(3.2 2.1 1.2)	(3.2 2.2 1.2)	(3.2 2.3 1.2)
(3.2 2.1 1.3)	(3.2 2.2 1.3)	(3.2 2.3 1.3)
(3.3 2.1 1.1)	(3.3 2.2 1.1)	(3.3 2.3 1.1)
(3.3 2.1 1.2)	(3.3 2.2 1.2)	(3.3 2.3 1.2)
(3.3 2.1 1.3)	(3.3 2.2 1.3)	(3.3 2.3 1.3)

(Red underlined are trichotomic orders of the form a > b > c like the Genuine Category class which is the main diagonal of the semiotic 3×3 matrix.) Therefore, the above table contains all possible trichotomic orders, i.e.

(a = b = c), (a > b > c), (a < b < c)

and all mixed forms.

2. As we have seen in the first part of this study (Toth 2009a), the introduction of contextures for the dyadic sub-signs changes the structural realities as presented by the reality thematics of the sing classes quite a bit:

$(1.1_{3,1}\ \underline{1.2_{1}\ 1.3_{3}})$	M<1, 3>-thematized M<3, 1>
$(2.1_{1}\ \underline{1.2_{1}\ 1.3_{3}})$	M<1, 3>-thematized O <1>
$(3.1_{3}\ \underline{1.2_{1}\ 1.3_{3}})$	M<1 ,3>-thematized I <3>
$(\underline{2.1_{1}\ 2.2_{2,1}}\ 1.3_{3})$	O<1, <2,1>>-tehematized M <3>
	I<3>, O<2, 1>-thematized M<3>
$(\underline{3.1_{3}}\ \underline{2.2_{2,1}}\ \underline{1.3_{3}})$	I<3>, M<3>-thematized O<2, 1>
	O<2, 1>, M<3>-thematized I<3>
$(\underline{3.1_{3}\ 3.2_{2}}\ 1.3_{3})$	I<3, 2>-thematized M<3>
$(2.1_{1}\ \underline{2.2_{2,1}\ 2.3_{2}})$	O<<2, 1>, 3>- thematized O<1>
$(3.1_{3}\ \underline{2.2_{2,1}\ 2.3_{2}})$	O<<2, 1>, 2>- thematized I<3>
$(\underline{3.1_{3}\ 3.2_{2}}\ 2.3_{2})$	I<3, 2>-thematized O<2>
$(3.1_{3}\ \underline{3.2_{2}\ 3.3_{3,2}})$	I<2, <3, 2>>-thematized I<2>

In the following, we give the additional structural realities of the reality thematics of the so-called "irregular" sign classes, in which the order of the Peircean sign classes has been abolished:

$(3.1_{3}\ 2.2_{1,2}\ 1.1_{1,3})$	×	$(\underline{1.1_{3,1}}\ 2.2_{2,1}\ \underline{1.3_{3}})$	M-O-M	<<3,1>, <2,1>, 3>
$(3.1_{3}\ 2.3_{2}\ 1.1_{1,3})$	×	$(\underline{1.1_{3,1}}\ 3.2_{2}\ \underline{1.3_{3}})$	M-I-M	<<3,1>, 2, 3>
$(3.1_{3}\ 2.3_{2}\ 1.2_{1})$	×	$(2.1_{1}\ 3.2_{2}\ 1.3_{3})$	O-I-M	<1, 2, 3>
$(3.2_{2}\ 2.1_{1}\ 1.1_{1,3})$	×	$(\underline{1.1_{3,1}\ 1.2_{1}}\ 2.3_{2})$	M-M-O	<<3,1>, 1, 2>
$(3.2_{2}\ 2.1_{1}\ 1.2_{1})$	×	$(\underline{2.1_{1}}\ 1.2_{1}\ \underline{2.3_{2}})$	O-M-O	<1, 1, 2>
$(3.2_{2}\ 2.1_{1}\ 1.3_{3})$	×	$(3.1_{3}\ 1.2_{1}\ 2.3_{2})$	I-M-O	<3, 1, 2>
$(3.2_{2}\ 2.2_{1,2}\ 1.1_{1,3})$	×	$(1.1_{3,1}\ \underline{2.2_{2,1}\ 2.3_{2}})$	M-O-O	<<3,1>, <2,1>, 2>
$(3.2_{2}\ 2.3_{2}\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 3.2_{2}\ 2.3_{2})$	M-I-O	<<3,1>, 2, 2>
$(3.2_{2}\ 2.3_{2}\ 1.2_{1})$	×	$(\underline{2.1_{1}}\ 3.2_{2}\ \underline{2.3_{2}})$	O-I-O	<1, 2, 2>
$(3.3_{2,3}\ 2.1_{1}\ 1.1_{1,3})$	×	$(\underline{1.1_{3,1}\ 1.2_{1}}\ 3.3_{3,2})$	M-M-I	<<3,1>, 1, <3,2>>
$(3.3_{2,3}\ 2.1_{1}\ 1.2_{1})$	×	$(2.1_{1}\ 1.2_{1}\ 3.3_{3,2})$	O-M-I	<1, 1, <3,2>>
$(3.3_{2,3}\ 2.1_{1}\ 1.3_{3})$	×	$(\underline{3.1_{3}}\ 1.2_{1}\ \underline{3.3_{3,2}})$	I-M-I	<3, 1, <3,2>>
$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 2.2_{2,1}\ 3.3_{3,2})$	M-O-I	<<3,1>, <2,1>, <3,2>>
$(3.3_{2,3}\ 2.2_{1,2}\ 1.2_{1})$	×	$(\underline{2.1_{1}\ 2.2_{2,1}}\ 3.3_{3,2})$	O-O-I	<1, <2,1>, <3,2>>
$(3.3_{2,3}\ 2.2_{1,2}\ 1.3_{3})$	×	$(\underline{3.1_{3}}\ 2.2_{2,1}\ \underline{3.3_{3,2}})$	I-O-I	<3, <2,1>, <3,2>>
$(3.3_{2,3}\ 2.3_{2}\ 1.1_{1,3})$	×	$(1.1_{3,1}\ \underline{3.2_{2}\ 3.2_{3,2}})$	M-I-I	<<3,1>, 2, <3,2>>
$(3.3_{2,3}\ 2.3_{2}\ 1.2_{1})$	×	$(2.1_{1}\ \underline{3.2_{2}\ 3.3_{3,2}})$	O-I-I	<1, 2, <3,2>>

As one sees easily, in the part-system of the 17 "irregular" sign classes, completely new reality structures appear that do not appear in the complementary system of the 10 Peircean sign classes nor in any other system or part-system for lower or higher n-adic m-otomic sign classes (cf. Toth 2007, pp. 214 ss.). Therefore, as suggested in my earlier work, the semiotic restriction for trichotomic values (3.a 2.b 1.c) with (a ≤ b ≤ c) has to be abolished, since the 10 Peircean sign classes are not only a set-theoretic, but also a reality-theoretic fragment of the complete representational system of n-contextural 3-adic 3-otomic semiotics.

Bibliography

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Polycontextural-semiotic reality theory. In: Electronic Journal of Mathematical Semiotics, 2009a

Toth, Alfred, Additive association. In: Electronic Journal of Mathematical Semiotics, 2009b

Decompositions of semiotic matrices

1. In one of the most intelligent books that has ever been written, in Rudolf Kaehr's "Skizze eines Gewebes rechnender Räume in denkender Leere", we read: "In transcomputationalen Systemen gibt es eine Vielheit von gleichen und selbigen Systemen, die Übergänge verschiedenster Ursprünge realisieren und die in verschiedene Emanationen engebettet sind. In einem klassischen binären System gehört jeder binäre Teilgraph als Teil zum System. M.a.W., ein Teilsystem lässt sich nicht von anderen Teilsystemen absondern oder isolieren. Deswegen nicht, weil es letztlich einen mit anderen Teilsystemen gemeinsamen Anfang hat (...). In polykontexturalen Systemen gibt es eine Vielheit selbiger und gleicher, doch nicht identischer Teilsysteme, die sich nicht mehr unter einem gemeinsamen binären Anfang subsumieren lassen (Kaehr 2004, pp. 141 s.).

Especially for semiotics, Kaehr (2009) has shown some decompositions of matrices by starting with contextuated sub-signs.

2. Let us first introduce a new semiotic 4×4 matrix (tetradic-trichotomic)

0.1	0.2	03	0.4
11	12	11	14
2.1	2.2	2.3	2.3
3..1	3.2	3.3	3.4

We can now decompose this matrix to its part-matices:

0.1	0.2	0.2	0.3	0.3	0.4
1.1	1.2	1.2	1.3	1.3	1.4

0.1	0.3	0.1	0.4	0.2	0.4
1.1	1.3	1.1	1.4	1.2	1.4

1.1	1.2	1.2	1.3	13	1.4
2.2	2.3	2.2	2.3	2.2	2.43
1.1	1.3	1.1	1.4	1.2	1.4
2.1	1.3	2.1	2.4	2.2	2.4
2.1	2.2	2.2	2.3	2.3	2.4
3.1	3.2	3.2	3.3	3.3	3.4
2.1	2.3	2.1	2.4	2.2	2.4
3.1	1.3	3.1	2.4	3.2	3.4
0.1	0.2	0.2	0.3	0.3	0.4
2.1	2.2	2.2	2.3	2.3	2.4
0.1	0.3	0.1	0.4	0.2	0.4
2.1	1.3	2.1	2.4	2.2	2.4
0.1	0.2	0.2	0.3	0.3	0.4
3.1	3.2	3.2	3.3	3.3	3.4
0.1	0.3	0.1	0.4	0.2	0.4
3.1	3.3	3.1	3.4	3.2	3.4
1.1	1.2	1.2	1.3	13	1.4
3.1	3.2	3.2	3.3	3.3	3.4

1.1	1.3		1.1	1.4		1.2	1.4
3.1	4.3		3.1	3.4		3.2	3.4

1.1	1.2		1.2	1.3		13	1.4
4.1	4.2		4.2	4.3		4.3	4.4

1.1	1.3		1.1	1.4		1.2	1.4
4.1	4.3		4.1	4.4		4.2	4.4

1.1	1.2		1.2	1.3		13	1.4
4.1	4.2		4.2	4.3		4.3	4.4

1.1	1.3		1.1	1.4		1.2	1.4
4.1	4.3		4.1	4.4		4.2	4.4

Hence, the above 4×4 matrix has 3 times 6 = 18 decompositional matrices.

3. Now let us have a look at the pre-semiotic tetradic trichotomic matrix introduced by Toth (2008):

$$\begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3 \end{pmatrix}$$

This matrix has the following 9 decompositions or part-matrices, respectively:

0.1	0.2		0.2	0.3		0.1	0.3
1.1	1.2		1.2	1.3		1.1	1.3

1.1	1.2	1.2	1.3	1.1	1.3
2.1	1.2	2.2	1.3	2.1	1.3

2.1	2.2	2.2	2.3	2.1	2.3
3.1	3.2	3.2	3.3	3.1	3.3

4. Finally, the usual 3×3 matrix has the following 6 decompositions:

$$\begin{pmatrix} 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3 \end{pmatrix}$$

1.1	1.2	1.2	1.3	1.1	1.3
2.1	2.2	2.2	2.3	2.1	2.3

2.1	2.2	2.2	2.3	2.1	2.3
3.1	3.2	3.2	3.3	3.1	3.3

1.1	1.2	1.2	1.3	1.1	1.3
3.1	3.2	3.2	3.3	3.1	3.3

An interesting question – raised indirectly by Ditterich (1990), who considered the Saussurean dyadic sign relation a part-relation of the Peircean triadic sign relation, is: With which of the 18 + 9 + 6 = 31 dyadic pair relations ($^2R \subset {}^3R \subset {}^4R \subset$ ...) is the Saussurean sign model identical? Since it is clear since Toth (1991) that the signifié is the media relation (M ≡ .1.), with which fundamental category corresponds the "image acoustique"? With (O ≡ .2.) – clearly no. With (I ≡ .3.) – most probably no. Therefore, the image acoustique is a semiosis (O → I ≡ .2. → .3.). Therefore, we can reconstruct the Saussurean sign relation as

$SSR = \{\langle x, y\rangle \mid x \in \{(1.1), (1.2), (1.3)\}, y = (2.3)\}$,

in enumerating form:

$$\begin{pmatrix} 1.1 & 1.2 \\ 2.2 & 2.3 \end{pmatrix} \begin{pmatrix} 1.2 & 1.3 \\ 2.2 & 2.3 \end{pmatrix} \begin{pmatrix} 1.1 & 1.3 \\ 2.2 & 2.3 \end{pmatrix},$$

from which the following dyadic sign relations can be constructed:

(1.1, 1.1)	(1.2, 1.2)	(1.1, 1.1)
(1.1, 1.2)	(1.2, 1.3)	(1.1, 1.3)
(1.1, 2.2)	(1.2, 2.2)	(1.1, 2.2)
(1.1, 2.3)	(1.2, 2.3)	(1.1, 2.3)
(1.2, 1.2)	(1.3, 1.3)	(1.3, 1.3)
(1.2, 2.2)	(1.3, 2.2)	(1.3, 2.2)
(1.2, 2.3)	(1.3, 2.3)	(1.3, 2.3)
(2.2, 2.2)	(2.2, 2.2)	(2.2, 2.2)
(2.2, 2.3)	(2.2, 2.3)	(2.2, 2.3)
(2.3, 2.3)	(2.3, 2.3)	(2.3, 2.3),

thus 10 "sign classes" each (and via dualization 10 corresponding "reality thematics".

Bibliography

Kaehr, Rudolf, Kaehr's "Skizze eines Gewebes rechnender Räume in denkender Leere". Glasgow 2004

Kaehr, Rudolf, Interactional operators in diamond semiotics. http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.pdf (2008)

The Saussurean sign model and its formal representation

1. The Saussurean sign model is dyadic (de Saussure 1967, pp. 76 ss.). It stays formally dyadic even if the signifié-relation could be proven a relation between a dyadic and a triadic relation (cf. Toth 1991).

2. Every n-ary relation has its closure in an (n+1)-ary relation (cf. Robertson 2005).

3. Every n-ary logical system is morphogrammatically incomplete and a fragment of an (n+1)-ary morphogrammatic system (Günther 1976, pp. 213 ss.).

4. Conclusion: Topologically, the dyadic Saussurean sign model must have its closure in a triadic sign model (2.). Logically, it is a fragment of a contextuated sign model with more than 2 contextures (3.), cf. esp. Kaehr 2009.

5. Because of (2.) we assume a triadic semiotic relation, but because of morphogrammatic reasons (3.), we have 4 contextures, so we better start with a tetradic semiotic matrix, in which the relation between sub-signs and inner environments (combinations of contextural indices) is not underbalanced.

6. Because of Bense's differentiation between categorial (c) and relational (r) numbers (1975, pp. 45 s., 65 ss.), we have $r \neq 0$, while categorial numbers are $c \geq 0$. So, iterations of zero-relations are excluded (*(0.0)), since pure objectivity has no subjective power (i.e. objects in the ontological space are incapable of entering relations). However, we have (0.1), (0.2), (0.3), (0.4).

7. We are now able to construct the following semiotic 4×4 (tetradic-tetratomic) matrix, in which we also enter the contextural numbers for a 4-contextural matrix.

$$\begin{pmatrix} 0.1_{3,4} & 0.2_{2,3} & 0.3_{2,4} & 0.4_{2,3,4} \\ 1.1_{1,3,4} & 1.2_{1,3} & 1.3_{1,4} & 1.4_{3,4} \\ 2.1_{1,3} & 2.2_{1,2,3} & 2.3_{1,2} & 2.4_{2,3} \\ 3.1_{1,4} & 3.2_{1,2} & 3.3_{1,2,4} & 3.4_{2,4} \end{pmatrix}$$

8. This matrix can be divided, or "decomposed" into the following 48 sub-matrices:

0.1	0.2	0.2	0.3	0.3	0.4
1.1	1.2	1.2	1.3	1.3	1.4
0.1	0.3	0.1	0.4	0.2	0.4
1.1	1.3	1.1	1.4	1.2	1.4
1.1	1.2	1.2	1.3	13	1.4
2.2	2.3	2.2	2.3	2.2	2.4
1.1	1.3	1.1	1.4	1.2	1.4
2.1	1.3	2.1	2.4	2.2	2.4
2.1	2.2	2.2	2.3	2.3	2.4
3.1	3.2	3.2	3.3	3.3	3.4
2.1	2.3	2.1	2.4	2.2	2.4
3.1	1.3	3.1	2.4	3.2	3.4
0.1	0.2	0.2	0.3	0.3	0.4
2.1	2.2	2.2	2.3	2.3	2.4
0.1	0.3	0.1	0.4	0.2	0.4
2.1	1.3	2.1	2.4	2.2	2.4
0.1	0.2	0.2	0.3	0.3	0.4
3.1	3.2	3.2	3.3	3.3	3.4

0.1	0.3	0.1	0.4	0.2	0.4
3.1	3.3	3.1	3.4	3.2	3.4
1.1	1.2	1.2	1.3	13	1.4
3.1	3.2	3.2	3.3	3.3	3.4
1.1	1.3	1.1	1.4	1.2	1.4
3.1	4.3	3.1	3.4	3.2	3.4
1.1	1.2	1.2	1.3	13	1.4
4.1	4.2	4.2	4.3	4.3	4.4
1.1	1.3	1.1	1.4	1.2	1.4
4.1	4.3	4.1	4.4	4.2	4.4
1.1	1.2	1.2	1.3	13	1.4
4.1	4.2	4.2	4.3	4.3	4.4
1.1	1.3	1.1	1.4	1.2	1.4
4.1	4.3	4.1	4.4	4.2	4.4

9. Each of the 4 sub-signs of these 48 2×2 sub-matrices can be contextuated now, whereby sub-signs of the form (a.b) → $(a.b)_{i,j}$ and sub-signs of the form (a.a) → $(a.a)_{i,,j,k}$.

10. Final summary: The Saussurean dyadic sign model can be mapped on 48 dyadic sign models as 3×3 sub-matrices in 4 contextures, based on the 3-adic Peircean sign model. If one constructs sign classes from these 4 sub-signs pro each of the 48 models, there is always (3-2) = 1 fundamental category lacking. The insight that the Saussurean signifiant-signifié-model is realized in only 1 of 48 possibilities, shows

that it is defective ("Le signifiant désigne l'image acoustique d'un mot". Le signifié désigne le concept, c'est-à-dire la représentation mentale d'une chose."), but also connected with the 47 matrices partly via sub-signs/semioses (morphisms) and/or inner environments (contextures). The application of contexturality theory to the Saussurean sign model also shows a potential for the enlargement of Saussurean semiotics and from there to linguistics.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Günther, Gotthard, Beiträge zur Grundlegung einer oprationsfähigen Dialektik. Vol. 1. Hamburg 1976

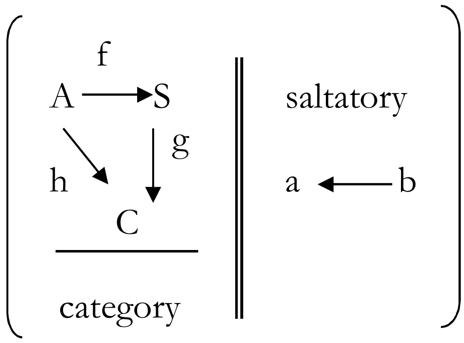
de Saussure, Ferdinand, Grundfragen der allgemeinen Sprachwissenschaft. Transl. by Herman Lommel. 2nd ed. Berlin 1967

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

Toth, Alfred, Bemerkungen zum Saussureschen Arbitraritätsgesetz und Zeichenmodell. In: Semiosis 63/64, 1991, pp. 43-62. Reprinted in: Eckardt, Michael und Lorenz Engell (eds.), Das Programm des Schönen. Ausgewählte Beiträge der Stuttgarter Schule zur Semiotik der Künste und der Medien. Weimar 2002, pp. 71-88

How many saltatories does a sign have?

1. Rudolf Kaehr (e.g. Kaehr 2009, p. 1) introducesd the basic element of diamond theory, the diamond category consisting out of category and its "saltatory" like follows:



Therefore, every sign class (3.a 2.b 1.c) has exactly one saltatory:

$(3.a)_{i,k} \rightarrow (2.b)_{k,l}$ ‖ $(3.a)_{k,i} \leftarrow (1.c)_{m,l}$

↘

$(1.c)_{l,m}$

2. The question is now, what is a semiotic category. For Kaehr 2009 (as well as for me), it is obviously a sign class (or reality thematic). Then, we have two possibilities how to treat the sub-sings: 1. as objects, 2. as morphisms. In the second class, therefore, we have a functor category with a few nice properties that have never been applied yet to semiotics (and which we spare for another publication). However, since a sign class is semiotic category, we do not get one, but 6 saltatories:

$(3.a_{i,k} \rightarrow 2.b_{k,l})$ ‖ $(3.a_{i,k} \leftarrow 2.b_{k,l})$

$(2.b_{k,l} \rightarrow 1.c_{l,m})$ ‖ $(2.b_{l,k} \leftarrow 1.c_{m,l})$

$(3.a_{i,k} \rightarrow 1.c_{l,m})$ ‖ $(3.a_{i,k} \leftarrow 1.c_{m,l})$,

corresponding to the following types of composition:

$(3.a_{i,j} \rightarrow 2.b_{k,l}) \lozenge (2.1_{k,l} \rightarrow 1.c_{m,n})$

$(1.c_{m,n} \rightarrow 2.b_{k,l}) \lozenge (2.b_{k,l} \rightarrow 3.a_{i,j})$

$(3.a_{i,j} \rightarrow 1.c_{m,n}) \lozenge (1.c_{m,n} \rightarrow 2.b_{k,l})$

$(2.b_{k,l} \rightarrow 1.c_{m,n}) \lozenge (1.c_{m,n} \rightarrow 3.a_{i,j})$

$(1.c_{m,n} \rightarrow 3.a_{i,j}) \lozenge (3.1_{i,j} \leftarrow 2.b_{k,l})$

$(2.b_{k,l} \rightarrow 3.a_{i,j}) \lozenge (3.a_{i,j} \leftarrow 1.c_{m,n})$, with matching conditions according to the maximal number 2 contextural indices, if C = 3 and of maximal number of 3 contextural indices, if C = 4 (maximal number in both cases with genuine sub-signs or identitive morphims/functors, resp., 0nly). However, in contextures C ≥ 3, we have 3!, 4!, 5!, etc. possible permutations of the contextural indices, so that from categorial indices alone, we have in C = 4 (3! = 6), in C = 5 (4! = 25) weitere Saltatorien.

Hence, summing up, every 3-adic n-contextural sign class has 3! = 6 permutations of their objects or morphisms, resp., plus n! permutations of their caegorial indices, thus together 3! · n! saltisitions.

Bibliography

Kaehr, Rudolf, Generalized diamonds. http://www.thinkartlab.com/pkl/media/Generalized_Diamonds/Generalized_Diamonds.pdf (2009)

Are there polycontextural signs?

1. After having published several dozens of articles about polycontextural semiotics, we finally come to the basic question if there are polycontextural signs. This may sound strange, but the question is necessary. Classical Peirce-Bensean semiotics has a system of reality which includes 10 levels, corresponding to the 10 reality thematics that are constructed by dualization from the 10 sign classes. Since each of the 10 sign classes has a subject-position, taken by the interpretant relation, it is not false to say that the 10 semiotic realities are contextures – and contextures each of which are monocontextural like the disseminated single contextures of polycontextural logic.

2. However, representatitives of polycontextural theory have often pointed out that semiotics is clearly a monocontextural system in which the logical Law of Identity (and the other 2-3 fundamental laws of classical thinking) are valid without restrictions. Now let us have a look at the 10 semiotic dual systems. Amongst them there is one sign class that is identical with its dualized structure:

$(3.1\ 2.2\ 1.3) = \times(3.1\ 2.2\ 1.3)$

True, this looks like identity, but compare this dual system with the following

$(3.1\ 2.3\ 1.3) \neq \times(3.1\ 3.2\ 1.3)$.

The latter disequation says:

$(3.1) \neq (3.1)$
$(2.3) \neq (3.2)$
$(1.3) \neq (1.3)$,

and we learn that $(3.1) = (1.3)°$ and $(1.3) = (3.1)°$ as is $(2.3) = (3.2)°$. What did we win by that? We win by that that we can replace the disequality sign by the equality sign and obtain either

$(3.1\ 2.2\ 1.3) \neq (3.1\ 2.2\ 1.3)$
or

$(3.1\ 2.3\ 1.3) = (3.1\ 3.2\ 1.3)$,

since we have already proven that

$(3.1) \neq (3.1)$
$(2.2) \neq (2.2)$

$(1.3) \neq (1.3)$.

It follows that classical semiotics has no identity and is thus polycontextural. The case is just so that the fundamental non-identity of classical semiotics is hidden behind a too low number of contextures involved. Since, if we go from C =1 up to C = 3, we have

$(3.1_{3}) \neq (3.1_{3})$
$(2.2_{1,2}) \neq (2.2_{2,1})$
$(1.3_{3}) \neq (1.3_{3})$

and for C = 4 even

$(3.1_{3,4}) \neq (3.1_{4,3})$
$(2.2_{1,2,4}) \neq (2.2_{4,2,1})$
$(1.3_{3,4}) \neq (1.3_{4,3})$

i.e. now, all arrows are turned around. So, from here, the question should not be if there are polycontextural signs, but if there are monocontextural signs. In classical semiotics, polycontexturality is hidden in the triadic-trichotomic structure of a seeming monocontexturality.

3. But let us ask the question what we do, when we write

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$

instead of

$(3.1\ 2.2\ 1.3)$.

Of course, one can say: We localize the sign in one or more contextures, whereby the genuine sub-signs, the identical morphisms, play a special role insofar as they are always located in +1 contexture compared to the other sub-signs. But can signs even be in contextures? What is in a contexture? - Kenograms and kenogram-sequences, so-called morphograms are in contextures. However, in kenograms, not only the contextural borders between sign and object (the three transcendences of the sign, respectively, cf. Toth 2009) are abolished, but also the law of materiality or sign-constancy (cf. Kronthaler 1992, pp. 292 ss.) is abolished (and replaced by structure-constancy). Kenograms are nothing but placeholders for later insertions of numbers, logical values or signs. So, if we have a thing like

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$,

then what we have here is an already filled (hidden) kenogram-structure, filled with sub-signs referring each of them to more than 1 contextures.

On the other side, in Toth (2003), I have tried to define signs directly on trito-numbers, i.e. polycontextural trito-structures, which have filled with quali-quantitative numbers. If you compare a thing like

(0000123)

with the contextuated sign relation above, then the huge difference becomes apparent. But let me avoid getting into more technical trouble and directly jump to the conclusion, which seems to get more and more evident anyway. As Kronthaler once correctly stated, the system of Mathematics of the Qualities (Kronthaler 1986) is, from the standpoint of quantitative mathematics, not even worth a groupoid. Now, a sign is defined on the basis of Peano numbers and the successor (and predecessor) relations according to Complete Induction (cf. Bense 1975, pp. 167 ss.; 1983, pp. 192 [on Peirce's "Axioms of Numbers"]). What then is a sign if it is no longer based on Peano numbers, but on a notion of number that is not even a groupoid? The answer is clear: **Nothing.** *Reduce the notion of sign deeper than on Peirce's fundamental categories, and you find yourself in a rain-forest, where there is absolutely no orientation any more possible.*

How should a sign, whose basic function is to substitute an object (and thereby establish the most important metaphysical border we know) be reduced to a "keno-sign" (Kronthaler 1992, p. 296), which cannot substitute and thus represent and which cannot even present because it is essentially nothing (a placeholder for anything), how could such a thing like a "keno-sign" even exist?

The conclusion of this paragraph is that something insane like a polycontextural sign (and thus a polycontextural semiotics) cannot exist. However, the conclusions of the former paragraphs were that semiotics is an essentially polycontextural system whose polycontexturality is just hidden for the border case of C = 2 (i.e. monocontexturality).

Now, we finally have the problem clearly lying before us. As nobody can seriously deny that semiotics – unlike logic and any other formal philosophical or mathematical theory - is based on a multiple and irreducible system of reality, nobody can deny, too, that a sign whose primary function is the substitution and representation of an object, can be based on a proto-logical concept which is unable to substitute and represent, which cannot even present (itself), because it is nothing but a placeholder. In the shortest possible way: **Since there is no induction of emptiness, there is no "keno-sign".**

As we see, we are able to contextuate signs and even prove that there is no eigenreality sensu stricto, because the order of the contextural indices (i, j, k) is turned around to (k, j, i), but what we really do, when we deal with polycontextural (or even monocontextural??) semiotics, is most highly unclear.

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Das Universum der Zeichen. Baden-Baden 1983

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986

Kronthaler, Engelbert, Zahl – Zeichen – Begriff. In: Semiosis 65-68, 1992, pp. 282-302

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Wie viele Kontexturgrenzen hat ein Zeichen? In: electronic Journal for Mathematical Semiotics, 2009

The 10 semiotic dual systems in 4 contextures and 3 number structures

1. This article has more the character of a handout and gives a complete oversight over the 10 Peircean sign classes, their 10 reality thematics in 1, 2, 3 and 4 semiotic contextures, whereby each time proto-, deutero- and trito-number structuress are differentiated. As I have already pointed out (Toth 2009), the term "structure" avoids the term field, since in non-identity based qualitative mathematics, there are no such things as rings or fields.

2. The 10 semiotic dual systems in 1 contexture

(3.1 2.1 1.1) × (1.1 1.2 1.3)
(3.1 2.1 1.2) × (2.1 1.2 1.3)
(3.1 2.1 1.3) × (3.1 1.2 1.3)
(3.1 2.2 1.2) × (2.1 2.2 1.3)
(3.1 2.2 1.3) × (3.1 2.2 1.3)
(3.1 2.3 1.3) × (3.1 3.2 1.3)
(3.2 2.2 1.2) × (2.1 2.2 2.3)
(3.2 2.2 1.3) × (3.1 2.2 2.3)
(3.2 2.3 1.3) × (3.1 3.2 2.3)
(3.3 2.3 1.3) × (3.1 3.2 3.3)

There is no differentiation into proto-, deutero- and trito-structures.

3. The 10 semiotic dual systems in 2 contextures

2-contextural dual-systems correspond to the complex dual systems introduced in Toth (2007, pp. 52 ss.). Instead of number 0 and 1 we used here the already introduced notation with + and -:

(±3.±1 ±2.±1 ±1.±1) × (±1.±1 ±1.±2 ±1 ±3)
(±3.±1 ±2.±1 ±1.±2) × (±2.±1 ±1.±2 ±1.±3)
(±3.±1 ±2.±1 ±1±.3) × (±3.±1 ±1.±2 ±1.±3)
(±3.±1 ±2.±2 ±1.±2) × (±2.±1 ±2.±2 ±1.±3)
(±3.±1 ±2.±2 ±1.±3) × (±3.±1 ±2.±2 ±1.±3)
(±3.±1 ±2.±3 ±1.±3) × (±3.±1 ±3.±2 ±1.±3)
(±3.±2 ±2.±2 ±1.±2) × (±2.±1 ±2.±2 ±2.±3)
(±3.±2 ±2.±2 ±1.±3) × (±3.±1 ±2.±2 ±2.±3)
(±3.±2 ±2.±3 ±1.±3) × (±3.±1 ±3.±2 ±2.±3)

$(\pm3.\pm3\ \ \pm2.\pm3\ \ \pm1.\pm3) \times (\pm3.\pm1\ \ \pm3.\pm2\ \ \pm3.\pm3)$

There is no differentiation into proto-, deutero- and trito-structures.

4. The 10 semiotic dual systems in 3 contextures

4.1. The 10 3-contextural dual systems on proto- and deutero- structure

$(3.1_{[[000],[001],[012]]}\ 2.1_{[0]}\ 1.1_{<[0],[[[000],[001],[012]]>}) \times$
$(1.1_{<[[000],[001],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.1_{[0]}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 1.2_{[0]}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.1_{[0]}\ 1.3_{[[000],[001],[012]]}) \times$
$(1.1_{<[[000],[001],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[012]]})$

$(3.1_{[[000],[001],[012]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]}\ 3.2_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]})\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]})\ 3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]})$
$(3.3_{<[[00],[01], [[000],[001],[012]]>}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[012]]}) \times$
$(3.1_{[[000],[001],[012]]}\ 3.2_{[[00],[01]]}\ 3.3_{<[[000],[001],[012]], [[00],[01]>})$

4.2. The 10 3-contextural dual systems on trito-structure

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.1_{[0]}\ 1.1_{<[0],[[000],[001],[010],[011][012]]>}) \times$
$(1.1_{<[[000],[001],[010],[011],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011][012]]}\ 2.1_{[0]}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 1.2_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.1_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(1.1_{<[[000],[001],[010],[011],[012]],[0]>}\ 1.2_{[0]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.1_{[[000],[001],[010],[011],[012]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]}\ 3.2_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.2_{[0]}) \times$
$(2.1_{[0]}\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]})\ 2.2_{<[[00],[01]], [0]>}\ 2.3_{[[00],[01]]})$

$(3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]})\ 3.2_{[[00],[01]]}\ 2.3_{[[00],[01]]})$

$(3.3_{<[[00],[01], [[000],[001],[010],[011],[012]]>}\ 2.3_{[[00],[01]]}\ 1.3_{[[000],[001],[010],[011],[012]]}) \times$
$(3.1_{[[000],[001],[010],[011],[012]]}\ 3.2_{[[00],[01]]}\ 3.3_{<[[000],[001],]010],[011],[012]], [[00],[01]>})$

5. The 10 semiotic dual systems in 4 contextures

The proto-structure of C = 4 has 4, the deutero-structure has 5, and the trito-structure of C = 4 has 15 qualitative numbers. Therefore, the dual system, (3.1 2.2 1.3) × (3.1 2.2 1.3) in 4 contexures looks

in the proto-structure like

$(3.1_{[[0000],[0001],[0012],[0123]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[0000],[0001],[0012],[0123]]}) \times$
$(3.1_{[[0000],[0001],[0012],[0123]]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[0000],[0001],[0012],[0123]]})$,

in the deutero-structure like

$(3.1_{[[0000],[0001],(0011)l[0012],[0123]]}\ 2.2_{<[0],[[00],[01]]>}\ 1.3_{[[0000],[0001],(0011), [0012],[0123]]}) \times$
$(3.1_{[[0000],[0001],(0011),[0012],[0123]]}\ 2.2_{<[[00],[01]], [0]>}\ 1.3_{[[0000],[0001],(00110012],[0123]]})$,

and in the trito-structure like

$(3.1_{[[0000],[0001],[0010],(0011)l[0012],[0100],[0101],[0102],[0110],[0111[,[0112],[0120[,[121] ,[122],[0123]]}\ 2.2_{<[0],[[00],[01]]>}$
$1.3_{[[0000],[0001],(0011), [0012],[0123]]}) \times$
$(3.1_{[[0000],[0001],[0010],(0011)l[0012],[0100],[0101],[0102],[0110],[0111[,[0112],[0120[,[121],[122],[0123]]}$
$1.3_{[[0000],[0001],(00110012],[0123]]})$,

6. Abbreviations

Especially in notations of sign relation for C ≥ 4, the index notation is awkward, although complete. Thus, according to

C = 2 (±3.±1 ±2.±1 ±1.±3) ≡ (${}^{0}_{1}3.{}^{0}_{1}1\ \ {}^{0}_{1}2.{}^{0}_{1}1\ \ {}^{0}_{1}1.{}^{0}_{1}3$),

we suggest the following notations:

C = 3 (${}^{0}_{3}3.{}^{0}_{3}1\ \ {}^{0}_{3}2.{}^{0}_{3}1\ \ {}^{0}_{3}1.{}^{0}_{3}3$) for proto- and deutero-structure
C = 3 (${}^{0}_{5}3.{}^{0}_{5}1\ \ {}^{0}_{5}2.{}^{0}_{5}1\ \ {}^{0}_{5}1.{}^{0}_{5}3$) for trito-structure

C = 4 (${}^{0}_{4}3.{}^{0}_{4}1\ \ {}^{0}_{4}2.{}^{0}_{4}1\ \ {}^{0}_{4}1.{}^{0}_{4}3$) for proto-structure
C = 4 (${}^{0}_{5}3.{}^{0}_{5}1\ \ {}^{0}_{5}2.{}^{0}_{5}1\ \ {}^{0}_{5}1.{}^{0}_{5}3$) for deutero-structure
C = 4 (${}^{0}_{15}3.{}^{0}_{15}1\ \ {}^{0}_{15}2.{}^{0}_{15}1\ \ {}^{0}_{15}1.{}^{0}_{15}3$) for trito-structure,

and so on for the higher contextures (e.g., C = 5: proto = 5; deutero = 7; trito = 203, according to the Sterling Numbers of the 2nd kind). However, the drawback of this short notation is that the contexture cannot be seen from the notation of the sign relation; cf.

C = 3 (${}^{0}_{5}3.{}^{0}_{5}1\ \ {}^{0}_{5}2.{}^{0}_{5}1\ \ {}^{0}_{5}1.{}^{0}_{5}3$) for trito-structure
C = 4 (${}^{0}_{5}3.{}^{0}_{5}1\ \ {}^{0}_{5}2.{}^{0}_{5}1\ \ {}^{0}_{5}1.{}^{0}_{5}3$) for deutero-structure,

but this problem can be solved by an index o.s.

Bibliography

Toth, Alfred, Grundlagen einer mathematischen Semiotik. Klagenfurt 2007
Toth, Alfred, Signs and qualitative numbers. In: Electronic Journal of Mathematical Semiotics, 2009

Signs and qualitative numbers

1. As Rudolf Kaehr (2009) has shown in an impressive article, it is not enough to introduce the Peircean fundamental categories Firstness, Secondness and Thirdness in order to scoop out the full mathematical potential that lies in sign relations. It is necessary, too, to introduce their inner environments:

Firstness:	Peirce:	A
	Kaehr:	A | a
Secondness:	Peirce:	$A \rightarrow B$
	Kaehr	$A \rightarrow B$ | c
Thirdness:	Peirce:	$A \rightarrow C$
	Kaehr:	$A \rightarrow C \mid b_1 \leftarrow b_2$

As one can see best under Thirdness, this means that with a morphism, also its corresponding hetero-morphism must be introduced. In his book "Toward Diamonds" (Kaehr 2007), Kaehr had illustrated the interplay between morphisms and hetero-morphisms with an auto-trip: I can only approach Stuttgart, when I am leaving Heilbronn at the same time. I.e., with each step foward, I also make a step backward. The steps backwards are the environment of the steps foward.

If we start with the semiotic 3×3 matrix and assume that signs work in 3 contextures, we obtain the following 3-contextural 3×3 matrix

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

and on its basis the 10 Peircean sign classes and their dual reality thematics, contextuated in 3 contextures:

$(3.1_3\ 2.1_1\ 1.1_{1,3}) \times (1.1_{3,1}\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.2_1) \times (2.1_1\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.1_1\ 1.3_3) \times (3.1_3\ 1.2_1\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3)$
$(3.1_3\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 1.3_3)$

$(3.2_2\ 2.2_{1,2}\ 1.2_1)$ × $(2.1_1\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$ × $(3.1_3\ 2.2_{2,1}\ 2.3_2)$
$(3.2_2\ 2.3_2\ 1.3_3)$ × $(3.1_3\ 3.2_2\ 2.3_2)$
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$ × $(3.1_3\ 3.2_2\ 3.3_{3,2})$

2. Now we will have a look at the qualitative numbers of the first 3 contextures. As it is known, qualitative numbers exist in three number areas (the possible term "number field" does not hold for qualitative numbers, which are not based on identity logic). They are called proto-, deutero- and trito-numbers. Therefore, the interface between the three number areas and thee 3 contextures are:

Proto	Deutero	Trito	
0	0	0	C1
00 01	00 01	00 01	C2
000 001 012	000 001 012	000 001 010 011 012	C3

While in C = 1 and C = 2 there is no difference between proto-, deutero- and trito-numbers, in C = 3, proto- and deutero-numbers are split up into 5 trito-numbers. This means: A sign class which lies in 1 contexture, e.g.

(3.a 2.b 1.c)

is trivially the same in all three qualitative number areas. A sign class which lies in 2 contextures, e.g. the complex sign classes introduced in Toth (2007, pp. 52 ss.), e.g.

(±3.±a ±2.±b ±1.±c)

is also the same in all three qualitative number areas, since this is the field of Aristotelian logic and complex number theory based on it.

However, a sign class which lies in 3 contextures, e.g.

$(3.1_3\ 2.2_{1,2}\ 1.2_1) \equiv (3.1_{[[000],[001],[010],[011],[012]}\ 2.2_{<[0],[00],[01]>}\ 1.2_{[o]})$

is "eineindeutig-mehrmöglich" (one-to-one pluri-valent) and corresponds exactly with the multi-ordinality of A. Korzybski" (Kronthaler 1986, p. 60). This means, from the correspondence between sign, resp. sub-sign and qualitative number, we have for (3.1_3 $2.2_{1,2}$ 1.2_1):

$(3.1_3) = 3.1_{[[000]}; 3.1_{[001]}; 3.1_{[010]}; 3.1_{[011]}; 3.1_{[012)}$
$(2.2_{1,2}) = 2.2_{[00)}, 2.2_{01]}$
$(1.2_1) = 1.2_{[0]}$

However, if we would write

[[000],[001],[010],[011],[012], <[0],[00],[01]>, [o])

instead of

(3.1_3 $2.2_{1,2}$ 1.2_1),

the notation of the sign-class with trito-numbers could mean the following sign relations in 3 contextures:

(3.1 2.2 1.2), (3.1 2.2 2.1), (1.3 2.2 1.2), (1.3 2.2 2.1),

since for converse sub-sign-relations we have

C((a.b)) = C ((a.b)°).

Also note that for all sub-signs with 2 or more indices, the respective qualitative numbers are members of ordered sets:

[[000],[001],[010],[011],[012], **<[0],[00],[01]>**, [o]),

while the sets of qualitative numbers per contexture are non-ordered:

[[000],[001],[010],[011],[012]] =
[[012], [011], [010], [001], [000]] =
[[011], [010], [012], [000], [001]] = ...

Therefore, the abolishment of eigenreality in polycontextural semiotics which can be numerically shown by the disequation

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \neq \times(3.1_3\ 2.2_{1,2}\ 1.3_3) = (3.1_3\ 2.2_{2,1}\ 1.3_3)$ with $(2.2_{1,2}) \neq (2.2_{2,1})$

shows that the dissolution of identity which abolishes eigenreality, is nothing else than the total-reflection of the ordered set of n-contextural indices for any n; cf. for n = 4:

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \neq \times(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) = (3.1_{3,4}\ 2.2_{4,2,1}\ 1.3_{3,4})$ with $(2.2_{1,2,4}) \neq (2.2_{4,2,1})$.

However, we also see that with increasing n:

n = 2: $(2.2_{1,2}) \neq (2.2_{2,1})$.
n = 3: with $(2.2_{1,2,4}) \neq (2.2_{4,2,1}) \neq (2.2_{4,1,2}) \neq (2.2_{2,1,4}) \neq (2.2_{2,4,1}) \neq (2.2_{4,1,2})$,

the qualitative-mathematical distance between identity-based monocontextural sign relations and identity-abolished polycontextural sign relations grows, and it grows exactly with n! = 1 · 2 · 3 · ... · n, so that we can measure the abyss or contextural border between an identity-based relation and a non-identity-based relation as

CB = (n! – 2).

Thus, CB gives us the number of permutations between a morphism and its heteromorphism. In the case of n =4, i.e. in a 5-contextural semiotic systems, we have (4! – 2) = 22, i.e.

$(a.b)_{i,j,k,l}$ .... $(a.b)_{l,k,j,i}$

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

(n! –2) = 22 permutations = contextural border (CB)

Bibliography

Kaehr, Rudolf, Toward Diamonds. Glasgow 2007

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

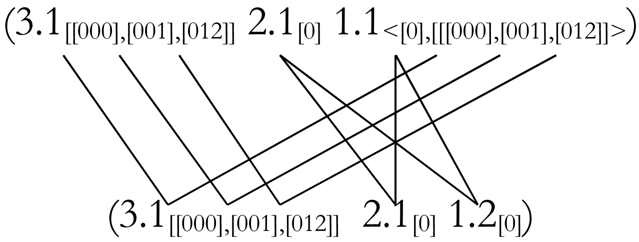
Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

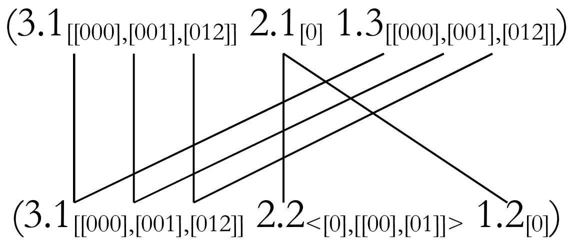
Connections of qualitative numbers in polycontextural sign classes

This article continues Toth (2009a, b). Since in qualitative mathematical systems, there are no set inclusions, but morphogrammatic fragments (Kronthaler 1986, Toth 2003), we show in this little contribution the connections between the successive pairs of the 10 Peircean sign classes in contexture C = 3 in trito-structure.

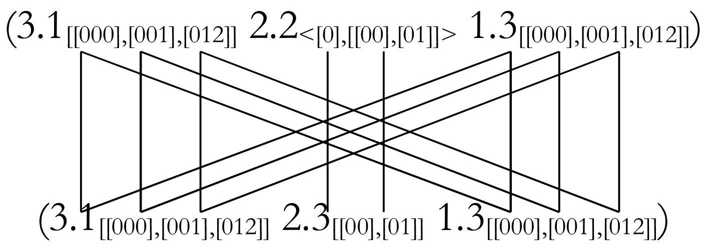
1st/2nd sign class



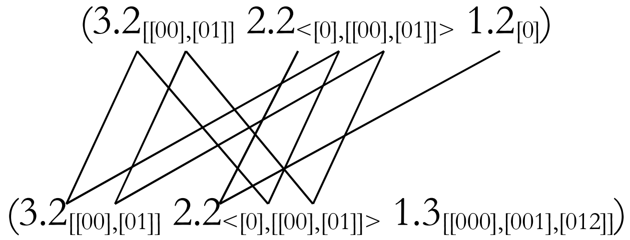
3rd/4th sign class



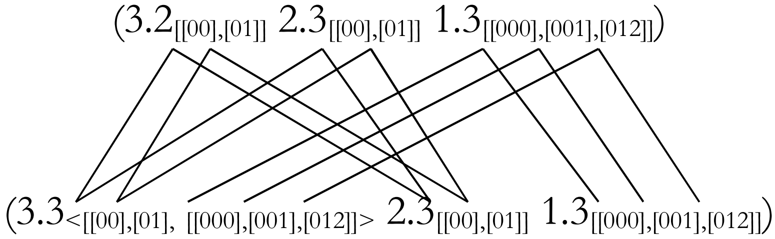
5th/6th sign class



7th/8th sign class



9th/10th sign class

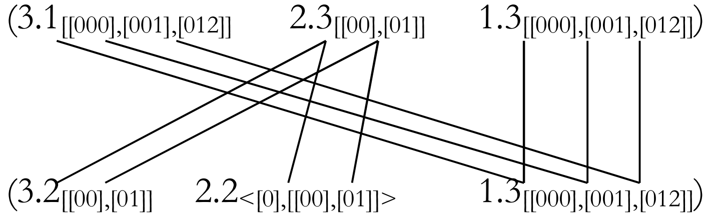


Especially, we also want to look at possible connections between the 6th and the 7th sign class, since we had found in Toth (2008) that there are neither static nor dynamic connections:

6th/7th sign class

(3.1	2.3	1.3)
∅	∅	∅
(3.2	2.2	1.3),

but now look at the connections via qualitative numbers:



Thus, qualitative (proto-, deutero-, trito-) numbers inherent in polycontextural sign establish a network between sign relations that is otherwise inaccessible.

Still note that [00] and [01] are the truth-value functions of classical logic. Moreover, these logical connections are at the same time, of course, semiotic connections. One could therefore say that semiotic connections between qualitative numbers are, depending on the respective contextures (2, 3, 4, ...) , at the same logical (2-, 3-, 4-valued) connections.

Bibliography

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Emanation und Immanation. In: Electronic Journal of Mathematical Semiotics, 2008

Toth, Alfred, Signs and qualitative numbers. In: Electronic Journal of Mathematical Semiotics, 2009a

Toth, Alfred, The 10 semiotic dual systems in 4 contextures and 3 number structures. In: Electronic Journal of Mathematical Semiotics, 2009b

Contextural operations on sub-signs

1. As Kronthaler (1986, pp. 36 ss.) and Toth (2003, pp. 36 ss.) have shown extensively, there are two kinds of operators on qualitative numbers: intra- and inter- (or trans-) operators. Intra-operators work inside of a contexture, inter-operators between contextures. As it is known from elementary semiotic arithmetic, sub-signs can be changed by adding the unit of a representation value (Rpv) to either the triadic or the trichotomic value. Thus, by simple addition or subtraction of Rpv = 1, one can produce the complete semiotic matrix, f. ex.

(1.1) + (0.1) = (1.2); (1.1) + (1.0) = (2.1); (1.1) + (1.1) + (1.0) = (3.2), etc.

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

2. Changing sub-signs by addition of Cv = +1

$1.1_{1,3}$	1.2_{1}	1.3_{3}
+1,0	+1,0	+1,0
$1.1_{2,3}$	1.2_{2}	1.3_{4}

(1.1) is now in the contextures of (3.3), (1.2) in the contexture of (2.3/3.4). The contexture of (1.3) is not defined over 3-cont. SR.

2.1_{1}	$2.2_{1,2}$	2.3_{2}
+1,0	+1,0	+1,0
2.1_{2}	$2.2_{2,2}$	2.3_{4}

(2.1) belongs now to the contexture of (2.3/3.2). (2.2) is excluded, since contextuated sub-signs cannot be self-referential! The new contexture of (2.3) is not defined over 3-cont. SR.

3.1_3 3.2_2 $3.3_{2,3}$

+1,0 +1,0 +1,0

3.1_4 3.2_3 $3.3_{3,3}$

The new contexture of (3.1) is not defined over 3-cont. SR. (3.2) is now in the contexture von (1.3/3.1). (3.3) would be again a case, where a sub-sign – in contrast with the abolishment of the logical law of identity – could be self-indentical.

2. Changing sub-signs by subtraction of Cv = +1

$1.1_{1,3}$ 1.2_1 1.3_3

-1,0 -1,0 -1,0

$1.1_{0,3}$ 1.2_0 1.3_2

(1.1) is now in the contextures of (1.3/3.1), (1.2) in the contexture 0, which is not defined. (1.3) is now in the constructure (1.2/2.1).

2.1_1 $2.2_{1,2}$ 2.3_2

-1,0 -1,0 -1,0

2.1_0 $2.2_{0,2}$ 2.3_1

A contexture 0 is not defined. $(2.2)_2$ is excluded, since identitive morphisms must have at least two (different) indices. (2.3) is not in the contexture of (1.2/2.1).

3.1_3 3.2_2 $3.3_{2,3}$

-1,0 -1,0 -1,0

3.1_2 3.2_1 $3.3_{1,3}$

(3.1) is now in the contexture of (2.3/(3.2), (3.2) in the contexture of (1.2/2.1), and (3.3) is in the contexture of (1.1).

As one sees, by simple addition or subtraction, one can reach any other of the several n-contextural matrices (cf. Toth 2009).

Bibliography

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Mein 1986

Toth, Alfred Die Hiochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Polycontextural matrices. In: Electronic Journal for Mathematical Semiotics, 2009

Connections of the 10 sign classes by contextural transgressions

In Toth (2009), I have shown how sub-signs change their contextures by adding or subtracting units of contexture values. In this article, I will show how the 10 Peircean sign classes hang together by inter-operators (trans-operators) based on adding or subtracting contexture values. Together with semiosis/retrosemiosic processes based on representation values or morphisms and probabilistic sign connections, we thus have here a third possibility for semiotic connections.

$(3.1_3\ 2.1_1\ 1.1_{1,3}) \quad \times \quad (1.1_{3,1}\ 1.2_1\ 1.3_3)$

[-, -, (0, -2)]

$(3.1_3\ 2.1_1\ 1.2_1) \quad \times \quad (2.1_1\ 1.2_1\ 1.3_3)$

[-, -, (+2, 0,)]

$(3.1_3\ 2.1_1\ 1.3_3) \quad \times \quad (3.1_3\ 1.2_1\ 1.3_3)$

[-, (0, +2), (-2, 0)]

$(3.1_3\ 2.2_{1,2}\ 1.2_1) \quad \times \quad (2.1_1\ 2.2_{2,1}\ 1.3_3)$

[-, -, (+2, 0)]

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \quad \times \quad (3.1_3\ 2.2_{2,1}\ 1.3_3)$

[-, (+1, -2), -]

$(3.1_3\ 2.3_2\ 1.3_3) \quad \times \quad (3.1_3\ 3.2_2\ 1.3_3)$

[(-1, 0), (-1, +2), (-2, 0)]

$(3.2_2\ 2.2_{1,2}\ 1.2_1) \quad \times \quad (2.1_1\ 2.2_{2,1}\ 2.3_2)$

[-, -, (+2, 0)]

$(3.2_2\ 2.2_{1,2}\ 1.3_3) \quad \times \quad (3.1_3\ 2.2_{2,1}\ 2.3_2)$

[-, (+1, -2), -]

$(3.2_2\ 2.3_2\ 1.3_3) \quad \times \quad (3.1_3\ 3.2_2\ 2.3_2)$

[(0, +3), -, -]

$(3.3_{2,3}\ 2.3_2\ 1.3_3) \quad \times \quad (3.1_3\ 3.2_2\ 3.3_{3,2})$

As one sees, two the transgression-structures are ambiguous:

[-, -, (+2, 0,)]

$(3.1_3\ 2.1_1\ 1.2_1) \sim (3.1_3\ 2.2_{1,2}\ 1.2_1) \sim (3.2_2\ 2.2_{1,2}\ 1.2_1)$

$(3.1_3\ 2.1_1\ 1.3_3) \sim (3.1_3\ 2.2_{1,2}\ 1.3_3) \sim (3.2_2\ 2.2_{1,2}\ 1.3_3)$

[-, (+1, -2), -]

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \sim (3.2_2\ 2.2_{1,2}\ 1.3_3)$

$(3.1_3\ 2.3_2\ 1.3_3) \sim (3.2_2\ 2.3_2\ 1.3_3)$

Bibliography

Toth, Alfred, Contextural operations on sub-signs. In: Electronic Journal of Mathematical Semiotics, 2009

Decimal equivalents for 3-contextural sign classes

Unlike a quantitative number, a qualitative number consists only in contextur 1 of one number. Already in C = 2, we have 2 qualitative numbers (00, 01), according to the two values of Aristotelian logic. Up to here, all three number structures (proto-, deutero- and trito-structure) are still the same. This changes from C = 3. Here, we have for proto- and deutero-structure 3 and for trito-structure 5 qualitative numbers. In C = 4, there are already 4, 5, and 15, and in C = 5, there are 5, 7, and 126 qualitative numbers. The idea that one Peano-number corresponds to more than one qualitative number is based on the Korzybski-principle of multi-ordinality, i.e. there are choices, but the characters of the choices and their number is strictly determined. Mathematics of the qualities is a system of living organisms and not of dead machines.

We will now look how they 9 sub-signs of the 3-contextural 3×3-matrix

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

are distributed over the 3 contextures of the qualitative numbers and their (decimal) Peano equivalents:

Proto	Deutero		Trito	Deci	
0	0	(1.1), (1.2), (2.1), (2.2)	0	0	C1
00 01	00 01	(2.2), (2.3), (3.2), (3.3)	00 01	0 1	C2
000 001 012	000 001 012	(1.1), (1.3), (3.1), (3.3)	000 001 010 011 012	0 1 3 4 5	C3

In the following we can now determine the 10 sign classes and their dual reality thematics by establishing intervals of Peano numbers over the qualitative numbers which correspond to the sub-signs as in the above table.

$(3.1_3\ 2.1_1\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$	→ I = [[0, 5], [0], [0, 5]]
$(3.1_3\ 2.1_1\ 1.2_1)$	×	$(2.1_1\ 1.2_1\ 1.3_3)$	→ I = [[0, 5, [0], [0]]
$(3.1_3\ 2.1_1\ 1.3_3)$	×	$(3.1_3\ 1.2_1\ 1.3_3)$	→ I = [[0, 5], [0], [0, 5]]
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0]]
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.1_3\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$	→ I = [[0, 1], [0, 1], [0]]
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.2_2\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 2.3_2)$	→ I = [0, 5], |0, 1], [0, 5]]
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 3.3_{3,2})$	→ I = [[0, 5], [0, 1], [0, 5]

Thus, the order of the intervals is.

$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$	→ I = [[0, 1], [0, 1], [0]]

$(3.1_3\ 2.1_1\ 1.2_1)$	×	$(2.1_1\ 1.2_1\ 1.3_3)$	→ I = [[0, 5], [0], [0]]

$(3.1_3\ 2.1_1\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$	→ I = [[0, 5], [0], [0, 5]]
$(3.1_3\ 2.1_1\ 1.3_3)$	×	$(3.1_3\ 1.2_1\ 1.3_3)$	→ I = [[0, 5], [0], [0, 5]]

$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0]]

$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.1_3\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.2_2\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 2.3_2)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 3.3_{3,2})$	→ I = [[0, 5], [0, 1], [0, 5]]

Hence, one recognizes that the 10 sign classes are divided in 5 classes according to their intervals of Peano numbers which are equivalents to the qualitative numbers corresponding to their contextures.

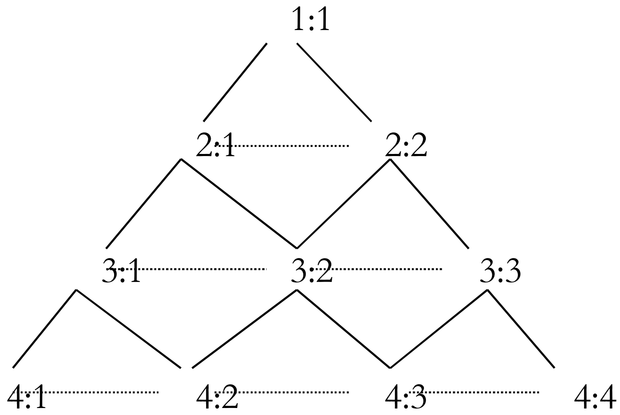
Bibliography

Toth, Alfred, Connections of the 10 sign classes by contextural transgressions. In: Electronic Journal of Mathematical Semiotics, 2009

Model of a 3-adic 3-contextural Proto-Semiotics

1. In Toth (2009a) and several other publications, 3-adic 3- and 4-contextural trito-semiotics has been introduced. For deutero-semiotics cf. Toth (2009b).

A proto-number is unambiguously determined by a pair of numbers (m:n), where m is the length of the kenogram sequence and n is the degree of accretion. Therefore, in the proto-structure, only one kenogram can be iterated at a time. This definition as well as the following figure are taken from Günther (1978, p. 258); vgl. Toth (2003, p. 16):



2. Unlike in deutero- and in trito-semiotics, it is impossible to write the contextural indices, if we define in (m:n) m as the (triadic or trichotomic) value and n as the frequency of this value. Like in deutero-semiotics, in proto.semiotics, too, the definitions of m^n and of (m:n), respectively, are of course not the same as in polycontextural theory, but the number graphs are the same, and this is how close we can come. We can therefore note the system of the 10 Peircean sign classes as basis-system of proto-semiotics as follows:

$(3.1_3\ 2.1_1\ 1.1_{1,3}) \rightarrow ((3:1)\ (2:1)\ (1:4))$
$(3.1_3\ 2.1_1\ 1.2_1) \rightarrow ((3:1)\ (2:2)\ (1:3))$
$(3.1_3\ 2.1_1\ 1.3_3) \rightarrow ((3:2)\ (2:1)\ (1:3))$
$(3.1_3\ 2.2_{1,2}\ 1.2_1) \rightarrow ((3:1)\ (2:3)\ (1:2))$
$(3.1_3\ 2.2_{1,2}\ 1.3_3) \rightarrow ((3:2)\ (2:2)\ (1:2))$
$(3.1_3\ 2.3_2\ 1.3_3) \rightarrow ((3:3)\ (2:1)\ (1:2))$
$(3.2_2\ 2.2_{1,2}\ 1.2_1) \rightarrow ((3:1)\ (2:4)\ (1:1))$
$(3.2_2\ 2.2_{1,2}\ 1.3_3) \rightarrow ((3:2)\ (2:3)\ (1:1))$
$(3.2_2\ 2.3_2\ 1.3_3) \rightarrow ((3:3)\ (2:2)\ (1:1))$
$(3.3_{2,3}\ 2.3_2\ 1.3_3) \rightarrow ((3:4)\ (2:1)\ (1:1))$

The mappings are bijective. However, also the contextures are clear, as long as always the same contextural indices are mapped to the same sub-signs.

Just one interesting structure which appears only in proto-semiotics: The proto-structures of the eigenreal sign class

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \quad \rightarrow \quad ((3{:}2)\ (2{:}2)\ (1{:}2))$

and of the categorial or "weak eigenreal" sign class (Bense 1992, p. 40)

$(3.3_{2,3}\ 2.2_{1,2}\ 1.1_{1,3}) \rightarrow \quad ((3{:}2)\ (2{:}2)\ (1{:}2))$

are identical! So, at least on proto-semiotic level, also the Genuine Category Class does show the structural feature of eigenreality.

Bibliography

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

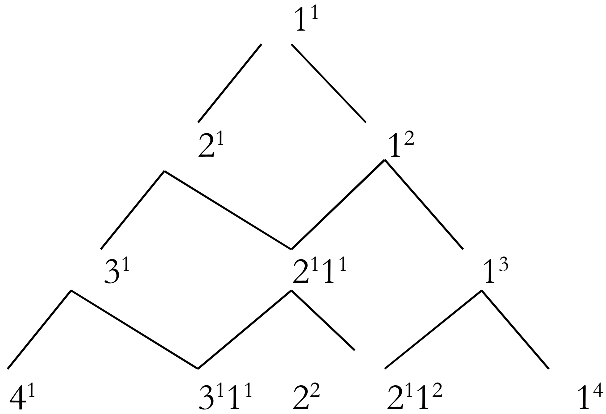
Toth, Alfred, New elements of theoretical semiotics (NETS), based on the work of Rudolf Kaehr. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Model of a 3-adic 3-contextural Deutero-Semiotics. In: Electronic Journal of Mathematical Semiotics, 2009b

Model of a 3-adic 3-contextural Deutero-Semiotics

1. In Toth (2009) and several other publications, 3-adic 3- and 4-contextural trito-semiotics has been introduced. However, in any polycontextural system, it is necessary to consider also the respective proto- and deutero-structures.

In a deutero-structure, each kenogram can be iterated, so that pair-set notation is not sufficient anymore. A deutero-sequence and its corresponding number are unambiguously determined by the number of partitions m^n, where m is the length of the iteration and n the number of different kenograms per length of iteration. This definition as well as the following figure are taken from Günther (1978, p. 258); vgl. Toth (2003, p. 16):



2. In semiotics, triadic-trichotomic values, can be written in a notation which resembles to power functions (cf. Toth 2007, p. 215; Toth 2003, pp. 36 ss.) which has been called frequency notation because the basis indicates the triadic value and the exponent the frequency of the trichotomic value. Hence, although semiotic frequency notation and logical deutero-notation are of course not the same, we get exactly the same kind of tree model for deutero-signs as for deutero-numbers. We can therefore note that system of the 10 Peircean sign classes and their dual reality thematics as basis-system of deutero-semiotics:

$(3^1_3\ 2^1_1\ 1^1_{1,3}) \times (1^1_{3,1}\ 1^2_1\ 1^3_3)$

$(3^1_3\ 2^1_1\ 1^2_1) \times (2^1_1\ 1^2_1\ 1^3_3)$

$(3^1_3\ 2^1_1\ 1^3_3) \times (3^1_3\ 1^2_1\ 1^3_3)$

$(3^1_3\ 2^2_{1,2}\ 1^2_1) \times 2^1_1\ 2^2_{2,1}\ 1^3_3)$

$(3^1_3\ 2^2_{1,2}\ 1^3_3) \times (3^1_3\ 2^2_{2,1}\ 1^3_3)$

$(3^1{}_3\ 2^3{}_2\ 1^3{}_3) \times (3^1{}_3\ 3^2{}_2\ 1^3{}_3)$

$(3^2{}_2\ 2^2{}_{1,2}\ 1^2{}_1) \times (2^1{}_1\ 2^2{}_{2,1}\ 2^3{}_2)$

$(3^2{}_2\ 2^2{}_{1,2}\ 1^3{}_3) \times (3^1{}_3\ 2^2{}_{2,1}\ 2^3{}_2)$

$(3^2{}_2\ 2^3{}_2\ 1^3{}_3) \times (3^1{}_3\ 3^2{}_2\ 2^3{}_2)$

$(3^3{}_{2,3}\ 2^3{}_2\ 1^3{}_3) \times (3^1{}_3\ 3^1{}_2\ 3^3{}_{3,2})$

Note that a further simplification $(1^1{}_{3,1}\ 1^2{}_1\ 1^3{}_3) \neq 1^6$ is impossible because of the different contextures involved.

Bibliography

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003
Toth Alfred, Grundleung einer mathematischen Semiotik. Klagenfurt 2009 (2009a)
Toth, Alfred, New elements of theoretical semiotics (NETS), based on the work of Rudolf Kaehr. In: Electronic Journal for Mathematical Semiotics, 2009b

The incompleteness of Peircean semiotics at the hand of intervals of natural numbers

1. That the Peirce-Bensean semiotic with its 10 sign classes, their 10 dual reality thematics are incomplete, was already shown in a number of article that are to be found in my book "Semiotic Structures and Processes" (Toth 2008). There two restriction laws that cause this incompleteness:

1.1. The Law of Triadicity

This means that a sign relation has to consist 1. of three relations x, y z and that 2. x, y, z have to be mapped on the fundamental categories 1, 2, 3 so that x, y and z are pairewise different. Therefore, semiotic relations like (3.a 3.b 1.c) or (2.b 2.b 2.b) are not considered sign classes. Note that this law does not exclude sign classes of the forms (3.a 1.c 2.b), (2.b 3.a 1.c), (2.b 1.c 3.a), (1.c 3.a 2.b), and (1.c 2.b 3.a) besides the "canonical" Peircean form (3.a 2.b 1.c), cf. Toth (2008, pp. 177 ss.).

1.2. The Law of Trichotomic Inclusion

This second restriction excludes sign relations like (3.3 2.2 1.1) or generally sign relations whose trichotomic values are smaller than the trichotomic values of the sub-sign before: (3.a 2.b 1.c) with ($a \leq b \leq c$). This law thus reduces the possible total of $3^3 = 27$ to 10 sign classes.

In Toth (2009), 1.) We had mapped the sub-signs of a 3-contextural 3×3 matrix to the first 3 contextures of qualitative numbers. 2.) We have given the decimal correspondences to these qualitative (proto-, deutero- and trito-) numbers:

Proto	Deutero		Trito	Deci	
0	0	(1.1), (1.2), (2.1), (2.2)	0	0	C1
00 01	00 01	(2.2), (2.3), (3.2), (3.3)	00 01	0 1	C2
000 001 012	000 001 012	(1.1), (1.3), (3.1), (3.3)	000 001 010 011 012	0 1 3 4 5	C3

In doing so, it was possible to determine the 10 sign classes and their dual reality thematics by intervals of Peano numbers over the qualitative numbers which correspond to the sub-signs as in the above table.

$(3.1_3\ 2.1_1\ 1.1_{1,3})$	×	$(1.1_{3,1}\ 1.2_1\ 1.3_3)$	→ I = [[0, 5], [0], [0, 5]]
$(3.1_3\ 2.1_1\ 1.2_1)$	×	$(2.1_1\ 1.2_1\ 1.3_3)$	→ I = [[0, 5, [0], [0]]
$(3.1_3\ 2.1_1\ 1.3_3)$	×	$(3.1_3\ 1.2_1\ 1.3_3)$	→ I = [[0, 5], [0], [0, 5]]
$(3.1_3\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0]]
$(3.1_3\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.1_3\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 1.3_3)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.2_2\ 2.2_{1,2}\ 1.2_1)$	×	$(2.1_1\ 2.2_{2,1}\ 2.3_2)$	→ I = [[0, 1], [0, 1], [0]]
$(3.2_2\ 2.2_{1,2}\ 1.3_3)$	×	$(3.1_3\ 2.2_{2,1}\ 2.3_2)$	→ I = [[0, 5], [0, 1], [0, 5]]
$(3.2_2\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 2.3_2)$	→ I = [0, 5], |0, 1], [0, 5]]
$(3.3_{2,3}\ 2.3_2\ 1.3_3)$	×	$(3.1_3\ 3.2_2\ 3.3_{3,2})$	→ I = [[0, 5], [0, 1], [0, 5]

The order of the intervals is as follows. We also introduce capitals in order to determine the types of intervals which we will look at after:

$(3.2_2\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 2.3_2)$ →	I = [[0, 1], [0, 1], [0]]	AAB
$(3.1_3\ 2.1_1\ 1.2_1) \times (2.1_1\ 1.2_1\ 1.3_3)$ →	I = [[0, 5], [0], [0]]	CBB
$(3.1_3\ 2.1_1\ 1.1_{1,3}) \times (1.1_{3,1}\ 1.2_1\ 1.3_3)$ →	I = [[0, 5], [0], [0, 5]]	CBC
$(3.1_3\ 2.1_1\ 1.3_3) \times (3.1_3\ 1.2_1\ 1.3_3)$ →	I = [[0, 5], [0], [0, 5]]	
$(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3)$ →	I = [[0, 5], [0, 1], [0]]	CAB
$(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3)$ →	I = [[0, 5], [0, 1], [0, 5]]	CAC
$(3.1_3\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 1.3_3)$ →	I = [[0, 5], [0, 1], [0, 5]]	
$(3.2_2\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 2.3_2)$ →	I = [[0, 5], [0, 1], [0, 5]]	
$(3.2_2\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 2.3_2)$ →	I = [[0, 5], [0, 1], [0, 5]]	
$(3.3_{2,3}\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 3.3_{3,2})$ →	I = [[0, 5], [0, 1], [0, 5]]	

First, one recognizes that the 10 sign classes are divided in only 5 classes according to their intervals of Peano numbers which are equivalents to the qualitative numbers corresponding to their contextures.

Second, looking at the types AAB, CBB, CBC, CAB, and CAC, we can easily find the lacking types:

AAB (BBC) (CCA) (CCB) (ABC)

(ABA)	(BCB)	CAC	CBC	(ACB)
(BAA)	CBB	(ACC)	(BCC)	(BAC)
			(BCA)	
			CAB	
			(CBA)	

The types in parenthesis are minimal. E.g., in analogy to AAB and CCA, there could be also stipulated a type CCB, etc. However, if we start with the above 5 types and their permutations, then we find in the 10 sign classes and their reality thematics actually 5/18 types, i.e. less then 1/3. But let us now look at the lacking types of intervals:

AAB = [[0, 1], [0, 1], [0]] → ABA = [[0, 1], [0], [0, 1]], BAA = [[0], [0, 1], [0, 1]]
CBB = [[0, 5], [0], [0]] → BBC = [[0], [0], [0, 5]], BCB = [[0], [0, 5], [0]]
CAC = [[0, 5], [0, 1], [0, 5]] → CCA = [[0, 5], [0, 5], [0, 1]],
ACC = [[0, 1], [0, 5], [0, 5]]
CBC = [[0, 5], [0], [0, 5]] → CCB = [[0, 5], [0, 5], [0]], BCC = [[0], [0, 5], [0, 5]]
CAB = [[0, 5], [0, 1], [0]] → ABC = [[0, 1], [0], [0, 5]], ACB = [[0, 1], [0, 5], [0]],
BAC = [[0], [0, 1], [0, 5]], BCA = [[0], [0, 5], [0, 1]],
CAB = [[0, 5], [0, 1], [0]], CBA = [[0, 5], [0], [0, 1]]

We can now try to reconstruct sign relations that are lacking in the Peirce-Bensean system, however, in the multi-ordinal possibility given by the ambiguousness of the qualitative numbers which have been mapped to the Peano numbers in our intervals:

ABA = [[0, 1], [0], [0, 1]]	=	e.g., (3.2 2.2 2.3)
BAA = [[0], [0, 1], [0, 1]]	=	e.g., (2.2 3.2 3.3)
BBC = [[0], [0], [0, 5]]	=	e.g., (2.2, 2.2, 3.1)
BCB = [[0], [0, 5], [0]	=	e.g., (2.2, 3.1, 2.2)
CCA = [[0, 5], [0, 5], [0, 1]]	=	e.g., (3.1, 3.1, 2.3)
ACC = [[0, 1], [0, 5], [0, 5]]	=	e.g., (2.3, 3.1, 3.1)
CCB = [[0, 5], [0, 5], [0]]	=	e.g., (3.1, 3.1, 2.2)
BCC = [[0], [0, 5], [0, 5]]	=	e.g., (2.2, 3.1, 3.1)
ABC = [[0, 1], [0], [0, 5]]	=	e.g., (3.2, 2.2, 3.1)
ACB = [[0, 1], [0, 5], [0]]	=	e.g., (3.2, 3.1, 2.2)
BAC = [[0], [0, 1], [0, 5]]	=	e.g., (2.2, 3.2, 3.1)
BCA = [[0], [0, 5], [0, 1]]	=	e.g., (2.2, 3.1, 3.2)
CAB = [[0, 5], [0, 1], [0]]	=	e.g., (3.1, 3.2, 2.2)
CBA = [[0, 5], [0], [0, 1]]	=	e.g., (3.1, 2.2, 3.2)

Although it is possible also to (re-)construct sign classes which obey the Law of Triadicity, we have given here choices in which this law is violated in order to show that all these types could never be realized without the abolishing of this law. Let us sum up: The abolishment of the Law of Trichotomic Inclusion enables to reach all the combinatory possible $3^3 = 27$ 3-adic sign relation. The abolisment of the Law of Triadicity enables to reach all the combinatory possible $9^3 = 243$ possible 3-adic sign relation whose members have not to be anymore pairwise different. The analysis of the intervals in this work therefore leads to exactly the same results as our former studies about sub-signs and semioses.

Bibliography

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Toth, Alfred, Decimal equivalents for 3-contextural sign classes. In: Electronic Journal of Mathematical Semiotics, 2009

Matching conditions of sub-signs in Stein sequences

1. As defined in Toth (2009), we start with the following aleatoric number sequence or Stein sequence whose elements are elements from the following finite set

{0, 1, 3, 4, 5}

1340543514531100151454531105435134054351
4531101514545311054453110151455435135435
3514531100151454531105453110145453110543
51340543514531101514545345311035

If we distribute the 9 sub-signs of the semiotic 3×3 matrix to those decimal numbers that correspond to the qualitative numbers of the contextures 1-3 (cf. Toth 2009), we get the following mappings:

0 ← (1.1), (1.2), (1.3), (2.1), (2.2), (2.3), (3.1), (3.2), (3.3)
1 ← (1.1), (1.3), (2.2), (2.3), (3.1), (3.2), (3.3)
2 ← Ø
3 ← (1.1), (1.3), (3.1), (3.3)
4 ← (1.1), (1.3), (3.1), (3.3)
5 ← (1.1), (1.3), (3.1), (3.3)

By these mappings we can substitute the above Stein number sequence by the following sequence of sub-signs of the semiotic 3×3 matrix:

(1.1), (1.3), (2.2), (2.3), (3.1), (3.2), (3.3), (1.1),
(.1 ≡ 1.) (.2 ≡ 2.) (.1 ≡ 3.) (.3 ≡ 1.)
(3. ≡ 2.) (.3 ≡ 3.) (.2 ≡ 3.)
(1.3), (3.1), (3.3), (1.1), (1.3), (3.1), (3.3), (1.1),
(.3 ≡ 3.) (.3 ≡ 1.) (.3 ≡ 3.) (.3 ≡ 1.)
(.1 ≡ 3.) (.1 ≡ 1.) (.1 ≡ 3.)

(1.2), (1.3), (2.1), (2.2), (2.3), (3.1), (3.2), (3.3)

(.2 ≡ 1.) (.1 ≡ 2.) (.3 ≡ 3.) (.3 ≡ 1.)

(.3 ≡ 2.) (.2 ≡ 2.)

As we see, the matching pairs of sub-signs start to come back soon. Moreover, as already pointed out in Toth (2009), the structure is somewhat iterative, because the order of the elements in the above mappings has left unchanged. Therefore, every sub-sign can match every sub-sign:

(1.1) ≡ (1.1)				
(1.1) ≡ (1.2)	(1.2) ≡ (1.2)			
(1.1) ≡ (1.3)	(1.2) ≡ (1.3)	(1.3) ≡ (1.3)		
(1.1) ≡ (2.1)	(1.2) ≡ (2.1)	(1.3) ≡ (2.1)	(2.1) ≡ (2.1)	
(1.1) ≡ (2.2)	(1.2) ≡ (2.2)	(1.3) ≡ (2.2)	(2.1) ≡ (2.2)	(2.2) ≡ (2.2)
(1.1) ≡ (2.3)	(1.2) ≡ (2.3)	(1.3) ≡ (2.3)	(2.1) ≡ (2.3)	(2.2) ≡ (2.3)
(1.1) ≡ (3.1)	(1.2) ≡ (3.1)	(1.3) ≡ (3.1)	(2.1) ≡ (3.1)	(2.2) ≡ (3.1)
(1.1) ≡ (3.2)	(1.2) ≡ (3.2)	(1.3) ≡ (3.2)	(2.1) ≡ (3.2)	(2.2) ≡ (3.2)
(1.1) ≡ (3.3)	(1.2) ≡ (3.3)	(1.3) ≡ (3.3)	(2.1) ≡ (3.3)	(2.2) ≡ (3.3)

(2.3) ≡ (2.3)			
(2.3) ≡ (3.1)	(3.1) ≡ (3.1)		
(2.3) ≡ (3.2)	(3.1) ≡ (3.2)	(3.2) ≡ (3.2)	
(2.3) ≡ (3.3)	(3.1) ≡ (3.3)	(3.2) ≡ (3.3)	(3.3) ≡ (3.3),

hence totally 55 matching conditions.

Well understood, these matching conditions concern solely sub-sign with environments of the form

$(a.b)_{i,j}$

and not

$(b.a)$ $(a.b)_{j,i}$

Thus, for the corresponding contextuated sub-sign of a Peano number of the form

-3,

we would have either $(a.b)_{j,i}$ or $(b.a)_{j,i}$.

Moreover, if i, j refer to 2 contextures, for –x (x ∈ {0, 1, 3, 4, 5) we have 4 possibilities for sub-signs:

(-a.b), (a.-b), (-a-,.-b).

Bibliography

Toth, Alfred, Sign relations from Stein number sequences. In: Electronic Journal of Mathematical Semiotics, 2009

Semiotic proto- and deutero-addition

1. How are sub-signs added (and subtracted)? According to Berger (1976), semiotic addition is union, e.g.

(2.1) + (2.1) = (2.2)
(2.2) + (2.3) = (2.3), etc.

2. A very interesting suggestion comes from Kaehr (2009), namely rejection, e.g.

(2.1) + (2.2) = (2.3)
(2.1) + (2.3) = (2.1)
(2.2) + (2.3) = (2.1)

However, the problem is that the sum is ambiguous if only one trichtomic value appears in the summands, e.g.

(2.1) + (2.1) = (2.2)? (2.3)?

3. In Toth (2009a), I have already suggested that one could add sub-signs with contextures.

3.1. A first possibility is to build the max both of the trichotomic values and of the contextures, thus max (a.b) and max(i,j), e.g.

$\max((2.1)_1, (2.2)_{1,2}) = (2.2)_2$

3.2. The second possibility is the union building not only from the sub-signs, but also from the contextures, e.g.

$(2.1)_1 + (2.2)_{1,2} = (2.2)_{1,2}$

4. Specifically for semiotic deutero-numbers, we have then for (2.1) and (2.2):

4.1. $(2^1{}_1) + (2^2{}_{1,2}) = (2^2{}_2)$

4.2. $(2^1{}_1) + (2^2{}_{1,2}) = (2^3{}_2)$

5. However, while addition and subtraction of semiotic trito-numbers do not cause any problems (Toth 2009b), the respective operations in the number structure of proto-signs do. Therefore, the question arises how we should best define proto-signs. If we define a proto-sign according to polycontextural theory as a pair of numbers

(m:n), where m determines the length and n the number of different kenos, then we would get the following semiotic "proto-matrix":

(2:1) (2:2) (2.2)

(2:2) (2:1) (2:2)

(2:2) (2:2) (2:1)

However, here, most of the non-genuine sub-signs coincide, since from the standpoint of a kenogramm, (2.3) = (1.2) = (0.1), etc. (Kronthaler's "Normal-formoperator).

However, another possibility to write the sub-signs as proto-signs is by interpreting m as semiotic value an n as the occurrence of this semiotic value in a sub-sign. Thus, e.g., in (1.1) 1 is the (triadic) value, and its occurrence is 2, since it is also trochotomic value. But in (1.2), the triadic value 1 occurs only once, and the triadic value 2 occurs only, too, i.e. we get (1:1) (2:1). In this case, the semiotic proto-matrix looks as follows:

(1:2) — (1:1) (2:1) (1:1) (3:1)

(2:1) (1:1) (2:2) — (2:1) (3:1)

(3:1) (1:1) (3:1) (2.:1) (3:2) —

As we see here, besides the genuine sub-signs, all sub-signs are **pairs** of proto-numbers. So, if we want to add (2.1) + (2.2), like above, we get the following strange result:

(2.1) + (2.2) = (2:1) (1:1) + (2.2) — = (2.3) (1.1) (?).

But if we add according to the first matrix:

(2:2) + (2:1) = (2:2),

then the sum says not more than it consists again of a single sub-sign with the same triadic value and as trichotomic value max ((2.1), (2.2)), since three different kenos at a length of 2 are not reachable for a "rejective" (*(2.3)). However, this would mean that the successor of (2:2) or (2.1) is identical with the successor of (2:1) or (2.2), namely (2:1) or (2.2), which is apparently nonsense.

Thus, let us attempt at adding proto-signs starting with the notation of successor.

(1:2) —   (1:1) (2:1)   (1:1) (3:1)

(2:1) (1:1)   (2:2) —   (2:1) (3:1)

(3:1) (1:1)   (3:1) (2.:1)   (3:2) —

In a "natural" way, a proto-sign (m:n) has basically two successors: 1) (m+1):n, and 2) m: (n+1). Now we see from the matrix

(1.2) —→ (1:1) (2:1)

(1.2) —→ (2:1) (1:1)

(1:2) —→ (2:2)

"Geometrically" these are thus the three possible successors of (1:2) —. That means, that the successor structures are:

(a.b) —→ (a:a) (b:a)

(a:b) —→ (b:a) (a:a)

(a:b) —→ (b:b),

or generally with a successor operator S(a) = (a+1):

(a.(a+1)) —→ (a:a) ((a+1):a)

(a:(a+1)) —→ ((a+1):a) (a:a)

(a:(a+1)) —→ ((a+1):(a+1)).

Bibliography

Berger, Wolfgang, Zur Algebra der Zeichenklassen. In Semiosis 4, 1986, pp. 20-24

Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2009)

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1976

Toth, Alfred, Contextural operations on sub-signs. In: Electronic Journal of Mathematical Semiotics, 2009a
Toth, Alfred, Decimal equivalents for 3-contextural sign classes. In: Electronic Journal of Mathematical Semiotics, 2009b

n-ads and n^{th} contextures

1. In Toth (2009), I had mapped the 9 sub-signs of the 3-contextural semiotic 3×3 matrix to the first 3 contextures of the system of qualitative numbers, here containing also the three number structures of proto-, deutero- and trito-numbers:

Proto	Deutero		Trito	Deci	
←	←	(1.1), (1.2),			
0	0	(2.1), (2.2)	0	0	C1
←	←	(2.2), (2.3),			
00	00	(3.2), (3.3)	00	0	
01	01		01	1	C2
←	←	(1.1), (1.3),			
000	000	(3.1), (3.3)	000	0	
001	001		001	1	
012	012		010	3	C3
			011	4	
			012	5	

However, if we disregard the identitive morphisms which appear in 2 contextures in a 3-contextural semiotics (in 3 contextures in a 4-contextural semiotic, etc.), we can easily see that there is connection between the value of a semiotic relation and its corresponding contexture:

K1:	0	(1.1), (1.2), (2.1)	monads
K2:	00, 01	(2.2), (2.3), (3.2)	dyads
K3:	000, 001, 010, 011, 012	(3.3), (3.1), (1.3)	triads

Form the way how the sub-signs are ordered now, one can see that n-ads belong to n-th contextures, with the exception that the dual sub-signs are always in the same contextures. The double appearance of the genuine sub-signs serves the decomposition of the respective matrices, cf. Günther (1979, pp. 231 ss.).

2. In a next step we have to ask what the differentiation between the three qualitative number structures mean for semiotics. Since all three number structures have to be mapped on the 9 sub-signs of the semiotic 3×3 matrix, it is a priori senseless to take over the definitions based on length and iteration/accretion of keno-symbols which work for qualitative numbers, but not for signs.

2.1. As a proto-sign we define a pair (m:n) consisting of a semiotic (i.e. triadic or trichotomic) value m and the occurrence of this value inside of a sign relation (dyad, triad). E.g., (2.1) = (2:1) (1:1); (2.2) = (2:2). As one sees, in most cases, sub-signs have to be represented by pairs of pairs of proto-signs rather than by pairs alone.

Therefore, the semiotic proto-matrix looks as follows:

(1:2) —	(1:1) (2:1)	(1:1) (3:1)
(2:1) (1:1)	(2:2) —	(2:1) (3:1)
(3:1) (1:1)	(3:1) (2.:1)	(3:2) —

2.2. As a deutero-sign we define an "exponential" function m^n consisting of a semiotic (i.e. triadic or trichotomic) value m and the occurrence of this value inside of a sign relation (dyad, triad). E.g., (2.1) = 2^1; (2.2) = 2^2. However, this is not just another writing of the pair-notation for proto-signs. There are two most important differences:

1. It is impossible to note the contextures (inner semiotic environments) to the proto-sign notation (e.g. 3.1_3 $2.2_{1,2}$ 1.3_3 ≠ (3:2 2:2 1:2)).

2. The deutero-sign notatio, already introduced in Toth (2007, p.215), allows a "ligature"-writing especially for reality thematics. (E.g. (3.1 2.3 1.3 × 3.1 3.2 1.3) = 3^2 1^1; (3.3 2.3 1.3 × 3.1 3.2 3.3) = 3^3, etc.). So, outside of well-defined sign classes and their bijective mappings to reality thematics, the fundamental-categorial or trito-structure may to be reconstructible form the deutero-structure. Unlike the trito-notation, the deutero-notation also allows to show the inner structures of thematizing and thematized realities in reality thematics (cf. Toth 2007, pp. 215 ss.).

2.3. As a trito-sign we define a regular numeric sign class together with its semiotic contextures in the form of inner semiotic environments (Kaehr 2008). Through dualization we get the corresponding reality thematics in which not only the order of the sub-signs and the prime-signs, but also the order of the contextural indices are turned around (semiotic diamond theory). E.g. (3.1_3 $2.2_{1,2}$ 1.3_3) × (3.1_3 $2.2_{2,1}$ 1.3_3).

2.4. E.g., we have for the notation of the sign class (3.1 2.1 1.3) in the proto-, deutero- and trito-strcuture:

Proto-SCl: ((3:2) (2:1) (1:3)) × ((3:2) (2:1) (1:3))

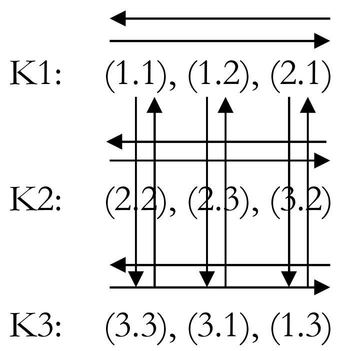
↓

Deutero-SCl: $(3^1\ 2^1\ 1^1) \times (3^1\ 1^2)$

↓

Trito-SCl: $(3.1_3\ 2.1_{1,2}\ 1.3_3) \times (3.1_3\ 1.2_{2,1}\ 1.3_3)$

3. In a third and last step, we can now determine the intra- and trans-successors and predecessors of every sub-sign per contexture and per qualitative number structure. However, in the case of semiotics, this is trivial, at least as long we stay in 3 contextures as we did up to now: Every sub-sign is at the same time the predecessor and the successor of every sub-sign.



Bibliography

Günther, Gotthard, Beiträge zur Grundlegung einer oprationsfähigen Dialektik.Vol. 2. Hamburg 1979

Kaehr, Rudolf, Toth's semiotic diamonds.
http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf
2009)

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007

Toth, Alfred, Decimal equivalents for 3-contextural sign classes. In: In: Electronic Journal of Mathematical Semiotics, 2009

Contextures, relations, and dimensions

1. In Toth (2009b), I have shown that there is a correspondence between semiotic contextures and n-adic relations insofar as monads correspond with C1, dyads with C2, and triads with C3:

Proto	Deutero		Trito	Deci	
0	0	(1.1), (1.2), (2.1), (2.2)	0	0	C1
00 01	00 01	(2.2), (2.3), (3.2), (3.3)	00 01	0 1	C2
000 001 012	000 001 012	(1.1), (1.3), (3.1), (3.3)	000 001 010 011 012	0 1 3 4 5	C3

K1:	0	(1.1), (1.2), (2.1)	monads
K2:	00, 01	(2.2), (2.3), (3.2)	dyads
K3:	000, 001, 010, 011, 012	(3.3), (3.1), (1.3)	triads

2. However, that is not all. We have to consider the structures of qualitative numbers for every contexture and every number structure. Then, we obtain

0	1, 2, 3	1-dim semiotics
00 01	(1.1), (2.2), (3.3) (1.2)/(2.1), (1.3)/(3.1), (2.3/(3.2)	2-dim semiotics

000	(1.1.1), (2.2.2), (3.3.3)	
001	(1.1.2), (1.1.3), (2.2.1), (2.2.3), (3.3.1), (3.3.2)	
010	(1.2.1), (1.3.1), (2.1.2), (2.3.2), (3.1.3), (3.2.3)	3-dim semiotics
011	(1.2.2), (1.3.3), (2.1.1), (2.3.3), (3.1.1), (3.2.2)	
012	(1.2.3), (1.3.2), (2.1.3), (2.3.1), (3.1.2), (3.2.1)	

What we thus get here, is a one-to-one correspondence not only between n.th contexture and n-adic sign relation, but of n.th contexture, n-adic sign relation and n.th dimension. The notion of semiotic dimension (unlike the use of the same word in the works of Ch. Morris) had been introduced in mathematical semiotics by me (Toth 1993, pp. 28 ss.). Therefore, 1-dimensional semiotics is linear semiotics in the geometrical sense of Bernays (1997, p.2), 2-dimensional semiotics is plain semiotics, and 3-dimensional semiotics is spatial semiotics (cf. Toth 2007, p. 11).

1-dimensional semiotics is the order of the three fundamental categories. 2-dimensional semiotics is Peirce-Bense-semiotics based on the dyadic constituency of sign classes and reality thematics. 3-dimensional semiotics is Stiebing-semiotics based on the triadic constituency of the sub-signs (cf. for all that, extensively, my two volumes "Mehrdimensionale Semiotik", Toth 2009a).

Since it is thus possible to identify n-th contexture and n-th dimension of a sign relation, artificial separations as well as specifications can be introduced by assigning contextural values to the sign relations of the three dimensions, which do not agree with the contextural values.

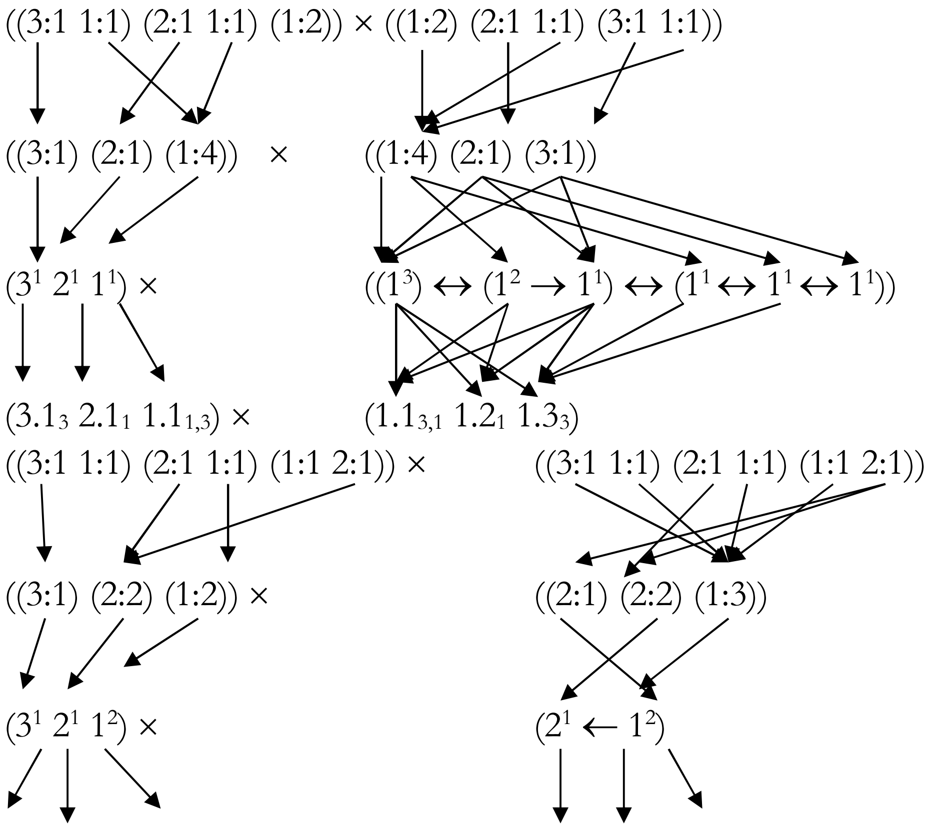
Bibliography

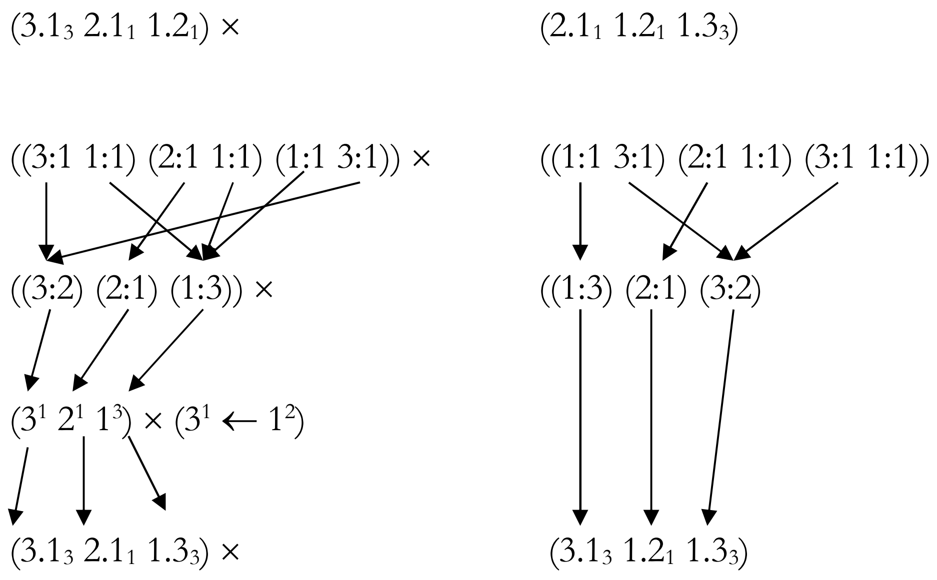
Bernays, Paul, Grundlagen der Geometrie. 14th Edition Stuttgart 1999
Toth, Alfred, Semiotik und Theoretische Linguistik. Tübingen 1993
Toth, Alfred Grundlegung einer mathematischen Semiotik. Klagenfurt 2007
Toth, Alfred, Mehrdimensionale Semiotik. 2 vols. Klagenfurt 2009 (2009a)
Toth, Alfred, n-ads and nth contextures. In: Electronic Journal for Mathematical Semiotics, 2009b

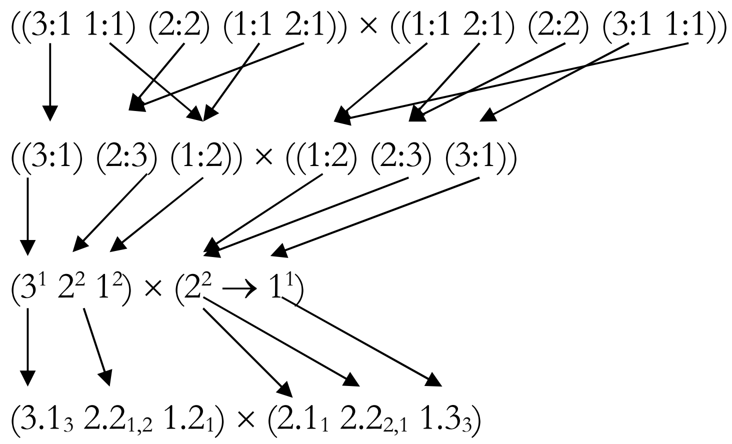
The mappings of semiotic proto-, deutero- and trito-structures

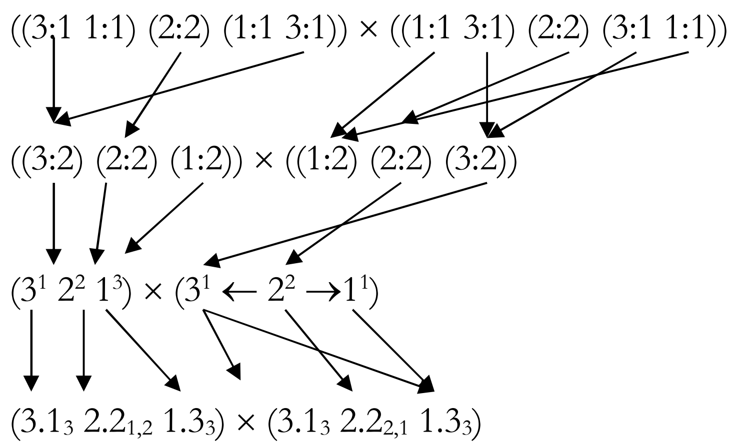
1. A sign is an element that substitutes an object by bridging World and Consciousness and thus ontological and semiotic space (cf. Bense 1975, pp. 45 s., 65 s.; Toth 2008, vol. 1, pp. 127 ss.). As such, the quantitative-mathematical notion of successor plays an eminent role (cf. Bense 1975, pp. 167 ss.; 1983, pp. 192 ss.). Therefore, a sign cannot be introduced as a sequence of kenograms, because kenograms are beyond the difference of consciousness and world, semiotic and ontological space, and also the notion of Peano succession does not hold. The consequence is that proto-, deutero- and trito-signs have to be defined independently from kenogrammatics and morphogrammatics. Especially the so-called Schadach-transformations that map Peano-numbers to proto-numbers, deutero-numbers, and trito-numbers (cf. Toth 2003, pp. 22 ss.), have to be redefined completely in order to work for signs that are on the level of qualitative numbers, but still signs, i.e. not on the same level of abstraction as kenosequences and morphograms. Moreover, we need mappings from proto- to deutero- and from deutero- to trito-structures.

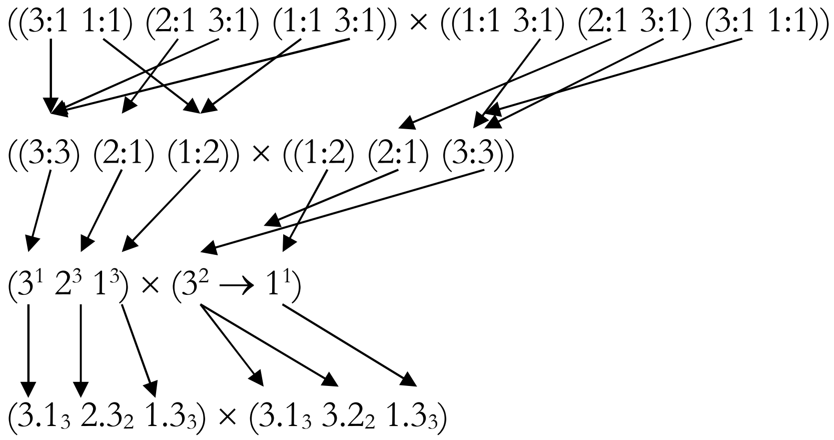
2. In the following, we will write, based on Toth (2009a, b, c, d), the 10 Peircean sign classes first as a series of proto-signs, then as a proto-class, then in its deutero-form, and finally in its trito-form, i.e. in its used numerical-fundamental-categorical form, to which the semiotic contextures (or inner semiotic environments) are added. The mapping between the respective levels are thus introduced "intuitively" first.

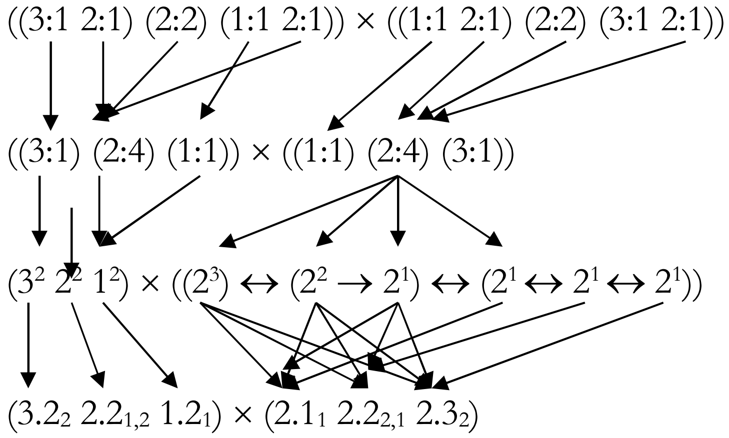


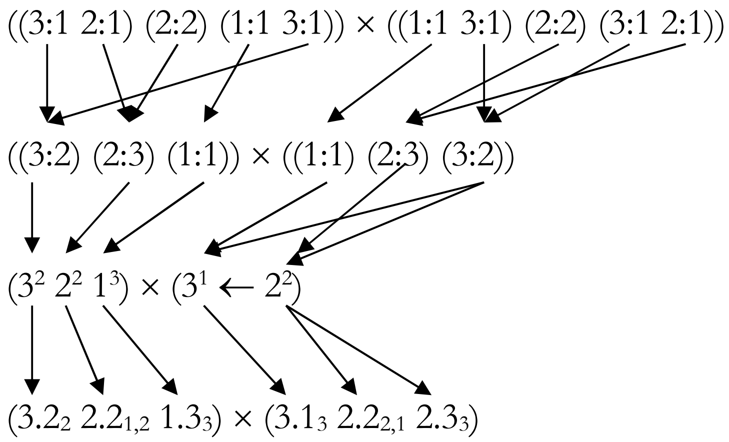

$(3.1_3\ 2.1_1\ 1.2_1) \times$
$(2.1_1\ 1.2_1\ 1.3_3)$
$((3{:}1\ 1{:}1)\ (2{:}1\ 1{:}1)\ (1{:}1\ 3{:}1)) \times$
$((1{:}1\ 3{:}1)\ (2{:}1\ 1{:}1)\ (3{:}1\ 1{:}1))$
$((3{:}2)\ (2{:}1)\ (1{:}3)) \times$
$((1{:}3)\ (2{:}1)\ (3{:}2)$
$(3^1\ 2^1\ 1^3) \times (3^1 \leftarrow 1^2)$
$(3.1_3\ 2.1_1\ 1.3_3) \times$
$(3.1_3\ 1.2_1\ 1.3_3)$

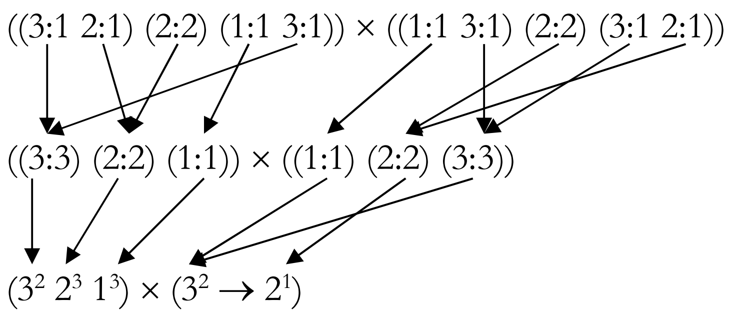

$((3{:}1\ 1{:}1)\ (2{:}2)\ (1{:}1\ 2{:}1)) \times ((1{:}1\ 2{:}1)\ (2{:}2)\ (3{:}1\ 1{:}1))$
$((3{:}1)\ (2{:}3)\ (1{:}2)) \times ((1{:}2)\ (2{:}3)\ (3{:}1))$
$(3^1\ 2^2\ 1^2) \times (2^2 \rightarrow 1^1)$
$(3.1_3\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 1.3_3)$


$((3{:}1\ 1{:}1)\ (2{:}2)\ (1{:}1\ 3{:}1)) \times ((1{:}1\ 3{:}1)\ (2{:}2)\ (3{:}1\ 1{:}1))$
$((3{:}2)\ (2{:}2)\ (1{:}2)) \times ((1{:}2)\ (2{:}2)\ (3{:}2))$
$(3^1\ 2^2\ 1^3) \times (3^1 \leftarrow 2^2 \rightarrow 1^1)$
$(3.1_3\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 1.3_3)$


((3:1 1:1) (2:1 3:1) (1:1 3:1)) × ((1:1 3:1) (2:1 3:1) (3:1 1:1))
((3:3) (2:1) (1:2)) × ((1:2) (2:1) (3:3))
$(3^1\ 2^3\ 1^3) \times (3^2 \rightarrow 1^1)$
$(3.1_3\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 1.3_3)$


((3:1 2:1) (2:2) (1:1 2:1)) × ((1:1 2:1) (2:2) (3:1 2:1))
((3:1) (2:4) (1:1)) × ((1:1) (2:4) (3:1))
$(3^2\ 2^2\ 1^2) \times ((2^3) \leftrightarrow (2^2 \rightarrow 2^1) \leftrightarrow (2^1 \leftrightarrow 2^1 \leftrightarrow 2^1))$
$(3.2_2\ 2.2_{1,2}\ 1.2_1) \times (2.1_1\ 2.2_{2,1}\ 2.3_2)$


((3:1 2:1) (2:2) (1:1 3:1)) × ((1:1 3:1) (2:2) (3:1 2:1))
((3:2) (2:3) (1:1)) × ((1:1) (2:3) (3:2))
$(3^2\ 2^2\ 1^3) \times (3^1 \leftarrow 2^2)$
$(3.2_2\ 2.2_{1,2}\ 1.3_3) \times (3.1_3\ 2.2_{2,1}\ 2.3_3)$


((3:1 2:1) (2:2) (1:1 3:1)) × ((1:1 3:1) (2:2) (3:1 2:1))
((3:3) (2:2) (1:1)) × ((1:1) (2:2) (3:3))
$(3^2\ 2^3\ 1^3) \times (3^2 \rightarrow 2^1)$

$(3.2_2\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 2.3_2)$

$((3{:}2)\ (2{:}1\ 3{:}1)\ (1{:}1\ 3{:}1)) \times ((1{:}1\ 3{:}1)\ (2{:}1\ 3{:}1)\ (3{:}2))$

$((3{:}4)\ (2{:}1)\ (1{:}1)) \times ((1{:}1)\ (2{:}1)\ (3{:}4))$

$(3^3\ 2^3\ 1^3) \times ((3^3) \leftrightarrow (3^2 \rightarrow 3^1) \leftrightarrow (3^1 \leftrightarrow 3^1 \leftrightarrow 3^1))$

$(3.3_{2,3}\ 2.3_2\ 1.3_3) \times (3.1_3\ 3.2_2\ 3.3_{2,3})$

Bibliography

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Das Universum der Zeichen. Baden-Baden 1983

Toth, Alfred, Die Hoczeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Semiotics- and Pre-Semiotics. 2 Vols. (2008)

Alfred, Toth, Semotic proto- and deutero-addition. In: Electronic Journal of Mathematical Semiotics, In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, The incompleteness of Peircean semiotics at the In: Electronic Journal for Mathematical Semiotics, 2009b

Toth, Alfred, Model of a 3-adic 3-contextural Proto-Semiotics. In: Electronic Journal of Mathematical Semiotics, 2009c

Toth, Alfred, Model of a 3-adic 3-contextural Deutero-Semiotics. In: Electronic Journal of Mathematical Semiotics, 2009d

A semiotics from tetradic prime-signs

1. Multi-dimensional semiotics can be constructed in several ways that exclude or complement one another, cf. my two volumes "Mehrdimensionale Semiotik" (Klagenfurt 2008). For example, it is possible to construct a 3-dim semiotics out of 2-adic or 3-adic prime-signs, a 4-dim semiotics out of 3-adic or 4-adic prime-signs (cf. the "tower" in Toth 2008b). Generally, a 1-dim semiotics is a line that contains the three fundamental categories. A 2-dim semiotics is the Peirce-Bensean semiotics which is constructed from the 2-adic prime-signs ((1.1), (1.2), (1.3), ..., (3.3)). An example for a 3-adic semiotics is Stiebings "sign cube", based on 3-adic prime-signs. However, instead of starting with 4-dimensional prime-signs of the form ((a.b) (c.d) (e.f) (g.h) and construct over them either a hypercube or the already mentioned "tower", we will start with the identification of semiotic contextures of the degree n, n-adic sign relations and 4-dimensional sign relations (cf. Toth 2009).

0	1, 2, 3	1-dim semiotics
00	(1.1), (2.2), (3.3)	2-dim semiotics
01	(1.2)/(2.1), (1.3)/(3.1), (2.3/(3.2)	
000	(1.1.1), (2.2.2), (3.3.3)	3-dim semiotics
001	(1.1.2), (1.1.3), (2.2.1), (2.2.3), (3.3.1), (3.3.2)	
010	(1.2.1), (1.3.1), (2.1.2), (2.3.2), (3.1.3), (3.2.3)	
011	(1.2.2), (1.3.3), (2.1.1), (2.3.3), (3.1.1), (3.2.2)	
012	(1.2.3), (1.3.2), (2.1.3), (2.3.1), (3.1.2), (3.2.1)	

2. If we now continue this list, we get the following table of the 15 trito-signs of the contextures of contexture and the

0000	(0.0.0.), (1.1.1.1) (2.2.2.2), (3.3.3.)
0001	(0001), (0002), (0003)
0010	(0010), (020), (0030)
0011	(0011), (0022), (0033)
0012	(0012), (0013), (0,14)
0100	(0100), (0200), (0300)
0101	(0101), (0202), (0303),
0102	(0102), (0103), (0203), (0204), (0302), (0304)
0110	(0110), (0220), (0330)
0111	(0111), (0222), (0333)
0112	(0112), (0113), (0221), (0223), (0331), (0332)
0120	(0120), (0130), (0210), (0230), (0310), (0320)
0121	(0121), (0131), (0212), (0232), (0313), (0323)
0122	(0122), (0133), (0211), (0233), (0311), (0322)

0123 (0123), (0132), (0213), (0231), (0312), (0321)

Thus, we obtain 18 qualitative numbers with 3 semiotic choices, 8 with 4 semiotic choices, and 48 with triadic choices, thus 74 qualitative tetradic sub-signs.

3. A tetradic sign class built from these tetradic sub-signs, lacks evidence of the first sight, but it is a necessary formal development out of 3-a semiotics. A 3-adic semiotic is restricted by two laws: 1. The law of triadicity, i.e., in a 3-adic semiotics all three positions are assigned three values (1, 2, 3) which must be pairwise different. 2. The trichotomic inclusion order: For any sign class (3.a 2.b 1.c), there is a ≤ b ≤ c.

4. Every n-adic sign class has n! permutations (cf. Toth 2008a). Therefore has any 4-adic sign class built according to the semiotic laws 3. 24 permutations.

5. As already pointed out in Toth (2009), it is possible to ascribe each of the tetradic sub-signs contextural indices, i.e. inner semiotic environments – although we are based here on a semiotic system, in which n.th contexture = n.th dimension. The following oversight over the 4-adic semiotic (numeric and categorical) matrices is taken off a recent by Rudolf Kaehr):

Numeric binary matrix

$$Sem^{(4,1)} \times Sem^{(4,1)} =$$

$$[(Sem^1 \times Sem^1), (Sem^2 \times Sem^2), (Sem^3 \times Sem^3), (Sem^4 \times Sem^4)]:$$

$$(Sem^1 \times Sem^1) = (1, 2, 3, x) \times (1, 2, 3, x)$$

$$(Sem^2 \times Sem^2) = (x, 2, 3, 4) \times (x, 2, 3, 4)$$

$$(Sem^3 \times Sem^3) = (1, 2, x, 4) \times (1, 2, x, 4)$$

$$(Sem^4 \times Sem^4) = (1, x, 3, 4) \times (1, x, 3, 4)$$

$$sem^1 \times sem^1 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_1 & 1.2_1 & 1.3_1 & 1.4 \\ 2 & 2.1_1 & 2.2_1 & 2.3_1 & 2.4 \\ 3 & 3.1_1 & 3.2_1 & 3.3_1 & 3.4 \\ 4 & 4.1 & 4.2 & 4.3 & 4.4 \end{pmatrix}$$

$$sem^2 \times sem^2 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_1 & 1.2_1 & 1.3_1 & 1.4 \\ 2 & 2.1_1 & 2.2_{1,2} & 2.3_{1,2} & 2.4_2 \\ 3 & 3.1_1 & 3.2_{1,2} & 3.3_{1,2} & 3.4_2 \\ 4 & 4.1 & 4.2_2 & 4.3_2 & 4.4_2 \end{pmatrix}$$

$$sem^3 \times sem^3 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1,3} & 1.2_{1,3} & 1.3_1 & 1.4_3 \\ 2 & 2.1_{1,3} & 2.2_{1,2,3} & 2.3_{1,2} & 2.4_{2,3} \\ 3 & 3.1_1 & 3.2_{1,2} & 3.3_{1,2} & 3.4_2 \\ 4 & 4.1_3 & 4.2_{3,2} & 4.3_2 & 4.4_{3,2} \end{pmatrix}$$

$$sem^4 \times sem^4 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1,3,4} & 1.2_{1,3} & 1.3_{1,4} & 1.4_{3,4} \\ 2 & 2.1_{1,3} & 2.2_{1,2,3} & 2.3_{1,2} & 2.4_{2,3} \\ 3 & 3.1_{1,4} & 3.2_{1,2} & 3.3_{1,2,4} & 3.4_{2,4} \\ 4 & 4.1_{3,4} & 4.2_{3,2} & 4.3_{2,4} & 4.4_{2,3,4} \end{pmatrix}$$

4 – contextural semiotic matrix

$$Sem^{(4,2)} = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & \mathbf{1.1_{1.3.4}} & 1.2_{1.3} & 1.3_{1.4} & 1.4_{3.4} \\ 2 & 2.1_{1.3} & \mathbf{2.2_{1.2.3}} & 2.3_{1.2} & 2.4_{2.3} \\ 3 & 3.1_{1.4} & 3.2_{1.2} & \mathbf{3.3_{1.2.4}} & 3.4_{2.4} \\ 4 & 4.1_{3.4} & 4.2_{3.2} & 4.3_{2.4} & \mathbf{4.4_{2.3.4}} \end{pmatrix}$$

Semiotic reduction matrix :

$$Sem^{(4,2)} = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1 & 1.2 & 1.3 & 1.4 \\ 2 & 2.1 & 2.2 & 2.3 & 2.4 \\ 3 & 3.1 & 3.2 & 3.3 & 3.4 \\ 4 & 4.1 & 4.2 & 4.3 & 4.4 \end{pmatrix}$$

Null

Bibliography

Kaehr, Rudolf, Sketchon semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)

Toth, Alfred, Semiotics and Pre-Semiotics. 2 vols. Klagenfurt 2008 (2008b)

Toth, Alfred, Contextures, relations, and dimensions.: In: Electronic Journal of Mathematical Semiotics, 2009

Semiotics from dyadic prime signs

1. Classical Peirce-Bensean semiotics is based on dyadic prime-signs:

PS = {(1.1), (1.2), (1.3), ..., (3.3)}

These dyads take two points in a Cartesian coordinate system.

If one would try to construct a semiotics based on monadic prime-signs, one would get any arbitrary line between the fundamental categories 1, 2 and 3.

In a 3-dimensional semiotics, prime-signs are triadic. The definition of a sign class in Stiebings sign-cube (Stiebing 1978, p. 77) is:

3-SCl = (a.3.b c.2.d e.1.f),

where (a, c, e) are the dimensional numbers, which can coincide according to Toth (2009) with the semiotic contextures.

Therefore, a 4-dimensional semiotics which is based on 4-adic prime-signs, can be defined in more than 1 way:

1. 4-SCl = (a.b.3.c) (d.e.2.f) (g.h.1.i).

where ((a, b), (d, e), (g, h)) is the set of semiotic dimensions. Because there are two variables for dimensional numbers, one can state that one is identical to the respective contexture.

2. 4-SCl = (a.3.b.c) (d.2.e.f) (g.1.h.i),

where (a, ...g) and (c, ..., i) are the sets of semiotic dimensions. Combinations are possible.

2. However, a hitherto never mentioned possibility to construct sign classes (and other semiotic relations) from dyadic prime-signs can be defined as follows:

SCl = ((a.b) (c.d)), ((e.f) (g.h)), ((i.k) (l.m))

Here, the dyads are themselves pairs of dyads. Which of the 4 variables per dyad is ascribed to triadic and trichotomic values and which the semiotic order is is completely arbitrary, e.g.

((a.b) (c.d)) = (3.1, 2.1) (triadicity, trichotomic inclusion)

((a.b) (c.d)) = (2.1, 3.1) (triadicity, no triochotomic inclusion)
((a.b) (c.d)) = (2.1, 2.1) (neither triadicity nor trichotomic inclusion)

If there are no semiotic restrictions, we have 9^9 = 387'420'489 combinations of dyads to pair of dyads. Another questions are: Let's say we ascribe

((3.b) (1./2./3.d)

or

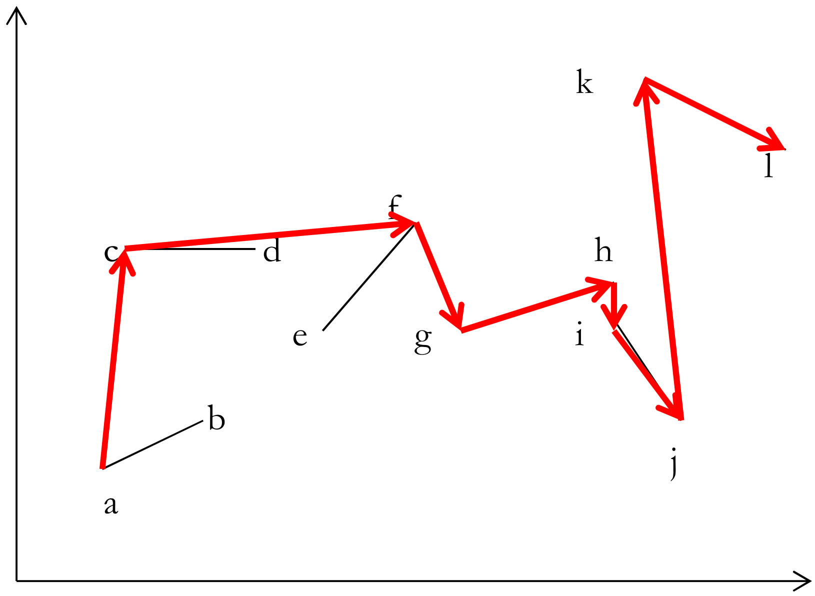
((a.3) (c. 1/2/3.),

what are (b, d) and (a, c) then? Dimensional numbers? Identical or non-identical to the respective contextures?

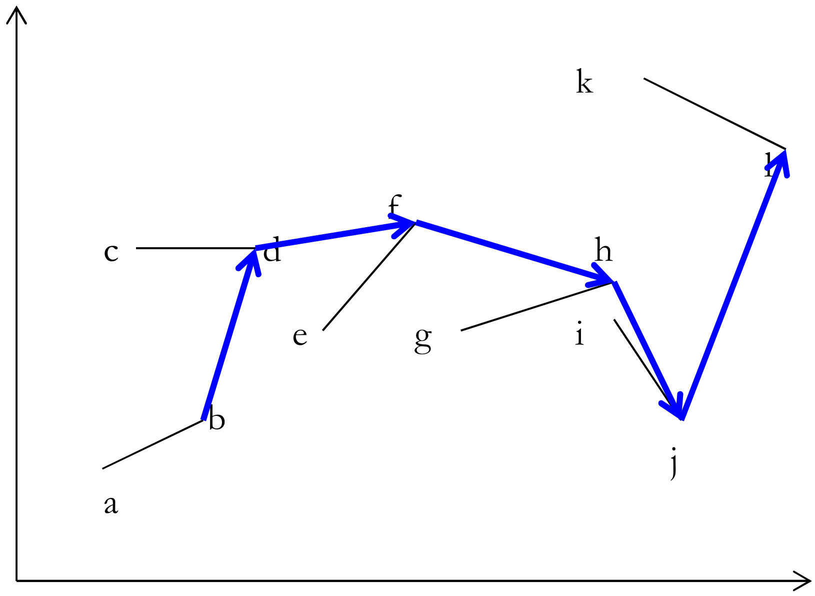
3. Dyads as (ordered) pairs of dyads take a line in a 2-dimensional Cartesian coordinate system. Let us start with

SCl = ((a.b) (c.d)), ((e.f) (g.h)), ((i.j) (k.l))

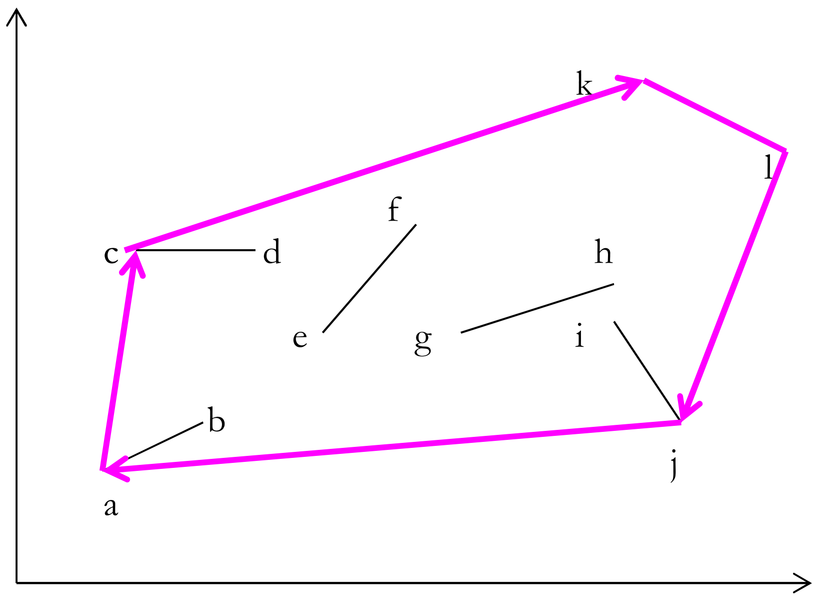
and draw the lines which correspond to the dyads-in-dyads fully arbitrarily, then we get, e.g.:



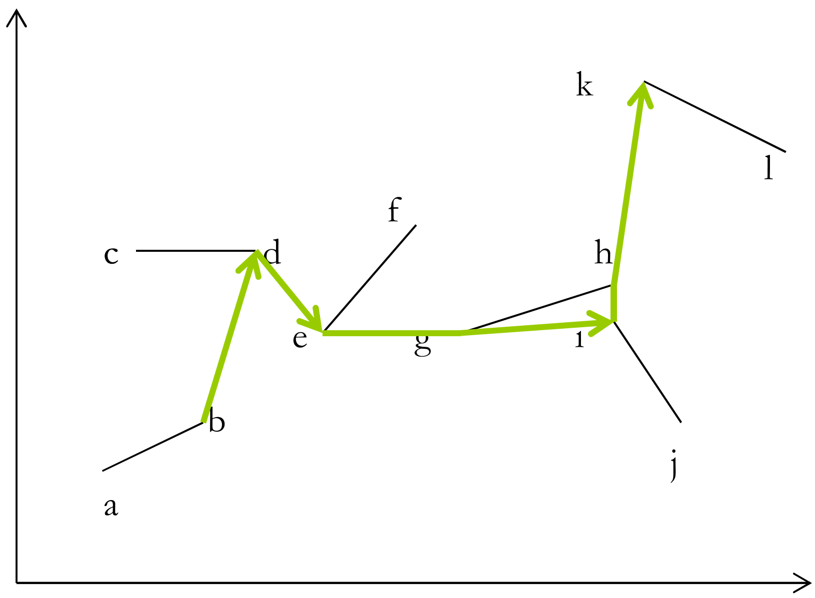
Outer graph of arbitrary sign class over dyads of dyads



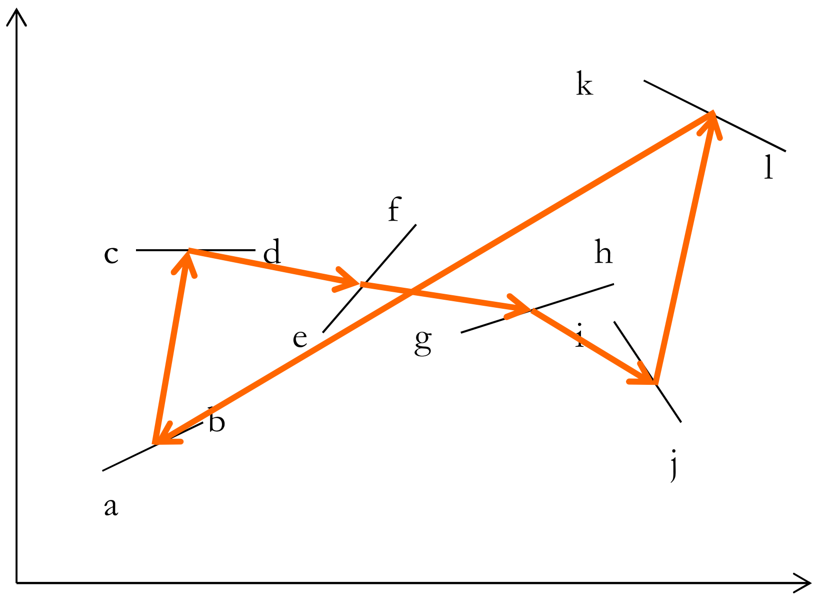
Inner graph of arbitrary sign class over dyads of dyads



Semiotic hull graph of arbitrary sign class over dyads of dyads



Semiotic kernel graph of arbitrary sign class over dyads of dyads



Semiotic relation-as-vertex-graph of arbitrary sign class over dyads of dyads

As one sees, sign relations defined over dyads of dyads lead to non-trivial graphs and highly interesting topological structures. By itself,

$^{2/2}$SCl = ((a.b) (c.d)), ((e.f) (g.h)), ((i.k) (l.m))

is but just a special case of similar sign relations defined over triads of triads

$^{3/3}$SCl = (((a.b.c) (d.e.f), (g.h.i)) ((j.k.l) (m.n.o) (p.q.r)), ((s.t.u) (v.w.x) (y, z, a))) =
(A, B, C), where A, B, C are triads over triads

or over tetrad of tetrads

$^{4/43}SCl$ = (A, B, C, D), where A, B, C, D are tetrads over tetrads,

generally

$^{n/n}SCl$ = (A, B, C, ... n), where A, ...n, are n-ads over n-ads.

Bibliography

Stiebing, Hans Michael, Zusammenfassungs- und Klassifikationsschemata von Wissenschaften und Theorien auf semiotischer und fundamentalkategorialer Basis. Diss. Stuttgart 1978

Toth, Alfred, Contextures, relations, and dimensions. In: In: Electronic Journal of Mathematical Semiotics, 2009

Polycontextural semiotic operations

1. Contextures and number structures

On the basis of Rudolf Kaehr's work (cf. bibliography), it is now possible, to reformulate the contexture-free polycontextural.-semiotic notations given in Toth (2003, pp. 36 ss.) in order to obtain a relatively complete organon of polycontextural semiotic operations which form, together with other topics, the heart of polycontexural semiotics. This will turn out to be of much bigger importance then the analysis of sub-signs or semioses.

2. The following table gives the three number structures of proto-, deutero- and trito-numbers for the first three contextures C1 – C3:

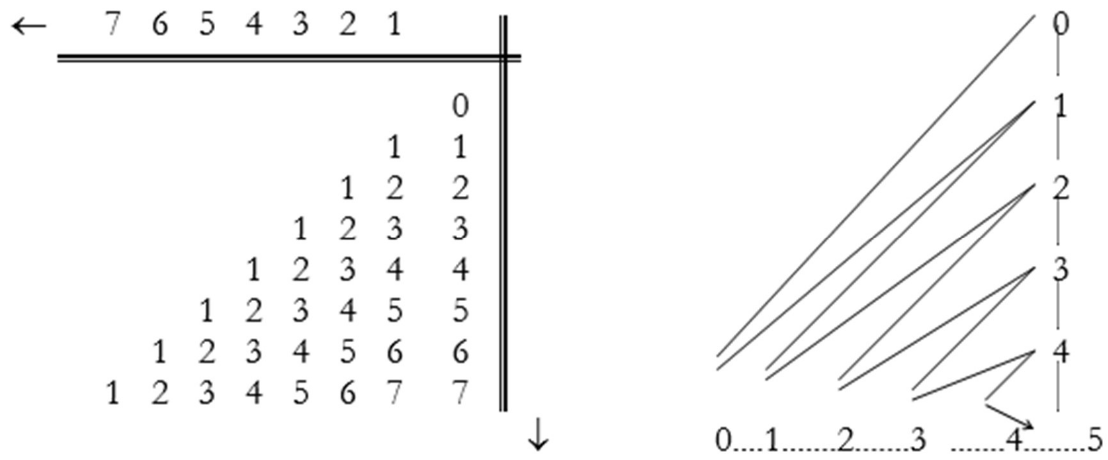
Proto	Deutero		Trito	Deci	
0	0	(1.1), (1.2), (2.1), (2.2)	0	0	C1
00	00	(2.2), (2.3), (3.2), (3.3)	00	0	
01	01		01	1	C2
000	000	(1.1), (1.3), (3.1), (3.3)	000	0	
001	001		001	1	
012	012		010	3	C3
			011	4	
			012	5	

As one sees easily, we have

Trito-Structure ⊂ Deutero-Strucure ⊂Protostructure, but

C1 ⊄ C2 ⊄ C2.

According to the decimal equivalents to the right, we also see 1) that the Peano number 2 cannot be represented by a kenogram, and 2) that many numbers are represented in different contextures and number structures. However, with that, the question arises how trito-numbers are to be introduced. Günther (1976-80, II, p. 261) had suggested that qualitative numbers are counted along two number axes which are orthogonal to one another. Therefore, trito-numbers are introduced, like proto- and deutero-numbers, but differemt from the Peano numbers, in a two-dimensional, planar way.



Summing up: In order to inaugurate a qualitative mathematics and a structural semiotics, proto- and deutero-numbers are not sufficient, because they still can be displayed in pure quantities, i.e. as pairs (m: n) and as partitions (m^n). Since this is not the case anymore for trito-numbers, they form the basis for qualitative mathematics and structural semiotics. However, one must not forget that the trito-numbers are just *differentiae specifiae* of the deutero-numbers, and the deutero-numbers just *differentiae specificae* of the proto-numbers (cf. in German Individuum-Art-Gattung).

2. Polycontextural operators

We differentiate between Intra- and Trans-operators (cf. Kronthaler 1986, pp. 37 ss.). Intra-operators connect qualitative numbers of the same quality, i.e. the same length, and cannot go out of a contexture. Trans-operators connect qualitative numbes of different qualities, i.e. length, and go between different contextures.

2.1. Intra-operators

2.1.1. Ein- und mehr-stellige Intra-Operatoren

As examples, trito-numbers are chosen, since several operators are non-trivial only for those. As examples for mappings of sub-signs and sign-relations to kenograms cf. Toth (2009).

2.1.1.1. Delete

Symbol: L^i. Deletes the i-th position, i.e. of w_i.
Example for i = 1: L^0(001023) = Ø01023

Example for i = 2: $L^{1,3}$(001023) = 0Ø1Ø23.

Example for i = m* (delete all positions):
L_6(001023) = ØØØØØØ.

2.1.1.2. Insert

Symbol: B^i_h. Inserts the value h in the place i.

Example for i = 3 and h = 2: B^3_2(001Ø23) = 001223.

Example for i = 1, j = 3, h = 0 and k = 0: $B^1_0{}^3_0$(0Ø1Ø23) = 001023.

Example for $B_{h, k, ..., l, m}$* (Insert h, k, ..., l, m into all places):
B_{001023}(ØØØØØØ) = 001023.

2.1.1.3. Nulling

Symbol: N^i. Nulling of the i-th position, d.h. $w_i \rightarrow 0$.

Example for i = 5: N^5(001023) = 001020.

Example for N^{ij} ($w_i \rightarrow 0$ und $w_j \rightarrow 0$), i = 4, j = 5: N^{45}(001023) = 001000.

Example for N_m* (Nulling of all positions): N_6(001023) = 000000.

2.1.1.4. Maximizing

Symbol: M^i. Maximizing w_i.

Example for i = 1: M^1(001023) = 011023.

Example for M^{ij} (Maximizing of w_i and w_j), i = 1, j = 2:
$M^{1,2}$(001023) = 012023.

Example for M_m* (maximizing of all positions): M_6(001023) = 012345.

2.1.1.5. Change of insertion

Symbol: $W^i{}_h$. $w_i \rightarrow h$.

Example for i = 3, h = 1: $W^i{}_h$(001023) = 001123.

Example for $W^i{}_h{}^j{}_k$ ($w_i \rightarrow h$ and $w_j \rightarrow k$), i = 3, h = 1, j = 5, k = 1:
$W^3{}_1{}^5{}_1$(001023) = 001121.

Example for $W_{h, k, \dots, l, m}$* (Change of insertion of all places): W_{012000}(001023) = 012000.

2.1.1.6. Transposition

Symbol: $T^i{}_h$. Transposition of w_i and w_h.

Example for i = 3, h = 4: $T^3{}_4$(001023) = 001203.

Example for $T^i{}_h{}^j{}_k$ ($w_i \rightarrow w_h$ and $w_j \rightarrow w_k$), i = 3, h = 4, j = 4, k = 5:
$T^3{}_4{}^4{}_5$(001023) = 001230.

For complete transposition cf. 2.1.1.7. Permutation.

2.1.1.7. Permutation

Symbol: $P_{i0 \dots im-1}$*. $w_0 \dots w_{m-1} \rightarrow w_{i0} \dots w_{im-1}$.

Example: P_{124530}(001023) = 012300.

2.1.1.8. Partial reflection

Symbol: $R^{\square\square\square\square\square\blacksquare\blacksquare\blacksquare}$. Partial reflection of the i positions, marked by "▪".

Examples: $R^{\square\square\square\blacksquare\blacksquare\blacksquare}$(001023) = 001320 = 001230.
$R^{\blacksquare\blacksquare\blacksquare\square\square\square}$(001023) = 010023.

Example for R_m* (total reflection): R_6(001023) = 320100 = 012300.

2.1.1.9. Quasi Intra Reflection

Symbol: ^{r}R□□□□□▪▪▪. Works like 2.1.1.8., however, not as mapping $K_m \rightarrow K_m$, but into the reflected contexture $K_m \rightarrow {}_mK$, i.e., normal form transformation which may be necessary after the reflection, works not on K_m, but on ${}_mK$.

Example: ^{r}R□□□▪▪▪(001023) = 0320100.

Example for $^{r}R_m$* (Quasi-Intra-Total-Reflection): $^{r}R_m$(001023) = 320100.

2.1.2. One-PLACED Intra Operators

2.1.2.1. Normal form Operator

Symbol: N: PN → PN, DN → DN, TN → TN (PN = .Proto-number, etc).

Example: N(2838538) = 0121321.

2.1.2.2. Constancy Operator

The constancy operator K_{zm} maps all kenograms onto $z_m \in K_m$ ab. Special cases are the operators L_m(chap. 2.1.1.1.), N_m(chap. 2.1.1.3) und M_m(chap. 2.1.1.4).

2.1.2.3. Reflectors

Symbol: R_m, $^{r}R_m$. $T_m \rightarrow {}_mT$, cf. chap. 2.1.1.8. and chap. 2.1.1.9.

2.1.2.4. Intra-Successor

2.1.2.4.1. Proto-Intra-Successor $^{i}{}_{P}N_m$

Examples:	p_m	0000	0001	0012
	p'_m	0001	0012	0123

2.1.2.4.2. Deutero-Intra-Successor $^{i}{}_{D}N_m$

Examples:	d_m	00012$\underline{3}$,	000111222$\underline{3}$,	0012$\underline{3}$
	d'_m	00112$\underline{2}$,	000111223$\underline{3}$,	0123$\underline{4}$

2.1.2.4.3. Trito-Intra-Successor ${}^{i}{}_{T}N_{m}$

Examples:	t_m	$0\underline{0}$	n	
	t'_m	$0\underline{1}$	t'_m	$0^1 \leftrightarrow \underline{1}^1$

t_m	$00\underline{0}$,	$00\underline{0}$,	$00\underline{0}$
t'_m	$01\underline{0}$,	$00\underline{1}$,	$01\underline{2}$

t_m	$000\underline{0}$,	$000\underline{0}$,	$000\underline{0}$,	$000\underline{0}$
t'_m	$001\underline{0}$,	$000\underline{1}$,	$001\underline{2}$,	$012\underline{3}$

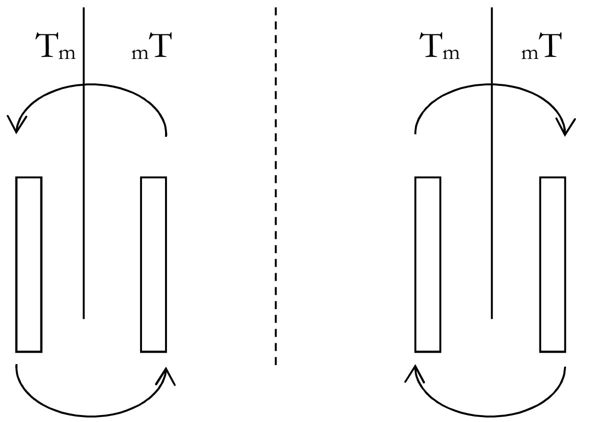
2.1.2.5. Intra-Predecessor

2.1.2.6. n-times Intra-Successor ${}^{i}N^{n}$ and -Predecessor ${}^{i}V^{n}$

If the successor ${}^{i}N_m$ or the predecessor ${}^{i}V_m$, respectively, work n-times after one another, then we have ${}^{i}N^{n}{}_{m}$ bzw. ${}^{i}V^{n}{}_{m}$ (Kronthaler 1986, p. 45).

2.1.2.7. Total Reflector ${}^{r}R_m$

Inside of the complete system



Successor Predecessor

for every kenogram „the number of its successor is even to the number of its predecessor and each time finite, if one counts only once. The application of the successor and predecessor operations is here unlimited, it can be applied infinite times after one another [...]. The Intra-operators, introduced up to now, especially successor and predecessor, are valid also in each of their reflected structures ${}_{m}K = {}^{r}R(K_m)$" (Kronthaler 1986, p. 48 s.):

Example: ↑Predecessor	000123	321000	Successor↑
	001234	432100	
↓Successor	012345	543210	Predecessor ↓

2.1.3. Multi-PLACED Intra-Operators

2.1.3.1. Intra-Addition +

2.1.3.1.1. Proto-Intra-Addition

Example:	5:1	00000
	5:3	00012
	5:4	00123

Another display uses the successor ${}^{i}N^{n}_{m}$. If one lets the indices away, we obtain:$p^s = p^i + p^j = N^j(p^i) = N^i(p^j)$.

Example: $p^1 + p^3 = N^3(p^1) = N^1(p^3) = N^3(00000) = N^1(00012) = 00123$.

2.1.3.1.2. Deutero-Intra-Addition

Example:	000111	+	000123	= 012345
N^1	000112		000112	V^1
N^2	000123		000111	V^2
N^3	001122		000012	V^3
N^4	001123		000011	V^4
N^5	001234		000001	V^5
N^6	012345		**000000**	V^6

2.1.3.1.3. Trito-Intra-Addition

Both methods, the ordinal and the one using the successor/predecessor auxiliary algorithm, correspond exactly to deutero-addition (Kronthaler 1986, p. 51; chap. 2.1.3.1.2.).

2.1.3.2. Intra-Subtraction –

Intra-Subtraction is the converse operation to Intra-Addition. For all three number structures, the same applys. Let be $i < j$. Then we get $d^j - d^i = d^{j-i} = V^n(d^j)$ with n from $V^n(d^i) = 0........0$ or $N^n(0........0) = d^i$ (Kronthaler 1986, p. 51).

2.1.3.3. Addition and subtraction in the system K_m — $_mK$

Example: $-0001203 \neq {}^rR(0001203) = 3021000$.

2.2. Trans-Operatoren

2.2.1. One- und multi-placed Trans-operators

2.2.1.1. Absorption

Symbol: $A^i_m = A(\ ^i\ )$. Absorbs the i-th position: $K_m \rightarrow K_{m-1}$, $m > 1$.
Example: $A^3(00102) = 0001$.

Symbol: $A^{i,j}_m = A(\ ^{i\ j}\ )$. Absorbs the i-th and j-th position: $K_m \rightarrow K_{m-2}$, $m > 2$.
Example: $A^{1\ 3}(00102) = 001$.

Symbol: $A^{i1,\ldots,in}_m$. Absorbs $i_1, \ldots, i_n$: $K_m \rightarrow K_{m-n}$, $n < m$.
Example: A(00102) = 01.

Symbol: $A^{(m-1)}{}_m$. Absorbs all but 1 position: $K_m \rightarrow K_1$.
Example: $A^{(4)}(00102) = 0$.

Symbol: A_m*. Total absorption: $K_m \rightarrow \bullet$ (Extinctor).
Example: $A_5(00102) = \bullet$.

2.2.1.2. Splitting

Symbol: $Z^{i,j}_m$. Splits a kenogram in two parts of lengths i and j, $i + j = m$:

$K_m \rightarrow K_i$, $K_m \rightarrow K_j$

Example: $Z^{2\ 3}(00102) \rightarrow 00$, $Z^{2\ 3}(00102) \rightarrow 012$

Z_m*: Splitting of the kenogram in single parts of length 1.

Example: $Z_5(00102)$ → 0, 0, 0, 0[1], 0

2.2.1.3. Iteration

Symbol: ${}_mI^i_j$. Iterates the i-th position j-times: $K_m \rightarrow K_{m+j}$.
Example: $I^2_3(00102) = 00111102$.

Symbol: ${}_mI^{i\ k}_{j\ l}$. Iterates the i-th position j-times and the k-th position l-times: $K_m \rightarrow K_{m+j+l}$.
Example: ${}_mI^{0\ 2}_{3\ 2}(00102) = 0000011102$.

Symbol: ${}_mI_{jo, \ldots, jm-1}$*. Iterator as a special case of a successor.
Example: $I_{31232}(00102) = 0000001110000222$.

2.2.2. One-PLACED Trans-operators

2.2.2.1. Trans-Successor ${}^tN_m$

2.2.2.1.1. Proto-Trans-Successor ${}^t{}_PN_m$

Example: ${}^t{}_PN(0012) = \varnothing 0012$ — 00012 / 00123

2.2.2.1.2. Deutero-Trans-Successor ${}^t{}_DN_m$

Example: ${}^t{}_DN(0012)$ — 00112 / 00122

2.2.2.1.3. Trito-Trans-Successor ${}^{t}_{T}N_{m}$

Example:

```
           01012
0102                    01012
           10102  ~
```

2.2.2.2. Trans-Predecessor ${}^{t}V_{m}$

2.2.2.2.1. Proto-Trans-Predecessor ${}^{t}_{P}V_{m}$

Example:

000123 → ~~0~~00123 ~ 0~~0~~0123 ~ 00~~0~~123

000123 → 00012~~3~~ ~ 0001~~2~~3 ~ 000~~1~~23

2.2.2.2.2. Deutero-Trans-Predecessor ${}^{t}_{D}V_{m}$ and

2.2.2.2.3. Trito-Trans-Predecessor ${}^{t}_{D}V_{m}$

Cf. Kronthaler (1986, pp. 59 ff.).

2.2.2.3. n-times Trans-Successor ${}^{t}N^{n}_{m}$ and

2.2.2.4. n-times Trans-Predecessor ${}^{t}V^{n}_{m}$

For ${}^{t}_{P}N^{n}_{m}$, ${}^{t}_{D}N^{n}_{m}$, ${}^{t}_{T}N^{n}_{m}$ and ${}^{t}_{P}V^{n}_{m}$, ${}^{t}_{D}V^{n}_{m}$, ${}^{t}_{T}V^{n}_{m}$ cf. Kronthaler (1986, pp. 62 ss.).

2.2.3. Multi-PLACED Trans-operators

2.2.3.1. Trans-Addition t

2.2.3.1.a. Absorptive Trans-Addition

2.2.3.1.a.1. Totally absorptive Trans-Addition

$$\left.\begin{array}{ll}\text{Left-Absorption:} & z_m + z_n \\ \text{Right-Absorption:} & z_n + z_m\end{array}\right\} = z_n$$

2.2.3.1.a.1.1. Canonical cases

01023 t 010 = 01023

01023 t 012 = 01023

2.2.3.1.a.1.2. Absorption under Splitting

If the Splitting has length 1, only the lengths of the summands n and m are taken in consideration, because we have:

[0] ~ [1] ~ [2] ~ ...

Another possibility to differentiate concerns the length of single Splitting-parts

(Kronthaler 1986, pp.67 ss.)::

Length 1: 0

0	1	0	2	3
0	1	2	3	4

Lenght 1–2–3:

0	1 0	2 3
0	0 0	1

impossible!

(Kronthaler 1986, p. 66).

2.2.3.1.a.2. Teilabsorptive Trans-Addition

Example: 0 [0 1] 0 2 = 0 0 1 0 2 0

t [0 1] [0]

absorbs 2 positions, juxtaposes 1 position: $T_{5+1} = T_6$.

left-absorptive: t_m t t_n = t_s right-absorptive: t_n t t_m = t'_s

In the following example, both cases be right-absorptive. What has been absorbed, is split:

| 0 0 1 0 2 3 | t | 0 1 0 | 2 | = 0 0 1 0 2 3 | 2 |

What is absorbing, is split:

| 0 0 1 0 | 2 3 | t | 0 1 0 | 2 | = | 0 | 0 1 0 | 2 | 2 3 |

2.2.3.1.b. Juxtapositive Trans-Addition

2.2.3.1.b.1. Canonical cases

2.2.3.1.b.1.1. Normal form juxtapositive t-Addition

Trito-numbers: | 0 1 0 2 | t | 0 0 1 2 3 0 | =

| 0 1 0 2 | 0 0 1 2 3 0 | ≠

| 0 0 1 2 3 0 | t | 0 1 0 2 | = | 0 0 1 2 3 0 | 0 1 0 2 |

Deutero-numbers: 00112 t 001123 = 00112001123 ~ 00001111223

Proto-numbers: 0012 t 001201 impossible, since 0 can be iterated!

2.2.3.1.b.1.2. Juxtaposition to the normal form of equivalent enograms

Example: 010 t 00 = 01000

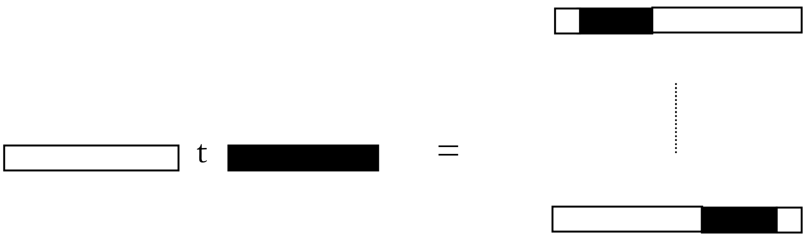
11

22 ~ 01033 ~ ...

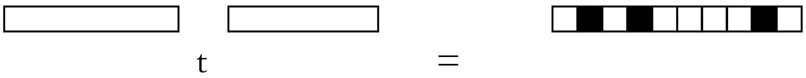
2.2.3.1.b.2. Splitting

2.2.3.1.b.2.1. One summand appears in normal form, the other is split arbitrarily (Kronthaler 1986, p. 67):

Splitting left (analogously right)



2.2.3.1.b.2.2. Both summands are split in arbitrary form (total splitting)



2.2.3.1.c. Juxtapositive Trans-Addition

Cf. Kronthaler (1986: 67).

2.2.3.1.d. t-Addition → i-Addition

Cf. Kronthaler (1986: 67f.)

2.2.3.1.1. Proto-Trans-Addition

2.2.3.1.1.1. Absorptive Proto-Trans-Addition

Total Absorption:

P_m		0	0	0	1	2	3	6:4	but cf.	0	0	0	1	2	3		6:4
	t			0	1	2		3:3	t			0	1	2	3	4	5:5
	=	0	0	0	1	2	3	6:4	=	impossible							

Partial Absorption:

```
Pn+m-1    0  0  0  1  2  3         6:4    and   0  0  0  1  2  3      6:4
                |                                        |
          t     0  1  2            3:3          t        0  1  2      3:3
          ------------------------------        --------------------------
          = 0  0  0  1  2  3  4  5  8:6         0  0  0  0 1 2  3 4   8:5
```

```
Pn+m-4      0 0  0  0  0  1  2  3 4  5             10:6
                 |  |  |  |
          t         0  0  1  2  3 4  5  6           8:7
          ----------------------------------------
          = 0  0  0  0  0  1  2  3  4 5  6 7 8 9   14:10  and

            0  0  0  0  0  1  2  3  4 5             10:6
                       || ||
          t            0  1  2  3  4 5              7:6
          ----------------------------------------
          = 0  0  0  0  0  0  1  2  3 4 5  6 7 8    14:9
```

```
     0  0  1  2  3    =  ______    00001234 }
     |  |    \           ------    00000123 }  P8
  t  0  0  0  1          ======    0000123  }
  ---------------        ______    0001234  }  P7

                         ------
                         ______    000123      P6
```

2.2.3.1.1.2. Juxtapositive Proto-Trans-Addition in the Minimal-Contexture P_{n+m}

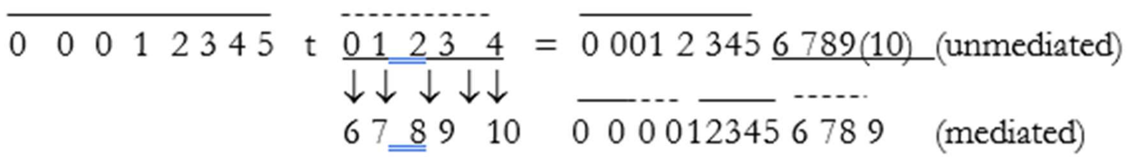
2.2.3.1.1.2.1. Mediated juxtapositive

```
     0  0  1  2  3    =  ______    00001234 }
     |  |    \           ------    00000123 }  P8
  t  0  0  0  1          ======    0000123  }
  ---------------        ______    0001234  }  P7

                         ------
                         ______    000123      P6
```

2.2.3.1.1.2.2. Unmediated juxtapositive

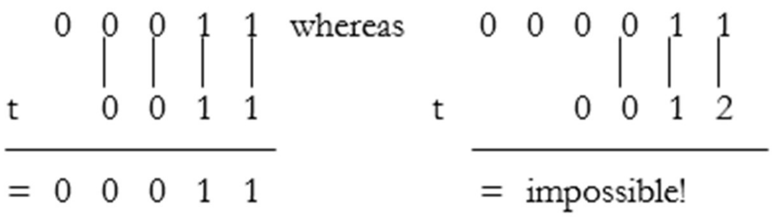
Example:



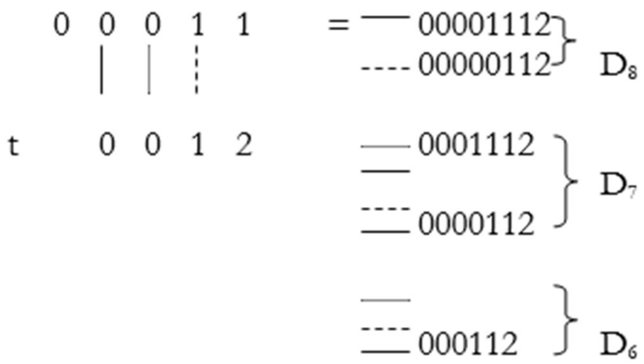
2.2.3.1.2. Deutero-Trans-Addition

2.2.3.1.2.1. Absorptive Deutero-Trans-Addition

Totally absorptive Deutero-Trans-Addition:



Partially absorptive Deutero-Trans-Addition:

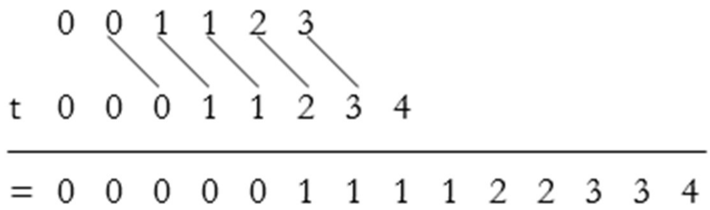


(There are still more possibilities left.)

2.2.3.1.2.2. Juxtapositive Deutero-Trans-Addition

2.2.3.1.2.2.1. Mediated juxtapositive Deutero-Trans-Addition

Example: (The connecting lines symbolize the addition of the corresponding iteration numbers.)



2.2.3.1.2.2.2. Unmediated juxtapositive Deutero-Trans-Addition

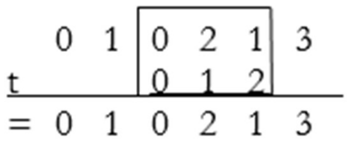
Cf.. Kronthaler (1986, p. 71).

2.2.3.1.3. Trito-Trans-Addition

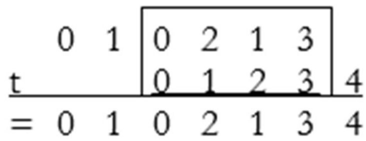
2.2.3.1.3.1. Absorptive Trito-t-Addition

Examples:

Totally absorptive Trito-Trans-: Addition:



Partially absorptive Trito-Trans-Addition:



2.2.3.1.3.2. Juxtapositive Trito-t-Addition

2.2.3.1.3.2.1. Canonical cases

2.2.3.1.3.2.1.1. Normalform-Juxtapositiv

Cf. Kronthaler (1986, p. 72).

2.2.3.1.3.2.1.2. Juxtaposition von Trito-Äquivalenzen

Example:

012 t 01 =	01201	= 01221	Repertoire: {0, ..., 4}	
	10	13		
	02	31	Choice:	01 12 23 34
	20	14 ~~41~~		02 13 24
	03	23		03 14
	30	32		04
	04 ~~40~~	24 ~~42~~		
	12	34 ~~43~~	+ permutations	

2.2.3.1.3.2.2. Splitting

For Splitting of one or two summands cf. Kronthaler (1986, p. 73).

2.2.3.2. Trans-Subtraction ⌐

2.2.3.2.1. Juxtapositive t-Subtraction (partial subtraction)

2.2.3.2.1.1. Total juxtapositive t-Subtraction

Example: 0010 ⌐ 01 = 001001.

2.2.3.2.1.2. Teiljuxtapositive t-Subtraction

2.2.3.2.1.2.1. In normal form

Example:

0 0 1 0 2 2 ⌐ 0 1 0 2 = 0 2 2 2 ~ 0 1 1 1

2.2.3.2.1.2.2. In einer zur Normalform äquivalenten Form

Example:

0 0 1 0 2 2 ⌐ 0 1 1 2 = 0 0 1 2

2.2.3.2.2. Total-t-Subtraction

2.2.3.2.2.1. Canonical case: Normal form-Subtraction

Example:

0 | 0 1 0 2 | 2 ⌐ | 0 1 0 2 | = 02 ~ 01

2.2.3.2.2.2. t-Subtraction of equivalent forms

Example:

0 0 1 | 0 2 2 | ⌐ | 0 1 1 | = 0 0 1

Bibliography

Günther, Gotthard, Beiträge zu einer Grundlegung einer operationsfähigen Dialektik. 3 vols. Hamburg 1976-80

Kaehr, Rudolf, 7 very important articles about Diamond Semiotics, available from http://rudys-diamond-strategies.blogspot.com/. Moreover: Xanadu's textemes, and Diamond Text Theory from http://www.thinkartlab.com/CCR/rudys-chinese-challenge.html (2008-2009)

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten, Frankfurt am Main 1996

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Contextures, relations, and dimensions. In: Electronic Journal of Mathematical Semiotics, Toth 2009

Contextuated and non-contextuated polycontextural semiotics

1. One concept of polycontextural semiotics in which the contextures are independent from the dimensions of the sign relations goes back to Kaehr (2008). Kaehr assigns each sub-sign of the 3×3 semiotic matrix their inner environments or contextures:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}$$

Here, we see that the numbers of the contextures are independent of the n-adic structure of the dyadic sub-signs. E.g., (1.2) and (2.1), (1.3) and (3.1), (2.3) and (3.2), generally: (a.b) and (a.b)° lie in the same contexture. However, this is only the case for the closed world of a sign class and for the also closed world of a reality thematics, but not between them, since by dualization, the order of the environments change; generally: $\times(a.b)_{i,j} = (b.a)_{j,i}$. We thus need TWO semiotic matrices, one for the subjective world of the signs and one for the objective world of their realities. The polycontextural still mediates between world an consciousness, but also states their difference at the same time!

From the following table

Monads	1, 3
Dyads	1, 2
Triads	2, 3

we see that monads can no only lie in C = 1, but also in C = 3, that dyads can not only lie in C = 1, but also in C = 2, and triads both in C = 2 and in C = 3. One has to be aware that all sub-signs are insofar dyadic as they are Cartesian products, but only 3 dyads are dyadic *sensu stricto*, namely Cartesian products with 2 as first factor. This situation points to a semiotic “particle”-dualism.

2. Another concept of polycontextural semiotics has been suggested by Toth (2003). The basic idea is here not, like in the former concept, to “cross-contextuate” the sub-signs and turning them in this way into polycontextural relations, but two assume that in polycontextural semiotics contextures and dimensions of a sign are identical.

Therefore, we have

1-contextural/1-dimensional semiotics

0	1, 2, 3

2-contextural/2-dimensional semiotics

00	(1.1), (2.2), (3.3)
01	(1.2)/(2.1), (1.3)/(3.1), (2.3)/(3.2)

3-contextural/3-dimensional semiotics

000	(1.1.1), (2.2.2), (3.3.3)
001	(1.1.2), (1.1.3), (2.2.1), (2.2.3), (3.3.1), (3.3.2)
010	(1.2.1), (1.3.1), (2.1.2), (2.3.2), (3.1.3), (3.2.3)
012	(1.2.3), (1.3.2), (2.1.3), (2.3.1), (3.1.2), (3.2.1)

4-contextural/4-dimensional semiotics

0000	(0.0.0.), (1.1.1.1) (2.2.2.2), (3.3.3.)
0001	(0001), (0002), (0003)
0010	(0010), (020), (0030)
0011	(0011), (0022), (0033)
0012	(0012), (0013), (0,14)
0100	(0100), (0200), (0300)
0101	(0101), (0202), (0303),
0102	(0102), (0103), (0203), (0204), (0302), (0304)
0110	(0110), (0220), (0330)
0111	(0111), (0222), (0333)
0112	(0112), (0113), (0221), (0223), (0331), (0332)
0120	(0120), (0130), (0210), (0230), (0310), (0320)
0121	(0121), (0131), (0212), (0232), (0313), (0323)
0122	(0122), (0133), (0211), (0233), (0311), (0322)
0123	(0123), (0132), (0213), (0231), (0312), (0321)

1-dimensional/1-contextural semiotics contains exactly the three fundamental categories of Peirce-Bensean semiotics. 2-dimensional/2-contextural semiotics contains exactly the 9 dyadic sub-signs which Bense had constructed as Cartesian Products out of Peirce's sequence of fundamental categories. 3-dimensional/3-contextural semiotics corresponds exactly to Stiebings's Sign-Cube (Stiebing 1977), and 4-dimensional/4-contextural semiotics is one of the many possibilities to construct a semiotics (or pre-semiotics) in which the contexture border between sign and object is abolished (Toth 2008a, b, c). As a kind of "proof" can be taken that the 1, 2, 4, and 15 choices of the qualitative numbers 1, 2, 3 and 4 deliver exactly the empty forms in which the respective 1-, 2-, 3-, and 4-dimensional sub-signs and not one less and not one more can be filled in. Thus, the general structures of the sub-signs of the 4 semiotics are:

1-dimensional/1-contextural semiotics:	(a), a ∈ {1, 2, 3}
2-dimensional/2-contextural semiotics:	(a.b), a, b ∈ {1, 2, 3}
3-dimensional/3-contextural semiotics:	(a.b.c), a, b, c ∈ {1, 2, 3}
4-dimensional/4-contextural semiotics:	(a.b.c.d), a, ..., d ∈ {1, 2, 3}

Ambiguous are the constructions of sign classes in 3-dimensional/3-contextural and in 4-dimensional/4-contextural semiotics:

1st possibility for interpretation of 3-adic sub-sign in 3-dim/3-cont sign classes:

(3.a.b 2.c.d 1.e.f) → (3.(a.b) 2.(c.d) 1.(e.f), where a, ..., f ∈ {1, 2, 3}

2nd possibility for interpretation of 3-adic sub-sign in 3-dim/3-cont sign classes:

(3.a.b 2.c.d 1.e.f) → ((3.a) .b) (2.c) .d) (1.e) .f), where a, ..., f ∈ {1, 2, 3}

Similar for 4-dim/4-cont sign classes. Special attention belongs to the question, if the Law of Triadicity has to be abolished, e.g.

(3.a.b 2.c.d 1.e.f) → (3.(3.b) 2.(2.d) 1.(1.f), or

(3.(3/2/1.b) 2.(2/1/3.d) 1.(1/2/3.f), and combinations.

The 2nd possibility may also be defined so, that (b, d, f) are the dimensional numbers: ((3.a) .b) (2.c) .d) (1.e) .f)),

whereby in this case dimensional number not have to be restricetd to 3; in the case of b = d = f, b, d, f > 3, we have a tower (Toth 2008b), which can be built as high as the Tower of Babylon where the growth of dimensions strops when the 3-rd dimension is reached.

The advantage of this second concept of polycontextural semiotics is not only that it is possible to differentiate between contextural and dimensional numbers, but since we have here

(1) n-adic sign relation = nth dimension = nth contexture,

the contextural indices (inner semiotic environments) can still be added in order to refine semiotic analysis or to enlarge semiotic complexity. We are thus capable of combining the two concepts of polycontextural semiotics presented in this article.

The fundamental reason, why they are two concepts, we can answer by having another look at the semiotic matrix:

	r1 →	r2 →	r3	
R1 ↓	$1.1_{1,3}$	1.2_{1}	1.3_{3}	Rx: Monad, Dyad, Triad in triadic value
R2 ↓	2.1_{1}	$2.2_{1,2}$	2.3_{2}	rx: Monad, Dyad, Triad in trich. value
R3	3.1_{3}	3.2_{2}	$3.3_{2,3}$	

Each of these sub-signs is a Cartesian product of PZ → PZ (= {1, 2, 3} → (1, 2, 3) and thus formally a dyad. However, semantically, only the genuine sub-signs (identitive morphisms) are relationally homogeneous, i.e. (1.1), (2.2), (3.3), while the rest is mixed between R1R2, R1R3, R2R3 and their converses, i.e. they are semantically everything else than dyads. This is, roughly speaking, the situation in monocontextural semiotics. The decisive step beyond this concept taken by polycontextural semiotics is thus that with abolishment of the logical law of identity the relational homogeneity of the genuine sub-signs, too, is taken away. Strictly speaking, from such a concept it follows that the assignment of contextural indices to sub-signs is (almost) completely arbitrary and the above model is just one solution (cf. Toth 2009). However, from that, it also follows, that the equality between dimensions and contextures is abolished (and that between n-relationality and n-dim., n-cont. anyway). In short, we have here

(2) n-adic sign relation ≠ nth dimension ≠ nth contexture

3. After our results have been presented so far, there is one more logical step to make, namely to combine the two models of a polycontextural semiotics, i.e. Kaehr's model (2008) and Toth's model (2003):

1-contextural/1-dimensional semiotics

0 $1_{1,3}$, $2_{1,2}$, $3_{2,3}$

2-contextural/2-dimensional semiotics

00 $(1.1)_{1,3}$, $(2.2)_{1,2}$, $(3.3)_{2,3}$
01 $(1.2)_{1}/(2.1)_{1}$, $(1.3)_{3}/(3.1)_{3}$, $(2.3)_{2}/(3.2)_{2}$

3-contextural/3-dimensional semiotics

Here, we have either (a.b.c) = ((a.b.) c) or (a (.b.c)) with right or left movement of the dimensional number. We will define (a.b.c) := ((a.b.) c).

000	$(1.1_{1,3}).1), (2.2_{1,2}).2), (3.3_{2,3}).3)$
001	$(1.1_{1,3}).2), (1.1_{1,3}).3), (2.2_{1,2}).1), (2.2_{1,2}).3), (3.3_{2,3}).1), (3.3_{2,3}).2)$
010	$(1.2_{1}).1), (1.3_{3}).1), (2.1_{1}).2), (2.3_{2}).2), (3.1_{3}).3), (3.2_{2}).3)$
012	$(1.2_{1}).3), (1.3_{3}).2), (2.1_{1}).3), (2.3_{2}).1), (3.1_{3}).2), (3.2_{2}).1)$

4-contextural/4-dimensional semiotics

Here, we use the assignment of contextural indices to the (dyadic) sub-signs of a 4×4 matrix by Kaehr (2008, p. 6), i.e. each ordered pair of dyads will be treated here as a (simple) dyad:

0000	$(0.0_{2,3,4}\ 0.0_{2,.3,4}), (1.1_{1,3,4}\ 1.1_{1,3,4})\ (2.2_{1,2,4}\ 2.2_{1,2,4}), (3.3_{1,2,4}\ 3.3_{1,2,4})$
0001	$(0.0_{2,3,4}\ 0.1_{1,4}), (0.0_{2,3,4}\ 0.2_{1,2}), (0.0_{2,3,4}\ 0.3_{2,4})$
0010	$(0.0_{2,3,4}\ 1.0_{1,4}), (0.0_{2,3,4}\ 2.0_{1,2}), (0.0_{2,3,4}\ 3.0_{2,4})$
0011	$(0.0_{2,3,4}\ 1.1_{1,3,4}), (0.0_{2,3,4}\ 2.2_{1,2,4}), (0.0_{2,3,4}\ 3.3_{2,3,4})$
0012	$(0.0_{2,3,4}\ 1.2_{2,4}), (0.0._{2,3,4}\ 1.3_{2,4}), (0.0_{2,3,4}\ 1.4_{3,4})$
$01_{1,4}\ 00_{1,1,4}$	$(0.1_{1,4}\ 0.0_{2,3,4}), (0.2_{1,2}\ 0.0_{2,3,4}), (0.3_{2,4}\ 0.0_{2,3,4})$
$01_{1,4}\ 01_{1,24}$	$(0.1_{1,4}\ 0.1_{1,4}), (0.2_{1,2}\ 0.2_{1,2}), (0.3_{2,4}\ 0.3_{2,4}),$
$01_{1,4}\ 02_{3.4.}$	$(0.1_{1,4}\ 0.2_{1,2}), (0.1_{1,4}\ 0.3_{2,4}), (0.2_{1,2}\ 0.3_{2,4}), (0.2_{1,2}\ 0.4_{2,3,}),$ $(0.3_{2,4}\ 0.2_{1,2}), (0.3_{2,4}\ 0.4_{2,3})$
$01_{1,4}\ 10_{1,4}$	$(0.1_{1,4}\ 1.0_{1,4}), (0.2_{1,2}\ 2.0_{1,2}), (0.3_{2,4}\ 3.0_{2,4})$
$01_{1,4}\ 11_{1,3,4}$	$(0.1_{1,4}\ 1.1_{1,3,4}), (0.2_{1,2}\ 2.2_{1,2,4}), (0.3_{2,4}\ 3.3_{2,3,4})$
$01_{1,4}\ 12$	$(0.1_{1,4}\ 1.2_{1,4}), (0.1_{1,4}\ 1.3_{2,4}), (0.2_{1,2}\ 2.1_{1,4}), (0.2_{1,2}\ 2.3_{2,4}),$ $(0.3_{2,4}\ 3.1_{3,4}), (0.3_{2,4}\ 3.2_{2,4})$
$01_{1,4}\ 20$	$(0.1_{1,4}\ 2.1_{1,4}), (0.1_{1,4}\ 3.0_{2,4}), (0.2_{1,2}\ 1.0_{1,4}), (0.2_{1,2}\ 3.0_{2,4}),$ $(0.3_{2,4}\ 1.0_{1,4}), (0.3_{2,4}\ 2.0_{2,1})$
$01_{1,4}\ 21_{1}$	$(0.1_{1,4}\ 2.1_{1,4}),)0.1_{1,4}\ 3.1_{3,4}), (0.2_{1,2}\ 1.2_{1}), (0.2_{1,2}\ 3.2_{1,2}),$ $(0.3_{2,4}\ 1.3_{2,4}), (0.3_{2,4}\ 2.3_{2,.4})$
$01_{1,4}\ 22_{1,}$	$(0.1_{1,4}\ 2.2_{1,2,4}), (0.1_{1,4}\ 3.3), (0.2_{1,2}\ 1.1_{1,3,4}), (0.2_{1,2}\ 3.3),$ $(0.3_{2,4}\ 1.1_{1,2,4}), (0.3_{2,4}\ 2.2_{1,2,4})$
$01_{1,4}\ 23_{1,2}$	$(0.1_{1,4}\ 2.3_{2,4}), (0.1_{1,4}\ 3.2_{24}), (0.2_{1,2}\ 1.3_{14}), (0.2_{1,2}\ 3.1_{3,4}),$ $(0.3_{2,4}\ 1.2_{1,4}), (0.3_{2,4}\ 2.1_{1,4})$

Bibliography

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2008)

Stiebing, Hans Michael, Zusammenfassungs- und Klassifikationsschemata von Wissenschaftennund Theorien auf semiotischer und fundamentalkategorialer Basis. Diss. Stuttgart 1978

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Semiotics and Pre-Semiotics. Klagenfurt 2008 (2008a)

Toth, Alfred, Der sympathische Abgrund. Klagenfurt 2008 (2008b)

Toth, Alfred, Entwurf einer objektiven Semiotik. Klagenfurt 2008 (2008c)
Toth, Alfred, Polycontextural matrices. In: Electronic Journal for Mathematical Semiotics, 2009

The polycontextural-semiotic matrices of a metaphysics of death

1. In his review article "Ideen zu einer Metaphysik des Todes", Gotthard Günther has shown that a 3-valued non-Aristotelian logic has also 3 identities:

1 = 2 first (classical) identity
2 = 3 second identity
1 = 3 third identity

and has pointed out that "es wäre erst noch zu untersuchen, ob der Fortfall der ersten Identität im Tode wirklich die ichhafte Identität des Individuums endgültig aufhebt" (Günther 1980 (1957), pp. 11 s.).

2. Since I have shown in Toth (2009) that polycontextural matrices are arbitrary in the limited sense that they have to obey the following abstract matrix-scheme (in the case of a semiotic 3×3 matrix):

$$\begin{pmatrix} (a.a)_{i,j} & (a.b)_{k} & (a.c)_{l} \\ (b.a)_{k} & (b.b)_{m,n} & (b.c)_{o} \\ (c.a)_{l} & (c.b)_{o} & (c.c)_{pq} \end{pmatrix}$$

whereby (i,j), (mn) and (pq) are linearly dependent,

we can state that the following two matrices of contextural indices

$$\begin{pmatrix} 1,3 & x & y \\ x^{\circ} & 1,2 & z \\ y^{\circ} & z^{\circ} & 2,3 \end{pmatrix} \qquad \begin{pmatrix} 2,3 & x & y \\ x^{\circ} & 1,2 & z \\ y^{\circ} & z^{\circ} & 1,3 \end{pmatrix}$$

represent the semiotic thematizations of classical identity (individuality), 1 = 2.

From this, is follows that the next two matrices represent the 1. non-classical identity:

$$\begin{pmatrix} 1,2 & x & y \\ x^\circ & 2,3 & z \\ y^\circ & z^\circ & 1,3 \end{pmatrix} \quad \begin{pmatrix} 1,3 & x & y \\ x^\circ & 2,3 & z \\ y^\circ & z^\circ & 1,2 \end{pmatrix}$$

and the last two matrix represent the 2. non-classical identity:

$$\begin{pmatrix} 1,2 & x & y \\ x^\circ & 1,3 & z \\ y^\circ & z^\circ & 2,3 \end{pmatrix} \quad \begin{pmatrix} 2,3 & x & y \\ x^\circ & 1,3 & z \\ y^\circ & z^\circ & 1,2 \end{pmatrix}$$

3. Now, we have for all 3-contextural matrices x, y ∈ {1, 2, 3}, so that we have for every single matrix $\pi(M) = 6$, thus totally 36 matrices. Amongst these 36 matrices, only 12 represent classical identity, and 24 represent thus the possibility uttered by Günther that the death is not the end of individuality. Since individuality can most abstractly described by aid of polycontextural semiotics, we get $24 \cdot 10 = 240$ polycontextural sign classes as the most fundamental organon of non-classical identity and survival of individuality. However, there is most of all one single sign class of foremost interest for us, the "eigenreal" sign class

(3.1 2.2 1.3) × (3.1 2.2 1.3)

whose reality thematic is dual-identical with its sign thematic (cf. Bense 1992).

From this sign class it is known since Walther (1982) that every sign class and reality thematic hang together with it in at least 1 sub-sign, and that this sign class determines the system of the 10 Peircean sign classes insofar as it partitions it in 3 systems of Trichotomic Triads plus the eigenreal dual system itself. Hence, it is sufficient to determine the 24 polycontextural instances of the eigenreal dual system in order to have the semiotic **determinant** of those semiotic partial systems which guarantee the survival of personal identity resp. individuality on the deepest, i.e. semiotic level. And since two of the three sub-signs are converse relations to one another, we can even reduce our 24 instances to 12 dual systems:

1. $(3.1_1 \quad 2.2_{1,3} \quad 1.3_2) \times (3.1_2 \quad 2.2_{3,1} \quad 1.3_1)$
2. $(3.1_1 \quad 2.2_{1,3} \quad 1.3_3) \times (3.1_3 \quad 2.2_{3,1} \quad 1.3_1)$
3. $(3.1_2 \quad 2.2_{1,3} \quad 1.3_1) \times (3.1_1 \quad 2.2_{3,1} \quad 1.3_2)$
4. $(3.1_2 \quad 2.2_{1,3} \quad 1.3_3) \times (3.1_3 \quad 2.2_{3,1} \quad 1.3_2)$
5. $(3.1_3 \quad 2.2_{1,3} \quad 1.3_1) \times (3.1_1 \quad 2.2_{3,1} \quad 1.3_3)$
6. $(3.1_3 \quad 2.2_{1,3} \quad 1.3_2) \times (3.1_2 \quad 2.2_{3,1} \quad 1.3_3)$

7.	$(3.1_1$	$2.2_{2,3}$	$1.3_2)$	×	$(3.1_2$	$2.2_{3,2}$	$1.3_1)$
8.	$(3.1_1$	$2.2_{2,3}$	$1.3_3)$	×	$(3.1_3$	$2.2_{3,2}$	$1.3_1)$
9.	$(3.1_2$	$2.2_{2,3}$	$1.3_1)$	×	$(3.1_1$	$2.2_{3,2}$	$1.3_2)$
10.	$(3.1_2$	$2.2_{2,3}$	$1.3_3)$	×	$(3.1_3$	$2.2_{3,2}$	$1.3_2)$
11.	$(3.1_3$	$2.2_{2,3}$	$1.3_1)$	×	$(3.1_1$	$2.2_{3,2}$	$1.3_3)$
12.	$(3.1_3$	$2.2_{2,3}$	$1.3_2)$	×	$(3.1_2$	$2.2_{3,2}$	$1.3_3)$

Bibliography

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Günther, Gotthard,. Beiträge zur Grundlegung einer operationsfähigen Dialektik. Vol. 3. Hamburg 1980

Toth, Alfred, Polycontextural matrices. In: Electronic Journal of Mathematical Semiotics. 2009

Walther, Elisabeth, Nachtrag zu Trichotomischen Triaden. In: Semiosis 27, 1982, pp. 15-20

Redundanzfreie Herstellung tetradisch-tetratomischer Zeichenklassen durch Abbildung tetradisch-dyadischer Relationen und ihrer Konversen auf das 4-kontexturale Trito-Zahlensystem

1. Bekanntlich wird in der Stuttgarter Semiotik die Frage, ob es 10 oder 27 Zeichenklassen gebe durch das Argument beantwortet, dass der trichotomische Stellenwert einer n-Ade kleiner oder gleich dem trichotomischen Stellenwert der (n+1)-Ade sein muss, oder formal:

Zkl = (3.a 2.b 1.c) mit $a \leq b \leq c$

Allerdings ist dies ein durch nichts zu begründendes ad-hoc-Gesetz. Vermutlich steckt dahinter die Idee, dass die durch Dualisation aus der Zeichentrichotomie entstehende Realitätsthematik der Zeichenthematik hauptwertig zurückstehen möge:

$(3.\underline{1}\ 2.\underline{2}\ 1.\underline{2}) \times (\underline{2}.1\ \underline{2}.2\ \underline{1}.3)$ 2-2-1
$(3.\underline{2}\ 2.\underline{1}\ 1.\underline{1}) \times (\underline{1}.1\ \underline{1}.2\ \underline{2}.3)$ 1-1-2

Hierfür könnte man inhaltlich argumentieren, dass die in der Trichotomie der Zeichenklasse eingebettete Realitätsthematik ebenso durch Dualisation erweckt wird wie umkehert die in der Trichotomie der Realiätätsthematik eingebettete Zeichenthematik. Da die Zeichenthematik jedoch immer den Subjektpol der Erkenntnisrelation und die Realitätsthematik den Objektpol angibt, ergibt sich mit der idealistischen Präponderanz des Subjektes über das Objekt exakt das Gesetz ($a \leq b \leq c$). Wir dürfen es in diesem Fall aber mit Günther getrost beerdigen: "Idealismus und Materialismus erscheinen nicht mehr als alternierende Welanschauungen, von denen entweder die eine oder die andere falsch sein muss, sondern als Entwicklungsstufen eines in sich folgerichtigen Denkprozesses (1991, S. xxvi f.).

2. Im folgenden gehen wir aus von der folgenden semiotischen 4×4-Matrix

$$\begin{pmatrix} 0.0 & 0.1 & 0.2 & 0.3 \\ 1.0 & 1.1 & 1.2 & 1.3 \\ 2.0 & 2.1 & 2.2 & 2.3 \\ 3.0 & 3.1 & 3.2 & 3.3 \end{pmatrix}$$

welche die dyadischen Relationen der Gestalt (a.b) über der Tetrade a, b ∈ {0, 1, 2, 3} zuzüglich ihrer Konversen (a.b)° enthält und bilden sie ab auf das qualitative 4-kontexturale Trito-System

	Protero	Deutero	Trito
K1	0	0	0
K2	00 01	00 01	00 01
K3	000 001 012	000 001 012	000 001 010 011 012
K4	0000 0001 0012 0123	0000 0001 0011 0012 0123	0000 0001 0010 0011 0012 0100 0101 0102 0110 0111 0112 0120 0121 0122 0123

Wir tun das, wie in Toth (2009) gezeigt, am besten, indem wir

(3.a 2.b 1.c 0.d) = (a, b, c, d)

setzten, denn diese Abbildung ist bijektiv. Damit erhalten wir

(3.0 2.0 1.0 0.0)
(3.0 2.0 1.0 0.1)
(3.0 2.0 1.0 0.2)
(3.0 2.0 1.0 0.3)

(3.0 2.0 1.1 0.0)
(3.0 2.0 1.1 0.1)
(3.0 2.0 1.1 0.2)

(3.0 2.0 1.1 0.3)

(3.0 2.0 1.2 0.0)
(3.0 2.0 1.2 0.1)
(3.0 2.0 1.2 0.2)
(3.0 2.0 1.2 0.3)

(3.0 2.0 1.3 0.0)
(3.0 2.0 1.3 0.1)
(3.0 2.0 1.3 0.2)
(3.0 2.0 1.3 0.3)

(3.0 2.1 1.0 0.0)
(3.0 2.1 1.0 0.1)
(3.0 2.1 1.0 0.2)
(3.0 2.1 1.0 0.3)

(3.0 2.1 1.1 0.0)
(3.0 2.1 1.1 0.1)
(3.0 2.1 1.1 0.2)
(3.0 2.1 1.1 0.3)

(3.0 2.1 1.2 0.0)
(3.0 2.1 1.2 0.1)
(3.0 2.1 1.2 0.2)
(3.0 2.1 1.2 0.3)

(3.0 2.1 1.3 0.0)
(3.0 2.1 1.3 0.1)
(3.0 2.1 1.3 0.2)
(3.0 2.1 1.3 0.3)

(3.0 2.2 1.0 0.0)
(3.0 2.2 1.0 0.1)
(3.0 2.2 1.0 0.2)
(3.0 2.2 1.0 0.3)

(3.0 2.2 1.1 0.0)
(3.0 2.2 1.1 0.1)
(3.0 2.2 1.1 0.2)
(3.0 2.2 1.1 0.3)

(3.0 2.2 1.2 0.0)
(3.0 2.2 1.2 0.1)

(3.0 2.2 1.2 0.2)
(3.0 2.2 1.2 0.3)

(3.0 2.2 1.3 0.0)
(3.0 2.2 1.3 0.1)
(3.0 2.2 1.3 0.2)
(3.0 2.2 1.3 0.3)

(3.0 2.3 1.0 0.0)
(3.0 2.3 1.0 0.1)
(3.0 2.3 1.0 0.2)
(3.0 2.3 1.0 0.3)

(3.0 2.3 1.1 0.0)
(3.0 2.3 1.1 0.1)
(3.0 2.3 1.1 0.2).
(3.0 2.3 1.1 0.3)

(3.0 2.3 1.2 0.0)
(3.0 2.3 1.2 0.1)
(3.0 2.3 1.2 0.2)
(3.0 2.3 1.2 0.3)

(3.0 2.3 1.3 0.0)
(3.0 2.3 1.3 0.1)
(3.0 2.3 1.3 0.2)
(3.0 2.3 1.3 0.3)

(3.1 2.1 1.1 0.1)
(3.2 2.2 1.2 0.2)
(3.3 2.3 1.3 0.3)

3. So, we have $16 \cdot 4 = 64 + 3 = 67$ sign classes for which clearly the "idealistic" restriction ($a \leq b \leq c$) does not hold. Moreover, this way of introducing 4-adic (or any-adic) sign classes is redundance free insofar as the last triple of sign classes are quasi abbreviations for the three more systems than the one that has been developed here explicitly.

Bibliography

Günther, Gotthard, Idee und Grundriss einer nicht-aristotelischen Logik. 3rd ed. Hamburg 1991

Toth, Alfred, Zeichenklasse und Kenogramm. In: Electronic Journal for Mathematical Semiotics, 2009

Kontexturell über- und unterbalancierte polykontextural-semiotische Matrizen

1. Der Ausgangspunkt der vorliegenden Abhandlung ist die Einsicht, dass das durch das Zeichen transzendierte Objekt nicht die einzige transzendente Grösse des Zeichen ist (vgl. Toth 2008), wie durchwegs angenommen wird. Wenn man sich überlegt, dass der Zeichenträger oder das Mittel des Zeichens aus einem Repertoire selektiert ist, von dem es sich, wenigstens als künstliches Zeichen, sowohl räumlich als auch zeitlich vollständig etablieren muss, so wird klar, dass bei diesem Übergang vom aktuellen Mittel zum realisierenden Mittel-Bezug die beiden Grössen einander transzendent geworden sind. Dasselbe gilt für das Verhältnis von Interpret und Interpretantenbezug: Peirce hatte ja gerade den Ausdruck Interpretant anstatt Interpret gewählt, weil sowohl der zeichenstiftende wie zeicheninterpretierende Interpret natürlich ausserhalb der triadischen Zeichenrelation bleiben.

2. Wir können damit hinsichtlich Transzendentalität der Fundamentalkategorien unterscheiden zwischen der minimalen, vollständig transzendenten Zeichenrelation $ZR_{3,3}$ und der minimalen, vollständig nicht-transzendenten Zeichenrelation $ZR_{6,6:}$

$ZR_{3,3}$ = (3.a 2.b 1.c)
$ZR_{6,3}$ = (3.a 2.b 1.c 0.d ⦿.e ◎.f)

Die Kategorie 0 als nicht-transzendente Kategorie für (.2.) wurde aus nostalgischen Gründen gewählt. Anstelle von ⦿ und ◎) hätten beliebige andere Symbole gewählt worden sein können. Wichtig ist einzig die Reihenfolge der transzendenten und nicht-transzendenten Kategorien in einer Zeichenrelation; sie ist allgemein:

$ZR_{allg.}$ = (3 → 2 → 1 → ⦿ → 0 → ◎)

3. Da die Existenz tetradischer, pentadischer usw. Zeichenrelationen formal nie in Frage gestellt worden war (vgl. Toth 2007, S. 179 ff.) und da man natürlich solche Zeichenklassen konstruieren kann, bei denen nur eine, zwei oder alle drei Fundamentalkategorien nicht nur transzendent, sondern auch nicht-transzendent vorkommen können, ergibt sich die folgende 4×4 semiotische Zeichenrelations-Matrix:

$ZR_{3,3}$	$ZR_{4,3}$	$ZR_{5,3}$	$ZR_{6,3}$
$ZR_{3,4}$	$ZR_{4,4}$	$ZR_{5,4}$	$ZR_{6,4}$
$ZR_{3,3}$	$ZR_{4,5}$	$ZR_{5,3}$	$ZR_{6,3}$
$ZR_{3,6}$	$ZR_{4,6}$	$ZR_{5,6}$	$ZR_{6,6}$

Die Matrix über Zeichenrelationen enthält also selbst wiederum Matrizen, und zwar solche der Gestalt n×n, m×n und n×m. In der folgenden Figur sind die zu einander transpositionellen Matrizen durch die gleiche Farbe markiert:

$ZR_{3,3}$	$ZR_{4,3}$	$ZR_{5,3}$	$ZR_{6,3}$
$ZR_{3,4}$	$ZR_{4,4}$	$ZR_{5,4}$	$ZR_{6,4}$
$ZR_{3,5}$	$ZR_{4,5}$	$ZR_{5,5}$	$ZR_{6,5}$
$ZR_{3,6}$	$ZR_{4,6}$	$ZR_{5,6}$	$ZR_{6,6}$

Demzufolge erhalten wir auch eine neue Eigenrealität

$ER = (ZR_{3,6}, ZR_{4,5}, ZR_{5,4}, ZR_{6,3})$

sowie eine neue Kategorienrealität

$KR = (ZR_{3,3}, ZR_{4,4}, ZR_{5,5}, ZR_{6,6})$.

4. In Toth (2008) wurden nun die total 16 semiotischen Dualsysteme, die über den $ZR_{3,3}$, ..., $ZR_{6,6}$ konstruierbar sind, ausführlich in Form von Zeichenklassen und Realitätsthematiken dargestellt, wobei die Systeme folgende Mengen von Repräsentationssystemen enthalten:

$S_{ZR3,3} = 10$
$S_{ZR4,4} = 35$
$S_{ZR5,5} = 64$
$S_{ZR6,6} = 95$

$S_{ZR4,3} = 15$	$S_{ZR5,4} = 53$
$S_{ZR3,4} = 20$	$S_{ZR4,5} = 60$
$S_{ZR5,3} = 21$	$S_{ZR6,4} = 64$
$S_{ZR3,5} = 35$	$S_{ZR4,6} = 95$

$S_{ZR6,3} = 28$ $S_{ZR6,5} = 100$

$S_{ZR3,6} = 56$ $S_{ZR5,6} = 95$

5. Wir bringen nun eine Übersicht über die einige der 16 Matrizen:

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.3_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix}_{3\times 3}$$

$$\begin{pmatrix} 0.0 & 0.1 & 0.2 & 0.3 \\ 1.0 & 1.1 & 1.2 & 1.3 \\ 2.0 & 2.1 & 2.2 & 2.3 \\ 3.0 & 3.1 & 3.2 & 3.3 \end{pmatrix}_{4\times 4}$$

$$\begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3 \end{pmatrix}_{3 \times 4}$$

$$\begin{pmatrix} 0.0 & 0.1 & 0.2 & 0.3 \\ 1.0 & 1.1 & 1.2 & 1.3 \\ 2.0 & .21 & 2.2 & 2.3 \end{pmatrix}_{4\times 3}$$

$$\begin{pmatrix} \circledcirc.1 & \circledcirc.2 & \circledcirc.4 \\ \odot.1 & \odot.2 & \odot.3 \\ 0.1 & 0.2 & 0.3 \\ 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3 \end{pmatrix}_{6\times 3}$$

$$\begin{pmatrix} 1.\circledcirc & 1.\odot & 1.0 & 1.1 & 1.2 & 1.3 \\ 2.\circledcirc & 2.\odot & 2.0 & 2.1 & 2.2 & 2.3 \\ 3.\circledcirc & 3.\odot & 3.0 & 3.1 & 3.2 & 3.3 \end{pmatrix}_{3\times 6}$$

Z.B. enthält die 3×6 Matrix folgende Struktur:

$1.\circledcirc$	$1.\odot$	1.0	1.1	1.2	1.3
$2.\circledcirc$	$2.\odot$	2.0	2.1	2.2	2.3
$3.\circledcirc$	$3.\odot$	3.0	3.1	3.2	3.3

Da in der rechten Blockmatrix die kleine semiotische Matrix auftaucht, können wir sie wieder wie oben mit Kontexturen indizieren. Nun erinnern wir uns aber daran, dass

(0: .2.), ($\odot$: .1.) und ($\circledcirc$: .3.)

die zusammengehörigen transzendental-nicht-transzendentalen Paare sind. Das bedeutet aber, dass die links von der vertikalen Trennlinie stehende Blockmatrix einfach die Blockmatrix der Realitätsthematik der rechts von der vertikalen Linie stehenden Blockmatrix der Zeichenthematik ist. In einem Zeichen wird ja die Realität eines Zeichens durch eine eigene Realitätsthematik vermittelt, die aus der Zeichenthematik dual gewonnen wird. Und in früheren Arbeiten hatten wir herausgefunden, dass die monokontexturale Semiotik an der dauernden Verwechslung von Inversion und Dualisation krankt: So ist (2.1) = (1.2)° und (2.1)° = (1.2), aber nur gdw. alle Subzeichen in der gleichen Matrix liegen, denn $\times(1.1_{1,3}) = (1.1)_{3,1}$, denn (1.1)° = (1.1):

$1.\circledcirc_{3'}$	$1.\odot_{1'}$	$1.0_{3,1}$	$1.1_{1,3}$	1.2_{1}	1.3_{3}
$2.\circledcirc_{2'}$	$2.\odot_{2,1}$	$2.0_{1''}$	2.1_{1}	$2.2_{1,2}$	2.3_{2}
$3.\circledcirc_{3,2}$	$3.\odot_{2''}$	3.0_{3}	3.1_{3}	3.2_{2}	$3.3_{2,3}$

R-Thematik Z-Thematik

vermitteltes Zeichen-Objekt

bzw.

Objekt-Zeichen

Alle n × m bzw. m× n Matrizen (mit n < oder > m) weisen also kategorielle Über- oder Unterbalancierung auf, und Über- und Unterbalancierung im Verhältnis der nicht-transzendenten Repräsentationen der zugehörigen Realitätsthematik des transzendenten Repräsentationsschemas zwischen Zeichen.- und Realitätsthematik.

Bibliographie

Toth, Alfred, Zwichen den Kontexturen. Klagenfurt 2008
Toth, Alfred Balancierte und unterbalancierte semotische Systmeme. In: Electronic Journal for Mathematical Semiotics, 2009

Transzendentale und nicht-transzendentale Zeichenklassen

1. Wie ich bereits in einigen Arbeiten, z.B. am ausführlichsten in Toth (2008a) gezeigt hatte, basiert der **Repräsentations**charakter eines Zeichens essentiell auf seinem **Substitutions**charakter. Ein Zeichen soll ja ein Objekt möglichst unabhängig von Raum und Zeit bezeichnen können. Die Gestalt eines Apfels als Icon sollte punkto Farbe des Apfels (Reifegrad, Sorte) und Form (Sorte) so allgemein sein, dass Äpfel auf allen Teilen der Welt, wo sie bekannt sind, bezeichnet werden können. Die praktische Anwendung eines Wegweisers als Index, der auf einen Ort verweist, wächst mit dem geographischen Abstand des Wegweisers zu diesem Ort. Die Wörter einer Sprache als Symbole sollten im ganzen Sprachgebiet verstanden werden können.

2. Der Unterschied zwischen Repräsentations- und Substitutionscharakter eines Zeichens ist wesentlich. Ein Zeichen repräsentiert nur ein Objekt, nicht aber ein Mittel oder einen Interpretanten. Genau genommen ist also das Zeichen qua Repräentativität monadisch (Leibniz), denn es wäre sinnlos, wenn das Zeichen etwa seinen Zeichenstifter mit-repräsentieren würde, dies ist nur in wenigen linguistischen Fällen so, nämlich bei den sogenannten Eponymen, wo das Zeichen einer Marke entspricht (sich eine "Davidoff" anzünden, mit einem "Zeppelin" fliegen (mit einem "Porsche" fahren), mit einem Ortseponym: einen "Cognac" ("Armagnac", "Montbazillac", "Tokaier", "Kretzer", etc.) trinken. Genau sinnlos wäre es, wenn das Zeichen sein Mittel repräsentiert, denn das wäre eine contradictio in adiecto, da das Mittel als Träger des Zeichens dient, da Zeichen, wenigstens als manifestierte, immer eines Mittels bedürfen, um geäussert bzw. wahrgenommen zu werden.

Vom Standpunkt des Substitutionscharakters her ist das Zeichen allerdings triadisch: Zunächst soll ein Objekt durch ein Zeichen vertreten werden. Das dient also etwa das altbekannte Taschentuch, das zu diesem Zwekc verknotet wird. Wenn aber jemand ein verknotetes Taschentuch findet, das nicht der Finder selbst verknotet hat, ist dieses Zeichen bedeutungslos, denn das Zeichen substituiert ebenfalls den Interpretanten, für den und durch den es in diesem Fall ein Zeichen für ein Anderes ist. Schliesslich substituiert das verknotete Taschentuch aus trivialen Gründen ebenfalls ein Mittel, denn dieses wird durch einen Mittel-Bezug substituiert, d.h. durch etwas nicht-Stoffliches. Das Stoffliche des Mittels wird sekundär, seine Funktion wird primär physikalisch (der Knoten sollte sich bis zum Erlöschen des Zeichens nicht in Luft auflösen, also z.B. so lange bestehen, bis das Referenzobjekt des Zeichens eingelöst ist, d.h. etwa der Anruf getätigt, das Essen aus dem Kühlschrank geholt ist, usw..

3. Vom Repräsentationscharakter des Zeichens her ergibt sich also folgendes triviales Schema:

Zeichen = Repr(Obj) (monadische Funktion)

Vom Substitutionscharakter des Zeichens her ergibt sich allerdings folgendes gar nicht-triviales Schema:

Zeichen = Subst(Mittel, Objekt, Interpretant) (triadische Funktion)

(Gibt es Zeichen, die dyadische Funktionen darstellen?)

Nun macht man sich schnell klar, dass es wieder die Substitutions- und nicht die Repräsentationsfunktion des Zeichens ist, die dafür verantwortlich ist, dass bei der Zeichengenese (Semiose) Objekt und Zeichen einander transzendent werden, denn auch bei natürlichen Zeichen repräsentiert ja etwa die Eisblume gewisse klimatische Parameter wie Temperatur oder Feuchtigkeit der Luft, allerdings kann in diesem Fall nicht die Rede davon sein, dass Zeichen (Eisblume) und Objekt (Klima) einander transzendent sind. Im Gegenteil ist die Eisblume Teil des Klimas, also sozusagen eine "Teilmenge" des Objektes, die sich von Objekt einzig dadurch unterscheidet, dass sie durch einen Interpretanten in einen Kausalzusammenhang zum Klima gebracht und dadurch in einem gewissen Sinne "interpretiert" wird.

Durch die triadische Substitutionsfunktion des Zeichens werden also 3 Objekte des ontischen Raumes (zur Unterscheidung von ontischem und semiotischem Raume vgl. Bense 1975, S. 45f., 65 ff.) zu 3 Kategorien des semiotischen Raumes für diese 3 Objekte sozusagen kopiert, wobei sich die Objekte und die Kopien einander paarweise als transzendent gegenüberstehen. Wir wollen hier vereinbaren, dass wir durchwegs den Standpunkt des Zeichens einnehmen, d.h. wir wollen nicht sagen, dass ein Zeichen seinem Objekt transzendent ist, sondern dass ein Objekt seinem Zeichen transzendent ist. Das bedeutet, dass wir unter einer transzendenten Zeichenklasse eine Zeichenklasse mit 6 Gliedern verstehen, nämlich die 3 Fundamentalkategorien des Peirceschen Zeichens zuzüglich ihrer 3 transzendenten Objekte. Dementsprechend meinen wir mit einer nicht-transzendenten Zeichenklasse einfach eine Peircesche Zeichenklasse mit den 3 Fundamentalkategorien:

Nicht-transzendente Zkl = (3.a 2.b 1.c)

Transzendente Zkl = (3.a 2.b 1.c 0.d ⦿.e ◎.f).

wobei also die Korrespondenzhierarchie zwischen transzendenten und nicht-transzendenten Objekten (Kategorien) wie folgt ist:

(3.a) → (2.b) → (1.c)
↓ ↓ ↓
(⦿.e) → (0.d) → (◎.f)

4. Diese Korrespondenzen ergeben sich also daraus, dass das jeweils obere Glied das untere **substituiert**. Allerdings stehen wir im Grunde jetzt vor einem Berg, da die oberen Glieder Relationen sind, aber die unteren Kategorien. Nach Bense (1975, S. 65 f.) haben daher die unteren Glieder nur Kategorialzahlen, die oberen aber zusätzlich Relationszahlen. Oder anders gesagt: Kategorien sind Relationen mit Relationszahl r = 0. Mit diesem "Trick" und der von Bense vorgeschlagenen Schreibweise $Z^r{}_k$ für "Zeichen" mit r ≥ 0, können wir unser Korrespondenzschema also viel besser wie folgt notieren:

$$\left.\begin{array}{lll} (Z^3{}_a) \rightarrow & (Z^2{}_b) \rightarrow & (Z^1{}_c) \\ \downarrow & \downarrow & \downarrow \\ (Z^0{}_a) \rightarrow & (Z^0{}_b) \rightarrow & (Z^0{}_c) \end{array}\right\} \quad a, b, c \in \{.1, .2, .3\}$$

somit haben wir das Problem gelöst; die Zeichen (⦿, 0, ◎) sind einfach Memoranda für die transzendentalen Entsprechungen von ((.1.), (.2.), (.3.)), aber im Grunde gibt es keinen Zwang ihrer Reihenfolge innerhalb einer transzendenten Zeichenrelation, d.h. wir haben

(3.a 2.b 1.c 0.d ⦿.e ◎.f) ~ (3.a 2.b 1.c ⦿.e 0.d ◎.f) ~ (3.a 2.b 1.c ◎.f ⦿.e 0.d) ~ (3.a ◎.f 2.b 1.c ⦿.e 0.d) ~ (0.d 3.a ◎.f 2.b ⦿.e 1.c) ~ etc.

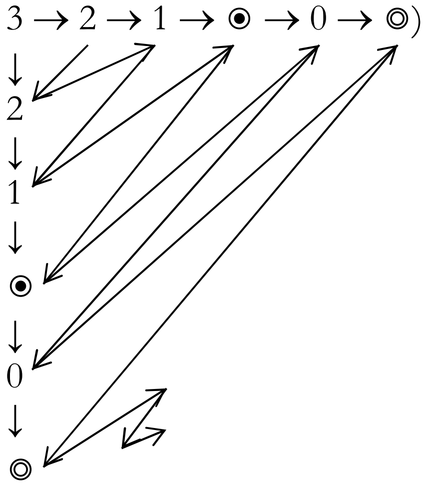
Wie man anhand der letzten zwei Äquivalenzen sieht, spricht rein formal sogar nichts dagegen, etwa die Ordnung (3.a → 2.b) oder die komplexe Ordnung (3.a → 2.b → 1.c) durch zwischengeschobene Kategorien mit r = 1 zu unterbrechen. Wie ich in Toth (2008b, c, d, e) gezeigt hatte, ergeben sich daraus (höchst interessante) semiotische "Zwischenzahlbereiche", die einiges mit den transzendentalen Zahlen der quantitativen Mathematik zu tun zu haben scheinen, die ja auch die lineare Reihe der natürlichen Zahlen in gewissen Sinne "unterbrechen", wobei der meistaus grösste Teil dieser transzendentalen Zahlen gar nicht bekannt ist. Ebenso unterbrechen die transzendenten (oder besser transzendentalen ?) qualitativen Kategorien die lineare Reihe der "Primzeichen", wobei auch diese semiotischen Zwischenzahlbereiche zum allergrössten Teil noch unbekannt sind.

5. Wir können damit hinsichtlich Transzendentalität der Zeichenklassen unterscheiden zwischen der minimalen, vollständig nicht-transzendenten Zeichenklasse $Zkl_{3,3}$ und der maximalen, vollständig transzendenten Zeichenklasse $ZR_{6,6}$.

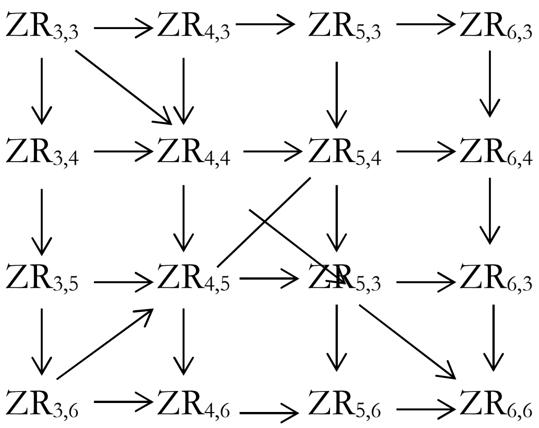
$Zkl_{3,3}$ = (3.a 2.b 1.c)

$Zkl_{6,3}$ = (3.a 2.b 1.c 0.d ⦿.e ◎.f)

Wie man sieht, gibt es jedoch im Gegensatz zur Linearität natürlicher und transzendenter Zahlen einen "flächigen Weg" zwischen $Zkl_{3,3}$ und $Zkl_{6,3,}$ und zwar den triadischen und den trichotomischen Werten nach:



Wenn man die semiotischen Zeichenzahlen und ihre Zwischen-Zahlen als quadratische Matrix ihrer Zeichenrelationen anordnet, so bekommt folgendes Schema, worin das horizontale und das vertikale Zeichen-Zahlen-Wachstum direkt aus den den semiotischen Systemen zugrunde liegenden Matrizen abgelesen werden kann:



Die Matrix über Zeichenrelationen enthält also selbst wiederum Matrizen, und zwar solche der Gestalt n×n, m×n und n×m. In der folgenden Figur sind die zu einander transpositionellen Matrizen durch die gleiche Farbe markiert:

$ZR_{3,3}$	$ZR_{4,3}$	$ZR_{5,3}$	$ZR_{6,3}$
$ZR_{3,4}$	$ZR_{4,4}$	$ZR_{5,4}$	$ZR_{6,4}$
$ZR_{3,5}$	$ZR_{4,5}$	$ZR_{5,5}$	$ZR_{6,5}$
$ZR_{3,6}$	$ZR_{4,6}$	$ZR_{5,6}$	$ZR_{6,6}$

Demzufolge erhalten wir auch eine neue Eigenrealität

$ER = (ZR_{3,6}, ZR_{4,5}, ZR_{5,4}, ZR_{6,3})$

sowie eine neue Kategorienrealität

$KR = (ZR_{3,3}, ZR_{4,4}, ZR_{5,5}, ZR_{6,6})$.

6. In Toth (2008f) wurden nun die total 16 semiotischen Systeme, die über den $ZR_{3,3}$, ..., $ZR_{6,6}$ konstruierbar sind, ausführlich in Form von Zeichenklassen und Realitätsthematiken dargestellt, wobei die Systeme folgende Mengen von Repräsentationssystemen enthalten:

(m×m): $S_{ZR3,3} = 10$; $S_{ZR4,4} = 35$; $S_{ZR5,5} = 64$; $S_{ZR6,6} = 95$

(m×n): $S_{ZR4,3} = 15$; $S_{ZR5,3} = 21$; $S_{ZR6,3} = 28$; $S_{ZR5,4} =53$; $S_{ZR6,4} = 64$; $S_{ZR6,5} = 100$

(n×m): $S_{ZR3,4} = 20$; $S_{ZR3,5} = 35$; $S_{ZR3,6} = 56$; $S_{ZR4,5} = 60$; $S_{ZR4,6} = 95$; $S_{ZR5,6} =95$

Die Frage, die sich jetzt natürlich stellt, ist diejenige der mengentheoretischen Inklusion resp. der qualitativen Fragment-Relation von $S_{x,y}$ mit $y < x$ (vgl. Toth 2003, S. 54 ff.). Es geht also im wesentlichen um folgende beiden mengentheoretischen Typen, die sich aus rein quantitativ imitieren lassen:

$M = \{0, 1, 3, 4, 5, 8\}$
$N = \{1, 3, 4, 5, 6\}$
$O = \{1, 3, 5, 8\}$

Zur Betrachtung unserer polykontexturalen Mengen schreiben wir

$O \subset M$ $\quad$ $O \sqsubset M$

$O \not\subset N$ $\quad$ $N \sqsubset M$, wobei das Zeichen $\subset$ die mengentheoretische Inklusion, das Zeichen $\sqsubset$ die polykontexturale Fragmentrelation bezeichnet.

Wenn wir Frakturbuchstaben für semiotische Systeme verwenden, können wir folgende drei Theoreme formulieren für transzendente, nicht-transzendente und gemischt transzendent-nicht-transzendente Zeichenklassen, deren Matrizen den Typen m×m, m × n und n×m entsprechen:

Theorem 1: $\mathcal{E}(Zkl_{n \times n}) \subset \mathcal{F}(Zkl_{n+m \times n+m})$ für $m \geq 0$.
(Alle diagonalen Systeme von Zeichenklassen sind abwärts ineinander enthalten.)

Theorem 2: $\mathcal{E}(Zkl_{n \times m}) \subset \mathcal{F}(Zkl_{n+i \times m+j})$ für $((n+i)\times(m+j)) \geq (n\times m)$.
(Nur solche Systeme von Zeichenklassen sind ineinander enthalten, bei denen sowohl das m als auch das n ineinander enthalten sind.)

Theorem 3: $\mathcal{E}(Zkl_{n \times m}) \sqsubset \mathcal{F}(Zkl_{n+i \times m+j})$ für $i \geq 0$, $j \geq 1$.
(Das System $\mathcal{F}$ darf also im m seiner Matrix mindestens 1 Element mehr enthalten.)

Im folgenden stellen wir die beiden semiotischen Systeme $ZR_{3,5}$ und $ZR_{4,6}$ einander gegenüber. Da die Bedingung $\mathcal{E}(Zkl_{n \times m}) \subset \mathcal{F}(Zkl_{n+i \times m+j})$ für $i \geq 0$, $j \geq 1$,für $j = 2$ erfüllt ist, gilt also Theorem 3, und es ist $\mathcal{E}(Zkl_{3 \times 5}) \sqsubset \mathcal{F}(Zkl_{4 \times 6})$. Wir deuten aus praktischen Gründen lediglich einige der Fragmentrelationen mit Verbindungslinien an.

3. $ZR_{3,5}$ = (3.a 2.b 1.c)	8. $ZR_{4,6}$ = (3.a 2.b 1.c O.d)
mit a, b, c, d, e ∈	mit a, b, c, d, e, f ∈
{.1, .2, .3, .O, .⊙}	{.1, .2, .3, .O, .⊙, .◎}

1 (3.0 2.0 1.0)	1 (3.0 2.0 1.0 0.0)
2 (3.0 2.0 1.⊙)	2 (3.0 2.0 1.0 0.⊙)
3 (3.0 2.0 1.1)	3 (3.0 2.0 1.0 0.◎)
4 (3.0 2.0 1.2)	4 (3.0 2.0 1.0 0.1)
5 (3.0 2.0 1.3)	5 (3.0 2.0 1.0 0.2)
6 (3.0 2.⊙ 1.⊙)	6 (3.0 2.0 1.0 0.3)
7 (3.0 2.⊙ 1.1)	7 (3.0 2.0 1.⊙ 0.⊙)
8 (3.0 2.⊙ 1.2)	8 (3.0 2.0 1.⊙ 0.◎)
9 (3.0 2.⊙ 1.3)	9 (3.0 2.0 1.◎ 0.◎)
10 (3.0 2.1 1.1)	10 (3.0 2.0 1.◎ 0.1)
11 (3.0 2.1 1.2)	11 (3.0 2.0 1.◎ 0.1)
12 (3.0 2.1 1.3)	12 (3.0 2.0 1.1 0.1)
13 (3.0 2.2 1.2)	13 (3.0 2.0 1.◎ 0.2)
14 (3.0 2.2 1.3)	14 (3.0 2.0 1.◎ 0.2)
15 (3.0 2.3 1.3)	15 (3.0 2.0 1.1 0.2)
16 (3.⊙ 2.⊙ 1.⊙)	16 (3.0 2.0 1.2 0.2)
17 (3.⊙ 2.⊙ 1.1)	17 (3.0 2.0 1.⊙ 0.3)
18 (3. ⊙ 2.1 1.1)	18 (3.0 2.0 1.◎ 0.3)
19 (3.⊙ 2.⊙ 1.2)	19 (3.0 2.0 1.1 0.3)
20 (3.⊙ 2.⊙ 1.3)	20 (3.0 2.0 1.2 0.3)
21 (3.⊙ 2.1 1.2)	21 (3.0 2.0 1.3 0.3)
22 (3.⊙ 2.1 1.3)	22 (3.0 2.⊙ 1.⊙ 0.⊙)
23 (3.⊙ 2.2 1.2)	23 (3.0 2.⊙ 1.⊙ 0.◎)
24 (3.⊙ 2.2 1.3)	24 (3.0 2.⊙ 1.⊙ 0.1)
25 (3.⊙ 2.3 1.3)	25 (3.0 2.⊙ 1.⊙ 0.2)
26 (3.1 2.1 1.1)	26 (3.0 2.⊙ 1.⊙ 0.3)
27 (3.1 2.1 1.2)	27 (3.0 2.⊙ 1.◎ 0.◎)
28 (3.1 2.1 1.3)	28 (3.0 2.⊙ 1.◎ 0.1)
29 (3.1 2.2 1.2)	29 (3.0 2.⊙ 1.1 0.1)
30 (3.1 2.2 1.3)	30 (3.0 2.⊙ 1.◎ 0.2)
31 (3.1 2.3 1.3)	31 (3.0 2.⊙ 1.1 0.2)
32 (3.2 2.2 1.2)	32 (3.0 2.⊙ 1.2 0.2)
33 (3.2 2.2 1.3)	33 (3.0 2.⊙ 1.◎ 0.3)
34 (3.2 2.3 1.3)	34 (3.0 2.⊙ 1.1 0.3)
35 (3.3 2.3 1.3)	35 (3.0 2.⊙ 1.2 0.3)
	36 (3.0 2.⊙ 1.3 0.3)
	37 (3.0 2.◎ 1.◎ 0.◎)
	38 (3.0 2.◎ 1.◎ 0.1)

39 (3.0 2.◎ 1.◎ 0.2)
40 (3.0 2.◎ 1.◎ 0.3)
41 (3.0 2.◎ 1.1 0.1)
42 (3.0 2.◎ 1.1 0.2)
43 (3.0 2.◎ 1.2 0.2)
44 (3.0 2.◎ 1.1 0.3)
45 (3.0 2.◎ 1.2 0.3)
46 (3.0 2.◎ 1.3 0.3)
47 (3.0 2.1 1.1 0.1)
48 (3.0 2.1 1.1 0.2)
49 (3.0 2.1 1.1 0.3)
50 (3.0 2.1 1.2 0.2)
51 (3.0 2.1 1.2 0.3)
52 (3.0 2.1 1.3 0.3)
53 (3.0 2.2 1.2 0.2)
54 (3.0 2.2 1.2 0.3)
55 (3.0 2.3 1.3 0.3)
56 (3.⊙ 2.⊙ 1.⊙ 0.⊙)
57 (3.⊙ 2.⊙ 1.⊙ 0.◎)
58 (3.⊙ 2.⊙ 1.⊙ 0.1)
59 (3.⊙ 2.⊙ 1.⊙ 0.2)
60 (3.⊙ 2.⊙ 1.⊙ 0.3)
61 (3.⊙2.⊙ 1.◎ 0.◎)
62 (3.⊙ 2.⊙ 1.◎ 0.1)
63 (3.⊙ 2.⊙ 1.1 0.1)
64 (3.⊙ 2.⊙ 1.◎ 0.2)
65 (3.⊙ 2.⊙ 1.1 0.2)
66 (3.⊙ 2.⊙ 1.2 0.2)
67 (3.⊙2.⊙ 1.1 0.3)
69 (3.⊙ 2.⊙ 1.2 0.3)
70 (3.⊙ 2.◎ 1.3 0.3)
71 (3.⊙ 2.1 1.1 0.1)
72 (3.⊙ 2.1 1.1 0.2)
73 (3.⊙ 2.1 1.1 0.3)
74 (3.⊙ 2.1 1.2 0.2)
75 (3.⊙ 2.1 1.2 0.3)
76 (3.⊙ 2.1 1.3 0.3)
77 (3.⊙ 2.2 1.2 0.2)

78 (3.⊙ 2.2 1.2 0.3)
79 (3.⊙ 2.2 1.3 0.3)
80 (3.⊙ 2.3 1.3 0.3)
81 (3.1 2.1 1.1 0.1)
82 (3.1 2.1 1.1 0.2)
83 (3.1 2.1 1.1 0.3)
84 (3.1 2.1 1.2 0.2)
85 (3.1 2.1 1.2 0.3)
86 (3.1 2.1 1.3 0.3)
87 (3.1 2.2 1.2 0.2)
88 (3.1 2.2 1.2 0.3)
89 (3.1 2.2 1.3 0.3)
90 (3.1 2.3 1.3 0.3)
91 (3.2 2.2 1.2 0.2)
92 (3.2 2.2 1.2 0.3)
93 (3.2 2.2 1.3 0.3)
94 (3.2 2.3 1.3 0.3)
95 (3.3 2.3 1.3 0.3)

Bibliographie

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Toth, Alfred, Die Hochzeit von Semiotik und Struktur. Klagenfurt 2003

Toth, Alfred, Wie viele Kontexturgrenzen hat ein Zeichen? In: Electronic Journal of Mathematical Semiotics, 2008a

Toth, Alfred, Die semiotischen Zahlbereiche. In: Electronic Journal of Mathematical Semiotics, 2008b

Toth, Alfred, Qualitative semiotische Zahlbereiche und Transzendenzen. In: Electronic Journal of Mathematical Semiotics, 2008c

Toth, Alfred, Semiotische Zwischenzahlbereiche. In: Electronic Journal of Mathematical Semiotics, 2008d

Toth, Alfred, Semiotische Zwischenzahlbereiche II. . In: Electronic Journal of Mathematical Semiotics, 2008e

Toth, Alfred, Balancierte und unbalancierte semiotische Systeme. In: Electronic Journal of Mathematical Semiotics, 2008f

Zu Kaehrs semiotischer Interaktion zwischen Sem_2 und Sem_1

1. Gegeben seien die folgenden beiden semiotischen Matrizen Sem1 und Sem2 (vgl. Kaehr 2009, S. 19):

$$Sem^1 = \begin{bmatrix} 1.1_1 & 1.2_1 & 1.3_1 \\ 2.1_1 & 2.2_1 & 2.3_1 \\ 3.1_1 & 3.2_1 & 3.3_1 \end{bmatrix}, \; Sem^2 = \begin{bmatrix} 3.3_2 & 3.4_2 & 3.5_2 \\ 4.3_2 & 4.4_2 & 4.5_2 \\ 5.3_2 & 5.4_2 & 5.5_2 \end{bmatrix}$$

Diese beiden triadischen Matrizen können zu einer pentadischen Matrix komponiert werden, indem das einzige Sem1 und Sem2 gemeinsame Subzeichen, (3.3), das in Sem1 in der Kontextur 1, in Sem2 in der Kontextur 2 auftritt, in seinen kontexturellen Formen konkateniert wird:

$$\begin{array}{ccccccc} 1.1 & 1.2 & 1.3 & & & & \\ 2.1 & 2.2 & 2.3 & & & & \\ 3.1 & 3.2 & 3.3_1 & \equiv & 3.3_1 & 3.4 & 3.5 \\ & & & & 4.3 & 4.4 & 4.5 \\ & & & & 5.3 & 5.4 & 5.5 \end{array}$$

Um eine vollständige pentadische Matrix zu erreichen, müssen nun die fehlenden Subzeichen (in der folgenden Matrix fett markiert) ergänzt werden:

$$\begin{pmatrix} 1.1 & 1.2 & 1.3 & & & \mathbf{1.4} & \mathbf{1.5} \\ 2.1 & 2.2 & 2.3 & & & \mathbf{2.4} & \mathbf{2.5} \\ 3.1 & 3.2 & 3.3_1 & \equiv & 3.3_2 & 3.4 & 3.5 \\ \mathbf{4.1.} & \mathbf{4.2} & & & 4.3 & 4.4 & 4.5 \\ \mathbf{5.1} & \mathbf{5.2} & & & 5.3 & 5.4 & 5.5 \end{pmatrix} \Rightarrow \begin{pmatrix} 1.1 & 1.2 & 1.3 & 1.4 & 1.5 \\ 2.1 & 2.2 & 2.3 & 2.4 & 2.5 \\ 3.1 & 3.2 & 3.3_{1,2} & 3.4 & 3.5 \\ 4.1. & 4.2 & 4.3 & 4.4 & 4.5 \\ 5.1 & 5.2 & 5.3 & 5.4 & 5.5 \end{pmatrix}$$

2. Ganz anders aber schaut Kaehrs Interaktionsmatrix zwischen Sem1 und Sem2 aus (Kaehr 2009, S. 22):

$$\text{inter}(\text{Sem}^{(5,3,2)}) = \begin{bmatrix} \text{MM} & 1 & 2 & 3 & 4 & 5 \\ 1 & 1.1_1 & 3.4_2 & 3.5_2 & 1.4 & 1.5 \\ 2 & 3.4_2 & 2.2_1 & 2.3_1 & 2.4 & 2.5 \\ 3 & 3.5_2 & 3.2_1 & 3.3_{1,2} & 3.4_2 & 3.5_2 \\ 4 & 4.1 & 4.2 & 3.4_2 & 4.4_2 & 4.5_2 \\ 5 & 5.1 & 5.2 & 3.5_2 & 5.4_s & 5.5_2 \end{bmatrix}$$

Hier fehlen also (1.2), (2.1), (1.3), (3.1) sowie (4.3) und (5.3); dafür treten (3.4), (3.5) dreimal (zweimal horizontal und einmal vertikal) auf.

Falls solche Matrizen existieren, haben sie enorme Konsequenzen für die Basistheorie der mathematischen Semiotik:

Die Reihen der Matrizen enthalten triadische Sprünge und Wiederholungen (1-3-3-4-5), (3-2-3-4-4), (3-2-3-3-3), etc. Anders ausgedrückt: Nachdem Triaden einer Zeichenrelation in der klassischen Semiotik durch die von unten nach oben gelesenen Reihen der Peirceschen 3×3-Matrix definiert sind:

$$\begin{pmatrix} 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3, \end{pmatrix}$$

wären die Pentaden der Kaehr-Matrix also so definiert, dass 1. das Prinzip der pentadischen Vollständigkeit der pentadischen Zeichenklasse wegfiele; dass 2. das Prinzip der paarweisen Verschiedenheit der fünf Subzeichen wegfiele, und dass 3. keine ersichtliche lienare Nachfolgerordnung in den Pentatomien der Pentaden mehr vorhanden wäre (z.B. 5.1 → 4.1, aber 2.2 → 3.4; 3.5 → 3.4, aber 2.3 → 3.5, etc.).

3. Kaehrs komponierte pentadische Matrix suggeriert grösstmögliche Arbitrariät bei der Kompositionen n-adischer und m-adischer zu (n+m-1)-adischer Matrizen und umgekehrt zur Dekomposition (n+m)-adischer Matrizen in (n-1-m)-adische und/oder (n-m-1)-adische Matrizen. Die einzige Anforderung an die "Richtigkeit" der komponierten Matrix wäre dann, dass die zueinander inversen Subzeichen die gleichen kontexturellen Indizes bekommen (z.B. (2.3) und (3.2), (1.5) und (5.1), etc.). In letzter Instanz führt diese Arbitrarität also dazu, dass in Übereinstimmung mit einer obigen Festellung die abstrakte Form einer pentadischen Zeichenklasse als

5-Zkl = (a.b c.d e.f g.h i.j)

mit a, ..., j ∈ {1, ..., 5}

anzusetzen ist. Da ferner die triadischen Hauptwerte a, c, e, g, i nicht mehr paarweise verschieden sein müssen, kann jede x-beliebige Folge von 6 Ziffern natürlicher Zahlen als pentadische Zeichenklasse interpretiert werden.

4. Eine Einschränkung für diese völlig verwilderte Menge von Zeichenklassen könnte man daraus entnehmen, dass man wie bei der triadischen Matrix die Reihen der pentadischen Matrix als "Haupt-Zeichenklassen" interpretiert und aus den pentatomischen Pentaden der dyadischen Subzeichen Regeln zur Komposition von Zeichenklassen ableitet. Und zwar so:

1. Die pentadischen Haupt-Zkln (Reihenvektoren) sind:

5.1	5.2	3.5	5.4	5.5
4.1	4.2	3.4	4.4	4.5
3.5	3.2	3.3	3.4	3.5
3.4	2.2	2.3	2.4	2.5
1.1	3.4	3.5	1.4	1.5

2. Die daraus abzuleitenden Regeln zum Bau von "Neben-Zkln" lauten:

2.1. 5-Zkl = (5.a 4.b 3.c 3.d 1.e):

(a = 1) → b = 1, c = 5, d = 4, e = 1
(a = 2) → b = c = d= 2, e = 4
(a = 5) → b = c = d = e = 5

2.2. (5-Zkl = (3.a 3.b 3.c 2.d 3.e)) → a = e = 5, b = 4, c = d = 3

Aus diesen etwas komplizierten Konstruktionsregeln für Zeichenklassen folgt jedenfalls, dass auch die Inklusionsordnung für Trichotomien (a ≤ b ≤ c) bei Pentatomien nur noch eingeschränkt gilt.

Bibliographie

Kaehr, Rudolf, Interpretations of the kenomic matrix.
http://www.thinkartlab.com/pkl/lola/Matrix/Matrix.pdf (2009)

Monokontexturale und polykontexturale Replizierung

1. Im Rahmen der monokontexturalen Semiotik spielen Replicas eine bedeutende Rolle bei der lokalen und temporalen Bestimmung von Zeichen, denn es "ist jede Realisierung eines Legizeichens immer eine Konkretisierung oder Individualisierung. Anders ausgedrückt: Jedes realisierte Legizeichen ist hinsichtlich seines Auftretens oder Vorkommens 'hier und jetzt' ein Sinzeichen" (Walther 1979, S. 88). Weil damit im Grunde Drittheiten auf Zweitheiten zurückgeführt werden, könnte man Replizierung auch als Singularisierung bezeichnen. Karl Hermann hat im Anschluss an Walther folgende Darstellung der 10 Zeichenklassen mit ihren zugehörigen Replicaklassen gefunden (Herrmann 1990, S. 97):

(3.1 2.1 1.1)

(3.1 2.1 1.2) ←(3.1 2.1 1.3)

(3.1 2.2 1.2) ←(3.1 2.2 1.3) ←(3.1 2.3 1.3)

(3.2 2.2 1.2) ← (3.2 2.2 1.3) ←(3.2 2.3 1.3) ← (3.3 2.3 1.3)

Mit der in Toth (2008, S. 159 ff.) entwickelten Methode der dynamisch-kategorietheoretischen Analyse ist es nun möglich, nicht nur die Replica-Klassen statisch, sondern auch die Prozesse der Replizierung dynamisch zu erfassen:

[[β°, id1], [α°, id1]]

[[β°, id1], [α°, α]] ←[[β°, id1], [α°, βα]]

[[β°, α], [α°, id2]] ←[[β°, α], [α°, β]] ← [[β°, βα], [α°, id3]]

[[β°, id2], [α°, id2]] ←[[β°, id2], [α°, β]] ← [[β°, β], [α°, id3]] ←[[β°, id3], [α°, id3]]

Im System der 10 Zeichenklassen gibt es somit die folgenden Replizierungstypen in kategorietheoretischer Notation:

[α ← βα]
<[id2 ← β], <[α ← βα], [β ← id3]>>
<[id2 ← β], <<[id2 ← β], [β ← id3]>, [β ← id3]>>,

sowie in numerisch-kategorialer Notation:

(2.1) ← (1.3)
<(2.2) ← (2.3), <(2.1) ← (1.3), (2.3) ← (3.3)>>
< (2.2) ← (2.3), <<(2.2) ← (2.3), (2.3) ← (3.3)>, (2.3) ← (3.3)>>.

2. Wenn wir von der monokontexturalen zur polykontexturalen Semiotik übergehen, stellt sich zunächst die Frage, ob sich Replizierung nur auf die Trichotomien der Subzeichen beschränkt oder auch die Kontexturen affiziert, in denen sich die Subzeichen befinden. Es sind also zwei Möglichkeiten denkbar:

1. $(2.3)_i \rightarrow (2.2)_i$
2. $(2.3)_i \rightarrow (2.2)_j$

Kaehr (2009, S. 8) ist offenbar der Ansicht, dass Singularisierung von semiotischen Prozessen einen Kontexturenwechsel impliziert. Er gibt folgendes Beispiel für die Replikation einer semiotischen Matrix:

$$repl_{1.1\ .1\ .1}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & \square & \square \\ S_1 & S_2 & \square \\ S_{1.1} & \square & S_3 \end{pmatrix}$$

Werden die beiden triadischen semiotischen Systeme

$$Sem^1 = \begin{bmatrix} 1.1_1 & 1.2_1 & 1.3_1 \\ 2.1_1 & 2.2_1 & 2.3_1 \\ 3.1_1 & 3.2_1 & 3.3_1 \end{bmatrix}, \quad Sem^2 = \begin{bmatrix} 3.3_2 & 3.4_2 & 3.5_2 \\ 4.3_2 & 4.4_2 & 4.5_2 \\ 5.3_2 & 5.4_2 & 5.5_2 \end{bmatrix}$$

zum folgenden pentadischen semiotischen System zusammengefasst:

$$Sem^{(5,3,2)} = \begin{bmatrix} MM & 1 & 2 & 3 & 4 & 5 \\ 1 & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 \\ 2 & 2.1 & 2.2 & 2.3 & 2.4 & 2.5 \\ 3 & 3.1 & 3.2 & 3.3 & 3.4 & 3.5 \\ 4 & 4.1 & 4.2 & 4.3 & 4.4 & 4.5 \\ 5 & 5.1 & 5.2 & 5.3 & 5.4 & 5.5 \end{bmatrix}$$

so erscheint in der folgenden replikativen semiotischen Matrix die eingebettete triadische Matrix Sem1 repliziert als Sem11, und die "Schaltstelle" der Einbettung der zwei Matrizen, das Subzeichen (3.3), bekommt nun zuzüglich zu seinem kontexturellen Index 1, der es mit der Matix Sem1 verbindet, und seinem kontexturellen Index 2, der mit es mit der Matirx Sem2 verbindet, eine zweite 1 als Zeichen der Replikation:

Replication of Sem $_1$**to Sem**$_{1.1}$

$$\text{repl}(\text{Sem}^{(5,3,2)}) = \begin{bmatrix} MM & 1 & 2 & 3 & 4 & 5 \\ 1 & 1.1_{1.1} & 1.2_{1.1} & 1.3_{1.1} & 1.4 & 1.5 \\ 2 & 2.1_{1.1} & 2.2_{1.1} & 2.3_{1.1} & 2.4 & 2.5 \\ 3 & 3.1_{1,1} & 3.2_{1.1} & 3.3_{1.1.2} & 3.4_2 & 3.5_2 \\ 4 & 4.1 & 4.2 & 4.3_2 & 4.4_2 & 4.5_2 \\ 5 & 5.1 & 5.2 & 5.3_2 & 5.4_2 & 5.5_2 \end{bmatrix}$$

Damit haben wir nun das nötige formale Instrumentarium beieinander, um polykontexturale Replizierungen zu konstruieren bzw. zu rekonstruieren:

1. Nicht-eingebettete 3-kontexturale Replikation

$(3.1_3\ 2.1_1\ 1.1_{1,3})$

$(3.1_3\ 2.1_1\ 1.2_1) \leftarrow (3.1_3\ 2.1_1\ 1.3_3)$

$(3.1_3\ 2.2_{1,2}\ 1.2_1) \leftarrow (3.1_3\ 2.2_{1,2}\ 1.3_3) \leftarrow (3.1_3\ 2.3_2\ 1.3_3)$

$(3.2_2\ 2.2_{1,2}\ 1.2_1) \leftarrow (3.2_2\ 2.2_{1,2}\ 1.3_3) \leftarrow (3.2_2\ 2.3_2\ 1.3_3) \leftarrow (3.3_{2,3}\ 2.3_2\ 1.3_3)$

2. In Sem1 eingebettete 3-kontexturale Replikation

$(3.1_{1,1}\ 2.1_{1,1}\ 1.1_{1,1})$

$(3.1_{1,1}\ 2.1_{1,1}\ 1.2_{1,1}) \leftarrow (3.1_{1,1,3}\ 2.1_{1,1,1}\ 1.3_{1,1,3})$

$(3.1_{1,1}\ 2.2_{1,1}\ 1.2_{1,1}) \leftarrow (3.1_{1,1,3}\ 2.2_{1,1}\ 1.3_{1,1,3}) \leftarrow (3.1_{1,1,3}\ 2.3_{1,1,2}\ 1.3_{1,1,3})$

$(3.2_{1,1}\ 2.2_{1,1}\ 1.2_{1,1}) \leftarrow (3.2_{1,1}\ 2.2_{1,1}\ 1.3_{1,1,3}) \leftarrow (3.2_{1,1}\ 2.3_{1,1,2}\ 1.3_{1,1,3})$
$\leftarrow (3.3_{1,1,2,3}\ 2.3_{1,1,2}\ 1.3_{1,1,3})$

3. **Singularisierung/Aktualisierung impliziert also Kontexturenwechsel.** Das ist wohl das erstaunlichste Ergebnis dieser Studie. Nun sind Ich und Du nach Günther (1975) qualitativ ebenso geschieden wie Diesseits und Jenseits. Man darf sich also fragen, ob die hier dargestellte polykontexturale Replikationstheorie

Anwendung im Bereich der linguistischen Deixis finden könnte, also dort, wo es darum geht, etwas oder jemand im Hier, Jetzt und als Ich/Du/Er sprachlich zu etablieren.

In Toth (1997, S. 83 ff.) hatte ich auf mehrere Typen von verletzter Deixis hingewiesen, die möglicherweise mit der hier vorgelegten Theorie untersucht werden können:

1. Verletzte Lokal-Deixis: *Ich bin dort in Mexiko.

2. Verletzte Temporal-Deixis: Bist du noch da? – *Nein ich bin schon weg.

3. Verletzte Personal-Deixis: Diese Nacht ist herrlich, mein Kind.
dein Freund/*Ihre Schwester.

4. Verletzte epistemische Deixis: *Ottokar weiss nicht, dass der Mond quadratisch ist.

5. Verletzte situative Deixis: Ich sehe, Du hast das Abendblatt in der Hand. *Hol es mir doch mal mehr.

6. Verletzte Deixis mit honoricis: *Hallo, hast Du Ihren Hut liegen lassen

(usw.)

Bibliographie

Kaehr, Rudolf, Interactional operators in diamond semiotics. http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.pdf (2009)

Herrmann, Karl, Zur Replica-Bildung im System der zehn Zeichenklassen. In: Semiosis 59/60, 1990, S. 95-101

Toth, Alfred, Entwurf einer semiotisch-relationalen Grammatik. Tübingen 1997

Toth, Alfred, Semiotische Prozesse und Systeme. Klagenfurt 2008

Walther, Elisabeth, Allgemeine Zeichenlehre. 2. Aufl. Stuttgart 1979

Durch Realisierung induzierter bifurkativer Kontexturen-wechsel

1. Replicas gehören zu den am wenigsten untersuchten Sorten von Zeichen. Darunter sind etwa Beispiele, Kopien, Belege, Abbilder usw. zu verstehen. Nach Peirce sind Replicas abgeleitete Zeichen, und zwar von Legizeichen-Klassen abgeleitete Sinzeichen-Klassen (Walther 1979, S. 88). D.h. nur Legizeichen-Klassen haben Replicas. Ferner ist, wie aus der folgenden Tabelle aus Walther (1979, S. 88) hervorgeht, die Abbildung von Replicas auf Zeichenklassen und umgekehrt nicht-eindeutig:

5.	(3.1 2.1 1.3)	→	2. (3.1 2.1 1.2)
6.	(3.1 2.2 1.3)	→	3. (3.1 2.2 1.2)
7.	(3.2 2.2 1.3)	→	4. (3.2 2.2 1.2)
8.	(3.1 2.3 1.3)	→	6. (3.1 2.2 1.3)
9.	(3.2 2.3 1.3)	→	7. (3.2 2.2 1.3)
10.	(3.3 2.3 1.3)	→	9. (3.2 2.2 1.3)

Noch wichtiger aber ist Peirce's eigene Feststellung, wonach man "das Sinzeichen, das als *realisiertes* Legizeichen verstanden wird, unterscheiden muss vom Sinzeichen, wie es in der Trichotomie des Mittelbezugs auf das Qualizeichen folgt" (Walther 1979, S. 88).

2. In Toth (2009b) wurde nachgewiesen, dass man in Übereinstimmung mit Kaehr (2009), der dieses Phänomen entdeckte, für replizierte Subzeichen eine eigene semiotische Matrix anzusetzen hat. Z.B., wenn wir von der üblichen monokontexturalen Matrix ausgehen:

$$\begin{pmatrix} 1.1_1 & 1.2_1 & 1.3_1 \\ 2.1_1 & 2.2_1 & 2.3_1 \\ 3.1_1 & 3.2_1 & 3.3_1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1.1_{1,1} & 1.2_{1,1} & 1.3_{1,1} \\ 2.1_{1,1} & 2.2_{1,1} & 2.3_{1,1} \\ 3.1_{1,1} & 3.2_{1,1} & 3.3_{1,1} \end{pmatrix}$$

Demzufolge lautet die Peirceaussage von der Verschiedenheit des Sinzeichens formal:

$(1.2)_1 \neq (1.2)_{1,1}$.

In Toth (2009a) war ferner gezeigt worden, dass dualisierte Subzeichen ebenfalls verschieden sind, z.B.

$(1.1)_{1,3} \neq \times(1.1)_{1,3} = (1.1)_{3,1}$.

Ungleich hinsichtlich dualer Strukturen allein (d.h. mit gleicher Kontextur) sind die Konversen innerhalb einer Kontextur, z.B.

$(1.2)_1 \neq (2.1)_1$.

Aus dem folgt nun aber: **Nicht nur muss man wegen der umgekehrten kontexturellen Indizes zwei semiotische Matrizen (für Zeichen- und Realitätstematik) ansetzen, sondern es bedarf mindestens zwei verschiedener Matrizen, je nachdem ob es sich um realisierte oder genuine trichotomisch zweitheitliche Subzeichen handelt.** Der Grund hierfür ist natürlich, dass in der polykontexturalen Semiotik der logische Identitätssatz ja nicht gilt, d.h. es ist nichts gleich, weder sich noch etwas anderem.

3. Da wir annehmen dürfen, dass die Unterscheidung zwischen genuiner und realisierter Zweitheit auch für die sog. Realitätsthematiken (vgl. Toth 2009c) gilt, ist also in der semiotischen Matrix die kreuzartig eingerahmte Teilmatrix betroffen:

$$\begin{pmatrix} 1.1_1 & 1.2_1 & 1.3_1 \\ 2.1_1 & 2.2_1 & 2.3_1 \\ 3.1_1 & 3.2_1 & 3.3_1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1.1_{1,1} & \boxed{1.2_{1,1}} & 1.3_{1,1} \\ \boxed{2.1_{1,1}} & \boxed{2.2_{1,1}} & \boxed{2.3_{1,1}} \\ 3.1_{1,1} & \boxed{3.2_{1,1}} & 3.3_{1,1} \end{pmatrix}$$

4. In Toth (2009b) war zudem zwischen eingebetteter und nicht-eingebetteter Replikation unterschieden worden. Wir wollen uns hier auf die in Sem1 eingebettete 3-kontexturale Replikation beschränken (Kaehr 2009).

In Sem1 eingebettete 3-kontexturale Replikation

$(3.1_{1,1}\ 2.1_{1,1}\ 1.1_{1,1})$

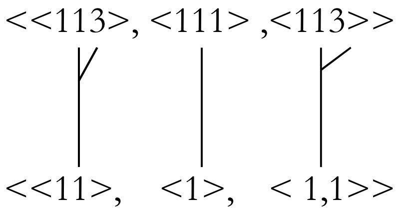
$(3.1_{1,1}\ 2.1_{1,1}\ 1.2_{1,1}) \leftarrow (3.1_{1,1,3}\ 2.1_{1,1,1}\ 1.3_{1,1,3})$

$(3.1_{1,1}\ 2.2_{1,1}\ 1.2_{1,1}) \leftarrow (3.1_{1,1,3}\ 2.2_{1,1}\ 1.3_{1,1,3}) \leftarrow (3.1_{1,1,3}\ 2.3_{1,1,2}\ 1.3_{1,1,3})$

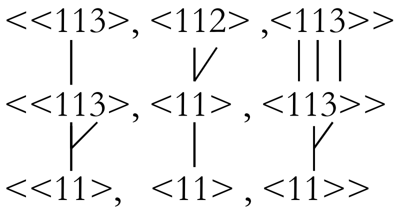
$(3.2_{1,1}\ 2.2_{1,1}\ 1.2_{1,1}) \leftarrow (3.2_{1,1}\ 2.2_{1,1}\ 1.3_{1,1,3}) \leftarrow (3.2_{1,1}\ 2.3_{1,1,2}\ 1.3_{1,1,3})$
$\leftarrow (3.3_{1,1,2,3}\ 2.3_{1,1,2}\ 1.3_{1,1,3})$

Dies sind nach dem retrosemiosischen Schema die 3 vollständigen Replika-Zyklen, wie sie Herrmann (1990) gefunden hatte. Wenn wir nun die kontexturellen Indizes betrachten:

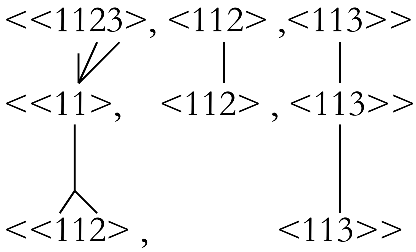
1. Zyklus



2. Zyklus



3. Zyklus



Realisierung impliziert also nicht nur trichotomischen Ersatz der Drittheit durch eine retrosemiosische Zweitheit, sondern gleichzeitig Öffnung der kenomischen Matrix durch Bifurkation für eine Iteration der Einheit der monokontexturalen Basis jeder polykontexturalen Matrix.[6]

Bibliographie

Kaehr, Rudolf, Interactional operators in diamond semiotics. http://www.thinkartlab.com/pkl/lola/Transjunctional%20Semiotics/Transjunctional%20Semiotics.pdf (2009)

Toth, Alfred, Zeichen und Zeichenklasse. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Bifurkationen und Zeichenzusammenhänge. In: Electronic Journal for Matematical Semiotics, 2009b

Toth, AlfredWas sind eigentlich Realitätsthematiken? In: Electronic Journal for Matematical Semiotics, 2009c

Walther, Elisabeth, Allgemeine Zeichenlehre. 2. Aufl. Stuttgart 1979

[6] Im übrigen finden wir im 3. Zyklus einen Fall von Absorption i.S. von inverser Bifurkation, sowie von Trifurkation.

Polykontexturale Superoperatoren in der Semiotik

1. Der Begriff des semiotischen “Superoperators” (Kaehr) setzt den Begriff der semiotischen Kontextur voraus, denn er vermittelt zwischen und nicht innerhalb von semiotischen Systemen. Nach Kaehr (2009) sind die wichtigsten Superoperatoren Identität, Permutation, Reduktion, Bifurkation und Replikation (Figur aus Kaehr 2009, S. 9):

Super – operators for the mapping of logical systems onto the matrix

$$\mathrm{Logic}^{(m)}: \left[\mathrm{Logic}^{(m)}\right]_{\mathrm{refl,\,act}} \xrightarrow[\mathrm{sops}]{} \left[\mathrm{Logic}^{(m)}\right]_{\mathrm{refl,\,act}}$$

$$\mathrm{sops} = \{\mathrm{id},\ \mathrm{perm},\ \mathrm{red},\ \mathrm{bif},\ \mathrm{repl}\}$$

$$\mathbf{id}: \forall i, j \in s(m): \quad \left(\mathrm{Logic}^{i.j}\right) \xrightarrow[\mathrm{id}]{} \left(\mathrm{Logic}^{i.j}\right)$$

$$\mathbf{perm}(i, j): \forall i, j \in s(m): \left(\mathrm{Logic}^{i}, \mathrm{Logic}^{j}\right) \xrightarrow[\mathrm{perm}]{} \left(\mathrm{Logic}^{j}, \mathrm{Logic}^{i}\right)$$

$$\mathbf{red}(i, j): \forall i, j \in s(m): \left(\mathrm{Logic}^{i}, \mathrm{Logic}^{j}\right) \xrightarrow[\mathrm{red}]{} \left(\mathrm{Logic}^{i}, \mathrm{Logic}^{i}\right)$$

$$\mathbf{bif}(i, j): \forall i, j \in s(m): \left(\mathrm{Logic}^{i}, \mathrm{Logic}^{j}\right) \xrightarrow[\mathrm{bif}]{} \left(\left(\mathrm{Logic}^{i} \,\|\, \mathrm{Logic}^{i}\right), \mathrm{Logic}^{j}\right)$$

$$\mathbf{repl}(i, j): \forall i, j \in s(m): \left(\mathrm{Logic}^{i}, \mathrm{Logic}^{j}\right) \xrightarrow[\mathrm{repl}]{} \left(\left(\mathrm{Logic}^{i} \,|\, \mathrm{Logic}^{i}\right), \mathrm{Logic}^{j}\right)$$

Als einziger dieser semiotischen Trans-Operatoren (wie man auch sagen könnte) wurde die Replikation, bereits von Peirce eingeführt, benutzt, womit die drittheitlichen trichotomischen Werte einer Zeichenklassen schrittweise vom Mittel- bis zum Interpretantenbezug abgebaut werden, bis überall nur noch zweitheitliche Bezüge aufscheinen, z.B.

(3.2 2.2 1.2)← (3.3 2.2 1.2) ← (3.3 2.3 1.2) ← (3.3 2.3 1.3) ≡
RRR(3.3 2.3 1.3) = (3.2 2.2 1.2)

Mit Hilfe von R oder der Replikation werden also Zeichenklassen in andere Zeichenklassen überführt, d.h. semiotische Transoperationen durchgeführt.
Unter semiotischen Identitätsoperatoren kann man Operatoren ι_1, ι_2, ι_3, ..., ι_n (im Falle der 10 peirceschen Zeichenklassen ist n = 10) verstehen, welche die Zeichenklassen auf sich selbst abbilden, z.B. ι_1(3.1 2.1 1.1) = (3.1 2.1 1.1).

Die bereits in Toth (2008, S. 177 ff.) eingeführten Permutationsoperatoren π_1, π_2, π_3, ..., π_n (im Falle von triadischen Zeichenklassen ist n = 6, da 3! = 6) sind eine spezielle Form der identischen Abbildungen von Zeichenklassen, da sie streng genommen nicht aus diesen Zeichenklassen hinausführen, z.B. $\pi_{1\text{-}6}$ (3.1 2.1 1.3) = {(3.1 2.1 1.3), (3.1 1.3 2.1), (2.1 1.3 3.1), (2.1 3.1 1.3, (1.3 2.13.1), (1.3 3.1 2.1)}

Reduktionsoperatoren, bisher unbekannt in der Semiotik, könnten z.B. dazu verwendet werden, um triadische peircesche Zeichenklassen auf dyadische Saussuresche Zeichengebilde zurückzuführen, z.B. ρ(3.1 2.1 1.3) = {(3.1, 2.1), (3.1, 1.3), (3.1 1.3)}.

2. Im Gegensatz zu den Identitäts-, Permutations- und Reduktions-Operationen wirken Bifurkation und Replikation primär an den kontexturellen Indizes:

3 – contextural semiotic matrix [repl, id, id]

$$Sem^{(3,2,2)}_{(repl,\,id,\,id)} = \begin{pmatrix} MM & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1_{1.1.3}} & \mathbf{1.2_{1.1}} & \mathbf{1.3_{3}} \\ 2_{1.2} & \mathbf{2.1_{1.1}} & \mathbf{2.2_{1.1.2}} & \mathbf{2.3_{2}} \\ 3_{2.3} & \mathbf{3.1_{3}} & \mathbf{3.2_{2}} & \mathbf{3.3_{2.3}} \end{pmatrix}$$

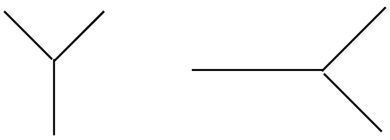
Wie man sieht, ist die Replikation eine Operation der Form $\mathcal{R}(a.b)_{i,j} = (a.b)_{i,ik}$. Das bedeutet aber, dass der erste kontexturelle Wert zum zweiten wird, indem eine Kopie seiner selbst an die erste Stelle gesetzt wird. Replikation wirkt also retrograd. In Übereinstimmung mit Peirce haben wir zusätzlich (a.3) → (a.2) (vgl. Walther 1979, S. 88 ff.). Allerdings bleibt, dann, wie die folgende Figur zeigt, mindestens 1 Zeichenklasse nicht ableitbar:

(3.1 2.1 1.1)
(3.1 2.1 1.2) ←(3.1 2.1 1.3)
(3.1 2.2 1.2) ←(3.1 2.2 1.3) ←(3.1 2.3 1.3)
(3.2 2.2 1.2) ← (3.2 2.2 1.3) ←(3.2 2.3 1.3) ← (3.3 2.3 1.3)

Wir wollen darum hier vorschlagen, unter Replikation die zusätzliche semiotische Ableitung (a.2) → (a.1) zu verstehen, d.h. Replikation ist wie folgt definiert:

$\mathcal{R}(a.b')_{i,j} := (a.b)_{i,ik}$. $b' \in \{3, 2\}$, $b \in \{1, 2\}$

3. Auch Bifurkation ist eine Operation von Kontexturenwechsel. Es ist interessant, dass eines der ersten peireceschen Zeichenmodelle bifurkativ ist: "A point upon which three lines of identity abut is a graph expressing relation of Teridentity" (Peirce ap. Brunning 1997, S. 257):



Teridentität beruht hier aber im Grunde darauf, dass die 3 äusseren Ecken des Graphen in der inneren, also einer 4. Ecke, zusammenfallen. Wird dann die 4. Ecke nicht gezählt (woraus sich ein tetradisches Zeichenmodell ergäbe), dann folgt, **dass Teridentität nichts anderes ist als Bifurkation.**

Bisher völlig unberücksichtigt blieb, dass es möglich ist, sämtliche 6 Permutationen einer Zeichenrelation in Form von Bifurkationen (Teridentitäten) darzustellen:

↗ (2.b)
π_1(3.a 2.b 1.c) = (3.a)
↘ (1.c)

↗ (1.c)
π_2(3.a 2.b 1.c) = (3.a)
↘ (2.b)

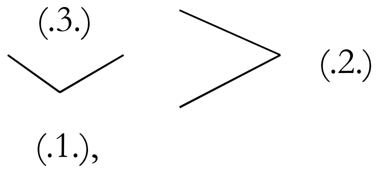
↗ (3.a)
π_3(3.a 2.b 1.c) = (2.b)
↘ (1.c)

↗ (1.c)
π_4(3.a 2.b 1.c) = (2.b)
↘ (3.a)

↗ (3.a)
π_5(3.a 2.b 1.c) = (1.c)
↘ (2.b)

↗ (2.b)
π_6(3.a 2.b 1.c) = (1.c)
↘ (3.a)

Eine **inverse Bifurkation** dürfte dem peirceschen Kreationsschema zugrunde liegen:



“das ein Zusammenwirken der ‘Erstheit’ und der ‘Drittheit’ zur Generierung der ‘Zweitheit’ verlangt” (Bense 1976, S. 107). Es wird hier ja gerade postuliert, dass nicht ein Objekt (.2.) durch einen Interpretanten (.3.) mit einem Mittel (.1.) bezeichnet wird, sondern dass ein Interpretant (.3.) ein Mittel (.1.) selektiert, um ein Objekt (bzw. einen Objektbezug (.2.)) zu generieren, der also relativ zu (.3.) und (.1.) etwas Neues darstellt, also aus folgenden zwei inversen Bifurkationen hergestellt werden kann:

	↙ (.3.)		↙ (.1.)
(.2.)		(.2.)	
	↖ (.1.)		↖ (.3.)

Nun ergibt sich aber eine überraschende Gemeinsamkeit zwischen einigen Typen von Bifurkation und inverser Bifurkation zur Replikation, die wir ja als retrograd (retrosemiosisch, degenerativ) bestimmt hatten: All jene Typen von Bifurkationen, die das folgende abstrakte Schema

↗ (c.d)

$\pi_i(3.a\ 2.b\ 1.c) = (a.b)$

↘ (e.f)

mit a < c und/oder a < e erfüllen, sind zugleich replikativ. Das sind also 4 der 6 möglichen Permutationen, nämlich π_3 bis und mit π_6.

4. Damit ergibt sich, die Frage, ob es tatsächlich korrekt ist, (1) die Permutationen der Zeichenklasse $(3.1_3\ 2.2_{1,2}\ 1.3_3)$ wie bisher (in der linken Kolonne) zu schreiben, oder ob sie nicht korrekter wie in der rechten Kolonne notiert werden müssen:

$\pi_1(3.1_3\ 2.1_1\ 1.3_3) \quad = \quad (3.1_3\ 2.1_1\ 1.3_3)$
$\pi_2(3.1_3\ 2.1_1\ 1.3_3) \quad = \quad (3.1_3\ 1.3_{1,3}\ 2.1_{1,1})$
$\pi_3(3.1_3\ 2.1_1\ 1.3_3) \quad = \quad (2.1_{1,1}\ 3.1_{1,3}\ 1.3_{1,3})$
$\pi_4(3.1_3\ 2.1_1\ 1.3_3) \quad = \quad (2.1_{1,1}\ 1.3_{1,3}\ 3.1_{1,3})$
$\pi_5(3.1_3\ 2.1_1\ 1.3_3) \quad = \quad (1.3_{1,3}\ 3.1_{1,3}\ 2.1_{1,1})$
$\pi_i(3.1_3\ 2.1_1\ 1.3_3) \quad = \quad (1.3_{1,3}\ 2.1_{1,1}\ 3.1_{1,3})$

(2) ergeben sich aus diesen Permutationstypen die folgenden Bifurkationstypen:

$\{(3.1_3\ 2.1_1\ 1.3_3)\}$
$\{(3.1_3\ 1.3_3\ 2.1_1), (3.1_3\ 1.3_1\ 2.1_1), (3.1_1\ 1.3_3\ 2.1_1)\}$
$\{(2.1_3\ 3.1_1\ 1.3_3), (2.1_1\ 3.1_1\ 1.3_1), ((2.1_1\ 3.1_1\ 1.3_1), ...\}$

$\mathcal{B}\ \pi_i(3.1_3\ 2.1_1\ 1.3_3)$ = $\{(2.1_3\ 1.3_1\ 3.1_3), (2.1_1\ 1.3_1\ 3.1_1),$
$(2.1_1\ 1.3_3\ 3.1_{1,}), ...\}$
$\{(1.3_3\ 3.1_1\ 2.1_3), (1.3_3\ 3.1_1\ 2.1_1),$
$(1.3_1\ 3.1_1\ 2.1_1)\}$
$\{(1.3_3\ 2.1_1\ 3.1_3), (1.3_{1,3}\ 2.1_{1,1}\ 3.1_{1,3}), ...\}$

Mit Hilfe der Einführung polykontexturaler Superoperatoren in die Semiotik ergeben sich überraschende Einsichten in die Semiosis und den Bau bekannter (aber monokontextural nicht genügend differenzierter) Zeichenschemata wie demjenigen der semiotischen Kreation. Speziell für die semiotischen Permutationssysteme wird hierdurch ein äusserst komplexer Ausschnitt aus dem Netz der semiotischen Kontexturen konstruierbar bzw. analysierbar, das enorme weitere Formalisierbarkeit erlaubt.

Bibliographie

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976

Brunning, Jacqueline, Genuine Triads and Teridentity. In: Houser, Nathan/Roberts, Don D./Van Evra, James, Studies in the Logic of Charles Sanders Peirce. Bloomington 1997, S. 252-263

Kaehr, Rudolf, Interpretations of the kenomic matrix. http://www.thinkartlab.com/pkl/lola/Matrix/Matrix.pdf (2009)

Toth, Alfred, Semiotische Prozesse und Systeme. Klagenfurt 2008

Walther, Elisabeth, Allgemeine Zeichenlehre. 2. Aufl. Stuttgart 1979

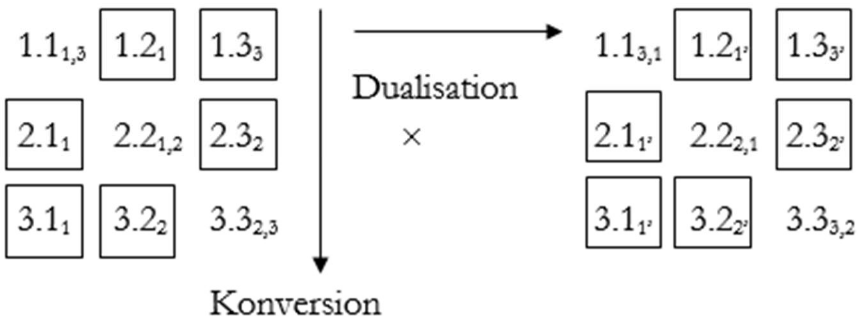
Konversion und Dualisation

In seiner letzten Vorlesung im Winter-Semester 1989/90 sagte Max Bense: "Das Legizeichen (1.3) ist der geringste Interpretant". Obwohl ich diese Aussage mit Max Bense ausführlich diskutieren konnte, ist mir erst dieser Tage die folgende Stelle aus Benses Buch "Vermittlung der Realitäten" wieder zu Gesicht gekommen: "Dieses abstrakte Legizeichen (...) ist das 'Mittel' höchster Semiotizität, 'Drittheit' des 'Mittels', und damit das Mittel höchster pragmatischer Verwendbarkeit, das die theoretische Phase jedes Erkenntnisprozesses einleitet (Bense 1976, S. 15).

Wenn man Benses Gedanken weiterführt, bekommt man statt der üblichen Trichotomien aus der kleinen semiotischen Matrix die folgenden Tetratomien:

1.1	1.2	1.3	3.1
2.1	2.2	2.3	3.2
3.1	3.2	3.3	3.3

Ferner kann man das Sinzeichen (1.2) als den kleinsten Objektbezug bestimmen (2.1). Die Verhältnisse (1.3) : (3.1), (2.3): (3.2), (3.3) : (3.3) und (1.2): (2.1) sind nun aber konverse und nicht duale Relationen, was man daraus sieht, dass sie dieselben Kontexturen zugeweisen bekommen, im Falle der 3-kontexturalen Matrix:



Man darf daraus schliessen, dass Benses Hinzunahme des jeweils einen dualen entsprechendes Gliedes einer anderen Trichotomie zu jeder Trichotomie und der dadurch bewerkstelligten Erweiterung der Trichotomien zu Tetraden erst der polykontexturalen Semiotik auf Subzeichenebene die Möglichkeit eröffnet, dass konverse Subzeichen in den gleichen semiotischen Kontexturen liegen.

Bibliographie

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1979

Der präsemiotisch-semiotische Übergang und der Aufbau der kontexturellen semiotischen Matrix

1. In meinen zwei Bänden "Semiotics und Pre-Semiotics" (Toth 2008) sowie in zahlreichen weiteren Arbeiten habe ich ohne semiotische Kontexturen zur Hilfe zu nehmen den präsemiotisch-semiotischen Übergang, den Max Bense als Adjazenzraum von ontologischem und semiotischem Raum (1975, S. 65 f.) bzw. von Nullheit zur Erstheit (1975, S. 45 f.) gekennzeichnet hatte, mit Hilfe der mathematischen Vererbungstheorie (vgl. Touretzky 1984) erklärt. Seit Rudolf Kaehr die kontexturellen semiotischen Matrizen (2008) sowie neuerdings Superopertoren (Transoperatoren) auch in die Semiotik eingeführt hat (2009), mag ich einen weiteren Erklärungsversuch der Erzeugung der semiotischen Matrix aus der präsemiotischen Triade von Sekanz, Semanz und Selektanz (vgl. Götz 1982, S. 4, 28).

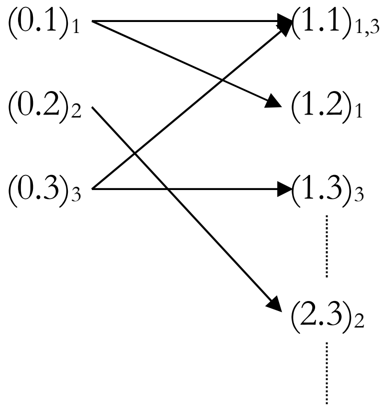
2. Den Fundamentalkategorien werden nach einem Vorschlag R. Kaehrs (2008) die kontexturellen Indizes der entsprechenden genuinen Subzeichen (im Sinne von iterierten Primzeichen) zugeschrieben

PZR = $((.1.)_{1,3}, (.2.)_{1,2}, (.3.)_{2,3})$,

so dass man den drei trichotomischen Gliedern der präsemiotischen Zeroness im Sinne Benses (1975, S. 65 f.) genuine kontextuelle Indizes zuschreiben dürfen wird

PZR = $((0.1)_1, (0.2)_2, (0.3)_3)$

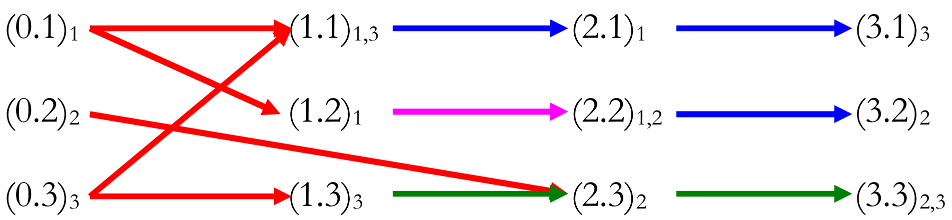
3. Nun gibt es aber eine Überraschung, denn nicht nur durchkreuzen die kontexturellen Abbildung vom präsemiotischen in den semiotischen Raum sämtliche auf der Vererbungstheorie basierenden Vorhersagen, sondern $(0.2)_2$ kann gar nicht wie alle übrigen Trichotomien von der Nullheit auf die Erstheit abgebildet werden:



In der unten stehenden Figur sind identische kontexturelle Abbildungen in rot, und Bifurkationen in blau. Grün ist die inverse Bifurkation. Lediglich

$(1.2)_1 \rightarrow (2.2)_{1,2}$

ist ein Fall von Touretzky-Vererbung.



Das ist nun also ein mit den Ergebnissen der polykontexturalen Logik kompatibles Schema der Semiose von der präsemiotischen Nullheit zur semiotischen Drittheit der Drittheit und damit die vollständige Rekonstruktion von Zeichengenese.

Da die von Kaehr beigebrachten Superoperationen der Identitätsabbildung und Reduktion einigermassen klar sein dürften und da die Bifurkation bereits in mehreren Arbeiten behandelt wurde, führe ich abschliessend die Unterscheidung von linker und rechter Replikation ein. Im Falle, dass bereits auf präsemiotischer Ebene mit Replikation gerechnet werden darf, fallen beide Typen, wie natürlich auch bei den semiotischen Fällen mit Monoindizierung, zusammen.

1. Replikation von links

$\mathcal{R}(1.1)_{1,3} = (1.1)_{1,1,3}$	$\mathcal{RR}(1.1)_{1,3} = (1.1)_{1,1,1,3}$
$\mathcal{R}(1.2)_{1} = (1.2)_{1,1}$	$\mathcal{RR}(1.2)_{1} = (1.1)_{1,1,1}$
$\mathcal{R}(1.3)_{3} = (1.3)_{3,3}$	$\mathcal{RR}(1.3)_{3} = (1.1)_{3,3.3}$
$\mathcal{R}(2.1)_{1} = (2.1)_{1,1}$	$\mathcal{RR}(2.1)_{1} = (2.1)_{1,1,1}$
$\mathcal{R}(2.2)_{1,2} = (2.2)_{1,1,2}$	$\mathcal{RR}(2.2)_{1,2} = (2.2)_{1,1,1,2}$
$\mathcal{R}(2.3)_{2} = (2.3)_{2,2}$	$\mathcal{RR}(2.3)_{2} = (2.3)_{2,2,2}$
$\mathcal{R}(3.1)_{3} = (3.1)_{3,3}$	$\mathcal{R}(3.1)_{3} = (3.1)_{3,3,3}$
$\mathcal{R}(3.2)_{2} = (3.2)_{2,2}$	$\mathcal{R}(3.2)_{2} = (3.2)_{2,2,2}$
$\mathcal{R}(3.3)_{2,3} = (3.3)_{2,2,3}$	$\mathcal{R}(3.3)_{2,3} = (3.3)_{2,2,2,3}$

2. Replikation von rechts

$\mathcal{R}(1.1)_{1,3} = (1.1)_{1,3,3}$	$\mathcal{R}\mathcal{R}(1.1)_{1,3} = (1.1)_{1,3,3,3}$
$\mathcal{R}(1.2)_{1} = (1.2)_{1,1}$	$\mathcal{R}\mathcal{R}(1.2)_{1} = (1.1)_{1,1,1}$
$\mathcal{R}(1.3)_{3} = (1.3)_{3,3}$	$\mathcal{R}\mathcal{R}(1.3)_{3} = (1.1)_{3,3,3}$
$\mathcal{R}(2.1)_{1} = (2.1)_{1,1}$	$\mathcal{R}\mathcal{R}(2.1)_{1} = (2.1)_{1,1,1}$
$\mathcal{R}(2.2)_{1,2} = (2.2)_{1,2,2}$	$\mathcal{R}\mathcal{R}(2.2)_{1,2} = (2.2)_{1,2,2,2}$
$\mathcal{R}(2.3)_{2} = (2.3)_{2,2}$	$\mathcal{R}\mathcal{R}(2.3)_{2} = (2.3)_{2,2,2}$
$\mathcal{R}(3.1)_{3} = (3.1)_{3,3}$	$\mathcal{R}(3.1)_{3} = (3.1)_{3,3,3}$
$\mathcal{R}(3.2)_{2} = (3.2)_{2,2}$	$\mathcal{R}(3.2)_{2} = (3.2)_{2,2,2}$
$\mathcal{R}(3.3)_{2,3} = (3.3)_{2,3,3}$	$\mathcal{R}(3.3)_{2,3} = (3.3)_{2,3,3,3}$

Besonders wenn mit Hilfe des Bifurkationsoperators gearbeitet wird, lass sich kontexturelle Strukturen von enormer Komplexität generieren.

Bibliographie

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Götz, Matthias, Schein Designs. Diss. Stuttgart 1982

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2008)

Kaehr, Rudolf, Interpretations of the kenomic matrix. http://www.thinkartlab.com/pkl/lola/Matrix/Matrix.pdf (2009)

Toth, Alfred, Semiotics and Pre-Semiotics. 2 Bde. Klagenfurt 2008

Touretzky, David, The Mathematics of Inheritence Systems. London 1984

Kontexturale Polysemie

Im folgenden soll kurz angedeutet werden, dass von den 4 von Kaehr (2009) eingeführten sog. Superoperatoren (da sie Kontexturgrenzen überschreiten, könnte man sie auch Inter- oder Trans-Operatoren nennen), vor allem durch Replikation und Bifurkation eine sehr grosse Zahl von semiotischen Strukturen mit kontexturaler Polysemie geschaffen wird. Wir gehen wie üblich aus von der 3-kontexturalen 3-adischen semiotischen Matrix, wo die (für diese Matrix) "kanonisierten" Indizes (eineindeutige Abbildung kontexturaler Indizes auf Subzeichen; gleiche Indizes für konverse, aber nicht für duale Suzeichen) abgelesen werden und mit den Polysemien in den unten stehenden Listen verglichen werden können.

1. Replikation von links

$\mathcal{R}(1.1)_{1,3} = (1.1)_{1,1,3}$
$\mathcal{R}(1.2)_{1} = (1.2)_{1,1}$
$\mathcal{R}(1.3)_{3} = (1.3)_{3,3}$

$\mathcal{R}(2.1)_{1} = (2.1)_{1,1}$
$\mathcal{R}(2.2)_{1,2} = (2.2)_{1,1,2}$
$\mathcal{R}(2.3)_{2} = (2.3)_{2,2}$

$\mathcal{R}(3.1)_{3} = (3.1)_{3,3}$
$\mathcal{R}(3.2)_{2} = (3.2)_{2,2}$
$\mathcal{R}(3.3)_{2,3} = (3.3)_{2,2,3}$

$\mathcal{R}\mathcal{R}(1.1)_{1,3} = (1.1)_{1,1,1,3}$
$\mathcal{R}\mathcal{R}(1.2)_{1} = (1.1)_{1,1,1}$
$\mathcal{R}\mathcal{R}(1.3)_{3} = (1.1)_{3,3.3}$

$\mathcal{R}\mathcal{R}(2.1)_{1} = (2.1)_{1,1,1}$
$\mathcal{R}\mathcal{R}(2.2)_{1,2} = (2.2)_{1,1,1,2}$
$\mathcal{R}\mathcal{R}(2.3)_{2} = (2.3)_{2,2,2}$

$\mathcal{R}(3.1)_{3} = (3.1)_{3,3,3}$
$\mathcal{R}(3.2)_{2} = (3.2)_{2,2,2}$
$\mathcal{R}(3.3)_{2,3} = (3.3)_{2,2,2,3}$

2. Replikation von rechts

$\mathcal{R}(1.1)_{1,3} = (1.1)_{1,3,3}$
$\mathcal{R}(1.2)_{1} = (1.2)_{1,1}$
$\mathcal{R}(1.3)_{3} = (1.3)_{3,3}$

$\mathcal{R}\mathcal{R}(1.1)_{1,3} = (1.1)_{1,3,3,3}$
$\mathcal{R}\mathcal{R}(1.2)_{1} = (1.2)_{1,1,1}$
$\mathcal{R}\mathcal{R}(1.3)_{3} = (1.3)_{3,3,3}$

$\mathcal{R}(2.1)_1 = (2.1)_{1,1}$ $\qquad$ $\mathcal{R}\mathcal{R}(2.1)_1 = (2.1)_{1,1,1}$

$\mathcal{R}(2.2)_{1,2} = (2.2)_{1,2,2}$ $\qquad$ $\mathcal{R}\mathcal{R}(2.2)_{1,2} = (2.2)_{1,2,2,2}$

$\mathcal{R}(2.3)_2 = (2.3)_{2,2}$ $\qquad$ $\mathcal{R}\mathcal{R}(2.3)_2 = (2.3)_{2,2,2}$

$\mathcal{R}(3.1)_3 = (3.1)_{3,3}$ $\qquad$ $\mathcal{R}\mathcal{R}(3.1)_3 = (3.1)_{3,3,3}$

$\mathcal{R}(3.2)_2 = (3.2)_{2,2}$ $\qquad$ $\mathcal{R}\mathcal{R}(3.2)_2 = (3.2)_{2,2,2}$

$\mathcal{R}(3.3)_{2,3} = (3.3)_{2,3,3}$ $\qquad$ $\mathcal{R}\mathcal{R}(3.3)_{2,3} = (3.3)_{2,3,3,3}$

3. Einige Bifurkationen

$\mathcal{B}(1.1)_{1,1,1,3} \rightarrow (1.1)_1\ [(1.2)_1],\ (1.1)_{1.1.3},\ (1.1)_{1,1},\ (1.1)_{1,3},\ (1.1)_{1,1,1},\ (1.1)_3\ [(1.3)_3]$

$\mathcal{B}(1.2)_{1,1,1} \rightarrow (1.2)_1,\ (1.2)_{1,1}$

$\mathcal{B}(1.3)_{3,3.3} \rightarrow (1.3)_3,\ (1.3)_{3,3}$

$\mathcal{B}(2.1)_{1,1,1} \rightarrow (2.1)_1,\ (2.1)_{1,1}$

$\mathcal{B}(2.2)_{1,1,1,2} \rightarrow (2.2)_1\ [(1.2)_{1,2}],\ (2.2)_{1.1.2},\ (2.2)_{1,1},\ (2.2)_{1,2},\ (2.2)_{1,1,1},\ (2.2)_2\ [(2.3)_2]$

$\mathcal{B}(2.3)_{2,2,2} \rightarrow (2.3)_2,\ (2.3)_{2,2}$

$\mathcal{B}(3.1)_{3,3,3} \rightarrow (3.1)_3,\ (3.1)_{3,3}$

$\mathcal{B}(3.2)_{2,2,2} \rightarrow (3.2)_2,\ (3.2)_{2,2}$

$\mathcal{B}(3.3)_{2,2,2,3} \rightarrow (3.3_2,\ (3.3)_{2,2.3},\ (3.3)_{2,2},\ (3.3)_{2,3},\ (3.3)_{2,2,2},\ (3.3)_3$

Wie man erkennt, sind aus genuinen Subzeichen durch Replikation und Bifurkation alle kontexturellen Werte herstellbar.

Bibliographie

Kaehr, Rudolf, Interpretations of the kenomic matrix. http://www.thinkartlab.com/pkl/lola/Matrix/Matrix.pdf (2009)

Diamantenschreibweise für kontexturierte Zeichenklassen

1. Polykontexturale Zeichenklassen verlangen nicht nur kategoriale, sondern auch antiparallel-kategoriale oder von Kaehr (2007) so genannte saltisitorische Notation. Beide, Kategorien und Saltatorien, können in einem Diamanten dargestellt werden. Zu meinem ersten Versuch einer semiotischen Diamententheorie vgl. Toth (2008b), zu fundamentaler Kritik Kaehr (2009a, b, c).

2. In Toth (2008a) hatte ich dynamische semiotische Kategorien eingeführt: Sie bilden nicht einfach Kategorien auf Subzeichen ab, wie etwa in

$(3.1\ 2.1\ 1.3) \rightarrow (\alpha°\beta°, \alpha°, \beta\alpha)$,

wo der triadisch verschachtelten Relation aus einer monadischen, dyadischen und triadischen Relation keinerlei Rechnung getragen wird, sondern sie übertragen die Phasenübergänge zwischen den semiosischen Relationen in die Morphismen, wie etwa in

$(3.1\ 2.1\ 1.3) \rightarrow ((3.2), (1.1); (2.1), (1.3)) \equiv [[\beta°, id1], [\alpha°, \beta\alpha]]$.

Wenn man aber (durch regressive Multiplikation) zu $(\alpha°\beta°, \alpha°, \beta\alpha)$ zurückkehren möchte, muss man entsprechend der Bezeichnungs-, Bedeutungs- und Gebrauchsfunktion des Zeichens das Paar natürlicher Transformationen um ein weiteres Glied in ein Tripel verwandeln, wie etwa im vorherigen Beispiel

$[[\beta°, id1], [\alpha°, \beta\alpha], [\beta\alpha, \alpha°\beta°]]$

2. Nun lautet aber die 3-konturale Fassung unserer Zeichenklasse

$(3.1_3\ 2.1_1\ 1.3_3)$.

Man kann sich dadurch behelfen, wie Kaehr (2009c) es tut, dass man einfach die Kategorien indiziert, wie man zuvor die Subzeichen indiziert hatte. Ferner kann man alles noch mehr vereinfachen, dass man alle Kategorien auf einen Morphismus α und seine Konverse $\alpha°$ reduziert; man hat dann $\alpha_{i,j}$ $/\alpha°_{i,j}$.

Ich sehe aber keinen Grund, auf die kategoriale Fundamentalunterscheidung zwischen

$(1 \rightarrow 2) \equiv \alpha$
$(2 \rightarrow 3) \equiv \beta$

zu verzichten, man muss ja die Komposition für jede Kategorie irgendwie nachweisen, und das geht so wohl am saubersten.

Auf jeden Fall sind wir jetzt soweit, dass wir die vollständige diamantentheore-tische Notation für eine polykontexturale Zeichenklasse angeben können. Als Beispiel stehe wiederum (3.1 2.1 1.3):

$$\left(\begin{array}{l} ((3.1)_3 \rightarrow ((2.1)_1 \rightarrow (1.3)_3) \equiv \\ [[\beta°, id1]_{(3\rightarrow 1)}, [\alpha°, \beta\alpha]_{(1\rightarrow 3)}, [\beta\alpha, \alpha°\beta°]_{(3\rightarrow 3')}] \\ \\ ((1.3)_3 \rightarrow ((2.1)_1 \rightarrow (3.1)_3) \equiv \\ [[\alpha, \alpha°\beta°]_{(3'\rightarrow 1')}, [\beta, id1]_{(1\rightarrow 3)}, [\alpha°\beta°, \beta\alpha]_{(3'\rightarrow 3)}] \end{array}\right)$$

Die allgemeine Form der diamantentheoretischen Notation für n-kontexturale Zeichenklassen mit n ≥ 2 ist daher:

$$\left(\begin{array}{l} ((3.a)_i \rightarrow ((2.b)_j \rightarrow (1.c)_k) \equiv \\ [[(3.2), (a.b)]_{(i\rightarrow j)}, [(2.1), (b.c)]_{(i\rightarrow k)}, [(1.3), (c.a)]_{(i\rightarrow k')}] \\ \\ ((1.a)_k \rightarrow ((2.b)_j \rightarrow (3.a)_i) \equiv \\ [[(c.1), (a.3)]_{3'\rightarrow 1'}, [(b.2), (1.c)]_{(1\rightarrow 3)}, [(a.d), (2.d)]_{(3'\rightarrow 3)}] \end{array}\right)$$

Bibliographie

Kaehr, Rudolf, Towards Diamonds. Glasgow 2007

Kaehr, Rudolf, Toth's semiotic diamonds. http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf (2009a)

Kaehr, Rudolf, Diamond Semiotics. http://rudys-diamond-strategies.blogspot.com/2008/12/diamond-semiotics.html (2009b)

Kaehr, Rudolf, Sketch on semiotics in diamonds. http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009c)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008 (2008a)

Toth, Alfred, In Transit. Klagenfurt 2008 (2008b)

Das vierfache Anfangen in der Semiotik

> Aristoteles beginnt, wie fast alle nach ihm, mit der Eins.
> Platon setzt auf die Zwei.
> Hegel, Heidegger und Peirce versuchen es mit der Drei.
> Pythagoras, Heidegger, Günther, Derrida halten es mit der Vier
> Es gibt keinen Ursprung; es gibt Vielheiten des Anfang(en)s."
>
> Rudolf Kaehr (2004, s.p.)

1. Übergänge erzeugen Orte, so wie sich zwischen Orten notwendigerweise Übergänge (Transitionen) ergeben. Nach polykontexturaler Auffassung verbindet eine Brücke nicht nur zwei Orte A und B über einen Abgrund, sondern der Abgrund ermöglicht erst die Verbindung von A und B, d.h. seine Überbrückung. Es geht also um das aus der klassischen Logik ausgeschlossene Zusammenspiel von Operatoren und Operanden, die sich nach polykontexturaler Sicht gegenseitig beeinflussen. Orte erzeugende Übergänge heissen hier Wiederholungen. Es gibt Wiederholungen des Alten und Wiederholungen des Neuen. Mit einem etwas gewöhnungsbedürftigen Terminus spricht Rudolf Kaehr von "kenomischen Disremptionen": "Diese Wiederholungen sind jedoch nicht nur in der Dimension der Generierung von Neuem, also der Evolution zu explizieren, sondern müssen zusätzlich bestimmt werden durch ihre komplementären Bestimmungen als 'emanative' Ausdifferenzierung mit ihren zwei Modi der Reduktion und der Komplikation auf einer jeweiligen Stufe der Evolution" (Kaehr 2004, s.p.).

Kaehr unterscheidet die folgenden 4 Möglichkeiten "doppelter Doppelbestimmung der Übergänge":

- Komplexitäts-aufbauend, durch Konstruktoren: evolutiv
- Komplexitäts-abbauend, durch Destruktoren: Monomorphienbildung
- Komplikations-aufbauend: Ausdifferenzierung durch Selbstabbildung
- Komplikations-abbauend: Reduktionen durch Selbstüberlagerungen

2. Auch wenn es keine semiotische Kenogrammatik geben kann, erweist sich, wie in diesem Artikel gezeigt werden soll, die Kaehrsche Unterscheidung der doppelten Doppelbestimmung der Übergänge als fruchtbar. Wir können o.B.d.A. in dem folgenden Kaehrschen Diagramm

$$\begin{array}{ccccc} & \downarrow & & \downarrow & \\ \leftrightarrow & \text{Objekt} & \leftrightarrow & \text{Objekt} & \leftrightarrow \\ & \downarrow & & \downarrow & \end{array}$$

“Objekt” durch “Zeichen” ersetzen und damit von doppelten Doppelbestimmungen in der Semiotik ausgehen.

2.1. Semiotische Konstruktoren

Als semiotische Konstruktoren fungieren prinzipiell die meisten der in Toth (2008, S. 11-19) angegebenen semiotischen Operatoren, speziell die bereits von Bense (1971) eingeführten drei basalen Operationen der Adjunktion, Superisation und Iteration.

2.2. Semiotische Destruktoren

Semiotische Destruktoren wurden zwar bisher nicht eingeführt worden, aber die Mechanismen des Zefalls von Zeichen sind aufgrund eines Kapitels in Arins Dissertation (Arin 1981, S. 328 ff.) rekonstruierbar und ausdifferenzierbar. Ebenfalls zu den Destruktoren gehören die in Toth (2008, S. 19) eingeführten Zerteilungen, vgl.

Zeichen: $Z_{mi,j} = Z(\cap_i \cap_j)$: Zerteilung in zwei Teile der Länge i und j; i + j = m
Beispiel: $Z_{2,4}$ (3.1 2.2 1.3) = (3.1); (2.2 1.3)
$Z_{2,4}$ (□□■ □■□ ■□□) = (□□■ □□□ □□□); (□□□ □■□ ■□□)

Z_m ist der Zerfall in lauter Einzelteile der Länge 1.

Beispiel: Z_6 (3.1 2.2 1.3) = 3; 1; 2; 2; 1; 3
Z_6 (■■ ■■ ■■) = ($■_3$); ($■_1$); ($■_2$); ($■_2$); ($■_1$); ($■_3$)

2.3. Semiotische Selbstabbildung

Auch was es bedeutet, wenn ein Zeichen ganz oder teilweise auf sich selbst abgebildet wird, ist bisher in der Semiotik unklar geblieben. Da ein Zeichen eine triadische Relation ist, können wir folgende Basis-Typen semiotischer Selbstabbildung unterscheiden:

- TR → TR ≡ (3.a 2.b 1.c) → (3.a 2.b 1.c)
(3.a 2.b 1.c) → (3.a 1.c 2.b)
(3.a 2.b 1.c) → (2.b 3.a 1.c)
(3.a 2.b 1.c) → (2.b 1.c 3.a)
(3.a 2.b 1.c) → (1.c 3.a 2.b)
(3.a 2.b 1.c) → (1.c 2.b 3.a), sowie Kombinationen.

- TR → DR ≡ (3.a 2.b 1.c) → {(3.a 1.c), (3.a 2.b), (2.b 3.a), (2.b 1.c), (1.c 3.a), (1.c 2.b)}

- TR → MR ≡ (3.a 2.b 1.c) ≡ {(3.a), (2.b), (1.c)}

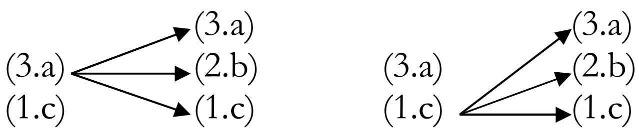
- DR → DR ≡ {(3.a 1.c), (3.a 2.b), (2.b 3.a), (2.b 1.c), (1.c 3.a), (1.c 2.b)} → {(3.a 1.c), (3.a 2.b), (2.b 3.a), (2.b 1.c), (1.c 3.a), (1.c 2.b)}

- DR → MR ≡ {(3.a 1.c), (3.a 2.b), (2.b 3.a), (2.b 1.c), (1.c 3.a), (1.c 2.b)} → {(3.a), (2.b), (1.c)}

- MR → MR ≡ {(3.a), (2.b), (1.c)} → {(3.a), (2.b), (1.c)}

Auch hier bleiben allerdings zahlreiche Möglichkeiten offen, die untersucht werden müssten; vgl. etwa bei DR → MR

(3.a 1.c) → {(3.a), (2.b), (1.c)} ≡



Ferner gibt es natürlich Dreier, Vierer- usw. Kombinationen.

2.4. Semiotische Selbstüberlagerung

Ebenfalls bisher undefiniert ist innerhalb der Semiotik der Begriff der Selbstüberlagerung von Zeichen. Wir können folgende fundamentale Typen unterscheiden:

$\Cap(1) \equiv \{\Cap(1, 1), \Cap(1, 2), \Cap(1, 3), \Cap(2, 1), \Cap(3, 1)\}$

$\Cap(2) \equiv \{\Cap(2, 2), \Cap(2, 3), \Cap(3, 2)\}$

$\Cap(3) \equiv \{\Cap(3, 3)\}$,

mit kategorialen Indizes:

$\Cap(1) \equiv \{ZR_{id1}, ZR_{\alpha}, ZR_{\beta\alpha}, ZR_{\alpha°}, ZR_{\alpha°\beta°}\}$

$\Cap(2) \equiv \{ZR_{id2}, ZR_{\beta}, ZR_{\beta°}\}$

$\Cap(3) \equiv \{ZR_{id3}\}$.

wobei ZR ∈ {MR, DR, TR}.

Auch hier gibt es natürlich mehrfache Selbstüberlagerungen. Durch Selbstüberlagerung könnte der bisher eher obsolete Begriff der semiotischen Absorption definiert werden, wobei offen ist, ob auch kürzere Relationen längere absorbieren können, wie dies bei den polykontexturalen Transoperatoren, die Kronthaler als "pathologisch" bezeichnete, der Fall ist (vgl. Kronthaler 1986, S. 65 ff.).

Bibliographie

Arin, Ertekin, Objekt- und Raumzeichen in der Architektur. Diss. Ing. Stuttgart 1981

Bense, Max, Zeichen und Design. Baden-Baden 1971

Kaehr, Rudolf, Entwurf einer Skizze eines Gewebes rechnender Räume in denkender Leere. Glasgow 2004

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt 1986

Toth, Alfred, Entwurf einer allgemeinen Zeichengrammatik. Klagenfurt 2008

Dissemination 4-kontexturaler Zeichenklassen in Intervallräumen von Repräsentationswerten

1. Jeder der 10 peirceschen Zeichenklassen kann ein Repräsentationswert in Form einer Kardinalzahl in einem abgeschlossenen Intervall von 9 bis 15 zugeordnet werden; diese Zuordnung ist nicht eindeutig, aber man kann dadurch repräsentationstheoretisch affine Zeichenklassen zusammenstellen. Im Gegensatz dazu ist die Abbildung von Kontexturwerten auf die Subzeichen von Zeichenklassen eindeutig; allerdings gibt es sehr viele verschiedene Verfahren dafür (vgl. Toth 2008). In dieser Arbeit gehen wir aus von der folgenden kontexturellen Belegung, die R. Kaehr (2008) vorgeschlagen hatte:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

Wie in Toth (2009) gezeigt, kann man nun aus Zeichenklassen, deren Subzeichen mehr als einen kontexturellen Index haben, mittels Kombinationen weitere Zeichenklassen bilden, deren Subzeichen nur je einen kontexturellen Index haben und anschliessend die Merkmalsmengen bestimmen, d.h. all diejenigen Zeichenklassen zu Mengen zusammenfassen, deren Subzeichen die gleichen kontexturellen Indizes in der gleichen Reihenfolge haben. Dadurch erhält man Gruppen von Mengen mit 1, 2, 3, 4, 5 und 6 Elementen, d.h. Zeichenklassen. Damit entsteht also neben der auf Repräsentationswerten basierenden Affinität eine zweite, kontexturelle Affinität, insofern repräsentationstheoretisch nicht-affine Zeichenklassen in der gleichen oder in gleichen Kontexturen disseminiert werden. Wir zeigen diese Disseminationen im folgenden mit einem Graphen in Form der 4 semiotischen Kontexturen in Abhängigkeit von den Repräsentationswerten. Die letzteren werden hier als Intervalle aufgefasst.

2. Die einzelnen Gruppen von Merkmalsmengen und ihre Disseminationsgraphen

2.1. Einergruppen

$\mathcal{M}_{211} \equiv \{(3.2_2\ 2.2_1\ 1.2_1)\}$ Rpw = 12

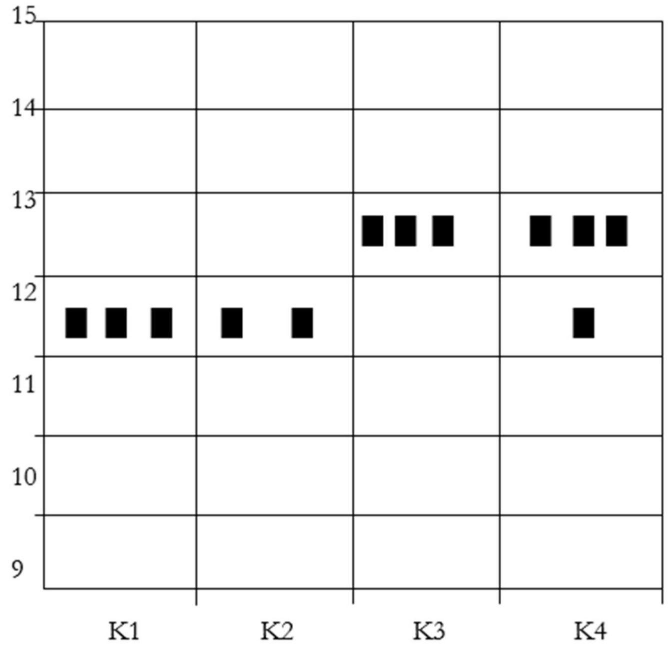
$\mathcal{M}_{214} \equiv \{(3.2_2\ 2.2_1\ 1.2_4)\}$ Rpw = 12

$\mathcal{M}_{221} \equiv \{(3.2_2\ 2.2_2\ 1.2_1)\}$ Rpw = 12

$\mathcal{M}_{241} \equiv \{(3.2_2\ 2.2_4\ 1.2_1)\}$ Rpw = 12

$\mathcal{M}_{334} \equiv \{(3.1_3\ 2.3_3\ 1.3_4)\}$ Rpw = 13

$m_{434} \equiv \quad \{(3.1_4\ 2.3_3\ 1.3_4)\}$ Rpw = 13

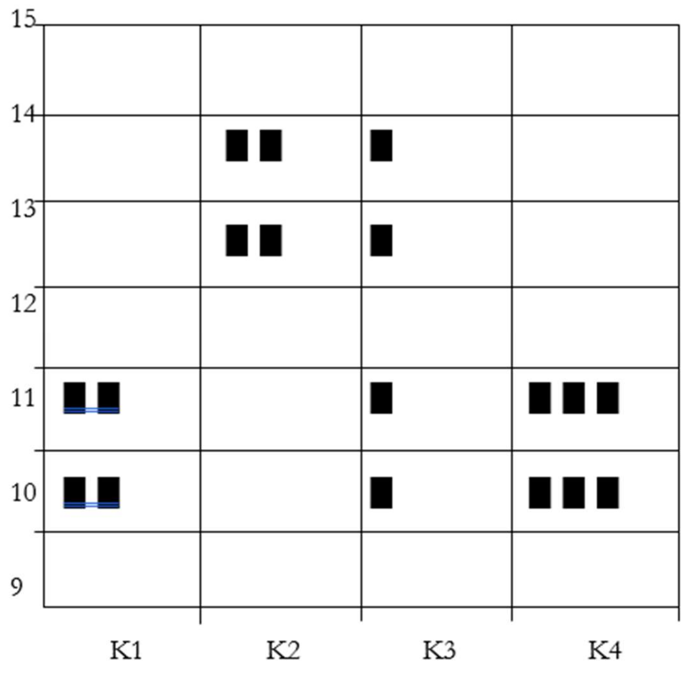


2.2. Zweiergruppen

$m_{223} \equiv \quad \{(3.2_2\ 2.2_2\ 1.3_3), (3.2_2\ 2.3_2\ 1.3_3)\}$ Rpw = 14/13

$m_{341} \equiv \quad \{(3.1_3\ 2.1_4\ 1.2_1), (3.1_3\ 2.2_4\ 1.2_1)\}$ Rpw = 10/11

$m_{441} \equiv \quad \{(3.1_4\ 2.1_4\ 1.2_1), (3.1_4\ 2.2_4\ 1.2_1)\}$ Rpw = 10/11



2.3. Dreiergruppen

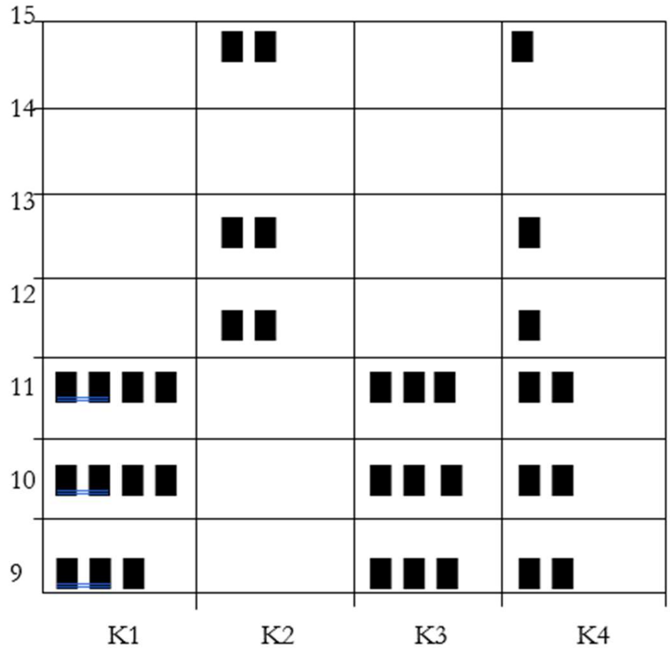
$m_{224} \equiv \{(3.2_2\ 2.2_2\ 1.2_4), (3.2_2\ 2.2_2\ 1.3_4), (3.3_2\ 2.3_2\ 1.3_4)\}$ Rpw = 12/13/15

$m_{311} \equiv \{(3.1_3\ 2.1_1\ 1.1_1), (3.1_3\ 2.1_1\ 1.2_1), (3.1_3\ 2.2_1\ 1.2_1)\}$ Rpw = 9/10/11

$m_{313} \equiv \{(3.1_3\ 2.1_1\ 1.1_3), (3.1_3\ 2.1_1\ 1.3_3), (3.1_3\ 2.2_1\ 1.3_3)\}$ Rpw = 9/11/12

$m_{411} \equiv \{(3.1_4\ 2.1_1\ 1.1_1), (3.1_4\ 2.1_1\ 1.2_1), (3.1_4\ 2.2_1\ 1.2_1)\}$ Rpw = 9/10/11

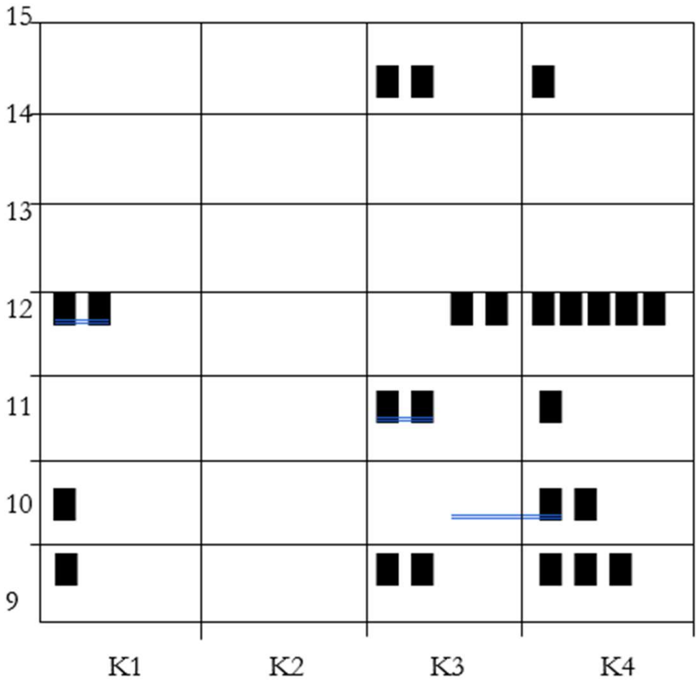
$m_{413} \equiv \{(3.1_4\ 2.1_1\ 1.1_3), (3.1_4\ 2.1_1\ 1.3_3), (3.1_4\ 2.2_1\ 1.3_3)\}$ Rpw = 9/11/12



2.4. Vierergruppen

$m_{343} \equiv \{(3.1_3\ 2.1_4\ 1.1_3), (3.1_3\ 2.1_4\ 1.3_3), (3.1_3\ 2.2_4\ 1.3_3), (3.3_3\ 2.3_4\ 1.3_3)$
Rpw = 9/11/12/15

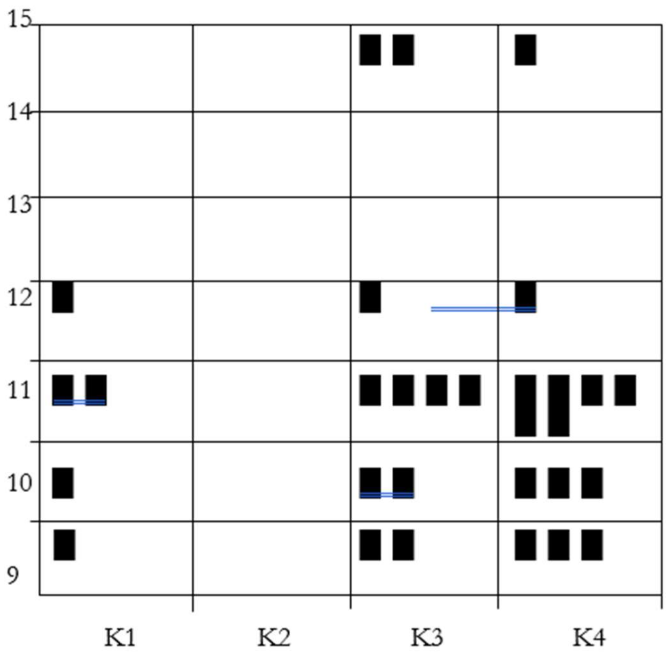
$m_{414} \equiv \{(3.1_4\ 2.1_1\ 1.1_4), (3.1_4\ 2.1_1\ 1.2_4), (3.1_4\ 2.2_1\ 1.3_4), (3.2_4\ 2.2_1\ 1.2_4)\}$
Rpw = 9/10/12/12



2.5. Fünfergruppen

$m_{314} \equiv$ $\{(3.1_3\ 2.1_1\ 1.1_4), (3.1_3\ 2.1_1\ 1.2_4), (3.1_3\ 2.1_1\ 1.3_4), (3.1_3\ 2.2_1\ 1.2_4),$
$(3.1_3\ 2.2_1\ 1.3_4)\}$ Rpw = 9/10/11/11/12

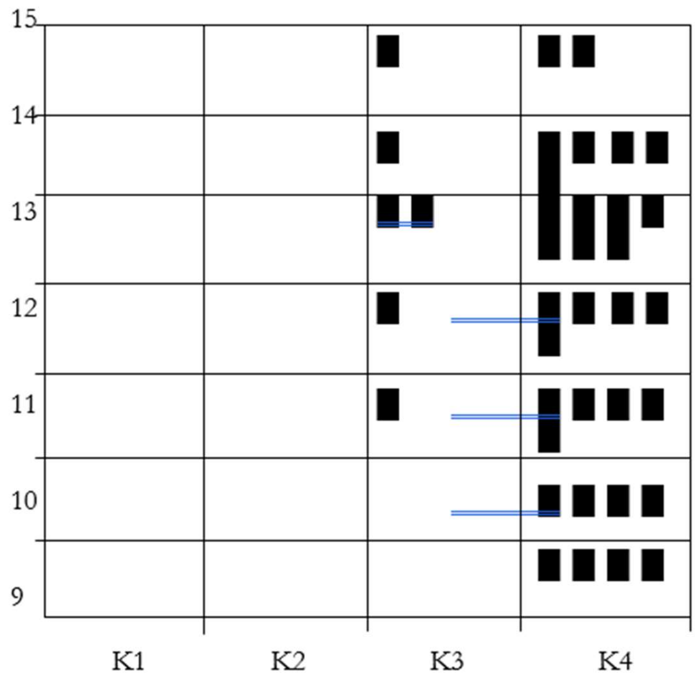
$m_{344} \equiv$ $\{(3.1_3\ 2.1_4\ 1.1_4), (3.1_3\ 2.1_4\ 1.2_4), (3.1_3\ 2.1_4\ 1.3_4), (3.1_3\ 2.2_4\ 1.2_4),$
$(3.3_3\ 2.3_4\ 1.3_4)\}$ Rpw = 9/10/11/11/15



2.6. Sechsergruppen

$\mathcal{M}_{443} \equiv$ {(3.1₄ 2.1₄ 1.3₃), (3.1₄ 2.2₄ 1.3₃), (3.1₄ 2.3₄ 1.3₃), (3.2₄ 2.2₄ 1.3₃), (3.2₄ 2.3₄ 1.3₃), (3.3₄ 2.3₄ 1.3₃)} Rpw = 11/12/13/13/14/15

$\mathcal{M}_{444} \equiv$ (3.1₄ 2.1₄ 1.1₄), (3.1₄ 2.1₄ 1.2₄), (3.1₄ 2.1₄ 1.3₄), (3.1₄ 2.2₄ 1.3₄), (3.2₄ 2.2₄ 1.3₄), (3.2₄ 2.3₄ 1.3₄)} Rpw = 9/10/11/12/13/14



Es ist also möglich, Zeichenklassen, die in mehr als 3 Kontexturen liegen, kontexturell "auseinanderzufalten", so dass jedes ihrer Subzeichen nur noch in einer Kontextur liegt und anschliessend diese Zeichenklassen zu kontexturellen Merkmalsmengen aufgrund gleicher kontextureller Werte in gleicher Ordnung zusammenzufassen und sei dann in Intervallräumen von Repräsentationswerten zu disseminieren, so dass jede der 6 Gruppen von Zeichenklassen auf einen spezifischen eigenen Verteilungsgraph abgebildet werden kann.

Bibliographie

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009c)

Toth, Alfred, The 10 semiotic dual systems in 4 contextures and 3 number structures. In: Electronic Journal for Mathematical Semiotics, 2009a

Toth, Alfred, Kontextuelle Affinität nicht-affiner Zeichenklassen. In: In: Electronic Journal for Mathematical Semiotics 2009b

Kontexturierte semiotische Kategorien

1. R. Kaehr (2008) hatte gezeigt, wie man Zeichenklassen kontexturiert, indem man ihre Subzeichen kontexturiert. Damit ist es natürlich möglich, auch die für die Subzeichen stehenden semiotischen Morphismen (Semiosen) zu kontextuieren. Da sich aufgrund der in Toth (2008, S. 159 ff.) eingeführten dynamischen semiotischen Kategorientheorie allerhand Schwierigkeiten einstellen können, soll die Funktionsweise kontexturierter semiotischer Kategorien anhand von 3- und 4-kontexturalen Semiosen dargestellt werden.

2. Zunächst kann man die kontexturierte semiotische Subzeichenmatrix in die kontexturierte semiotische Kategorienmatrix transformieren, indem man die Subzeichen durch die entsprechenden Kategorien ersetzt (vgl. z.B. Toth 1997, S. 21 ff.):

$$\begin{pmatrix} 1.1_{1,3} & 1.2_{1} & 1.3_{3} \\ 2.1_{1} & 2.2_{1,2} & 2.3_{2} \\ 3.1_{3} & 3.2_{2} & 3.3_{2,3} \end{pmatrix} \longrightarrow$$

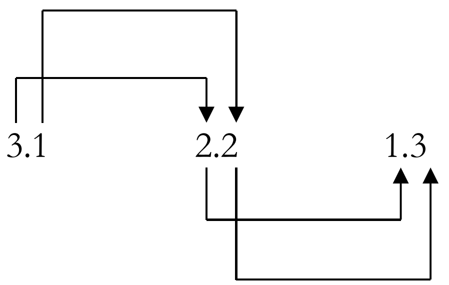
$$\begin{pmatrix} id_{1,3} & \alpha_{1} & \beta\alpha_{3} \\ \alpha°_{1} & id_{1,2} & \beta_{2} \\ \alpha°\beta°_{3} & \beta°_{2} & id_{2,3} \end{pmatrix}$$

Nun besteht die Essenz der in Toth (2008, S. 159 ff.) eingeführten dynamischen Kategorien darin, dass in Zeichenrelationen nicht die Subzeichen 1:1 durch Kategorien ersetzt werden, wie dies zuvor der Fall war, sondern dass der Tatsache Rechnung getragen wird, dass die triadische Zeichenrelation eine Relation über einer monadischen, einer dyadischen und einer triadischen Relation, d.h. eine Relation über Relationen bzw. eine “verschachtelte” Relation ist.

Wenn wir also z.B. die Zeichenklasse (3.1 2.2 1.3) nehmen, werden wir sie nicht statisch durch [α°β°, id2, βα] kategorial darstellen, sondern nach dem folgenden Muster:

(3.1 2.2 1.3) → [[3.2, 1.2], [2.1, 2.3]],

d.h.



Da die Kontexturenwerte jedes Subzeichens in keiner Beziehung zu den Kontexturwerten ihrer Primzeichen stehen, müssen die Kontexturenwerte der dynamisch zusammengesetzten Morphismen (Semiosen, Subzeichen) aus der obigen kategorialen Matrix bestimmt werden. Für das obige Beispiel bekommen wir also für die entsprechende 3-kontexturale Zeichenklasse:

$(3.1_{3}\ 2.2_{1,2}\ 1.3_{3}) \rightarrow [[3.2_{2}, 1.2_{1}], [2.1_{1}, 2.3_{2}]]$

Und für die entsprechende 4-kontexturale Zeichenklasse:

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \rightarrow [[3.2_{2,4}, 1.2_{1,4}], [2.1_{1,4}, 2.3_{2,4}]]$

Demzufolge können wir paarweise Transformationen (vgl. Toth 2009) wie folgt darstellen:

$$
\begin{array}{l}
(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ 1.2_{4,1}\ 1.3_{4,3}) \\
\qquad\ \downarrow \qquad\qquad\qquad\qquad\quad \downarrow \\
\underbrace{(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})}
\end{array}
$$

$(2.1_{1,4} \rightarrow 2.2_{1,2,4}) \rightarrow [2.2_{1,2,4}, 1.2_{1,4}] = [id_{1,2,4}, \alpha_{1,4}]$

$$
\begin{array}{l}
(3.1_{3,4}\ 2.1_{1,4}\ 1.1_{1,3,4}) \times (1.1_{4,3,1}\ 1.2_{4,1}\ 1.3_{4,3}) \\
\qquad\ \downarrow \qquad\qquad\qquad\qquad\quad \downarrow \\
\underbrace{(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ 2.2_{4,2,1}\ 1.3_{4,3})}
\end{array}
$$

$[(2.1_{1,4}\ 1.1_{1,3,4}) \rightarrow (1.1_{1,3,4}\ 1.3_{3,4})] \rightarrow [[[2.1_{1,4}, 1.1_{1,3,4}], [2.1_{1,4}, 1.3_{3,4}]], [[1.1_{1,3,4}], [1.3_{3,4}]]]$
$= [[[\alpha°_{1,4}, id1_{1,3,4}], [\alpha°_{1,4}, \beta\alpha_{3,4}]], [[id1_{1,3,4}, \beta\alpha_{3,4}]]]$

Bibliographie

Kaehr, Rudolf, Sketch on semiotics in diamonds. In: http://www.thinkartlab.com/pkl/lola/Semiotics-in-Diamonds/Semiotics-in-Diamonds.html (2009)

Toth, Alfred, Entwurf einer semiotisch-relationalen Grammatik. Tübingen 1997

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Toth, Alfred, Untersuchungen zu Zeichenobjekten II. In: Electronic Journal for Mathematical Semiotics, 2009

Ein weiterer semiotischer Erhaltungssatz

1. Zu einer Zeichenrelation und ihrer Realitätsthematik gehört nach Bense auch "die begrifflich fixierte Differenzierung zwischen 'Ontizität' und 'Semiotizität', die das Verhältnismäßige unserer Welterfahrung regelt" (Bense 1979, S. 19), und darüber orientiert das "Theorem über Ontizität und Semiotizität": "Mit wachsender Semiotizität steigt auch die Ontizität der Repräsentation an" (Bense 1976, S. 60). Auf diesem Hintergrund formuliert Bense dann in Analogie zu den Erhaltungssätzen der Physik einen semiotischen "Erhaltungssatz": "Insbesondere muss in diesem Zusammenhang das duale Symmetrieverhältnis zwischen den einzelnen Zeichenklassen und ihren entsprechenden Realitätsthematiken hervorgehoben werden. Dieses Symmetrieverhältnis besagt, dass man im Prinzip nur die 'Realität' bzw. die Realitätsverhältnisse metasemiotisch präsentieren kann, die man semiotisch zu repräsentieren vermag. Daher sind die Repräsentationswerte (d.h. die Summen der fundamentalen Primzeichen-Zahlen) einer Zeichenklasse invariant gegenüber der dualen Transformation der Zeichenklasse in ihre Realitätsthematik. Dieser semiotische 'Erhaltungssatz' kann dementsprechend als eine Folge des schon in *Vermittlung der Realitäten* (1976, p. 60 u. 62) ausgesprochenen Satzes [angesehen werden], dass mit der wachsenden Semiotizität der Repräsentativität in gleichem Maße auch ihre Ontizität ansteigt" (Bense 1981, S. 259).

2. Nun hatte ich bereits in Toth (2008) den Begriff der semiotischen Priorität eingeführt, der auf die Ordnung der thematisierenden oder thematisierten Subzeichen der durch eine Realitätsthematik präsentierten strukturellen Realität abhebt. In einer triadisch-trichotomischen (monokontexturalen) Semiotik können folgende 6 Typen unterschieden werden. Mit "X" wird jeweils das Thematisat, mit "A" und "B" werden die Thematisanten bezeichnet:

1. $(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3}) = (X \leftarrow (AB))$
2. $(2.1\ 3.1\ 1.3) \times (3.1\ \underline{1.3\ 1.2}) = (X \leftarrow (BA))$
3. $(3.1\ 1.3\ 2.1) \times (\underline{1.2}\ 3.1\ \underline{1.3}) = (A \rightarrow X \leftarrow B)$
4. $(2.1\ 1.3\ 3.1) \times (\underline{1.3}\ 3.1\ \underline{1.2}) = (B \rightarrow X \leftarrow A)$
5. $(1.3\ 3.1\ 2.1) \times (\underline{1.2\ 1.3}\ 3.1) = ((AB) \rightarrow X)$
6. $(1.3\ 2.1\ 3.1) \times (\underline{1.3\ 1.2}\ 3.1) = ((BA) \rightarrow X)$.

1. und 2. werden auch als Linksthematisate, 5. und 6. als Rechtsthematisate und 3. und 4. als "Sandwich-Thematisationen" bezeichnet (vgl. Toth 2007, S. 214 ff.).

Wenn man nun die Zeichenklassen kontexturiert, wobei K = 4 sei, erhält man die folgenden 6 Typen polykontexturaler Zeichenklassen:

1. $(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ \underline{1.2}_{4,1}\ \underline{1.3}_{4,3}) = (X \leftarrow (AB))$
2. $(2.1_{1,4}\ 3.1_{3,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ \underline{1.3}_{4,3}\ \underline{1.2}_{4,1}) = (X \leftarrow (BA))$

3. $(3.1_{3,4}\ 1.3_{3,4}\ 2.1_{1,4}) \times (\underline{1.2}_{4,1}\ 3.1_{4,3}\ \underline{1.3}_{4,3}) = (A \rightarrow X \leftarrow B)$
4. $(2.1_{1,4}\ 1.3_{3,4}\ 3.1_{3,4}) \times (\underline{1.3}_{4,3}\ 3.1_{4,3}\ \underline{1.2}_{4,1}) = (B \rightarrow X \leftarrow A)$
5. $(1.3_{3,4}\ 3.1_{3,4}\ 2.1_{1,4}) \times (\underline{1.2}_{4,1}\ \underline{1.3}_{4,3}\ 3.1_{4,3}) = ((AB) \rightarrow X)$
6. $(1.3_{3,4}\ 2.1_{1,4}\ 3.1_{3,4}) \times (\underline{1.3}_{4,3}\ \underline{1.2}_{4,1}\ 3.1_{4,3}) = ((BA) \rightarrow X)$.

Wie man feststellt, gilt zwar hinsichtlich der Ordnung der kontexturalen Indizes

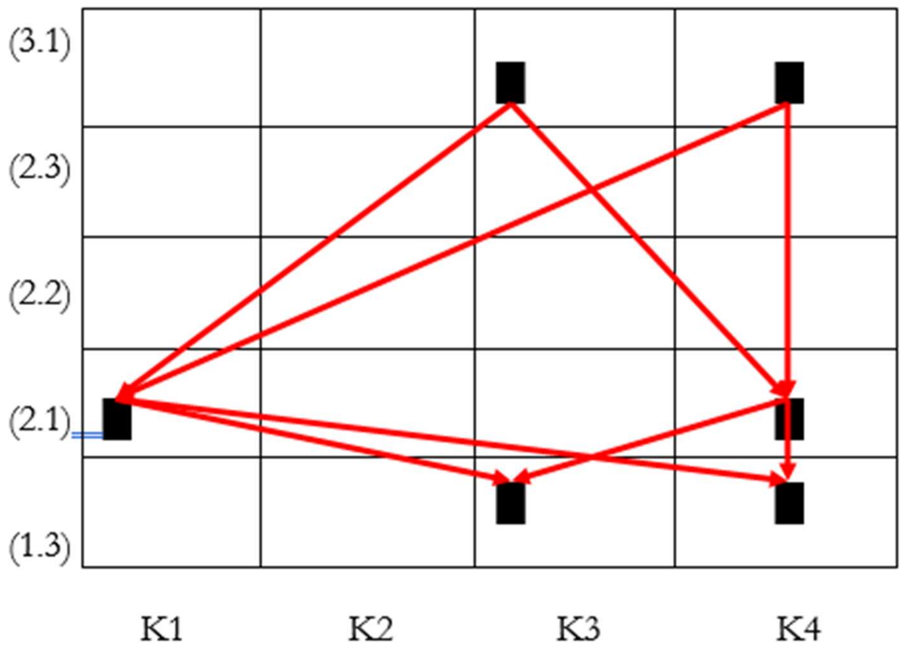
$\times(3.a_{i,j,k}\ 2.b_{l,m,n}\ 1.c_{o,p,q}) = (c.1_{q,p,o}\ b.2_{n,m,l}\ a.3_{k,j,i})$ $(i,...,p \in \{\varnothing, 1, 2, 3, 4\})$,

d.h. die Reihenfolge der Kontexturen wird umgekehrt und damit die logische Identität der Subzeichen aufgehoben, aber die Reihenfolge der Kontexturen entspricht der Prioritätenhierchie der thematisierten und thematisierenden Subzeichen:

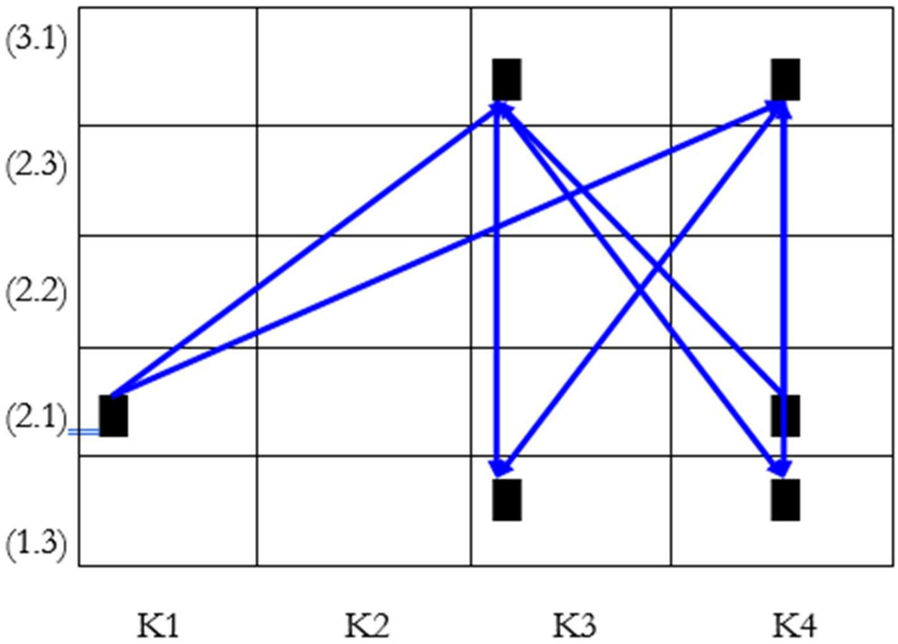
$(3.1_{4,3}\ \underline{1.2}_{4,1}\ \underline{1.3}_{4,3}) = (X \leftarrow (A < B))$ ~ $1 < 3$
$(3.1_{4,3}\ \underline{1.3}_{4,3}\ \underline{1.2}_{4,1}) = (X \leftarrow (B > A))$ ~ $3 > 1$
$(\underline{1.2}_{4,1}\ 3.1_{4,3}\ \underline{1.3}_{4,3}) = (A^{<} \rightarrow X \leftarrow B^{>})$ ~ $1 < 3$
$(\underline{1.3}_{4,3}\ 3.1_{4,3}\ \underline{1.2}_{4,1}) = (B^{>} \rightarrow X \leftarrow A^{<})$ ~ $3 > 1$
$(\underline{1.2}_{4,1}\ \underline{1.3}_{4,3}\ 3.1_{4,3}) = ((A < B) \rightarrow X)$ ~ $1 < 3$
$(\underline{1.3}_{4,3}\ \underline{1.2}_{4,1}\ 3.1_{4,3}) = ((B > A) \rightarrow X)$ ~ $3 > 1$

Die Ergebnisse können in den folgenden 6 Graphen dargestellt werden:

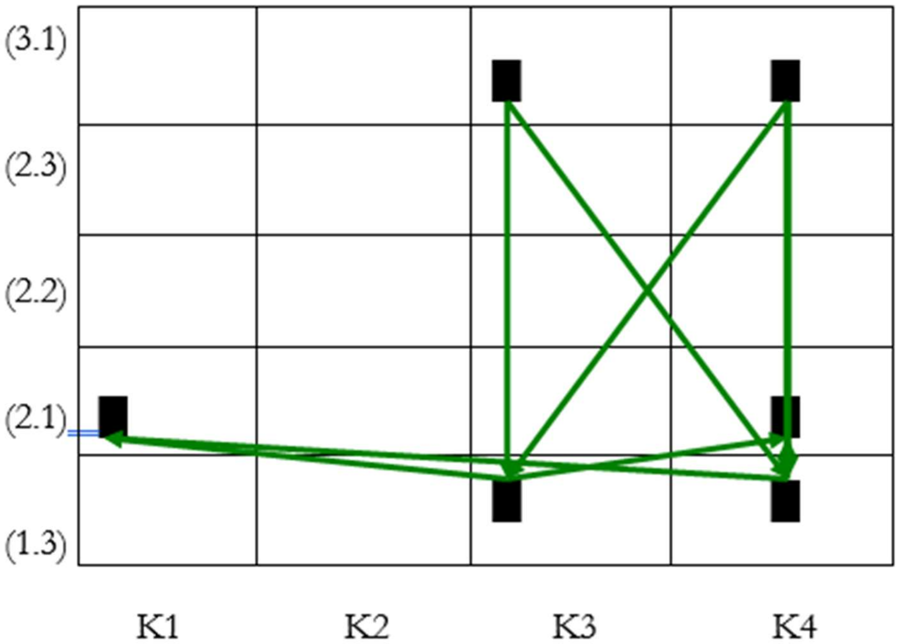
1. $(3.1_{3,4}\ 2.1_{1,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ \underline{1.2}_{4,1}\ \underline{1.3}_{4,3}) = (X \leftarrow (AB))$



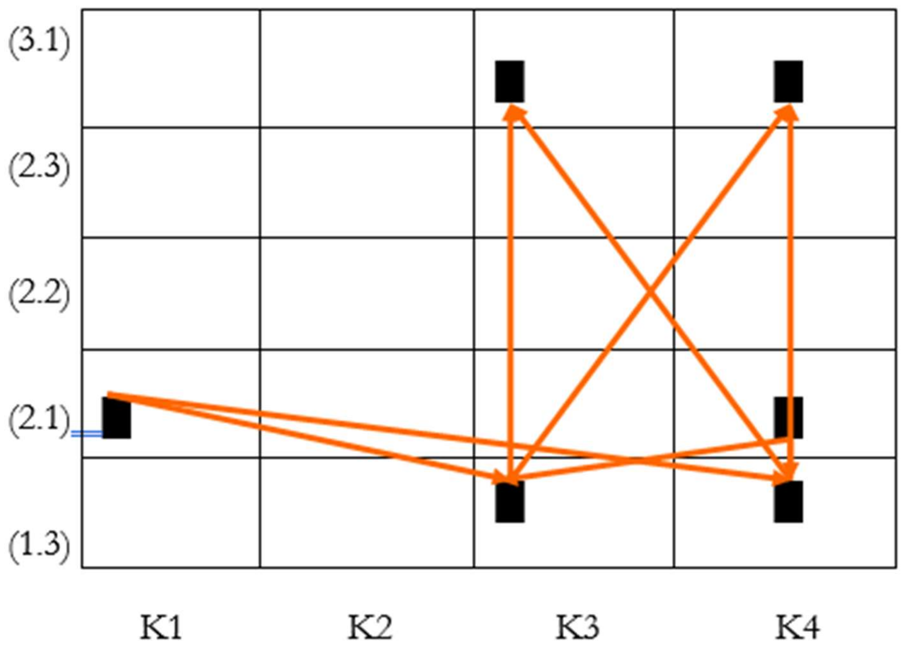
2. $(2.1_{1,4}\ 3.1_{3,4}\ 1.3_{3,4}) \times (3.1_{4,3}\ \underline{1.3}_{4,3}\ \underline{1.2}_{4,1}) = (X \leftarrow (BA))$



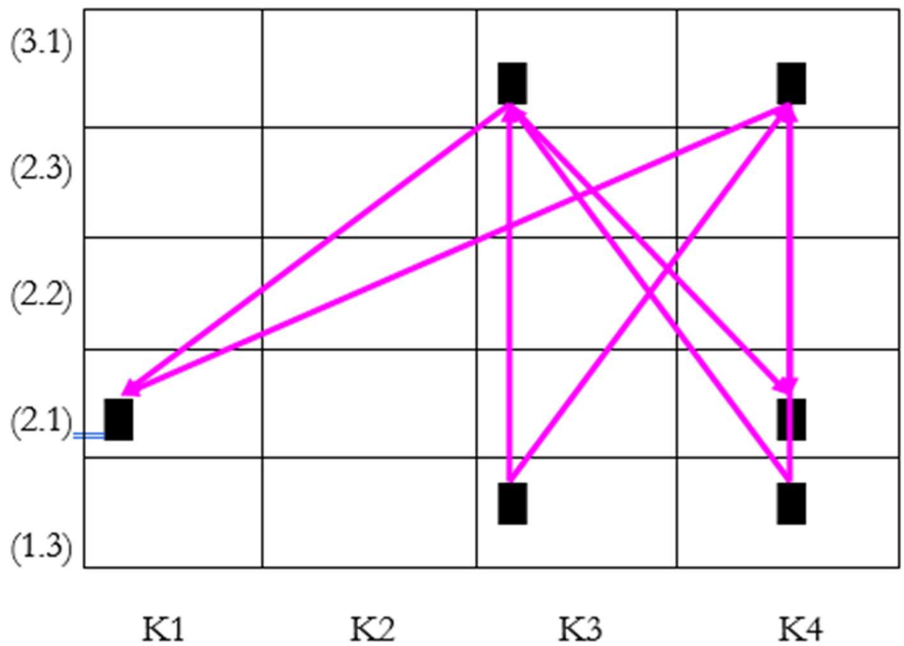
3. $(3.1_{3,4}\ 1.3_{3,4}\ 2.1_{1,4}) \times (\underline{1.2}_{4,1}\ 3.1_{4,3}\ \underline{1.3}_{4,3}) = (A \rightarrow X \leftarrow B)$



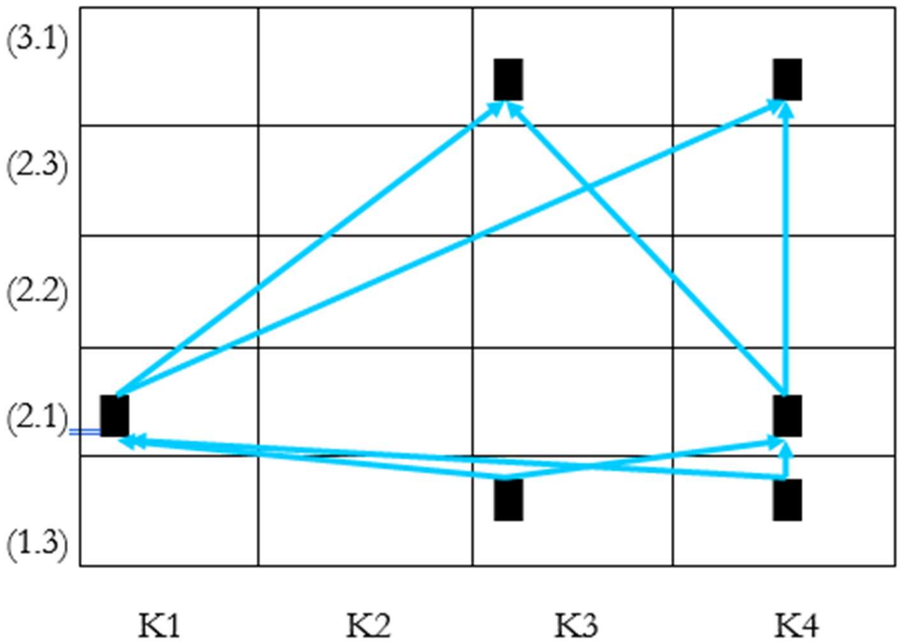
4. $(2.1_{1,4}\ 1.3_{3,4}\ 3.1_{3,4}) \times (\underline{1.3}_{4,3}\ 3.1_{4,3}\ \underline{1.2}_{4,1}) = (B \rightarrow X \leftarrow A)$



5. $(1.3_{3,4}\ 3.1_{3,4}\ 2.1_{1,4}) \times (\underline{1.2}_{4,1}\ \underline{1.3}_{4,3}\ 3.1_{4,3}) = ((AB) \rightarrow X)$



6. $(1.3_{3,4}\ 2.1_{1,4}\ 3.1_{3,4}) \times (\underline{1.3_{4,3}\ 1.2_{4,1}}\ 3.1_{4,3}) = ((BA) \rightarrow X)$



Es gilt also im Anschluss an Bense (1981, S. 259) folgendes semiotische

Theorem: Die in der Ordnung der Kontexturen einer Zeichenklasse feststellbare semiotische Priorität ist invariant gegenüber der dualen Transformation der Zeichenklasse in ihre Realitätsthematik.

Bibliographie

Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007, 2. Aufl. 2008 (2008a)

Toth, Alfred, Priority in thematized realities. In: Electronic Journal of Mathematical Semiotics, 2008b

Zur semiotischen quantitativ-qualitativen Arithmetik

1. Nach Toth (2008, S. 155 ff.) gelten für Subzeichen folgende quantitative arithmetische Gesetze:

(a.b) + 1 = (a+1.b), falls a < 3
(a.b+1), falls b < 3

(a.b) + 2 = (a+2.b), falls a = 1
(a.b+2), falls b = 1

(a.b) + 3 = (a+1.b+2), falls a < 3 und b = 1
(a+2.b+1), falls a = 1 und b < 3

(a.b) + 4 = (a+2.b+2), falls a = 1 und b = 1

2. Wenn wir nun statt von der quantitativen von der qualitativen Matrix ausgehen:

△	◭	▲
□	◨	■
○	◑	●

dann bekommen wir die folgenden qualitativen arithmetischen Gesetze:

(a.b) ∈ {△, ◭, □, ◨}→ (a.b) +1 ∈ {▲, ■, ○, ◑, ●}

(a.b) ∈ {△}→ (a.b) +2 ∈ {▲, ○}

(a.b) ∈ {△, ◭, □}→ (a.b) +3 ∈ {■, ◑}

(a.b) ∈ {△}→ (a.b) +4 = ●

Für die Konkatenation zweier Dyaden zu Triaden (vgl. Walther 1979, S. 79) gilt wegen

(3.a 2.b 1.c) mit a ≤ b ≤ c

a ∈ {○, ◑, ●}

a = ○ → b ∈ {□, ◨, ■}

a = ◑ → b ∈ {◨, ■}

a = ● → b ∈ {■}

b = □ → b ∈ {△, ◭, ▲}

b = ◨ → b ∈ {◭, ▲}

b = ■ → b ∈ {▲}

Bibliographie

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Toth, Alfred, Allgemeine Zeichenlehre. 2. Aufl. Stuttgart 1979

Transzendentale Semiotiken

1. Von ihrer ganzen Konzeption her ist die peircesche Semiotik nicht-transzendental: Eine "absolut vollständige Diversität von 'Welten' und 'Weltstücken', von 'Sein' und 'Seiendem' ist einem Bewusstsein, das über triadischen Zeichenrelationen fungiert, prinzipiell nicht repräsentierbar" (Bense 1979, S. 59), aber Peirce hält "den Unterschied zwischen dem Erkenntnisobjekt und –subjekt fest, indem er beide Pole durch ihr Repräsentiert-Sein verbindet" (Walther 1989, S. 76). Bense fasste wie folgt zusammen: "Wir setzen damit einen eigentlichen (d.h. nicht-transzendentalen) Erkenntnisbegriff voraus, dessen wesentlicher Prozeß darin besteht, faktisch zwischen (erkennbarer) 'Welt' und (erkennendem) 'Bewusstsein' zwar zu unterscheiden, aber dennoch eine reale triadische Relation, die 'Erkenntnisrelation', herzustellen" (Bense 1976, S. 91).

In ihrem Geiste erweist sich damit die Peirce-Semiotik durch und durch als ein amerikanisches Produkt, "denn transzendentale Probleme des Himmels und des ewigen Lebens sind ‚un-American'" (Günther 2000, S. 240, Fn. 22), oder, sehr schön ausgedrückt: „Erlkönigs Töchter tanzen nicht am Rande der Highways, und Libussa und ihre Gefährtinnen wiegen sich nicht in den Baumwipfeln der riesigen Wälder der Neuen Welt" (2000, S. 217), denn es ist die Intuition des Pragmatismus, „zu ignorieren, dass der Mensch in früheren Kulturen schon gedacht hat" (2000, S. 241). Dies liegt daran, „dass nichts in Amerika, was aus der spirituellen Tradition der Alten Welt stammt, mit grösserer Verständnislosigkeit registriert wird, als die metaphysische Entwertung des Diesseits" (2000, S. 149).

2. Bense fasst denn das Zeichen auch explizit als Funktion auf, um die „Disjunktion zwischen Welt und Bewusstsein" zu überbrücken (1975, S. 16). Von diesem pragmatistischen Standpunkt auch kommt also streng genommen die Frage nach den von Zeichen bezeichneten oder sie substituierenden Objekten gar nicht auf, denn „Seinsthematik [kann] letztlich nicht anders als durch Zeichenthematik motiviert und legitimiert werden" (Bense 1981, S. 16), so dass "Objektbegriffe nur hinsichtlich einer Zeichenklasse relevant sind und nur relativ zu dieser Zeichenklasse eine semiotische Realitätsthematik besitzen, die als ihr Realitätszusammenhang diskutierbar und beurteilbar ist" (Bense 1976, S. 109). Bense (1981, S. 11) brachte dies auf die Formel: "Gegeben ist, was repräsentierbar ist". Von diesem nicht-transzendentalen Standpunkt aus sind also Zeichen schlicht und einfach deswegen notwendig, weil wir ohne sie die Welt der Objekte gar nicht wahrnehmen könnten. Anderseits kommt, wie gesagt, bei dieser Konzeption niemand auf die Idee, nach den bezeichneten Objekten zu fragen, denn durch die Definition des Zeichens ist zum vornherein klar, dass wir diese nie erreichen können: sie erreichen uns nur durch die Filter unserer Perzeption und Apperzeption, d.h. immer interpretiert und damit als Zeichen. Die Sehnsucht des Soldaten, der allein in der Kaserne sitzt und das Photo seiner Geliebten küsst, im Stillen hoffend, es möge sich doch in die reale

Person verwandeln, ist also in einer Peirce-Benseschen Semiotik gänzlich ausgeschlossen. Trotzdem findet sich das Motiv, die Brücke zwischen dem Diesseits der Zeichen und dem Jenseits ihrer Objekte zu überschreiten, in der Weltliteratur zu allen Zeiten bis in die Gegenwart.

3. In Toth (2009a) wurde eine nicht-transzendentale Semiotik auf der Basis einer qualitativen Zahlenrelation vorgeschlagen. Die grundlegende Überlegung ist dabei, dass die Primzeichenrelation

PZR = (.1., .2., .3.)

sowohl die quantitative Nachfolgerelation der Ordnungsrelation

$(.1.) \rightarrow (.2.) \rightarrow (.3.)$

als auch die qualitative Vorgängerrelation der Selektionsrelation

(.1) > (.2.) > (.3.)

in sich vereinigt, d.h. zugleich quantitativ und qualitativ ist:

PZR = $(.1.) \lesseqgtr (.2.) \lesseqgtr (.3.)$.

Damit kann die quantitative semiotische Matrix durch eine qualitative ersetzt werden:

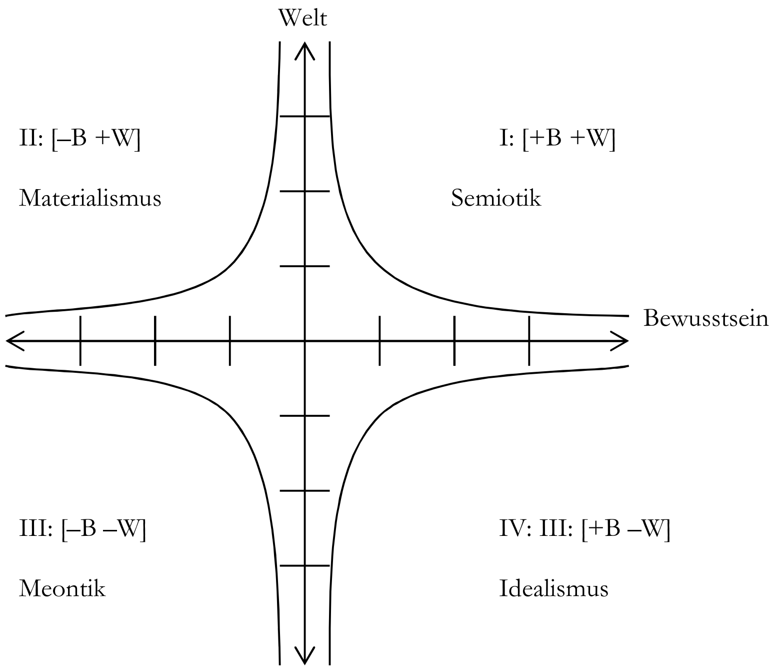
$$\begin{pmatrix} (1.1) & (1.2) & (1.3) \\ (2.1) & (2.2) & (2.3) \\ (3.1) & (3.2) & (3.3) \end{pmatrix} \Longrightarrow \begin{pmatrix} \text{△} & \text{◭} & \text{▲} \\ \text{□} & \text{◨} & \text{■} \\ \text{○} & \text{◑} & \text{●} \end{pmatrix}$$

Hier werden also die Grenzen zwischen Quantität und Qualität, aber keine eigentlichen semiotischen Kontexturen unterschieden.

4. Der erste Versuch einer "polykontexturalen" Semiotik geht auf Toth (2000) zurück und wurde in Toth (2008b) vollständig präsentiert. Sie geht davon aus, dass die Primzeichenrelation parametrisierbar ist:

PZR = $(\pm 3.\pm a\ \pm 2.\pm b\ \pm 1.\pm c)$

Der grundlegende Gedanke dahinter ist Benses Definition des Zeichens als Funktion zwischen Welt und Bewusstsein, d.h. zwischen Objekt und Subjekt. Wenn man nun die Objektspositionen der Zeichenrelation negativ parametrisiert, erhält man idealistische, wenn man die Subjektspositionen negativ parametrisiert, materialistische und wenn man sowohl die Subjekts- als auch die Objektspositionen negativ parametrisiert, meontische Zeichenklassen. Das Peircesche Zeichen wird damit zum Spezialfall des durchwegs positiv parametrisierten Zeichens, d.h. eines Zeichens, bei dem sowohl die Subjekts- als auch die Objektspositionen positiv parametrisiert sind. Trägt man nun diese 4 Zeichenfunktionen in ein kartesisches Koordinatensystem ein, so erhält man eine Hyperbel mit 4 Ästen, die entweder zur Welt-Achse, zur Bewusstseins-Achse, zu beiden oder zu keinen von beiden asymptotisch ist:



Es ist nun einfach, Zeichenklassen (bzw. Realitätsthematiken) zu konstruieren, die in Bezug auf die Parametrisierung der Sub- bzw. Primzeichen inhomogen sind, z.B.

(+3.-a +2.+b –1.-c).

Hat nur ein einziges Primzeichen ein anderes Vorzeichen als die übrigen Primzeichen einer Zeichenrelation, so liegt die entsprechende Zeichenfunktion in mindestens 2 Quadranten. Diese Quadranten können als "semiotische Kontexturen" definiert werden, weil die parametrisch inhomogenen Zeichenfunktionen jeweils die "Niemandslandbereiche" zwischen den asymptotischen Hyperbelästen und Ordinate/ Abszisse durchschneiden, d.h. durch mathematisch und semiotisch undefiniertes Gebiet führen. Solche Zeichenklassen weisen damit Mischformen semiotischer (im engeren Sinne), idealistischer, materialistischer oder meontischer Zeichenfunktionen auf.

5. Während dies bisherigen Versuche einer transzendentalen Semiotik entweder von den Qualitäten oder den Kontexturen ausgingen, geht der folgende Versuch, dem in Toth (2008c, d) drei Bände gewidmet wurden, von der benseschen Unterscheidung zwischen ontologischem und semiotischem Raum aus (Bense 1975, S. 45 f., 65 f.). Der Grundgedanke ist, dass bereits die Objekte, sobald sie wahrgenommen werden,

in Bezug auf ihre Form, Gestalt oder Funktion wahrgenommen werden. Dies bedeutet, dass es eine Ebene der Präsemiotik gibt, die der eigentlichen Semiose, d.h. der Transformation eines Objektes in ein Zeichen vorangeht und deren Trichotomie von Götz (1982, S. 5, 28) mit "Sekanz – Semanz – Selektanz" bezeichnet wurde und die sich bei der Zeichengenese auf die semiotischen Trichotomien, wie sie durch die Subzeichen und ihre Semiosen repräsentiert werden, vererbt. Bense setzt daher zwischen dem ontologischen Raum der Objekte und dem semiotischen Raum der Zeichen einen Zwischenraum an der "disponiblen" Objekte an und charakterisiert ihn kategoriell mit "Nullheit". Diese Nullheit ergänzt nun die Peirce Triade von Erst-, Zweit- und Drittheit zu einer Tetrade, in die das Objekt als kategorielles Objekt in die präsemiotische Zeichenrelation eingebettet ist:

PrZR = (3.a 2.b 1.c 0.d)

Während also (3.a), (2.b) und (1.c) nicht-transzendente Kategorien sind, ist (0.d) das ursprünglich dem Zeichen transzendente Objekte, dessen Transzendenz in dieser Einbettung freilich aufgehoben ist:

PrZR = (3.a 2.b 1.c ‖ 0.d) → PrZR = (3.a 2.b 1.c ╫ 0.d),

wobei das Zeichen ‖ für die Kontexturengrenze zwischen Zeichen und Objekt und das Zeichen ╫ für deren Durchbrechung steht.

6. Während die bisherigen Versuche vom Standpunkt der Polykontexturalitätstheorie nicht als polykontextural eingestuft werden, weil der logische Identitätssatz in allen diesen transzendentalen Semiotiken immer noch Gültigkeit hat, geht der Versuch einer "echten" Polykontexturalisierung der Semiotik auf einige jüngste Arbeiten von Rudolf Kaehr zurück (z.B. Kaehr 2008). Hier wird davon ausgegangen, dass die (monokontexturale) Peircesche Zeichenrelation

ZR = (3.a 2.b 1.c)

ein 1-kontexturaler Sonderfall der n-kontextural disseminierten Semiotiken ist. Die Kontexturen, in denen sich eine Zeichenklasse befinden kann, werden als Indizes den Subzeichen zugewiesen, d.h. nicht die ganze Zeichenklasse, sondern ihre Subzeichen werden kontexturell markiert. Damit kann eine Zeichenklasse natürlich in mehreren Kontexturen gleichzeitig erscheinen, was sogar der Normalfall ist. Grundsätzlich ist nach Günther (1979, S. 229 ff.) die Zuweisung von Kontexturen zu Subzeichen weitgehend frei. Es muss lediglich beachtet werden, dass genuine Subzeichen, d.h. identitive semiotische Morphismen immer in mindestens 2 Kontexturen stehen, weil die Kontexturen auf der Basis quadratischer Matrizen verteilt werden und sich deren Blöcke in den Hauptdiagonalen schneiden. Zum Beispiel könnte eine 4-kontexturale Zeichenklasse wie folgt aussehen:

$ZR = (3.a_{i,j,k}\ 2.b_{l,m,n}\ 1.c_{o,p,q})$,

wobei i, ..., q ∈ {∅, 1, 2, 3, 4}. ∅ besagt dabei lediglich, dass ein j ∈ {i, ..., q} auch unbesetzt sein kann, wie etwa im Falle der folgenden Zeichenklassen:

$3\text{-}ZR = (3.1_{3}\ 2.2_{1,2}\ 1.2_{1})$
$4\text{-}ZR = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$

Bei der 4-kontexturalen Zeichenklasse liegen also die nicht-genuinen Subzeichen in 2 und das genuine Subzeichen in 3 Kontexturen, wobei die 4. Kontextur allen Subzeichen gemein ist. Bei der 3-kontexturalen Zeichenklasse gibt es dagegen keine Kontextur, in der alle Subzeichen liegen.

Bei dieser echt-polykontexturalen Semiotik ist nun das logische Identitätsgesetz wahrhaft aufgehoben, was am besten am Verhalten von Subzeichen, die mehr als einen kontexturalen Index tragen, bei Dualisierung sieht:

$\times(3.1_{3}\ 2.2_{1,2}\ 1.3_{3}) = (3.1_{3}\ 2.2_{2,1}\ 1.3_{3})$.

Es gibt hier also wegen $(2.2_{1,2}) \neq (2.2_{2,1})$ keine Eigenrealität mehr. Dies bedeutet im Einklang mit Bense (1992), dass wesentlichste Teile der Semiotik zusammenbrechen. Ferner sind in Kaehr's Semiotik die Theoreme der Objekttranszendenz des Zeichens und der Zeichenkonstanz, die nach Kronthaler (1992) eine monokontexturale Semiotik limitieren, immer noch gültig, so dass also auch diese Semiotik trotz der entfallenden Identität der Zeichen zwischen Zeichen- und Realitätsthematik (bzw. der Irresistibilität der Zeichen durch die Dualisation) nicht wirklich polykontextural ist.

7. Als kleinen Einschub wollen wir hier kurz reflektieren, was Polykontexturalität im Zusammenhang mit Semiotik überhaupt bedeutet. Ein Zeichen, in dem die Zeichenkonstanz aufgehoben und durch Strukturkonstanz ersetzt ist, ist ein Morphogramm. In dieser Form können zwar problemlos Zeichenklassen und Realitätsthematiken notiert (vgl. Toth 2003), aber keine konkreten Zeichen verwendet werden. Ein verknotetes Taschentuch, das sich über Nacht verwandelt, kann keine Zeichenfunktion haben. Zeichen, die der Kommunikation mit der Gesellschaft, d.h. nicht nur zum privaten Gebrauch dienen, müssen wiedererkennbar sein, d.h. an materiale Konstanz gebunden sein. Ohne Materialkonstanz keine Zeichenkonstanz und ohne Zeichenkonstanz keine Zeichen. Was man also immer unter einer polykontexturalen Semiotik versteht: das Limitationstheorem der Zeichenkonstanz kann man nicht ausser Kraft setzen ohne die gesamte Pragmatik der Zeichenverwendung zu zerstören.

Dagegen ist, es wie an den obigen Modellen mit Ausnahme desjenigen von Kaehr gezeigt, möglich, nur das Limitationstheorem der Objekttranszendenz ausser Kraft zu setzen. Damit darf aber nicht gemeint sein, dass Zeichen und Objekt ununterscheidbar werden. Ununterscheidbar sind sie genau dann, wenn der logische Identitätssatz aufgehoben ist. Wie wir aber gesehen haben, ist dieser Satz nirgendwo ausser in der kaehrschen Konzeption aufgehoben. Das Bestehenbleiben des Identitätssatzes garantiert damit die Unterscheidbarkeit von Zeichen und Objekt und macht sozusagen nicht ihre metaphysische Identität, sondern nur ihre Positionen austauschbar, etwa so, wie es im "Bildnis des Dorian Gray" von Oscar Wilde geschildert ist. Dort verändert sich ja das Bild, d.h. das Zeichen, statt des Objektes, d.h. statt Dorian. Der Vorgang ist allerdings erstens reversibel, denn am Ende des Romans erscheint das Bild verändert und nicht Dorian, und zweitens können die Diener sehr wohl zwischen dem Bild und dem vor ihm liegenden Leiche Dorian's unterscheiden. Wie gezeigt wurde, kann man in der Semiotik die Grenzen zwischen Zeichen und Objekt aufheben, indem man

1. die quantitativen Subzeichen durch qualitative Subzeichen ersetzt.

2. die Subzeichen parametrisiert und die Zeichenfunktion vom 1. Quadranten eines kartesischen Koordinatensystems in allen 4 Quadranten einzeichnet, was sich in natürlicher Weise aus der Benseschen Konzeption der Zeichenfunktion als einer hyperbolischen Funktion ergibt, die sowohl zur Welt- als auch zur Bewusstseins-Achse asymptotisch ist.

3. das Objekt des ontologischen Raumes als kategoriales Objekt in die triadische Zeichenrelation des semiotischen Raumes einbettet und dadurch einen Zwischenbereich erhält, der die Nullheit im Sinne Benses als vierte Fundamentalkategorie innerhalb einer tetradischen präsemiotischen Zeichenrelation enthält

Bei der kaehrschen Konzeption wird, wie bereits mehrfach gesagt, zwar die Identitätsrelation zwischen Zeichenklasse und Reaitätsthematik aufgehoben, aber nicht die Transzendenz des Objektes eines Zeichens. Es ist ferner nicht klar, welchen Status die Realitätsthematiken in der kaehrschen Semiotik haben. Auf jeden Fall können sie nicht mehr den Objektpol der Erkenntnisrelation thematisieren und so den Subjektpol der Zeichenthematik komplementieren, wie dies in der peirceschen Semiotik der Fall ist (vgl. Gfesser 1990, S. 133). Statt sich zu fragen: "Are there signs anyway?", wie es Kaehr in einer neuen Arbeit tut (Kaehr 2009), sollte man hier vielleicht besser fragen: "Are there objects anyway?". Denn wo sind in der polykontexturalen Ontologie die Objekte? Subjekt und Objekt sind ja austauschbar, und wenn hier der Begriff Objekt, an dem Günther festhält, noch irgendwelchen Sinn macht, dann ganz sicher nicht im Sinne des Gegen-standes, dem be-geg-net werden kann. Da das Kenogramm per definitionem immateriell ist, kann es auf kenogrammatischer Ebene auf jeden Fall keine Objekte geben. Es fragt sich daher nur, ob es dann Subjekte gibt, nicht nur deshalb, weil die beiden Begriffe einander ja voraussetzen, sondern weil der Begriff des Subjektes aus Sinn und Bedeutung, genauer: der Fähigkeit zur Interpretation definiert ist. Und da es Interpretation nur durch Zeichen gibt, müssten also Kenogramme der Interpretation und damit der Repräsentation fähig sein – aber gerade das sind sie ja per definitionem nicht. Statt Objekten würde man also auf kenogrammatischer Ebene Zeichen erwarten, aber Zeichen setzen, wie weiter oben bemerkt, das Prinzip der Induktion der Ordinalzahlen und das Prinzip der reversen Induktion der selektiven Kategorien voraus und können daher keine Kenogramme sein. Während das Zeichen die Gruppenaxiome erfüllt (Toth 2008a, S. 37 ff.), erfüllen die Kenogramme nicht einmal die Anforderung an ein Gruppoid. Will man zusätzlich zu den formalen Theorie der Quantität eine formale Theorie der Qualitäten errichten, dann ist es also der falsche Weg, die Quantitäten noch von ihrem letzten Rest an Zeichenhaftigkeit (oder Subzeichenhaftigkeit) zu befreien, sondern man sollte ihnen die Fähigkeit zur Interpretation geben, denn Qualitäten können nur durch Zeichen unterschieden werden – die Frage, was 1 Apfel und 1 Birne gäbe, ist, wie sattsam bekannt ist, in einer Theorie der Quantitäten eben nicht beantwortbar. Eine "Mathematik der Qualitäten" (Kronthaler 1986) muss daher eine qualitativ interpretierbare und das heisst eine semiotische Mathematik und keine Keno- oder Morphogrammatik sein, denn diese mag wohl die tiefsten formalen Strukturen sowohl von Quantitäten als auch von Qualitäten thematisieren, aber sie zu repräsentieren und mit ihnen tatsächlich zu RECHNEN, vermag sie nicht.

8. In diesem abschliessenden Kapitel wollen wir uns fragen, ob es sinnvoll wäre, die vier transzendentalen Semiotiken, d.h. die drei von uns begründeten und die eine von Kaehr begründete, miteinander zu kombinieren. Bei vier Modellen ergeben sich also sechs mögliche Kombinationen:

8.1. Qualitative Semiotik und parametrisierte Semiotik

PZR = $(.1.) \lesseqgtr (.2.) \lesseqgtr (.3.)$
SZR = {△, ◭, ▲, □, ◨, ■, ○, ◑, ●} } →
PZR = (±3.±a ±2.±b ±1.±c)

SZR = {±△, ±◭, ±▲, ±□, ±◨, ±■, ±○, ±◑, ±●}

Mit dieser Definition der Subzeichenrelation können die Qualitäten des Zeichens, wie ihre entsprechenden Quantitäten, in verschiedenen Kontexturen aufscheinen. Dies ist eine Konsequenz aus der Theorie der parametrisierten Zeichen, bringt aber nichts grundsätzlich Neues.

8.2. Qualitative Semiotik und Einbettungstheorie

SZR = {△, ◭, ▲, □, ◨, ■, ○, ◑, ●}
PrZR = {3.a 2.b 1.c 0.d)

Es bleibt, die kategoriale Nullheit durch drei Qualitäten (d ∈ {.1, .2, .3}) zu repräsentieren. Nach Toth (2009b) sind das

(⊓), (⊔), (⊏) bzw. (⊓*), (⊔*), (⊏*),

wobei die gestirnten nur bei Realitätsthematiken entsprechend dem zwar tetradischen, aber trichotomischen Zeichenmodell vorkommen.

Bei der Kombination bekommen wir also

SZR = {△, ◭, ▲, □, ◨, ■, ○, ◑, ●, ⊓, ⊔, ⊏}

Diese Relation ist allerdings insofern heterogen, als die ersten neun Qualitäten für Relationen, die letzten drei Qualitäten aber für eine Kategorie stehen. In Toth (2008e) wurde daher argumentiert, dass es nicht nur die Objekttranszendenz, sondern auch eine Transzendenz (oder Introszendenz) des Interpretanten und eine Transzendenz (oder Ultraszendenz) des Mittels gibt und dass eine vollständige transzendentale Zeichenrelation daher aus 6 Glieder besteht:

TrZR = {3.a 2.b 1.c 0.d ◎.e ⊙.f},

worin also (0.d) das 0-relationale kategoriale Objekt, (◎.e) den 0-relationalen kategorialen Interpreten und (⊙.f) das 0-relationale kategoriale Mittel bezeichnen. Genauso wie die letzten zwei, ist also bereits (0.d) eine Qualität, so dass die Ersetzung der präsemiotischen Trichotomie durch ⊓, ⊔, ⊏ nichts mehr als eine Schreibkonvention ist.

8.3. Qualitative Semiotik und kaehrsche Semiotik

Sie bestünde einfach darin, dass man SZR durch Kontexturen indiziert, also etwa im Falle einer 3-kontexturalen Semiotik:

K-SZR = SZR = {$△_{1,3}$, $◭_{1}$, $▲_{3}$, $□_{1}$, $◨_{1,2}$, $■_{2}$, $○_{3}$, $◑_{2}$, $●_{2,3}$}

8.4. Parametrisierte Semiotik und Einbettungstheorie

ZR = (±3.±a ±2.±b ±1.±c)

Diese im 2. Band von Toth (2008d) bereits behandelte Semiotik geht aus von

Pr-ZR = (±3.±a ±2.±b ±1.±c ±0.±d)

8.5. Parametrisierte Semiotik und Kaehr-Semiotik

Ausgangdefinition wäre im 3-kontexturalen Fall eine Zeichendefinition der folgenden Form

$K\text{-}ZR = ((\pm 3.\pm a)_{i,j,k}\ (\pm 2.\pm b)_{l,m,n}\ (\pm 1.\pm c)_{o,p,q})$ mit i, ..., 1 $\in \{\varnothing, 1, 2, 3\}$

8.6. Einbettungstheorie und Kaehr-Semiotik

Ausgangsdefinition der Zeichenrelation wäre im 4-kontexturalen Fall, der in diesem Fall wegen der Tetradizität der Zeichenklassen minimal ist:

$K\text{-}Pr\text{-}ZR = (3.a_{i,j,k}\ 2.b_{l,m,n}\ 1.c_{o,p,q}\ 0.d_{r,s,t})$ mit i, ..., t $\in \{\varnothing, 0, 1, 2, 3\}$

Zusammenfassend lässt sich feststellen, dass die Kombinationen 8.1 bis 8.6 gegenüber den Haupttypen transzendentaler Semiotik, die durch Elimination des Theorems der Objekttranszendenz ausgezeichnet sind, zwar Verfeinerungen des formalen semiotischen Apparates, aber keine metaphysischen Neurungen erbringen.

Abschliessend sei denjenigen, die keinen Nutzen in einer transzendentalen Semiotik sehen oder für die dieses Thema in den Bereich der Magie gehört, mit Günther zugerufen: "Das neue Thema der Philosophie ist die Theorie der Kontexturalgrenzen, die die Wirklichkeit durchschneiden" (Günther, Der Tod des Idealismus und die letzte Mythologie, hrsg. von Rudolf Kaehr, S. 47).

Bibliographie

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975
Bense, Max, Vermittlung der Realitäten. Baden-Baden 1976
Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979
Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981
Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992
Gfesser, Karl, Bemerkungen zum "Zeichenband". In: Walther, Elisabeth und Bayer, Udo (Hrsg.), Zeichen von Zeichen für Zeichen. Baden-Baden 1990, S. 129-141.

Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. Bd. 2. Hamburg 1979
Günther, Gotthard, Die amerikanische Apokalypse. München 2000
Günther, Gotthard, Der Tod des Idealismus und die letzte Mythologie. Undat. Fragm., hrsg. von Rudolf Kaehr: http://www.thinkartlab.com/pkl/tod-ideal.htm
Götz, Matthias, Schein Design. Diss. Stuttgart 1982
Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)

Kaehr, Rudolf, Polycontexturality of signs?
http://www.thinkartlab.com/pkl/lola/PolySigns/PolySigns.pdf (2009)
Toth, Alfred, Monokontexturale und polykontexturale Semiotik. In: Bernard, Jeff and Gloria Withalm (Hrsg.), Myths, Rites, Simulacra. Proceedings of the 10th International Symposium of the Austrian Association for Semiotics, University of Applied Arts Vienna, December 2000. Vol. I: Theory and Foundations & 7th Austro-Hungarian Semio-Philosophical Colloquium. Vienna: Institute for Socio-Semiotic Studies, S. 117-134 (= Applied Semiotics, vol. 18)
Toth, Alfred, Grundlegung einer polykontexturalen Semiotik. In: Grundlagenstudien aus Kybernetik und Geisteswissenschaft 44, 2003, S. 139-149
Toth, Alfred, Grundlegung einer mathematischen Semiotik. Klagenfurt 2007, 2. Aufl. 2008 (2008a)
Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2008 (2008b)
Toth, Alfred, Der sympathische Abgrund. Klagenfurt 2008 (2008c)
Toth, Alfred, Semiotics and Pre-Semiotics. 2 Bde. Klagenfurt 2008 (2008d)
Toth, Alfred, Wie viele Kontexturgrenzen hat ein Zeichen? In: Electronic Journal for Mathematical Semiotics, 2009a
Toth, Alfred, Das Zeichen als qualitative Zahlenrelation. In: Electronic Journal for Mathematical Semiotics, 2009b
Toth, Alfred, Die qualitativen polykontextural-semiotischen Funktionen. In: Electronic Journal for Mathematical Semiotics, 2009c
Walther, Elisabeth, Charles Sanders Peirce – Leben und Werk. Baden-Baden 1989

Qualitative parametrische Semiotik

1. Die in Toth (2009) eingeführte qualitative Semiotik geht davon aus, dass in der Primzeichenrelation die drei Relata quantitativ durch die Nachfolger- oder "Posterioritäts"-Beziehung (Bense 1979, S. 60):

$PZR_{quan} = (.1.) < (.2.) < (.3.)$

sowie qualitativ durch die Selektions-Beziehung (Bense 1979, S. 60)

$PZR_{qual} = (.1.) > (.2.) > (.3.)$

geordnet ist, wobei zur Abkürzung für diese quantitativ-qualitative Ordnung kurz

$PZR = (.1.) \lesseqgtr (.2.) \lesseqgtr (.3.)$

geschrieben wird.

2. Dagegen geht die in Toth (2000) eingeführte parametrische Semiotik von der benseschen Vorstellung des Zeichens als einer Vermittlungsfunktion zwischen Welt und Bewusstsein aus (Bense 1975, S. 16), wobei sich wegen der Transzendenz der Zeichenfunktion zu ihrem bezeichneten Objekt einerseits und wegen der Introszendenz zu ihrem bezeichnenden Interpretanten anderseits eine doppelte Asymptose ergibt, welche die Zeichenfunktion als Hyperbel auffassen lässt, die die Gleichung

$y = x^{-1}$

erfüllt. Nimmt man ferner die negative Funktion

$y = -x^{-1}$

hinzu, erhält man eine Hyperbeldarstellung mit Ästen in allen 4 Quadranten eines kartesischen Koordinatensystems. Diese Quadranten bzw. die Zeichenfunktionen in ihnen sind folglich definiert durch
Quadrant I: [+x, +y]
Quadrant II: [-x, + y]
Quadrant III: [-x, -y]
Quadrant IV: [+x, -y]

Macht man sich bewusst, dass sich die "Welt"-Koordinate eines Zeichens auf das Objekt des Zeichens und die "Bewusstseins-"Koordinate auf das Subjekt des

Zeichens bezieht, können wir die Parametrisierungen der vier Quadranten auch wie folgt notieren:

Quadrant I: [+S, +O]
Quadrant II: [-S, + O]
Quadrant III: [-S, -O]
Quadrant IV: [+S, -O]

Inneherhalb einer Zeichenklasse, die sich aus drei dyadischen Subzeichen zusammensetzt, muss daher jedes Subzeichen einzeln mit Hilfe des Parameters [±S±O] bestimmt werden. Wir bekommen damit

ZR = (±3.±a ±2.±b ±1.±c)

Aus dieser parametrischen Zeichenrelation lässt sich eine enorm grosse Menge von Zeichenklassen und Realitätsthematiken gewinnen, vgl. Toth (2008, S. 57 ff.).

3. Will man nun aber die qualitative und die parametrische Semiotik miteinander vereinigen, d.h. sucht man eine gemeinsame Definition für die folgenden beiden Zeichendefinitionen

ZR_{qual} = {△, ◭, ▲, □, ◨, ■, ○, ◑, ●}
ZR_{par} = (±3.±a ±2.±b ±1.±c),

so kommt man zunächst auf folgende aufzählende Definition einer parametrischen Semiotik:

$ZR_{par-qual}$ = {±△, ±◭, ±▲, ±□, ±◨, ±■, ±○, ±◑, ±●}

Damit steht man aber vor einem Problem: Die semiotischen Qualitäten sind per Subzeichen definiert. Was aber tut man in Fällen wie

SZ = (+a.-b)
SZ = (-a.+b),

wo die Primzeichen eines Subzeichens verschieden parametrisiert sind? Hierzu ist es nötig, die Qualitäten neu zu definieren, und zwar per kartesische Produkte aus den Primzeichen. In Anlehnung an frühere Arbeiten (z.B. Toth 2008) schlagen wir folgendes System vor:

PRZ_{qual} = (|, I, |)

Dann bekommen wir folgende semiotische Matrix

	]	I	[
]	]]	]I	][
I	I]	II	I[
[	[ ]	[I	[[

Die Symbole für die semiotischen Qualitäten sind insofern suggestiv angesetzt, als] auf Linksabgeschlossenheit (quantitativ: kein Vorgänger) und [auf Rechtsabgeschlossenheit (quantitativ: kein Nachfolger) verweist. I weist dagegen sowohl auf Vorgänger wie Nachfolger, d.h. auf Links- und Rechtsoffenheit hin.

Die Korrespondenzen zwischen den per Subzeichen und der per Primzeichen definierten Symbolen sind:

△~]]	□ ~ I]	○ ~ [ ]
◭~]I	◨ ~ II	◑ ~ [I
▲~][	■ ~ I[	● ~ [[

Nun kann man parametrisieren, wobei wir wieder die Korrespondenzen angeben:

±△ → {+]+], -]+], +], -], -]-]}
±◭ → {+]+I, −]+I, +]-I, -]-I}
±▲ → {+]+[, −]+[, +]-[, -]-[}
±□ → {+I+], -I+], +I-], -I-]}
±◨ → {+I+I, -I+I, +I-I, -I-I}
±■ → {+I+[, -I+[, +I-[, -I-[}
±○ → {+[+], -[+], +[-], -[-]}
±◑ → {+[+I, -[+I, +[-I, -[-I}
±● → {+[+[, -[+[, +[-[, -[-[}

Als Beispiel zeigen wir die qualitativ-parametrische Notation der folgenden Zeichenklassen:

(3.-1 –2.2 1.3) → (+[-], -I+], +]+[)
(-3.1 2.-2 –1.-3) → (-[+], +I-], -]-[)
(3.1 2.2 –1.3) → (+[+], +I+], -]+[)

Bibliographie

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979

Toth, Alfred, Ein Notationssystem für semiotische Vermittlung. In: Electronic Journal for Mathematical Semiotics, 2008

Toth, Alfred, Das Zeichen als qualitative Zahlenrelation. In: Electronic Journal for Mathematical Semiotics, 2009

Parametrische kontexturale Semiotik

1. Unter einer kontexturalen Semiotik verstehe ich eine Semiotik, deren Zeichenklassen sich aus Subzeichen zusammensetzen, die sich in mehr als einer semiotischen Kontextur befinden können. Demnach ist eine kontexturale Zeichenklasse eine solche, die in mehr als 1 Kontextur liegt. Im Falle einer 1-kontexturalen Zeichenklasse liegt daher ein Grenzfall vor, in dem alle Subzeichen in derselben Kontextur, nämlich in K = 1, liegen. Die kontexturale Semiotik basiert auf der folgenden Definition des Zeichens, wobei im folgenden Beispiel von 4 Kontexturen ausgegangen wird:

$ZR = ((3.a)_{i,j,k}, (2.b)_{l,m,n}, (1.c_{o,po,q}))$ mit i, ..., q $\in \{\emptyset, 1, 2, 3, 4\}$ für K = 4

Der Begriff der semiotischen Kontextur bezieht sich dabei auf deren Einführung durch Kaehr (2008). Eine 4-kontexturale Semiotik beruht beispielsweise auf der folgenden semiotischen Matrix:

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

2. Der Begriff der semiotischen Kontextur war jedoch bereits 2000 von mir in die Semiotik eingeführt worden (vgl. ausführlich Toth 2008, S. 57 ff.). In der seinerzeitigen Konzeption wurde darin die Abbildung der hyperbolischen Zeichenfunktionen

$y = x^{-1} \qquad y = -x^{-1}$

auf die komplexe Gaußsche Zahlenebene verstanden. Dadurch enthält Zeichenklassen der Form

$ZR = (\pm 3.\pm a \;\; \pm 2.\pm b \;\; \pm 1.\pm c)$.

Der hiermit verbundene Begriff der semiotischen Kontextur besteht darin, dass die vier Quadranten eines kartesischen Koordinatensystems durch die Parameterbestimmungen (±Subjekt ±Objekt) für jedes Subzeichen einer Zeichenklasse bzw. Realitätsthematik voneinander logisch und erkenntnistheoretisch abgesondert sind, dabei aber eine zyklische Relation bilden. Die einzelnen Quadranten wurden somit als semiotische Kontexturen bestimmt, insofern der Parameterbereich [+S, +O] als semiotische Kontextur, der Parameterbereich [-S, +O] als materialistische

Kontextur, der Parameterbereich [-S, -O] als meontische Kontextur, und der Parameterbereich [+S, -O] als idealistische Kontextur interpretiert werden können.

3. In dieser Arbeit wollen wir uns fragen, ob die beiden Zeichendefinitionen

$ZR = ((3.a)_{i,j,k}, (2.b)_{l,m,n}, (1.c_{o,po,q}))$ mit i, ..., q $\in \{\varnothing, 1, 2, 3, 4\}$ für K = 4

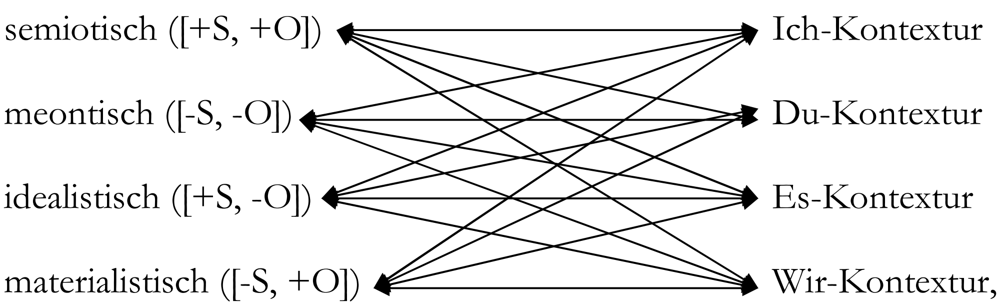
und

$ZR = (\pm 3.\pm a \ \ \pm 2.\pm b \ \ \pm 1.\pm c)$

sinnvoll zu folgender Zeichendefinition

$ZR = ((\pm 3.\pm a)_{i,j,k} \ (\pm 2.\pm b)_{l,m,n} \ (\pm 1.\pm c)_{o,p,q})$ mit i, ..., 1 $\in \{\varnothing, 1, 2, 3\}$

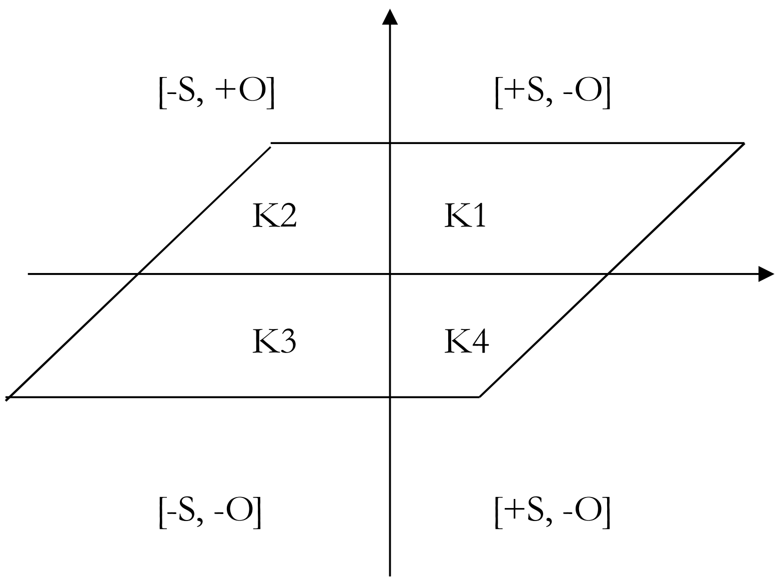
vereinheitlicht werden können. Wir hätten dann eine doppelt kontexturierte Zeichenfunktion, die nicht nur qua Parametrisierung zwischen der “Disjunktion von Welt und Bewusstsein” (Bense 1975, S. 16) vermittelt, sondern auch, nach einem Vorschlag R. Kaehrs (2009, S. 14), zwischen einer Ich-, einer Du-, einer Es- und einer Wir-Kontextur. Damit ergeben sich für ein doppelt-kontexturiertes Zeichen also die folgenden Kombinationen:



d.h.

$[+S, +O]_{ICH}$, $[+S, +O]_{DU}$, $[+S, +O]_{ES}$, $[+S, +O]_{WIR}$
$[-S, -O]_{ICH}$, $[-S, -O]_{DU}$, $[-S, -O]_{ES}$, $[-S, -O]_{WIR}$
$[+S, -O]_{ICH}$, $[+S, -O]_{DU}$, $[+S, -O]_{ES}$, $[+S, -O]_{WIR}$
$[-S, +O]_{ICH}$, $[-S, +O]_{DU}$, $[-S, +O]_{ES}$, $[-S, +O]_{WIR}$

4. Ein Modell einer solchen doppelt-kontexturierten Semiotik könnte wie folgt aussehen:



Jeder Punkt auf der waagrechten Ebene ist mit einem Punkt im kartesischen Koordinatensystem durch ein Paar von Punkten der Form (+x, +y) verbunden. Da wir die Punkte auf den Kontexturen vorderhand einfach durch K1, K2, K3, K4 bestimmen, können diese Punkte also als Indizes zu jedem Subzeichen treten, wobei die Subzeichen dann durch die allgemeine Form

$(a.b)_{i,j,k}$ mit $i, j, k \in \{1, 2, 3, 4\}$

bestimmt sind. Wir haben dann also

$(1.1)_{1,2,3,4}$, $(1.2)_{1,2,3,4}$, $(1.3)_{1,2,3,4}$, ... $(3.3)_{1,2,3,4}$.

Die parametrische kontexturale Semiotik erlaubt damit die Konstruktion der komplexen Semiotik über der Gaußschen Zahlenebene und die kontexturelle Spezifizierung der komplexen Zeichen durch deren Zuweisung zu den epistemologischen Relationen subjektives und objektives Subjekt sowie subjektives und objektives Objekt.

Bibliographie

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979
Kaehr, Rudolf, Diamond Semiotics. http://www.thinkartlab.com/pkl/lola/Diamond%20Semiotics/Diamond%20Semiotics.pdf (2008)
Kaehr, Rudolf, Xanadu's textems. http://www.thinkartlab.com/pkl/lola/Xanadu-textemes/Xanadu-textemes.pdf (2009)
Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2008

Die Substituierbarkeit von Subzeichen durch qualitative semiotische Funktionen

1. Gemäss Toth (2008c, S. 7 ff.) lässt sich eine abstrakte polykontextural-semiotische tetradisch-relationale Repräsentationsklasse, bestehend aus Zeichenklasse und dualer Realitätsematik, wie folgt notieren

PDS = ((((.0.), (.1.)), (.2.)), (.3.)) × (((.3.), ((.2.), ((.1.), (.0.)))).

Während nun eine logische 4-stellige Relation 6 2-stellige, 4 3-stellige und 1 4-stellige Partialrelation enthält (gemäss den Newtonschen Binominalkoeffizienten), enthält eine semiotische 4-stellige Relation die folgenden 4 + 15 + 24 + 24 = 67 qualitativen Partialrelationen:

monadische Partialrelationen: (.0.), (.1.), (.2.), (.3.).

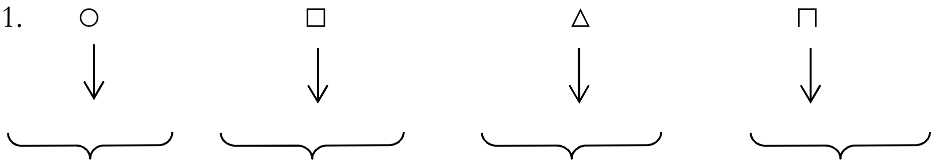
dyadische Partialrelationen: (⊓), (⊔), (⊏), (⊓*), (⊔*), (⊏*), (△), (◭), (▲), (□), (◨), (■), (○), (◑), (●).

triadische Partialrelationen: (0., 2., 1.), (0., 1., 2.), (1., 2., 0.), (1., 0., 2), (2., 1., 0.), (2., 0., 1), (3., 2., 1.), (3., 1., 2.), (2., 3., 1.), (2., 1., 3.), (1., 3., 2.), (1., 2., 3), (0., 3., 2.), (0., 2., 3.), (2., 3., 0.), (2., 0., 3.), (3., 2., 0.), (3., 0., 2.), (0., 3., 1.), (0., 1., 3.), (1., 3., 0.), (1., 0., 3.), (3., 1., 0.), (3., 0., 1.).

tetradische Partialrelationen: (3., 2., 1., 0.), (2., 3., 1., 0.), (2., 1., 3., 0.), (1., 2., 3., 0.), (3., 1., 2., 0.), (1., 3., 2., 0.), (2., 3., 0., 1.), (3., 2., 0., 1.), (2., 1., 0., 3.), (1., 2., 0., 3.), (3., 1., 0., 2.), (1., 3., 0., 2.), (2., 0., 3., 1.), (3., 0., 2., 1.), (2., 0., 1., 3.), (1., 0., 2., 3.), (3., 0., 1., 2.), (1., 0., 3., 2.), (0., 2., 3., 1.), (0., 3., 2., 1.), (0., 1., 2., 3.), (0., 2., 1., 3.), (0., 3., 1., 2.), (0., 1., 3., 2.).

Die drei dyadischen Relationen (⊓*), (⊔*) und (⊏*) treten allerdings ausschliesslich in Realitätsthematiken auf. In einer polykontexturalen Semiotik, in der die Grenze zwischen Zeichen und Objekt aufgehoben ist, sind also sämtliche Partialrelationen miteinander austauschbar. Während dies für die oben aufgeführten monadischen, dyadischen, triadischen und tetradischen Partialrelationen untereinander ohne weiteres einsichtig ist, zeigen wir in der vorliegenden Arbeit die Ersetzung der dyadischen Subzeichen polykontexturaler Zeichenklassen und Realitätsthematiken durch triadische monokontexturale Voll- und triadische polykontexturale qualitative Partialrelationen mit Hilfe der in Toth (2008d) eingeführten semiotischen Funktionen. Durch diese Substitutionen wird eine enorme Menge von semiotischen Verbindungen zwischen Zeichenklassen sichtbar gemacht, die bis anhin unzugänglich blieben (vgl. Toth 2008a, S. 28 ff.) und damit natürlich auch ein Teil jenes unsichtbaren "semiotic web", in das sämtliche kommunikativen, kreativen und repräsentativen Prozesse eingebunden sind.

1.



(○) = f(⊓, △, □)
(○) = f(⊓, □, △)
(○) = f(⊔, △, □)
(○) = f(⊔, ◭, □)
(○) = f(⊔, ◭, ◨)
(○) = f(⊔, □, △)
(○) = f(⊔, □, ◭)
(○) = f(⊔, ◨, ◭)
(○) = f(⊏, △, □)
(○) = f(⊏, ◭, □)
(○) = f(⊏, ◭, ◨)
(○) = f(⊏, ▲, □)
(○) = f(⊏, ▲, ◨)
(○) = f(⊏, ▲, ■)
(○) = f(⊏, □, △)
(○) = f(⊏, □, ◭)
(○) = f(⊏, □, ▲)
(○) = f(⊏, ◨, ◭)
(○) = f(⊏, ◨, ▲)
(○) = f(⊏, ■, ▲)
(○) = f(△, ⊓, □)
(○) = f(△, ⊔, □)
(○) = f(△, ⊏, □)
(○) = f(△, □, ⊓)
(○) = f(△, □, ⊔)
(○) = f(△, □, ⊏)
(○) = f(◭, ⊔, □)
(○) = f(◭, ⊔, ◨)
(○) = f(◭, ⊏, □)
(○) = f(◭, ⊏, ◨)
(○) = f(◭, ▲, ⊏*)
(○) = f(◭, □, ⊔)
(○) = f(◭, □, ⊏)
(○) = f(◭, ◨, ⊔)
(○) = f(◭, ◨, ⊏)
(○) = f(◭, ⊏*, ▲)
(○) = f(▲, ⊏, □)
(○) = f(▲, ⊏, ◨)
(○) = f(▲, ⊏, ■)
(○) = f(▲, ◭, ⊏*)
(○) = f(▲, □, ⊏)
(○) = f(▲, ◨, ⊏)
(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)

(□) = f(⊓, △, ○)
(□) = f(⊔, △, ○)
(□) = f(⊔, ◭, ○)
(□) = f(⊔, ○, △)
(□) = f(⊔, ○, ◭)
(□) = f(⊏, △, ○)
(□) = f(⊏, ◭, ○)
(□) = f(⊏, ▲, ○)
(□) = f(⊏, ○, △)
(□) = f(⊏, ○, ◭)
(□) = f(⊏, ○, ▲)
(□) = f(△, ⊓, ○)
(□) = f(△, ⊔, ○)
(□) = f(△, ⊏, ○)
(□) = f(△, ○, ⊓)
(□) = f(△, ○, ⊔)
(□) = f(△, ○, ⊏)
(□) = f(◭, ⊔, ○)
(□) = f(◭, ⊏, ○)
(□) = f(◭, ▲, ⊏*)
(□) = f(◭, ▲, ⊔*)
(□) = f(◭, ⊔*, ▲)
(□) = f(◭, ⊏*, ▲)
(□) = f(◭, ○, ⊔)
(□) = f(◭, ○, ⊏)
(□) = f(▲, ⊏, ○)
(□) = f(▲, ◭, ⊔*)
(□) = f(▲, ◭, ⊏*)
(□) = f(▲, ⊔*, ◭)
(□) = f(▲, ⊔*, ◨)
(□) = f(▲, ◨, ⊔*)
(□) = f(▲, ◨, ⊏*)
(□) = f(▲, ⊏*, ◭)
(□) = f(▲, ⊏*, ◨)
(□) = f(▲, ○, ⊏)
(□) = f(⊔*, ◭, ▲)
(□) = f(⊔*, ▲, ◭)
(□) = f(⊔*, ▲, ◨)
(□) = f(⊔*, ◨, ▲)
(□) = f(⊔*, ◨, ■)
(□) = f(⊔*, ■, ◨)
(□) = f(◨, ▲, ⊔*)
(□) = f(◨, ▲, ⊏*)
(□) = f(◨, ⊔*, ▲)
(□) = f(◨, ⊔*, ■)
(□) = f(◨, ■, ⊔*)

(△) = f(⊓, □, ○)
(△) = f(⊓, ○, □)
(△) = f(⊔, □, ○)
(△) = f(⊔, ○, □)
(△) = f(⊏, □, ○)
(△) = f(⊏, ○, □)
(△) = f(⊓*, ◭, ▲)
(△) = f(⊓*, ▲, ◭)
(△) = f(◭, ⊓*, ▲)
(△) = f(◭, ▲, ⊓*)
(△) = f(◭, ▲, ⊔*)
(△) = f(◭, ▲, ⊏*)
(△) = f(◭, ⊔*, ▲)
(△) = f(◭, ⊏*, ▲)
(△) = f(▲, ⊓*, ◭)
(△) = f(▲, ◭, ⊓*)
(△) = f(▲, ◭, ⊔*)
(△) = f(▲, ◭, ⊏*)
(△) = f(▲, ⊔*, ◭)
(△) = f(▲, ⊏*, ◭)
(△) = f(⊔*, ◭, ▲)
(△) = f(⊔*, ▲, ◭)
(△) = f(□, ⊓, ○)
(△) = f(□, ⊔, ○)
(△) = f(□, ⊏, ○)
(△) = f(□, ○, ⊓)
(△) = f(□, ○, ⊔)
(△) = f(□, ○, ⊏)
(△) = f(⊏*, ◭, ▲)
(△) = f(⊏*, ▲, ◭)
(△) = f(○, ⊓, □)
(△) = f(○, ⊔, □)
(△) = f(○, ⊏, □)
(△) = f(○, □, ⊓)
(△) = f(○, □, ⊔)
(△) = f(○, □, ⊏)

(⊓) = f(△, □, ○)
(⊓) = f(△, ○, □)
(⊓) = f(□, △, ○)
(⊓) = f(□, ○, △)
(⊓) = f(○, △, □)
(⊓) = f(○, □, △)

(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(○) = f(□, ⊏, △)
(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)
(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)
(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◭, ⊔)
(○) = f(◨, ◭, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)

(□) = f(◨, ■, ⊏*)
(□) = f(◨, ⊏*, ▲)
(□) = f(◨, ⊏*, ■)
(□) = f(■, ⊔*, ◨)
(□) = f(■, ◨, ⊔*)
(□) = f(■, ◨, ⊏*)
(□) = f(■, ⊏*, ◨)
(□) = f(⊏*, ◭, ▲)
(□) = f(⊏*, ▲, ◭)
(□) = f(⊏*, ▲, ◨)
(□) = f(⊏*, ◨, ▲)
(□) = f(⊏*, ◨, ■)
(□) = f(⊏*, ■, ◨)
(□) = f(○, ⊓, △)
(□) = f(○, ⊔, △)
(□) = f(○, ⊔, ◭)
(□) = f(○, ⊏, △)
(□) = f(○, ⊏, ◭)
(□) = f(○, ⊏, ▲)
(□) = f(○, △, ⊓)
(□) = f(○, △, ⊔)
(□) = f(○, △, ⊏)
(□) = f(○, ◭, ⊔)
(□) = f(○, ◭, ⊏)
□) = f(○, ▲, ⊏)

(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

2.

○	□	△	⊔
(○) = f(⊓, △, □)	(□) = f(⊓, △, ○)	(△) = f(⊓, □, ○)	(⊔) = f(△, □, ○)
(○) = f(⊓, □, △)	(□) = f(⊔, △, ○)	(△) = f(⊓, ○, □)	(⊔) = f(△, ○, □)
(○) = f(⊔, △, □)	(□) = f(⊔, ◭, ○)	(△) = f(⊔, □, ○)	(⊔) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(□) = f(⊔, ○, △)	(△) = f(⊔, ○, □)	(⊔) = f(◭, ◨, ○)
(○) = f(⊔, ◭, ◨)	(□) = f(⊔, ○, ◭)	(△) = f(⊏, □, ○)	(⊔) = f(◭, ◨, ◑)
(○) = f(⊔, □, △)	(□) = f(⊏, △, ○)	(△) = f(⊏, ○, □)	(⊔) = f(◭, ○, □)
(○) = f(⊔, □, ◭)	(□) = f(⊏, ◭, ○)	(△) = f(⊓*, ◭, ▲)	(⊔) = f(◭, ○, ◨)
(○) = f(⊔, ◨, ◭)	(□) = f(⊏, ▲, ○)	(△) = f(⊓*, ▲, ◭)	(⊔) = f(◭, ◑, ◨)
(○) = f(⊏, △, □)	(□) = f(⊏, ○, △)	(△) = f(◭, ⊓*, ▲)	(⊔) = f(□, △, ○)
(○) = f(⊏, ◭, □)	(□) = f(⊏, ○, ◭)	(△) = f(◭, ▲, ⊓*)	(⊔) = f(□, ◭, ○)
(○) = f(⊏, ◭, ◨)	(□) = f(⊏, ○, ▲)	(△) = f(◭, ▲, ⊔*)	(⊔) = f(□, ○, △)
(○) = f(⊏, ▲, □)	(□) = f(△, ⊓, ○)	(△) = f(◭, ▲, ⊏*)	(⊔) = f(□, ○, ◭)
(○) = f(⊏, ▲, ◨)	(□) = f(△, ⊔, ○)	(△) = f(◭, ⊔*, ▲)	(⊔) = f(◨, ◭, ○)
(○) = f(⊏, ▲, ■)	(□) = f(△, ⊏, ○)	(△) = f(◭, ⊏*, ▲)	(⊔) = f(◨, ◭, ◑)
(○) = f(⊏, □, △)	(□) = f(△, ○, ⊓)	(△) = f(▲, ⊓*, ◭)	(⊔) = f(◨, ○, ◭)
(○) = f(⊏, □, ◭)	(□) = f(△, ○, ⊔)	(△) = f(▲, ◭, ⊓*)	(⊔) = f(◨, ◑, ◭)
(○) = f(⊏, □, ▲)	(□) = f(△, ○, ⊏)	(△) = f(▲, ◭, ⊔*)	(⊔) = f(○, △, □)
(○) = f(⊏, ◨, ◭)	(□) = f(◭, ⊔, ○)	(△) = f(▲, ◭, ⊏*)	(⊔) = f(○, ◭, □)
(○) = f(⊏, ◨, ▲)	(□) = f(◭, ⊏, ○)	(△) = f(▲, ⊔*, ◭)	(⊔) = f(○, ◭, ◨)
(○) = f(⊏, ■, ▲)	(□) = f(◭, ▲, ⊏*)	(△) = f(▲, ⊏*, ◭)	(⊔) = f(○, □, △)
(○) = f(△, ⊓, □)	(□) = f(◭, ▲, ⊔*)	(△) = f(⊔*, ◭, ▲)	(⊔) = f(○, □, ◭)
(○) = f(△, ⊔, □)	(□) = f(◭, ⊔*, ▲)	(△) = f(⊔*, ▲, ◭)	(⊔) = f(○, ◨, ◭)
(○) = f(△, ⊏, □)	(□) = f(◭, ⊏*, ▲)	(△) = f(□, ⊓, ○)	(⊔) = f(◑, ◭, ◨)
(○) = f(△, □, ⊓)	(□) = f(◭, ○, ⊔)	(△) = f(□, ⊔, ○)	(⊔) = f(◑, ◨, ◭)
(○) = f(△, □, ⊔)	(□) = f(◭, ○, ⊏)	(△) = f(□, ⊏, ○)	
(○) = f(△, □, ⊏)	(□) = f(▲, ⊏, ○)	(△) = f(□, ○, ⊓)	
(○) = f(◭, ⊔, □)	(□) = f(▲, ◭, ⊔*)	(△) = f(□, ○, ⊔)	
(○) = f(◭, ⊔, ◨)	(□) = f(▲, ◭, ⊏*)	(△) = f(□, ○, ⊏)	
(○) = f(◭, ⊏, □)	(□) = f(▲, ⊔*, ◭)	(△) = f(⊏*, ◭, ▲)	
(○) = f(◭, ⊏, ◨)	(□) = f(▲, ⊔*, ◨)	(△) = f(⊏*, ▲, ◭)	
(○) = f(◭, ▲, ⊏*)	(□) = f(▲, ◨, ⊔*)	(△) = f(○, ⊓, □)	
(○) = f(◭, □, ⊔)	(□) = f(▲, ◨, ⊏*)	(△) = f(○, ⊔, □)	
(○) = f(◭, □, ⊏)	(□) = f(▲, ⊏*, ◭)	(△) = f(○, ⊏, □)	
(○) = f(◭, ◨, ⊔)	(□) = f(▲, ⊏*, ◨)	(△) = f(○, □, ⊓)	
(○) = f(◭, ◨, ⊏)	(□) = f(▲, ○, ⊏)	(△) = f(○, □, ⊔)	
(○) = f(◭, ⊏*, ▲)	(□) = f(⊔*, ◭, ▲)	(△) = f(○, □, ⊏)	

(○) = f(▲, ⊏, □)

(○) = f(▲, ⊏, ◨)

(○) = f(▲, ⊏, ■)

(○) = f(▲, ◭, ⊏*)

(○) = f(▲, □, ⊏)

(○) = f(▲, ◨, ⊏)

(○) = f(▲, ◨, ⊏*)

(○) = f(▲, ■, ⊏)

(○) = f(▲, ⊏*, ◭)

(○) = f(▲, ⊏*, ◨)

(○) = f(▲, ⊏*, ◑)

(○) = f(▲, ◑, ⊏*)

(○) = f(□, ⊓, △)

(○) = f(□, ⊔, △)

(○) = f(□, ⊔, ◭)

(○) = f(□, ⊏, △)

(○) = f(□, ⊏, ◭)

(○) = f(□, ⊏, ▲)

(○) = f(□, △, ⊓)

(○) = f(□, △, ⊔)

(○) = f(□, △, ⊏)

(○) = f(□, ◭, ⊔)

(○) = f(□, ◭, ⊏)

(○) = f(□, ▲, ⊏)

(○) = f(◨, ⊔, ◭)

(○) = f(◨, ⊏, ◭)

(○) = f(◨, ⊏, ▲)

(○) = f(◨, ◭, ⊔)

(○) = f(◨, ◭, ⊏)

(○) = f(◨, ▲, ⊏)

(○) = f(◨, ▲, ⊏*)

(○) = f(◨, ■, ⊏*)

(○) = f(◨, ⊏*, ▲)

(○) = f(◨, ⊏*, ■)

(○) = f(■, ⊏, ▲)

(○) = f(■, ▲, ⊏)

(○) = f(■, ◨, ⊏*)

(○) = f(■, ⊏*, ◨)

(○) = f(■, ⊏*, ◑)

(○) = f(■, ◑, ⊏*)

(○) = f(⊏*, ◭, ▲)

(○) = f(⊏*, ▲, ◭)

(○) = f(⊏*, ▲, ◨)

(○) = f(⊏*, ▲, ◑)

(○) = f(⊏*, ◨, ▲)

(○) = f(⊏*, ◨, ■)

(□) = f(⊔*, ▲, ◭)

(□) = f(⊔*, ▲, ◨)

(□) = f(⊔*, ◨, ▲)

(□) = f(⊔*, ◨, ■)

(□) = f(⊔*, ■, ◨)

(□) = f(◨, ▲, ⊔*)

(□) = f(◨, ▲, ⊏*)

(□) = f(◨, ⊔*, ▲)

(□) = f(◨, ⊔*, ■)

(□) = f(◨, ■, ⊔*)

(□) = f(◨, ■, ⊏*)

(□) = f(◨, ⊏*, ▲)

(□) = f(◨, ⊏*, ■)

(□) = f(■, ⊔*, ◨)

(□) = f(■, ◨, ⊔*)

(□) = f(■, ◨, ⊏*)

(□) = f(■, ⊏*, ◨)

(□) = f(⊏*, ◭, ▲)

(□) = f(⊏*, ▲, ◭)

(□) = f(⊏*, ▲, ◨)

(□) = f(⊏*, ◨, ▲)

(□) = f(⊏*, ◨, ■)

(□) = f(⊏*, ■, ◨)

(□) = f(○, ⊓, △)

(□) = f(○, ⊔, △)

(□) = f(○, ⊔, ◭)

(□) = f(○, ⊏, △)

(□) = f(○, ⊏, ◭)

(□) = f(○, ⊏, ▲)

(□) = f(○, △, ⊓)

(□) = f(○, △, ⊔)

(□) = f(○, △, ⊏)

(□) = f(○, ◭, ⊔)

(□) = f(○, ◭, ⊏)

(□) = f(○, ▲, ⊏)

(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

3. ○ □ △ ⊏

○ ↓	□ ↓	△ ↓	⊏ ↓
(○) = f(⊓, △, □)	(□) = f(⊓, △, ○)	(△) = f(⊓, □, ○)	(⊏) = f(△, □, ○)
(○) = f(⊓, □, △)	(□) = f(⊔, △, ○)	(△) = f(⊓, ○, □)	(⊏) = f(△, ○, □)
(○) = f(⊔, △, □)	(□) = f(⊔, ◭, ○)	(△) = f(⊔, □, ○)	(⊏) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(□) = f(⊔, ○, △)	(△) = f(⊔, ○, □)	(⊏) = f(◭, ◨, ○)
(○) = f(⊔, ◭, ◨)	(□) = f(⊔, ○, ◭)	(△) = f(⊏, □, ○)	(⊏) = f(◭, ◨, ◑)
(○) = f(⊔, □, △)	(□) = f(⊏, △, ○)	(△) = f(⊏, ○, □)	(⊏) = f(◭, ○, □)
(○) = f(⊔, □, ◭)	(□) = f(⊏, ◭, ○)	(△) = f(⊓*, ◭, ▲)	(⊏) = f(◭, ○, ◨)
(○) = f(⊔, ◨, ◭)	(□) = f(⊏, ▲, ○)	(△) = f(⊓*, ▲, ◭)	(⊏) = f(◭, ◑, ◨)
(○) = f(⊏, △, □)	(□) = f(⊏, ○, △)	(△) = f(◭, ⊓*, ▲)	(⊏) = f(▲, □, ○)
(○) = f(⊏, ◭, □)	(□) = f(⊏, ○, ◭)	(△) = f(◭, ▲, ⊓*)	(⊏) = f(▲, ◨, ○)
(○) = f(⊏, ◭, ◨)	(□) = f(⊏, ○, ▲)	(△) = f(◭, ▲, ⊔*)	(⊏) = f(▲, ◨, ◑)
(○) = f(⊏, ▲, □)	(□) = f(△, ⊓, ○)	(△) = f(◭, ▲, ⊏*)	(⊏) = f(▲, ■, ○)
(○) = f(⊏, ▲, ◨)	(□) = f(△, ⊔, ○)	(△) = f(◭, ⊔*, ▲)	(⊏) = f(▲, ■, ◑)
(○) = f(⊏, ▲, ■)	(□) = f(△, ⊏, ○)	(△) = f(◭, ⊏*, ▲)	(⊏) = f(▲, ■, ●)
(○) = f(⊏, □, △)	(□) = f(△, ○, ⊓)	(△) = f(▲, ⊓*, ◭)	(⊏) = f(▲, ○, □)
(○) = f(⊏, □, ◭)	(□) = f(△, ○, ⊔)	(△) = f(▲, ◭, ⊓*)	(⊏) = f(▲, ○, ◨)
(○) = f(⊏, □, ▲)	(□) = f(△, ○, ⊏)	(△) = f(▲, ◭, ⊔*)	(⊏) = f(▲, ○, ■)
(○) = f(⊏, ◨, ◭)	(□) = f(◭, ⊔, ○)	(△) = f(▲, ◭, ⊏*)	(⊏) = f(▲, ◑, ◨)
(○) = f(⊏, ◨, ▲)	(□) = f(◭, ⊏, ○)	(△) = f(▲, ⊔*, ◭)	(⊏) = f(▲, ◑, ■)
(○) = f(⊏, ■, ▲)	(□) = f(◭, ▲, ⊏*)	(△) = f(▲, ⊏*, ◭)	(⊏) = f(▲, ●, ■)
(○) = f(△, ⊓, □)	(□) = f(◭, ▲, ⊔*)	(△) = f(⊔*, ◭, ▲)	(⊏) = f(□, △, ○)
(○) = f(△, ⊔, □)	(□) = f(◭, ⊔*, ▲)	(△) = f(⊔*, ▲, ◭)	(⊏) = f(□, ◭, ○)
(○) = f(△, ⊏, □)	(□) = f(◭, ⊏*, ▲)	(△) = f(□, ⊓, ○)	(⊏) = f(□, ▲, ○)
(○) = f(△, □, ⊓)	(□) = f(◭, ○, ⊔)	(△) = f(□, ⊔, ○)	(⊏) = f(□, ○, △)
(○) = f(△, □, ⊔)	(□) = f(◭, ○, ⊏)	(△) = f(□, ⊏, ○)	(⊏) = f(□, ○, ◭)
(○) = f(△, □, ⊏)	(□) = f(▲, ⊏, ○)	(△) = f(□, ○, ⊓)	(⊏) = f(□, ○, ▲)

(○) = f(◭, ⊔, □)
(○) = f(◭, ⊔, ◨)
(○) = f(◭, ⊏, □)
(○) = f(◭, ⊏, ◨)
(○) = f(◭, ▲, ⊏*)
(○) = f(◭, □, ⊔)
(○) = f(◭, □, ⊏)
(○) = f(◭, ◨, ⊔)
(○) = f(◭, ◨, ⊏)
(○) = f(◭, ⊏*, ▲)
(○) = f(▲, ⊏, □)
(○) = f(▲, ⊏, ◨)
(○) = f(▲, ⊏, ■)
(○) = f(▲, ◭, ⊏*)
(○) = f(▲, □, ⊏)
(○) = f(▲, ◨, ⊏)
(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)
(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(○) = f(□, ⊏, △)
(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)
(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)
(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◭, ⊔)
(○) = f(◨, ◭, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)

(□) = f(▲, ◭, ⊔*)
(□) = f(▲, ◭, ⊏*)
(□) = f(▲, ⊔*, ◭)
(□) = f(▲, ⊔*, ◨)
(□) = f(▲, ◨, ⊔*)
(□) = f(▲, ◨, ⊏*)
(□) = f(▲, ⊏*, ◭)
(□) = f(▲, ⊏*, ◨)
(□) = f(▲, ○, ⊏)
(□) = f(⊔*, ◭, ▲)
(□) = f(⊔*, ▲, ◭)
(□) = f(⊔*, ▲, ◨)
(□) = f(⊔*, ◨, ▲)
(□) = f(⊔*, ◨, ■)
(□) = f(⊔*, ■, ◨)
(□) = f(◨, ▲, ⊔*)
(□) = f(◨, ▲, ⊏*)
(□) = f(◨, ⊔*, ▲)
(□) = f(◨, ⊔*, ■)
(□) = f(◨, ■, ⊔*)
(□) = f(◨, ■, ⊏*)
(□) = f(◨, ⊏*, ▲)
(□) = f(◨, ⊏*, ■)
(□) = f(■, ⊔*, ◨)
(□) = f(■, ◨, ⊔*)
(□) = f(■, ◨, ⊏*)
(□) = f(■, ⊏*, ◨)
(□) = f(⊏*, ◭, ▲)
(□) = f(⊏*, ▲, ◭)
(□) = f(⊏*, ▲, ◨)
(□) = f(⊏*, ◨, ▲)
(□) = f(⊏*, ◨, ■)
(□) = f(⊏*, ■, ◨)
(□) = f(○, ⊓, △)
(□) = f(○, ⊔, △)
(□) = f(○, ⊔, ◭)
(□) = f(○, ⊏, △)
(□) = f(○, ⊏, ◭)
(□) = f(○, ⊏, ▲)
(□) = f(○, △, ⊓)
(□) = f(○, △, ⊔)
(□) = f(○, △, ⊏)
(□) = f(○, ◭, ⊔)
(□) = f(○, ◭, ⊏)
(□) = f(○, ▲, ⊏)

(△) = f(□, ○, ⊔)
(△) = f(□, ○, ⊏)
(△) = f(⊏*, ◭, ▲)
(△) = f(⊏*, ▲, ◭)
(△) = f(○, ⊓, □)
(△) = f(○, ⊔, □)
(△) = f(○, ⊏, □)
(△) = f(○, □, ⊓)
(△) = f(○, □, ⊔)
(△) = f(○, □, ⊏)

(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ○, ◑)
(⊏) = f(■, ○, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)
(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)
(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)
(⊏) = f(◑, ◨, ▲)
(⊏) = f(◑, ■, ▲)
(⊏) = f(●, ▲, ■)
(⊏) = f(●, ■, ▲)

(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

4.

○	□	◭	⊔
↓	↓	↓	↓
(○) = f(⊓, △, □)	(□) = f(⊓, △, ○)	(◭) = f(⊔, □, ○)	(⊔) = f(△, □, ○)
(○) = f(⊓, □, △)	(□) = f(⊔, △, ○)	(◭) = f(⊔, ◨, ○)	(⊔) = f(△, ○, □)
(○) = f(⊔, △, □)	(□) = f(⊔, ◭, ○)	(◭) = f(⊔, ◨, ◑)	(⊔) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(□) = f(⊔, ○, △)	(◭) = f(⊔, ○, □)	(⊔) = f(◭, ◨, ○)
(○) = f(⊔, ◭, ◨)	(□) = f(⊔, ○, ◭)	(◭) = f(⊔, ○, ◨)	(⊔) = f(◭, ◨, ◑)
(○) = f(⊔, □, △)	(□) = f(⊏, △, ○)	(◭) = f(⊔, ◑, ◨)	(⊔) = f(◭, ○, □)
(○) = f(⊔, □, ◭)	(□) = f(⊏, ◭, ○)	(◭) = f(⊏, □, ○)	(⊔) = f(◭, ○, ◨)
(○) = f(⊔, ◨, ◭)	(□) = f(⊏, ▲, ○)	(◭) = f(⊏, ◨, ○)	(⊔) = f(◭, ◑, ◨)
(○) = f(⊏, △, □)	(□) = f(⊏, ○, △)	(◭) = f(⊏, ◨, ◑)	(⊔) = f(□, △, ○)
(○) = f(⊏, ◭, □)	(□) = f(⊏, ○, ◭)	(◭) = f(⊏, ○, □)	(⊔) = f(□, ◭, ○)
(○) = f(⊏, ◭, ◨)	(□) = f(⊏, ○, ▲)	(◭) = f(⊏, ○, ◨)	(⊔) = f(□, ○, △)
(○) = f(⊏, ▲, □)	(□) = f(△, ⊓, ○)	(◭) = f(⊏, ◑, ◨)	(⊔) = f(□, ○, ◭)
(○) = f(⊏, ▲, ◨)	(□) = f(△, ⊔, ○)	(◭) = f(⊓*, △, ▲)	(⊔) = f(◨, ◭, ○)
(○) = f(⊏, ▲, ■)	(□) = f(△, ⊏, ○)	(◭) = f(⊓*, ▲, △)	(⊔) = f(◨, ◭, ◑)
(○) = f(⊏, □, △)	(□) = f(△, ○, ⊓)	(◭) = f(△, ⊓*, ▲)	(⊔) = f(◨, ○, ◭)
(○) = f(⊏, □, ◭)	(□) = f(△, ○, ⊔)	(◭) = f(△, ▲, ⊓*)	(⊔) = f(2-2, ◑, ◭)

(○) = f(⊏, □, ▲)
(○) = f(⊏, ◨, ◭)
(○) = f(⊏, ◨, ▲)
(○) = f(⊏, ■, ▲)
(○) = f(△, ⊓, □)
(○) = f(△, ⊔, □)
(○) = f(△, ⊏, □)
(○) = f(△, □, ⊓)
(○) = f(△, □, ⊔)
(○) = f(△, □, ⊏)
(○) = f(◭, ⊔, □)
(○) = f(◭, ⊔, ◨)
(○) = f(◭, ⊏, □)
(○) = f(◭, ⊏, ◨)
(○) = f(◭, ▲, ⊏*)
(○) = f(◭, □, ⊔)
(○) = f(◭, □, ⊏)
(○) = f(◭, ◨, ⊔)
(○) = f(◭, ◨, ⊏)
(○) = f(◭, ⊏*, ▲)
(○) = f(▲, ⊏, □)
(○) = f(▲, ⊏, ◨)
(○) = f(▲, ⊏, ■)
(○) = f(▲, ◭, ⊏*)
(○) = f(▲, □, ⊏)
(○) = f(▲, ◨, ⊏)
(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)
(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(○) = f(□, ⊏, △)
(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)
(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)

(□) = f(△, ○, ⊏)
(□) = f(◭, ⊔, ○)
(□) = f(◭, ⊏, ○)
(□) = f(◭, ▲, ⊏*)
(□) = f(◭, ▲, ⊔*)
(□) = f(◭, ⊔*, ▲)
(□) = f(◭, ⊏*, ▲)
(□) = f(◭, ○, ⊔)
(□) = f(◭, ○, ⊏)
(□) = f(▲, ⊏, ○)
(□) = f(▲, ◭, ⊔*)
(□) = f(▲, ◭, ⊏*)
(□) = f(▲, ⊔*, ◭)
(□) = f(▲, ⊔*, ◨)
(□) = f(▲, ◨, ⊔*)
(□) = f(▲, ◨, ⊏*)
(□) = f(▲, ⊏*, ◭)
(□) = f(▲, ⊏*, ◨)
(□) = f(▲, ○, ⊏)
(□) = f(⊔*, ◭, ▲)
(□) = f(⊔*, ▲, ◭)
(□) = f(⊔*, ▲, ◨)
(□) = f(⊔*, ◨, ▲)
(□) = f(⊔*, ◨, ■)
(□) = f(⊔*, ■, ◨)
(□) = f(◨, ▲, ⊔*)
(□) = f(◨, ▲, ⊏*)
(□) = f(◨, ⊔*, ▲)
(□) = f(◨, ⊔*, ■)
(□) = f(◨, ■, ⊔*)
(□) = f(◨, ■, ⊏*)
(□) = f(◨, ⊏*, ▲)
(□) = f(◨, ⊏*, ■)
(□) = f(■, ⊔*, ◨)
(□) = f(■, ◨, ⊔*)
(□) = f(■, ◨, ⊏*)
(□) = f(■, ⊏*, ◨)
(□) = f(⊏*, ◭, ▲)
(□) = f(⊏*, ▲, ◭)
(□) = f(⊏*, ▲, ◨)
(□) = f(⊏*, ◨, ▲)
(□) = f(⊏*, ◨, ■)
(□) = f(⊏*, ■, ◨)
(□) = f(○, ⊓, △)
(□) = f(○, ⊔, △)
(□) = f(○, ⊔, ◭)

(◭) = f(△, ▲, ⊔*)
(◭) = f(△, ▲, ⊏*)
(◭) = f(△, ⊔*, ▲)
(◭) = f(△, ⊏*, ▲)
(◭) = f(▲, ⊓*, △)
(◭) = f(▲, △, ⊓*)
(◭) = f(▲, △, ⊔*)
(◭) = f(▲, △, ⊏*)
(◭) = f(▲, ⊔*, △)
(◭) = f(▲, □, ⊔*)
(◭) = f(▲, ⊏*, △)
(◭) = f(▲, ⊏*, □)
(◭) = f(▲, ⊏*, ○)
(◭) = f(▲, ○, ⊏*)
(◭) = f(⊔*, ▲, □)
(◭) = f(⊔*, ▲, △)
(◭) = f(⊔*, □, ▲)
(◭) = f(□, ⊔, ○)
(◭) = f(□, ⊏, ○)
(◭) = f(□, ▲, ⊔*)
(◭) = f(□, ▲, ⊏*)
(◭) = f(□, ⊔*, ▲)
(◭) = f(□, ⊏*, ▲)
(◭) = f(□, ○, ⊔)
(◭) = f(□, ○, ⊏)
(◭) = f(◨, ⊔, ○)
(◭) = f(◨, ⊔, ◑)
(◭) = f(◨, ⊏, ○)
(◭) = f(◨, ⊏, ◑)
(◭) = f(◨, ○, ⊔)
(◭) = f(◨, ○, ⊏)
(◭) = f(◨, ◑, ⊔)
(◭) = f(◨, ◑, ⊏)
(◭) = f(⊏*, △, ▲)
(◭) = f(⊏*, ▲, △)
(◭) = f(⊏*, ▲, □)
(◭) = f(⊏*, ▲, ○)
(◭) = f(⊏*, □, ▲)
(◭) = f(⊏*, ○, ▲)
(◭) = f(○, ⊔, □)
(◭) = f(○, ⊔, ◨)
(◭) = f(○, ⊏, □)
(◭) = f(○, ⊏, ◨)
(◭) = f(○, ▲, ⊏*)
(◭) = f(○, □, ⊔)
(◭) = f(○, □, ⊏)

(⊔) = f(○, △, □)
(⊔) = f(○, ◭, □)
(⊔) = f(○, ◭, ◨)
(⊔) = f(○, □, △)
(⊔) = f(○, □, ◭)
(⊔) = f(○, ◨, ◭)
(⊔) = f(◑, ◭, ◨)
(⊔) = f(◑, ◨, ◭)

(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◮, ⊔)
(○) = f(◨, ◮, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◮, ▲)
(○) = f(⊏*, ▲, ◮)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

(□) = f(○, ⊏, △)
(□) = f(○, ⊏, ◮)
(□) = f(○, ⊏, ▲)
(□) = f(○, △, ⊓)
(□) = f(○, △, ⊔)
(□) = f(○, △, ⊏)
(□) = f(○, ◮, ⊔)
(□) = f(○, ◮, ⊏)
(□) = f(○, ▲, ⊏)

(◮) = f(○, ◨, ⊔)
(◮) = f(○, ◨, ⊏)
(◮) = f(○, ⊏*, ▲)
(◮) = f(◑, ⊔, ◨)
(◮) = f(◑, ⊏, ◨)
(◮) = f(◑, ◨, ⊔)
(◮) = f(◑, ◨, ⊏)

5. ○ → (○) = f(⊓, △, □)
(○) = f(⊓, □, △)
(○) = f(⊔, △, □)
(○) = f(⊔, ◮, □)
(○) = f(⊔, ◮, ◨)
(○) = f(⊔, □, △)

□ → (□) = f(⊓, △, ○)
(□) = f(⊔, △, ○)
(□) = f(⊔, ◮, ○)
(□) = f(⊔, ○, △)
(□) = f(⊔, ○, ◮)
(□) = f(⊏, △, ○)

◮ → (◮) = f(⊔, □, ○)
(◮) = f(⊔, ◨, ○)
(◮) = f(⊔, ◨, ◑)
(◮) = f(⊔, ○, □)
(◮) = f(⊔, ○, ◨)
(◮) = f(⊔, ◑, ◨)

⊏ → (⊏) = f(△, □, ○)
(⊏) = f(△, ○, □)
(⊏) = f(◮, □, ○)
(⊏) = f(◮, ◨, ○)
(⊏) = f(◮, ◨, ◑)
(⊏) = f(◮, ○, □)

(○) = f(⊔, □, ◭)
(○) = f(⊔, ◨, ◭)
(○) = f(⊏, △, □)
(○) = f(⊏, ◭, □)
(○) = f(⊏, ◭, ◨)
(○) = f(⊏, ▲, □)
(○) = f(⊏, ▲, ◨)
(○) = f(⊏, ▲, ■)
(○) = f(⊏, □, △)
(○) = f(⊏, □, ◭)
(○) = f(⊏, □, ▲)
(○) = f(⊏, ◨, ◭)
(○) = f(⊏, ◨, ▲)
(○) = f(⊏, ■, ▲)
(○) = f(△, ⊓, □)
(○) = f(△, ⊔, □)
(○) = f(△, ⊏, □)
(○) = f(△, □, ⊓)
(○) = f(△, □, ⊔)
(○) = f(△, □, ⊏)
(○) = f(◭, ⊔, □)
(○) = f(◭, ⊔, ◨)
(○) = f(◭, ⊏, □)
(○) = f(◭, ⊏, ◨)
(○) = f(◭, ▲, ⊏*)
(○) = f(◭, □, ⊔)
(○) = f(◭, □, ⊏)
(○) = f(◭, ◨, ⊔)
(○) = f(◭, ◨, ⊏)
(○) = f(◭, ⊏*, ▲)
(○) = f(▲, ⊏, □)
(○) = f(▲, ⊏, ◨)
(○) = f(▲, ⊏, ■)
(○) = f(▲, ◭, ⊏*)
(○) = f(▲, □, ⊏)
(○) = f(▲, ◨, ⊏)
(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)
(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(○) = f(□, ⊏, △)

(□) = f(⊏, ◭, ○)
(□) = f(⊏, ▲, ○)
(□) = f(⊏, ○, △)
(□) = f(⊏, ○, ◭)
(□) = f(⊏, ○, ▲)
(□) = f(△, ⊓, ○)
(□) = f(△, ⊔, ○)
(□) = f(△, ⊏, ○)
(□) = f(△, ○, ⊓)
(□) = f(△, ○, ⊔)
(□) = f(△, ○, ⊏)
(□) = f(◭, ⊔, ○)
(□) = f(◭, ⊏, ○)
(□) = f(◭, ▲, ⊏*)
(□) = f(◭, ▲, ⊔*)
(□) = f(◭, ⊔*, ▲)
(□) = f(◭, ⊏*, ▲)
(□) = f(◭, ○, ⊔)
(□) = f(◭, ○, ⊏)
(□) = f(▲, ⊏, ○)
(□) = f(▲, ◭, ⊔*)
(□) = f(▲, ◭, ⊏*)
(□) = f(▲, ⊔*, ◭)
(□) = f(▲, ⊔*, ◨)
(□) = f(▲, ◨, ⊔*)
(□) = f(▲, ◨, ⊏*)
(□) = f(▲, ⊏*, ◭)
(□) = f(▲, ⊏*, ◨)
(□) = f(▲, ○, ⊏)
(□) = f(⊔*, ◭, ▲)
(□) = f(⊔*, ▲, ◭)
(□) = f(⊔*, ▲, ◨)
(□) = f(⊔*, ◨, ▲)
(□) = f(⊔*, ◨, ■)
(□) = f(⊔*, ■, ◨)
(□) = f(◨, ▲, ⊔*)
(□) = f(◨, ▲, ⊏*)
(□) = f(◨, ⊔*, ▲)
(□) = f(◨, ⊔*, ■)
(□) = f(◨, ■, ⊔*)
(□) = f(◨, ■, ⊏*)
(□) = f(◨, ⊏*, ▲)
(□) = f(◨, ⊏*, ■)
(□) = f(■, ⊔*, ◨)
(□) = f(■, ◨, ⊔*)
(□) = f(■, ◨, ⊏*)

(◭) = f(⊏, □, ○)
(◭) = f(⊏, ◨, ○)
(◭) = f(⊏, ◨, ◑)
(◭) = f(⊏, ○, □)
(◭) = f(⊏, ○, ◨)
(◭) = f(⊏, ◑, ◨)
(◭) = f(⊓*, △, ▲)
(◭) = f(⊓*, ▲, △)
(◭) = f(△, ⊓*, ▲)
(◭) = f(△, ▲, ⊓*)
(◭) = f(△, ▲, ⊔*)
(◭) = f(△, ▲, ⊏*)
(◭) = f(△, ⊔*, ▲)
(◭) = f(△, ⊏*, ▲)
(◭) = f(▲, ⊓*, △)
(◭) = f(▲, △, ⊓*)
(◭) = f(▲, △, ⊔*)
(◭) = f(▲, △, ⊏*)
(◭) = f(▲, ⊔*, △)
(◭) = f(▲, □, ⊔*)
(◭) = f(▲, ⊏*, △)
(◭) = f(▲, ⊏*, □)
(◭) = f(▲, ⊏*, ○)
(◭) = f(▲, ○, ⊏*)
(◭) = f(⊔*, ▲, □)
(◭) = f(⊔*, ▲, △)
(◭) = f(⊔*, □, ▲)
(◭) = f(□, ⊔, ○)
(◭) = f(□, ⊏, ○)
(◭) = f(□, ▲, ⊔*)
(◭) = f(□, ▲, ⊏*)
(◭) = f(□, ⊔*, ▲)
(◭) = f(□, ⊏*, ▲)
(◭) = f(□, ○, ⊔)
(◭) = f(□, ○, ⊏)
(◭) = f(◨, ⊔, ○)
(◭) = f(◨, ⊔, ◑)
(◭) = f(◨, ⊏, ○)
(◭) = f(◨, ⊏, ◑)
(◭) = f(◨, ○, ⊔)
(◭) = f(◨, ○, ⊏)
(◭) = f(◨, ◑, ⊔)
(◭) = f(◨, ◑, ⊏)
(◭) = f(⊏*, △, ▲)
(◭) = f(⊏*, ▲, △)
(◭) = f(⊏*, ▲, □)

(⊏) = f(◭, ○, ◨)
(⊏) = f(◭, ◑, ◨)
(⊏) = f(▲, □, ○)
(⊏) = f(▲, ◨, ○)
(⊏) = f(▲, ◨, ◑)
(⊏) = f(▲, ■, ○)
(⊏) = f(▲, ■, ◑)
(⊏) = f(▲, ■, ●)
(⊏) = f(▲, ○, □)
(⊏) = f(▲, ○, ◨)
(⊏) = f(▲, ○, ■)
(⊏) = f(▲, ◑, ◨)
(⊏) = f(▲, ◑, ■)
(⊏) = f(▲, ●, ■)
(⊏) = f(□, △, ○)
(⊏) = f(□, ◭, ○)
(⊏) = f(□, ▲, ○)
(⊏) = f(□, ○, △)
(⊏) = f(□, ○, ◭)
(⊏) = f(□, ○, ▲)
(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ▲, ◑)
(⊏) = f(■, ▲, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)
(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)

(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)
(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)
(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◭, ⊔)
(○) = f(◨, ◭, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

(□) = f(■, ⊏*, ◨)
(□) = f(⊏*, ◭, ▲)
(□) = f(⊏*, ▲, ◭)
(□) = f(⊏*, ▲, ◨)
(□) = f(⊏*, ◨, ▲)
(□) = f(⊏*, ◨, ■)
(□) = f(⊏*, ■, ◨)
(□) = f(○, ⊓, △)
(□) = f(○, ⊔, △)
(□) = f(○, ⊔, ◭)
(□) = f(○, ⊏, △)
(□) = f(○, ⊏, ◭)
(□) = f(○, ⊏, ▲)
(□) = f(○, △, ⊓)
(□) = f(○, △, ⊔)
(□) = f(○, △, ⊏)
(□) = f(○, ◭, ⊔)
(□) = f(○, ◭, ⊏)
(□) = f(○, ▲, ⊏)

(◭) = f(⊏*, ▲, ○)
(◭) = f(⊏*, □, ▲)
(◭) = f(⊏*, ○, ▲)
(◭) = f(○, ⊔, □)
(◭) = f(○, ⊔, ◨)
(◭) = f(○, ⊏, □)
(◭) = f(○, ⊏, ◨)
(◭) = f(○, ▲, ⊏*)
(◭) = f(○, □, ⊔)
(◭) = f(○, □, ⊏)
(◭) = f(○, ◨, ⊔)
(◭) = f(○, ◨, ⊏)
(◭) = f(○, ⊏*, ▲)
(◭) = f(◑, ⊔, ◨)
(◭) = f(◑, ⊏, ◨)
(◭) = f(◑, ◨, ⊔)
(◭) = f(◑, ◨, ⊏)

(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)
(⊏) = f(◑, ◨, ▲)
(⊏) = f(◑, ■, ▲)
(⊏) = f(●, ▲, ■)
(⊏) = f(●, ■, ▲)

6.

○ ↓	□ ↓	▲ ↓	⊏ ↓
(○) = f(⊓, △, □)	(□) = f(⊓, △, ○)	(▲) = f(⊏, □, ○)	(⊏) = f(△, □, ○)
(○) = f(⊓, □, △)	(□) = f(⊔, △, ○)	(▲) = f(⊏, ◨, ○)	(⊏) = f(△, ○, □)
(○) = f(⊔, △, □)	(□) = f(⊔, ◭, ○)	(▲) = f(⊏, ◨, ◑)	(⊏) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(□) = f(⊔, ○, △)	(▲) = f(⊏, ■, ○)	(⊏) = f(◭, ◨, ○)
(○) = f(⊔, ◭, ◨)	(□) = f(⊔, ○, ◭)	(▲) = f(⊏, ■, ◑)	(⊏) = f(◭, ◨, ◑)
(○) = f(⊔, □, △)	(□) = f(⊏, △, ○)	(▲) = f(⊏, ■, ●)	(⊏) = f(◭, ○, □)
(○) = f(⊔, □, ◭)	(□) = f(⊏, ◭, ○)	(▲) = f(⊏, ○, □)	(⊏) = f(◭, ○, ◨)
(○) = f(⊔, ◨, ◭)	(□) = f(⊏, ▲, ○)	(▲) = f(⊏, ○, ◨)	(⊏) = f(◭, ◑, ◨)
(○) = f(⊏, △, □)	(□) = f(⊏, ○, △)	(▲) = f(⊏, ○, ■)	(⊏) = f(▲, □, ○)
(○) = f(⊏, ◭, □)	(□) = f(⊏, ○, ◭)	(▲) = f(⊏, ◑, ◨)	(⊏) = f(▲, ◨, ○)
(○) = f(⊏, ◭, ◨)	(□) = f(⊏, ○, ▲)	(▲) = f(⊏, ◑, ■)	(⊏) = f(▲, ◨, ◑)
(○) = f(⊏, ▲, □)	(□) = f(△, ⊓, ○)	(▲) = f(⊏, ●, ■)	(⊏) = f(▲, ■, ○)
(○) = f(⊏, ▲, ◨)	(□) = f(△, ⊔, ○)	(▲) = f(⊓*, △, ◭)	(⊏) = f(▲, ■, ◑)
(○) = f(⊏, ▲, ■)	(□) = f(△, ⊏, ○)	(▲) = f(⊓*, ◭, △)	(⊏) = f(▲, ■, ●)
(○) = f(⊏, □, △)	(□) = f(△, ○, ⊓)	(▲) = f(△, ⊓*, ◭)	(⊏) = f(▲, ○, □)
(○) = f(⊏, □, ◭)	(□) = f(△, ○, ⊔)	(▲) = f(△, ◭, ⊓*)	(⊏) = f(▲, ○, ◨)
(○) = f(⊏, □, ▲)	(□) = f(△, ○, ⊏)	(▲) = f(△, ◭, ⊔*)	(⊏) = f(▲, ○, ■)
(○) = f(⊏, ◨, ◭)	(□) = f(◭, ⊔, ○)	(▲) = f(△, ◭, ⊏*)	(⊏) = f(▲, ◑, ◨)
(○) = f(⊏, ◨, ▲)	(□) = f(◭, ⊏, ○)	(▲) = f(△, ⊏*, ◭)	(⊏) = f(▲, ◑, ■)
(○) = f(⊏, ■, ▲)	(□) = f(◭, ▲, ⊏*)	(▲) = f(◭, ⊓*, △)	(⊏) = f(▲, ●, ■)
(○) = f(△, ⊓, □)	(□) = f(◭, ▲, ⊔*)	(▲) = f(◭, △, ⊓*)	(⊏) = f(□, △, ○)
(○) = f(△, ⊔, □)	(□) = f(◭, ⊔*, ▲)	(▲) = f(◭, △, ⊔*)	(⊏) = f(□, ◭, ○)
(○) = f(△, ⊏, □)	(□) = f(◭, ⊏*, ▲)	(▲) = f(◭, △, ⊏*)	(⊏) = f(□, ▲, ○)
(○) = f(△, □, ⊓)	(□) = f(◭, ○, ⊔)	(▲) = f(◭, ⊔*, △)	(⊏) = f(□, ○, △)
(○) = f(△, □, ⊔)	(□) = f(◭, ○, ⊏)	(▲) = f(◭, ⊔*, □)	(⊏) = f(□, ○, ◭)
(○) = f(△, □, ⊏)	(□) = f(▲, ⊏, ○)	(▲) = f(◭, □, ⊔*)	(⊏) = f(□, ○, ▲)
(○) = f(◭, ⊔, □)	(□) = f(▲, ◭, ⊔*)	(▲) = f(◭, □, ⊏*)	(⊏) = f(◨, ◭, ○)
(○) = f(◭, ⊔, ◨)	(□) = f(▲, ◭, ⊏*)	(▲) = f(◭, ⊏*, △)	(⊏) = f(◨, ◭, ◑)
(○) = f(◭, ⊏, □)	(□) = f(▲, ⊔*, ◭)	(▲) = f(◭, ⊏*, □)	(⊏) = f(◨, ▲, ○)
(○) = f(◭, ⊏, ◨)	(□) = f(▲, ⊔*, ◨)	(▲) = f(◭, ⊏*, ○)	(⊏) = f(◨, ▲, ◑)
(○) = f(◭, ▲, ⊏*)	(□) = f(▲, ◨, ⊔*)	(▲) = f(◭, ○, ⊏*)	(⊏) = f(◨, ○, ◭)
(○) = f(◭, □, ⊔)	(□) = f(▲, ◨, ⊏*)	(▲) = f(⊔*, △, ◭)	(⊏) = f(◨, ○, ▲)
(○) = f(◭, □, ⊏)	(□) = f(▲, ⊏*, ◭)	(▲) = f(⊔*, ◭, △)	(⊏) = f(◨, ◑, ▲)
(○) = f(◭, ◨, ⊏)	(□) = f(▲, ○, ⊏)	(▲) = f(⊔*, ◭, □)	(⊏) = f(■, ▲, ○)
(○) = f(◭, ⊏*, ▲)	(□) = f(⊔*, ◭, ▲)	(▲) = f(⊔*, □, ◭)	(⊏) = f(■, ▲, ◑)
(○) = f(▲, ⊏, □)	(□) = f(⊔*, ▲, ◭)	(▲) = f(⊔*, □, ◨)	(⊏) = f(■, ▲, ●)
(○) = f(▲, ⊏, ◨)	(□) = f(⊔*, ▲, ◨)	(▲) = f(⊔*, ◨, □)	(⊏) = f(■, ○, ▲)
(○) = f(▲, ⊏, ■)	(□) = f(⊔*, ◨, ▲)	(▲) = f(□, ⊏, ○)	(⊏) = f(■, ◑, ▲)
(○) = f(▲, ◭, ⊏*)	(□) = f(⊔*, ◨, ■)	(▲) = f(□, ◭, ⊔*)	(⊏) = f(■, ●, ▲)
(○) = f(▲, □, ⊏)	(□) = f(⊔*, ■, ◨)	(▲) = f(□, ◭, ⊏*)	(⊏) = f(○, △, □)
(○) = f(▲, ◨, ⊏)	(□) = f(◨, ▲, ⊔*)	(▲) = f(□, ⊔*, ◭)	(⊏) = f(○, ◭, □)

(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)
(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(○) = f(□, ⊏, △)
(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)
(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)
(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◭, ⊔)
(○) = f(◨, ◭, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)

(□) = f(◨, ▲, ⊏*)
(□) = f(◨, ⊔*, ▲)
(□) = f(◨, ⊔*, ■)
(□) = f(◨, ■, ⊔*)
(□) = f(◨, ■, ⊏*)
(□) = f(◨, ⊏*, ▲)
(□) = f(◨, ⊏*, ■)
(□) = f(■, ⊔*, ◨)
(□) = f(■, ◨, ⊔*)
(□) = f(■, ◨, ⊏*)
(□) = f(■, ⊏*, ◨)
(□) = f(⊏*, ◭, ▲)
(□) = f(⊏*, ▲, ◭)
(□) = f(⊏*, ▲, ◨)
(□) = f(⊏*, ◨, ▲)
(□) = f(⊏*, ◨, ■)
(□) = f(⊏*, ■, ◨)
(□) = f(○, ⊓, △)
(□) = f(○, ⊔, △)
(□) = f(○, ⊔, ◭)
(□) = f(○, ⊏, △)
(□) = f(○, ⊏, ◭)
(□) = f(○, ⊏, ▲)
(□) = f(○, △, ⊓)
(□) = f(○, △, ⊔)
(□) = f(○, △, ⊏)
(□) = f(○, ◭, ⊔)
(□) = f(○, ◭, ⊏)
(□) = f(○, ▲, ⊏)

(▲) = f(□, ⊔*, ◨)
(▲) = f(□, ◨, ⊔*)
(▲) = f(□, ◨, ⊏*)
(▲) = f(□, ⊏*, ◭)
(▲) = f(□, ⊏*, ◨)
(▲) = f(□, ○, ⊏)
(▲) = f(◨, ⊏, ○)
(▲) = f(◨, ⊏, ◑)
(▲) = f(◨, ⊔*, □)
(▲) = f(◨, □, ⊔*)
(▲) = f(◨, □, ⊏*)
(▲) = f(◨, ⊏*, □)
(▲) = f(◨, ⊏*, ○)
(▲) = f(◨, ○, ⊏)
(▲) = f(◨, ○, ⊏*)
(▲) = f(◨, ◑, ⊏)
(▲) = f(■, ⊏, ○)
(▲) = f(■, ⊏, ◑)
(▲) = f(■, ⊏, ●)
(▲) = f(■, ○, ⊏)
(▲) = f(■, ◑, ⊏)
(▲) = f(■, ●, ⊏)
(▲) = f(⊏*, △, ◭)
(▲) = f(⊏*, ◭, △)
(▲) = f(⊏*, ◭, □)
(▲) = f(⊏*, ◭, ○)
(▲) = f(⊏*, □, ◭)
(▲) = f(⊏*, □, ◨)
(▲) = f(⊏*, ◨, □)
(▲) = f(⊏*, ◨, ○)
(▲) = f(⊏*, ○, ◭)
(▲) = f(⊏*, ○, ◨)
(▲) = f(⊏*, ○, ◑)
(▲) = f(⊏*, ◑, ○)
(▲) = f(○, ⊏, □)
(▲) = f(○, ⊏, ◨)
(▲) = f(○, ⊏, ■)
(▲) = f(○, ◭, ⊏*)
(▲) = f(○, □, ⊏)
(▲) = f(○, ◨, ⊏)
(▲) = f(○, ◨, ⊏*)
(▲) = f(○, ■, ⊏)
(▲) = f(○, ⊏*, ◭)
(▲) = f(○, ⊏*, ◨)
(▲) = f(○, ⊏*, ◑)
(▲) = f(○, ◑, ⊏*)

(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)
(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)
(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)
(⊏) = f(◑, ◨, ▲)
(⊏) = f(◑, ■, ▲)
(⊏) = f(●, ▲, ■)
(⊏) = f(●, ■, ▲)

(○) = f(◑, ▲, ⊏*)	(▲) = f(◑, ⊏, ◨)
(○) = f(◑, ■, ⊏*)	(▲) = f(◑, ⊏, ■)
(○) = f(◑, ⊏*, ▲)	(▲) = f(◑, ◨, ⊏)
(○) = f(◑, ⊏*, ■)	(▲) = f(◑, ■, ⊏)
(○) = f(◑, ⊏*, ●)	(▲) = f(◑, ⊏*, ○)
(○) = f(◑, ●, ⊏*)	(▲) = f(◑, ○, ⊏*)
(○) = f(●, ⊏*, ◑)	(▲) = f(●, ⊏, ■)
(○) = f(●, ◑, ⊏*)	(▲) = f(●, ■, ⊏)

7.

○	◨	◭	⊔
(○) = f(⊓, △, □)	(◨) = f(⊔, ◭, ○)	(◭) = f(⊔, □, ○)	(⊔) = f(△, □, ○)
(○) = f(⊓, □, △)	(◨) = f(⊔, ◭, ◑)	(◭) = f(⊔, ◨, ○)	(⊔) = f(△, ○, □)
(○) = f(⊔, △, □)	(◨) = f(⊔, ○, ◭)	(◭) = f(⊔, ◨, ◑)	(⊔) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(◨) = f(⊔, ◑, ◭)	(◭) = f(⊔, ○, □)	(⊔) = f(◭, ◨, ○)
(○) = f(⊔, ◭, ◨)	(◨) = f(⊏, ◭, ○)	(◭) = f(⊔, ○, ◨)	(⊔) = f(◭, ◨, ◑)
(○) = f(⊔, □, △)	(◨) = f(⊏, ◭, ◑)	(◭) = f(⊔, ◑, ◨)	(⊔) = f(◭, ○, □)
(○) = f(⊔, □, ◭)	(◨) = f(⊏, ▲, ○)	(◭) = f(⊏, □, ○)	(⊔) = f(◭, ○, ◨)
(○) = f(⊔, ◨, ◭)	(◨) = f(⊏, ▲, ◑)	(◭) = f(⊏, ◨, ○)	(⊔) = f(◭, ◑, ◨)
(○) = f(⊏, △, □)	(◨) = f(⊏, ○, ◭)	(◭) = f(⊏, ◨, ◑)	(⊔) = f(□, △, ○)
(○) = f(⊏, ◭, □)	(◨) = f(⊏, ○, ▲)	(◭) = f(⊏, ○, □)	(⊔) = f(□, ◭, ○)
(○) = f(⊏, ◭, ◨)	(◨) = f(⊏, ◑, ◭)	(◭) = f(⊏, ○, ◨)	(⊔) = f(□, ○, △)
(○) = f(⊏, ▲, □)	(◨) = f(⊏, ◑, ▲)	(◭) = f(⊏, ◑, ◨)	(⊔) = f(□, ○, ◭)
(○) = f(⊏, ▲, ◨)	(◨) = f(◭, ⊔, ○)	(◭) = f(⊓*, △, ▲)	(⊔) = f(◨, ◭, ○)
(○) = f(⊏, ▲, ■)	(◨) = f(◭, ⊔, ◑)	(◭) = f(⊓*, ▲, △)	(⊔) = f(◨, ◭, ◑)
(○) = f(⊏, □, △)	(◨) = f(◭, ⊏, ○)	(◭) = f(△, ⊓*, ▲)	(⊔) = f(◨, ○, ◭)
(○) = f(⊏, □, ◭)	(◨) = f(◭, ⊏, ◑)	(◭) = f(△, ▲, ⊓*)	(⊔) = f(2-2, ◑, ◭)
(○) = f(⊏, □, ▲)	(◨) = f(◭, ○, ⊔)	(◭) = f(△, ▲, ⊔*)	(⊔) = f(○, △, □)
(○) = f(⊏, ◨, ◭)	(◨) = f(◭, ○, ⊏)	(◭) = f(△, ▲, ⊏*)	(⊔) = f(○, ◭, □)
(○) = f(⊏, ◨, ▲)	(◨) = f(◭, ◑, ⊔)	(◭) = f(△, ⊔*, ▲)	(⊔) = f(○, ◭, ◨)
(○) = f(⊏, ■, ▲)	(◨) = f(◭, ◑, ⊏)	(◭) = f(△, ⊏*, ▲)	(⊔) = f(○, □, △)
(○) = f(△, ⊓, □)	(◨) = f(▲, ⊏, ○)	(◭) = f(▲, ⊓*, △)	(⊔) = f(○, □, ◭)
(○) = f(△, ⊔, □)	(◨) = f(▲, ⊏, ◑)	(◭) = f(▲, △, ⊓*)	(⊔) = f(○, ◨, ◭)
(○) = f(△, ⊏, □)	(◨) = f(▲, ⊔*, □)	(◭) = f(▲, △, ⊔*)	(⊔) = f(◑, ◭, ◨)
(○) = f(△, □, ⊓)	(◨) = f(▲, □, ⊔*)	(◭) = f(▲, △, ⊏*)	(⊔) = f(◑, ◨, ◭)
(○) = f(△, □, ⊔)	(◨) = f(▲, □, ⊏*)	(◭) = f(▲, ⊔*, △)	
(○) = f(△, □, ⊏)	(◨) = f(▲, ⊏*, □)	(◭) = f(▲, □, ⊔*)	
(○) = f(◭, ⊔, □)	(◨) = f(▲, ⊏*, ○)	(◭) = f(▲, ⊏*, △)	
(○) = f(◭, ⊔, ◨)	(◨) = f(▲, ○, ⊏)	(◭) = f(▲, ⊏*, □)	
(○) = f(◭, ⊏, □)	(◨) = f(▲, ○, ⊏*)	(◭) = f(▲, ⊏*, ○)	
(○) = f(◭, ⊏, ◨)	(◨) = f(▲, ◑, ⊏)	(◭) = f(▲, ○, ⊏*)	
(○) = f(◭, ▲, ⊏*)	(◨) = f(⊔*, ▲, □)	(◭) = f(⊔*, ▲, □)	
(○) = f(◭, □, ⊔)	(◨) = f(⊔*, □, ▲)	(◭) = f(⊔*, ▲, △)	

(○) = f(◭, □, ⊏)
(○) = f(◭, ◨, ⊔)
(○) = f(◭, ◨, ⊏)
(○) = f(◭, ⊏*, ▲)
(○) = f(▲, ⊏, □)
(○) = f(▲, ⊏, ◨)
(○) = f(▲, ⊏, ■)
(○) = f(▲, ◭, ⊏*)
(○) = f(▲, □, ⊏)
(○) = f(▲, ◨, ⊏)
(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)
(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(○) = f(□, ⊏, △)
(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)
(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)
(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◭, ⊔)
(○) = f(◨, ◭, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(◨) = f(⊔*, □, ■)
(◨) = f(⊔*, ■, □)
(◨) = f(□, ▲, ⊔*)
(◨) = f(□, ▲, ⊏*)
(◨) = f(□, ⊔*, ▲)
(◨) = f(□, ⊔*, ■)
(◨) = f(□, ■, ⊔*)
(◨) = f(□, ■, ⊏*)
(◨) = f(□, ⊏*, ▲)
(◨) = f(□, ⊏*, ■)
(◨) = f(■, ⊔*, □)
(◨) = f(■, □, ⊔*)
(◨) = f(■, □, ⊏*)
(◨) = f(■, ⊏*, □)
(◨) = f(■, ⊏*, ○)
(◨) = f(■, ○, ⊏*)
(◨) = f(⊏*, ▲, □)
(◨) = f(⊏*, ▲, ○)
(◨) = f(⊏*, □, ▲)
(◨) = f(⊏*, □, ■)
(◨) = f(⊏*, ■, □)
(◨) = f(⊏*, ■, ○)
(◨) = f(⊏*, ○, ▲)
(◨) = f(⊏*, ○, ■)
(◨) = f(○, ⊔, ◭)
(◨) = f(○, ⊏, ◭)
(◨) = f(○, ⊏, ▲)
(◨) = f(○, ◭, ⊔)
(◨) = f(○, ◭, ⊏)
(◨) = f(○, ▲, ⊏)
(◨) = f(○, ▲, ⊏*)
(◨) = f(○, ■, ⊏*)
(◨) = f(○, ⊏*, ▲)
(◨) = f(○, ⊏*, ■)
(◨) = f(◑, ⊔, ◭)
(◨) = f(◑, ⊏, ◭)
(◨) = f(◑, ⊏, ▲)
(◨) = f(◑, ◭, ⊔)
(◨) = f(◑, ◭, ⊏)
(◨) = f(◑, ▲, ⊏)
(◭) = f(⊔*, □, ▲)
(◭) = f(□, ⊔, ○)
(◭) = f(□, ⊏, ○)
(◭) = f(□, ▲, ⊔*)
(◭) = f(□, ▲, ⊏*)
(◭) = f(□, ⊔*, ▲)
(◭) = f(□, ⊏*, ▲)
(◭) = f(□, ○, ⊔)
(◭) = f(□, ○, ⊏)
(◭) = f(◨, ⊔, ○)
(◭) = f(◨, ⊔, ◑)
(◭) = f(◨, ⊏, ○)
(◭) = f(◨, ⊏, ◑)
(◭) = f(◨, ○, ⊔)
(◭) = f(◨, ○, ⊏)
(◭) = f(◨, ◑, ⊔)
(◭) = f(◨, ◑, ⊏)
(◭) = f(⊏*, △, ▲)
(◭) = f(⊏*, ▲, △)
(◭) = f(⊏*, ▲, □)
(◭) = f(⊏*, ▲, ○)
(◭) = f(⊏*, □, ▲)
(◭) = f(⊏*, ○, ▲)
(◭) = f(○, ⊔, □)
(◭) = f(○, ⊔, ◨)
(◭) = f(○, ⊏, □)
(◭) = f(○, ⊏, ◨)
(◭) = f(○, ▲, ⊏*)
(◭) = f(○, □, ⊔)
(◭) = f(○, □, ⊏)
(◭) = f(○, ◨, ⊔)
(◭) = f(○, ◨, ⊏)
(◭) = f(○, ⊏*, ▲)
(◭) = f(◑, ⊔, ◨)
(◭) = f(◑, ⊏, ◨)
(◭) = f(◑, ◨, ⊔)
(◭) = f(◑, ◨, ⊏)

(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

8.

○ ↓	◨ ↓	◭ ↓	⊏ ↓
(○) = f(⊓, △, □)	(◨) = f(⊔, ◭, ○)	(◭) = f(⊔, □, ○)	(⊏) = f(△, □, ○)
(○) = f(⊓, □, △)	(◨) = f(⊔, ◭, ◑)	(◭) = f(⊔, ◨, ○)	(⊏) = f(△, ○, □)
(○) = f(⊔, △, □)	(◨) = f(⊔, ○, ◭)	(◭) = f(⊔, ◨, ◑)	(⊏) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(◨) = f(⊔, ◑, ◭)	(◭) = f(⊔, ○, □)	(⊏) = f(◭, ◨, ○)
(○) = f(⊔, ◭, ◨)	(◨) = f(⊏, ◭, ○)	(◭) = f(⊔, ○, ◨)	(⊏) = f(◭, ◨, ◑)
(○) = f(⊔, □, △)	(◨) = f(⊏, ◭, ◑)	(◭) = f(⊔, ◑, ◨)	(⊏) = f(◭, ○, □)
(○) = f(⊔, □, ◭)	(◨) = f(⊏, ▲, ○)	(◭) = f(⊏, □, ○)	(⊏) = f(◭, ○, ◨)
(○) = f(⊔, ◨, ◭)	(◨) = f(⊏, ▲, ◑)	(◭) = f(⊏, ◨, ○)	(⊏) = f(◭, ◑, ◨)
(○) = f(⊏, △, □)	(◨) = f(⊏, ○, ◭)	(◭) = f(⊏, ◨, ◑)	(⊏) = f(▲, □, ○)
(○) = f(⊏, ◭, □)	(◨) = f(⊏, ○, ▲)	(◭) = f(⊏, ○, □)	(⊏) = f(▲, ◨, ○)
(○) = f(⊏, ◭, ◨)	(◨) = f(⊏, ◑, ◭)	(◭) = f(⊏, ○, ◨)	(⊏) = f(▲, ◨, ◑)
(○) = f(⊏, ▲, □)	(◨) = f(⊏, ◑, ▲)	(◭) = f(⊏, ◑, ◨)	(⊏) = f(▲, ■, ○)
(○) = f(⊏, ▲, ◨)	(◨) = f(◭, ⊔, ○)	(◭) = f(⊓*, △, ▲)	(⊏) = f(▲, ■, ◑)
(○) = f(⊏, ▲, ■)	(◨) = f(◭, ⊔, ◑)	(◭) = f(⊓*, ▲, △)	(⊏) = f(▲, ■, ●)
(○) = f(⊏, □, △)	(◨) = f(◭, ⊏, ○)	(◭) = f(△, ⊓*, ▲)	(⊏) = f(▲, ○, □)
(○) = f(⊏, □, ◭)	(◨) = f(◭, ⊏, ◑)	(◭) = f(△, ▲, ⊓*)	(⊏) = f(▲, ○, ◨)
(○) = f(⊏, □, ▲)	(◨) = f(◭, ○, ⊔)	(◭) = f(△, ▲, ⊔*)	(⊏) = f(▲, ○, ■)
(○) = f(⊏, ◨, ◭)	(◨) = f(◭, ○, ⊏)	(◭) = f(△, ▲, ⊏*)	(⊏) = f(▲, ◑, ◨)
(○) = f(⊏, ◨, ▲)	(◨) = f(◭, ◑, ⊔)	(◭) = f(△, ⊔*, ▲)	(⊏) = f(▲, ◑, ■)
(○) = f(⊏, ■, ▲)	(◨) = f(◭, ◑, ⊏)	(◭) = f(△, ⊏*, ▲)	(⊏) = f(▲, ●, ■)
(○) = f(△, ⊓, □)	(◨) = f(▲, ⊏, ○)	(◭) = f(▲, ⊓*, △)	(⊏) = f(□, △, ○)
(○) = f(△, ⊔, □)	(◨) = f(▲, ⊏, ◑)	(◭) = f(▲, △, ⊓*)	(⊏) = f(□, ◭, ○)

(○) = f(△, ⊏, □)

(○) = f(△, □, ⊓)

(○) = f(△, □, ⊔)

(○) = f(△, □, ⊏)

(○) = f(◭, ⊔, □)

(○) = f(◭, ⊔, ◨)

(○) = f(◭, ⊏, □)

(○) = f(◭, ⊏, ◨)

(○) = f(◭, ▲, ⊏*)

(○) = f(◭, □, ⊔)

(○) = f(◭, □, ⊏)

(○) = f(◭, ◨, ⊔)

(○) = f(◭, ◨, ⊏)

(○) = f(◭, ⊏*, ▲)

(○) = f(▲, ⊏, □)

(○) = f(▲, ⊏, ■)

(○) = f(▲, ◭, ⊏*)

(○) = f(▲, □, ⊏)

(○) = f(▲, ◨, ⊏)

(○) = f(▲, ◨, ⊏*)

(○) = f(▲, ■, ⊏)

(○) = f(▲, ⊏*, ◭)

(○) = f(▲, ⊏*, ◨)

(○) = f(▲, ⊏*, ◑)

(○) = f(▲, ◑, ⊏*)

(○) = f(□, ⊓, △)

(○) = f(□, ⊔, △)

(○) = f(□, ⊔, ◭)

(○) = f(□, ⊏, △)

(○) = f(□, ⊏, ◭)

(○) = f(□, ⊏, ▲)

(○) = f(□, △, ⊓)

(○) = f(□, △, ⊔)

(○) = f(□, △, ⊏)

(○) = f(□, ◭, ⊔)

(○) = f(□, ◭, ⊏)

(○) = f(□, ▲, ⊏)

(○) = f(◨, ⊔, ◭)

(○) = f(◨, ⊏, ◭)

(○) = f(◨, ⊏, ▲)

(○) = f(◨, ◭, ⊔)

(○) = f(◨, ◭, ⊏)

(○) = f(◨, ▲, ⊏)

(○) = f(◨, ▲, ⊏*)

(○) = f(◨, ■, ⊏*)

(○) = f(◨, ⊏*, ▲)

(◨) = f(▲, ⊔*, □)

(◨) = f(▲, □, ⊔*)

(◨) = f(▲, □, ⊏*)

(◨) = f(▲, ⊏*, □)

(◨) = f(▲, ⊏*, ○)

(◨) = f(▲, ○, ⊏)

(◨) = f(▲, ○, ⊏*)

(◨) = f(▲, ◑, ⊏)

(◨) = f(⊔*, ▲, □)

(◨) = f(⊔*, □, ▲)

(◨) = f(⊔*, □, ■)

(◨) = f(⊔*, ■, □)

(◨) = f(□, ▲, ⊔*)

(◨) = f(□, ▲, ⊏*)

(◨) = f(□, ⊔*, ▲)

(◨) = f(□, ■, ⊔*)

(◨) = f(□, ■, ⊏*)

(◨) = f(□, ⊏*, ▲)

(◨) = f(□, ⊏*, ■)

(◨) = f(■, ⊔*, □)

(◨) = f(■, □, ⊔*)

(◨) = f(■, □, ⊏*)

(◨) = f(■, ⊏*, □)

(◨) = f(■, ⊏*, ○)

(◨) = f(■, ○, ⊏*)

(◨) = f(⊏*, ▲, □)

(◨) = f(⊏*, ▲, ○)

(◨) = f(⊏*, □, ▲)

(◨) = f(⊏*, □, ■)

(◨) = f(⊏*, ■, □)

(◨) = f(⊏*, ■, ○)

(◨) = f(⊏*, ○, ▲)

(◨) = f(⊏*, ○, ■)

(◨) = f(○, ⊔, ◭)

(◨) = f(○, ⊏, ◭)

(◨) = f(○, ⊏, ▲)

(◨) = f(○, ◭, ⊔)

(◨) = f(○, ◭, ⊏)

(◨) = f(○, ▲, ⊏)

(◨) = f(○, ▲, ⊏*)

(◨) = f(○, ■, ⊏*)

(◨) = f(○, ⊏*, ▲)

(◨) = f(○, ⊏*, ■)

(◨) = f(◑, ⊔, ◭)

(◨) = f(◑, ⊏, ◭)

(◨) = f(◑, ⊏, ▲)

(◭) = f(▲, △, ⊔*)

(◭) = f(▲, △, ⊏*)

(◭) = f(▲, ⊔*, △)

(◭) = f(▲, □, ⊔*)

(◭) = f(▲, ⊏*, △)

(◭) = f(▲, ⊏*, □)

(◭) = f(▲, ⊏*, ○)

(◭) = f(▲, ○, ⊏*)

(◭) = f(⊔*, ▲, □)

(◭) = f(⊔*, ▲, △)

(◭) = f(⊔*, □, ▲)

(◭) = f(□, ⊔, ○)

(◭) = f(□, ⊏, ○)

(◭) = f(□, ▲, ⊔*)

(◭) = f(□, ▲, ⊏*)

(◭) = f(□, ⊏*, ▲)

(◭) = f(□, ○, ⊔)

(◭) = f(□, ○, ⊏)

(◭) = f(◨, ⊔, ○)

(◭) = f(◨, ⊔, ◑)

(◭) = f(◨, ⊏, ○)

(◭) = f(◨, ⊏, ◑)

(◭) = f(◨, ○, ⊔)

(◭) = f(◨, ○, ⊏)

(◭) = f(◨, ◑, ⊔)

(◭) = f(◨, ◑, ⊏)

(◭) = f(⊏*, △, ▲)

(◭) = f(⊏*, ▲, △)

(◭) = f(⊏*, ▲, □)

(◭) = f(⊏*, ▲, ○)

(◭) = f(⊏*, □, ▲)

(◭) = f(⊏*, ○, ▲)

(◭) = f(○, ⊔, □)

(◭) = f(○, ⊔, ◨)

(◭) = f(○, ⊏, □)

(◭) = f(○, ⊏, ◨)

(◭) = f(○, ▲, ⊏*)

(◭) = f(○, □, ⊔)

(◭) = f(○, □, ⊏)

(◭) = f(○, ◨, ⊔)

(◭) = f(○, ◨, ⊏)

(◭) = f(○, ⊏*, ▲)

(◭) = f(◑, ⊔, ◨)

(◭) = f(◑, ⊏, ◨)

(◭) = f(◑, ◨, ⊔)

(◭) = f(◑, ◨, ⊏)

(⊏) = f(□, ▲, ○)

(⊏) = f(□, ○, △)

(⊏) = f(□, ○, ◭)

(⊏) = f(□, ○, ▲)

(⊏) = f(◨, ◭, ○)

(⊏) = f(◨, ◭, ◑)

(⊏) = f(◨, ▲, ○)

(⊏) = f(◨, ▲, ◑)

(⊏) = f(◨, ○, ◭)

(⊏) = f(◨, ○, ▲)

(⊏) = f(◨, ◑, ◭)

(⊏) = f(◨, ◑, ▲)

(⊏) = f(■, ▲, ○)

(⊏) = f(■, ▲, ◑)

(⊏) = f(■, ▲, ●)

(⊏) = f(■, ○, ▲)

(⊏) = f(■, ◑, ▲)

(⊏) = f(■, ●, ▲)

(⊏) = f(○, △, □)

(⊏) = f(○, ◭, □)

(⊏) = f(○, ◭, ◨)

(⊏) = f(○, ▲, □)

(⊏) = f(○, ▲, ◨)

(⊏) = f(○, ▲, ■)

(⊏) = f(○, □, △)

(⊏) = f(○, □, ◭)

(⊏) = f(○, □, ▲)

(⊏) = f(○, ◨, ◭)

(⊏) = f(○, ◨, ▲)

(⊏) = f(○, ■, ▲)

(⊏) = f(◑, ◭, ◨)

(⊏) = f(◑, ▲, ◨)

(⊏) = f(◑, ▲, ■)

(⊏) = f(◑, ◨, ◭)

(⊏) = f(◑, ◨, ▲)

(⊏) = f(◑, ■, ▲)

(⊏) = f(●, ▲, ■)

(⊏) = f(●, ■, ▲)

(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

(◨) = f(◑, ◭, ⊔)
(◨) = f(◑, ◭, ⊏)
(◨) = f(◑, ▲, ⊏)

9.

○	◨	▲	⊏
(○) = f(⊓, △, □)	(◨) = f(⊔, ◭, ○)	(▲) = f(⊏, □, ○)	(⊏) = f(△, □, ○)
(○) = f(⊓, □, △)	(◨) = f(⊔, ◭, ◑)	(▲) = f(⊏, ◨, ○)	(⊏) = f(△, ○, □)
(○) = f(⊔, △, □)	(◨) = f(⊔, ○, ◭)	(▲) = f(⊏, ◨, ◑)	(⊏) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(◨) = f(⊔, ◑, ◭)	(▲) = f(⊏, ■, ○)	(⊏) = f(◭, ◨, ○)
(○) = f(⊔, ◭, ◨)	(◨) = f(⊏, ◭, ○)	(▲) = f(⊏, ■, ◑)	(⊏) = f(◭, ◨, ◑)
(○) = f(⊔, □, △)	(◨) = f(⊏, ◭, ◑)	(▲) = f(⊏, ■, ●)	(⊏) = f(◭, ○, □)
(○) = f(⊔, □, ◭)	(◨) = f(⊏, ▲, ○)	(▲) = f(⊏, ○, □)	(⊏) = f(◭, ○, ◨)
(○) = f(⊔, ◨, ◭)	(◨) = f(⊏, ▲, ◑)	(▲) = f(⊏, ○, ◨)	(⊏) = f(◭, ◑, ◨)
(○) = f(⊏, △, □)	(◨) = f(⊏, ○, ◭)	(▲) = f(⊏, ○, ■)	(⊏) = f(▲, □, ○)
(○) = f(⊏, ◭, □)	(◨) = f(⊏, ○, ▲)	(▲) = f(⊏, ◑, ◨)	(⊏) = f(▲, ◨, ○)
(○) = f(⊏, ◭, ◨)	(◨) = f(⊏, ◑, ◭)	(▲) = f(⊏, ◑, ■)	(⊏) = f(▲, ◨, ◑)
(○) = f(⊏, ▲, □)	(◨) = f(⊏, ◑, ▲)	(▲) = f(⊏, ●, ■)	(⊏) = f(▲, ■, ○)
(○) = f(⊏, ▲, ◨)	(◨) = f(◭, ⊔, ○)	(▲) = f(⊓*, △, ◭)	(⊏) = f(▲, ■, ◑)

(○) = f(⊏, ▲, ■)
(○) = f(⊏, □, △)
(○) = f(⊏, □, ◭)
(○) = f(⊏, □, ▲)
(○) = f(⊏, ◨, ◭)
(○) = f(⊏, ◨, ▲)
(○) = f(⊏, ■, ▲)
(○) = f(△, ⊓, □)
(○) = f(△, ⊔, □)
(○) = f(△, ⊏, □)
(○) = f(△, □, ⊓)
(○) = f(△, □, ⊔)
(○) = f(△, □, ⊏)
(○) = f(◭, ⊔, □)
(○) = f(◭, ⊔, ◨)
(○) = f(◭, ⊏, □)
(○) = f(◭, ⊏, ◨)
(○) = f(◭, ▲, ⊏*)
(○) = f(◭, □, ⊔)
(○) = f(◭, □, ⊏)
(○) = f(◭, ◨, ⊔)
(○) = f(◭, ◨, ⊏)
(○) = f(◭, ⊏*, ▲)
(○) = f(▲, ⊏, □)
(○) = f(▲, ⊏, ■)
(○) = f(▲, ◭, ⊏*)
(○) = f(▲, □, ⊏)
(○) = f(▲, ◨, ⊏)
(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)
(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(○) = f(□, ⊏, △)
(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)

(◨) = f(◭, ⊔, ◑)
(◨) = f(◭, ⊏, ○)
(◨) = f(◭, ⊏, ◑)
(◨) = f(◭, ○, ⊔)
(◨) = f(◭, ○, ⊏)
(◨) = f(◭, ◑, ⊔)
(◨) = f(◭, ◑, ⊏)
(◨) = f(▲, ⊏, ○)
(◨) = f(▲, ⊏, ◑)
(◨) = f(▲, ⊔*, □)
(◨) = f(▲, □, ⊔*)
(◨) = f(▲, □, ⊏*)
(◨) = f(▲, ⊏*, □)
(◨) = f(▲, ⊏*, ○)
(◨) = f(▲, ○, ⊏)
(◨) = f(▲, ○, ⊏*)
(◨) = f(▲, ◑, ⊏)
(◨) = f(⊔*, ▲, □)
(◨) = f(⊔*, □, ▲)
(◨) = f(⊔*, □, ■)
(◨) = f(⊔*, ■, □)
(◨) = f(□, ▲, ⊔*)
(◨) = f(□, ▲, ⊏*)
(◨) = f(□, ⊔*, ▲)
(◨) = f(□, ■, ⊔*)
(◨) = f(□, ■, ⊏*)
(◨) = f(□, ⊏*, ▲)
(◨) = f(□, ⊏*, ■)
(◨) = f(■, ⊔*, □)
(◨) = f(■, □, ⊔*)
(◨) = f(■, □, ⊏*)
(◨) = f(■, ⊏*, □)
(◨) = f(■, ⊏*, ○)
(◨) = f(■, ○, ⊏*)
(◨) = f(⊏*, ▲, □)
(◨) = f(⊏*, ▲, ○)
(◨) = f(⊏*, □, ▲)
(◨) = f(⊏*, □, ■)
(◨) = f(⊏*, ■, □)
(◨) = f(⊏*, ■, ○)
(◨) = f(⊏*, ○, ▲)
(◨) = f(⊏*, ○, ■)
(◨) = f(○, ⊔, ◭)
(◨) = f(○, ⊏, ◭)
(◨) = f(○, ⊏, ▲)
(◨) = f(○, ◭, ⊔)

(▲) = f(⊓*, ◭, △)
(▲) = f(△, ⊓*, ◭)
(▲) = f(△, ◭, ⊓*)
(▲) = f(△, ◭, ⊔*)
(▲) = f(△, ◭, ⊏*)
(▲) = f(△, ⊏*, ◭)
(▲) = f(◭, ⊓*, △)
(▲) = f(◭, △, ⊓*)
(▲) = f(◭, △, ⊔*)
(▲) = f(◭, △, ⊏*)
(▲) = f(◭, ⊔*, △)
(▲) = f(◭, ⊔*, □)
(▲) = f(◭, □, ⊔*)
(▲) = f(◭, □, ⊏*)
(▲) = f(◭, ⊏*, △)
(▲) = f(◭, ⊏*, □)
(▲) = f(◭, ⊏*, ○)
(▲) = f(◭, ○, ⊏*)
(▲) = f(⊔*, △, ◭)
(▲) = f(⊔*, ◭, △)
(▲) = f(⊔*, ◭, □)
(▲) = f(⊔*, □, ◭)
(▲) = f(⊔*, □, ◨)
(▲) = f(⊔*, ◨, □)
(▲) = f(□, ⊏, ○)
(▲) = f(□, ◭, ⊔*)
(▲) = f(□, ◭, ⊏*)
(▲) = f(□, ⊔*, ◭)
(▲) = f(□, ⊔*, ◨)
(▲) = f(□, ◨, ⊔*)
(▲) = f(□, ◨, ⊏*)
(▲) = f(□, ⊏*, ◭)
(▲) = f(□, ⊏*, ◨)
(▲) = f(□, ○, ⊏)
(▲) = f(◨, ⊏, ○)
(▲) = f(◨, ⊏, ◑)
(▲) = f(◨, ⊔*, □)
(▲) = f(◨, □, ⊔*)
(▲) = f(◨, □, ⊏*)
(▲) = f(◨, ⊏*, □)
(▲) = f(◨, ⊏*, ○)
(▲) = f(◨, ○, ⊏)
(▲) = f(◨, ○, ⊏*)
(▲) = f(◨, ◑, ⊏)
(▲) = f(■, ⊏, ○)
(▲) = f(■, ⊏, ◑)

(⊏) = f(▲, ■, ●)
(⊏) = f(▲, ○, □)
(⊏) = f(▲, ○, ◨)
(⊏) = f(▲, ○, ■)
(⊏) = f(▲, ◑, ◨)
(⊏) = f(▲, ◑, ■)
(⊏) = f(▲, ●, ■)
(⊏) = f(□, △, ○)
(⊏) = f(□, ◭, ○)
(⊏) = f(□, ▲, ○)
(⊏) = f(□, ○, △)
(⊏) = f(□, ○, ◭)
(⊏) = f(□, ○, ▲)
(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ▲, ◑)
(⊏) = f(■, ▲, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)
(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)
(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)
(⊏) = f(◑, ◨, ▲)
(⊏) = f(◑, ■, ▲)
(⊏) = f(●, ▲, ■)

(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)
(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◭, ⊔)
(○) = f(◨, ◭, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

(◨) = f(○, ◭, ⊏)
(◨) = f(○, ▲, ⊏)
(◨) = f(○, ▲, ⊏*)
(◨) = f(○, ■, ⊏*)
(◨) = f(○, ⊏*, ▲)
(◨) = f(○, ⊏*, ■)
(◨) = f(◑, ⊔, ◭)
(◨) = f(◑, ⊏, ◭)
(◨) = f(◑, ⊏, ▲)
(◨) = f(◑, ◭, ⊔)
(◨) = f(◑, ◭, ⊏)
(◨) = f(◑, ▲, ⊏)

(▲) = f(■, ⊏, ●)
(▲) = f(■, ○, ⊏)
(▲) = f(■, ◑, ⊏)
(▲) = f(■, ●, ⊏)
(▲) = f(⊏*, △, ◭)
(▲) = f(⊏*, ◭, △)
(▲) = f(⊏*, ◭, □)
(▲) = f(⊏*, ◭, ○)
(▲) = f(⊏*, □, ◭)
(▲) = f(⊏*, □, ◨)
(▲) = f(⊏*, ◨, □)
(▲) = f(⊏*, ◨, ○)
(▲) = f(⊏*, ○, ◭)
(▲) = f(⊏*, ○, ◨)
(▲) = f(⊏*, ○, ◑)
(▲) = f(⊏*, ◑, ○)
(▲) = f(○, ⊏, □)
(▲) = f(○, ⊏, ◨)
(▲) = f(○, ⊏, ■)
(▲) = f(○, ◭, ⊏*)
(▲) = f(○, □, ⊏)
(▲) = f(○, ◨, ⊏)
(▲) = f(○, ◨, ⊏*)
(▲) = f(○, ■, ⊏)
(▲) = f(○, ⊏*, ◭)
(▲) = f(○, ⊏*, ◨)
(▲) = f(○, ⊏*, ◑)
(▲) = f(○, ◑, ⊏*)
(▲) = f(◑, ⊏, ◨)
(▲) = f(◑, ⊏, ■)
(▲) = f(◑, ◨, ⊏)
(▲) = f(◑, ■, ⊏)
(▲) = f(◑, ⊏*, ○)
(▲) = f(◑, ○, ⊏*)
(▲) = f(●, ⊏, ■)
(▲) = f(●, ■, ⊏)

(⊏) = f(●, ■, ▲)

10.

○	■	▲	⊏
↓	↓	↓	↓
(○) = f(⊓, △, □)	(■) = f(⊏, ▲, ○)	(▲) = f(⊏, □, ○)	(⊏) = f(△, □, ○)
(○) = f(⊓, □, △)	(■) = f(⊏, ▲, ◑)	(▲) = f(⊏, ◨, ○)	(⊏) = f(△, ○, □)
(○) = f(⊔, △, □)	(■) = f(⊏, ▲, ●)	(▲) = f(⊏, ◨, ◑)	(⊏) = f(◭, □, ○)
(○) = f(⊔, ◭, □)	(■) = f(⊏, ○, ▲)	(▲) = f(⊏, ■, ○)	(⊏) = f(◭, ◨, ○)

(○) = f(⊔, ◭, ◨)
(○) = f(⊔, □, △)
(○) = f(⊔, □, ◭)
(○) = f(⊔, ◨, ◭)
(○) = f(⊏, △, □)
(○) = f(⊏, ◭, □)
(○) = f(⊏, ◭, ◨)
(○) = f(⊏, ▲, □)
(○) = f(⊏, ▲, ◨)
(○) = f(⊏, ▲, ■)
(○) = f(⊏, □, △)
(○) = f(⊏, □, ◭)
(○) = f(⊏, □, ▲)
(○) = f(⊏, ◨, ◭)
(○) = f(⊏, ◨, ▲)
(○) = f(⊏, ■, ▲)
(○) = f(△, ⊓, □)
(○) = f(△, ⊔, □)
(○) = f(△, ⊏, □)
(○) = f(△, □, ⊓)
(○) = f(△, □, ⊔)
(○) = f(△, □, ⊏)
(○) = f(◭, ⊔, □)
(○) = f(◭, ⊔, ◨)
(○) = f(◭, ⊏, □)
(○) = f(◭, ⊏, ◨)
(○) = f(◭, ▲, ⊏*)
(○) = f(◭, □, ⊔)
(○) = f(◭, □, ⊏)
(○) = f(◭, ◨, ⊔)
(○) = f(◭, ◨, ⊏)
(○) = f(◭, ⊏*, ▲)
(○) = f(▲, ⊏, □)
(○) = f(▲, ⊏, ■)
(○) = f(▲, ◭, ⊏*)
(○) = f(▲, □, ⊏)
(○) = f(▲, ◨, ⊏)
(○) = f(▲, ◨, ⊏*)
(○) = f(▲, ■, ⊏)
(○) = f(▲, ⊏*, ◭)
(○) = f(▲, ⊏*, ◨)
(○) = f(▲, ⊏*, ◑)
(○) = f(▲, ◑, ⊏*)
(○) = f(□, ⊓, △)
(○) = f(□, ⊔, △)
(○) = f(□, ⊔, ◭)
(■) = f(⊏, ◑, ▲)
(■) = f(⊏, ●, ▲)
(■) = f(▲, ⊏, ○)
(■) = f(▲, ⊏, ◑)
(■) = f(▲, ⊏, ●)
(■) = f(▲, ○, ⊏)
(■) = f(▲, ◑, ⊏)
(■) = f(▲, ●, ⊏)
(■) = f(⊔*, □, ◨)
(■) = f(⊔*, ◨, □)
(■) = f(□, ⊔*, ◨)
(■) = f(□, ◨, ⊔*)
(■) = f(□, ◨, ⊏*)
(■) = f(□, ⊏*, ◨)
(■) = f(◨, ⊔*, □)
(■) = f(◨, □, ⊔*)
(■) = f(◨, □, ⊏*)
(■) = f(◨, ⊏*, □)
(■) = f(◨, ⊏*, ○)
(■) = f(◨, ○, ⊏*)
(■) = f(⊏*, □, ◨)
(■) = f(⊏*, ◨, □)
(■) = f(⊏*, ◨, ○)
(■) = f(⊏*, ○, ◨)
(■) = f(⊏*, ○, ◑)
(■) = f(⊏*, ◑, ○)
(■) = f(○, ⊏, ▲)
(■) = f(○, ▲, ⊏)
(■) = f(○, ◨, ⊏*)
(■) = f(○, ⊏*, ◨)
(■) = f(○, ⊏*, ◑)
(■) = f(○, ◑, ⊏*)
(■) = f(◑, ⊏, ▲)
(■) = f(◑, ▲, ⊏)
(■) = f(◑, ⊏*, ○)
(■) = f(◑, ○, ⊏*)
(■) = f(●, ⊏, ▲)
(■) = f(●, ▲, ⊏)
(▲) = f(⊏, ■, ◑)
(▲) = f(⊏, ■, ●)
(▲) = f(⊏, ○, □)
(▲) = f(⊏, ○, ◨)
(▲) = f(⊏, ○, ■)
(▲) = f(⊏, ◑, ◨)
(▲) = f(⊏, ◑, ■)
(▲) = f(⊏, ●, ■)
(▲) = f(⊓*, △, ◭)
(▲) = f(⊓*, ◭, △)
(▲) = f(△, ⊓*, ◭)
(▲) = f(△, ◭, ⊓*)
(▲) = f(△, ◭, ⊔*)
(▲) = f(△, ◭, ⊏*)
(▲) = f(△, ⊏*, ◭)
(▲) = f(◭, ⊓*, △)
(▲) = f(◭, △, ⊓*)
(▲) = f(◭, △, ⊔*)
(▲) = f(◭, △, ⊏*)
(▲) = f(◭, ⊔*, △)
(▲) = f(◭, ⊔*, □)
(▲) = f(◭, □, ⊔*)
(▲) = f(◭, □, ⊏*)
(▲) = f(◭, ⊏*, △)
(▲) = f(◭, ⊏*, □)
(▲) = f(◭, ⊏*, ○)
(▲) = f(◭, ○, ⊏*)
(▲) = f(⊔*, △, ◭)
(▲) = f(⊔*, ◭, △)
(▲) = f(⊔*, ◭, □)
(▲) = f(⊔*, □, ◭)
(▲) = f(⊔*, □, ◨)
(▲) = f(⊔*, ◨, □)
(▲) = f(□, ⊏, ○)
(▲) = f(□, ◭, ⊔*)
(▲) = f(□, ◭, ⊏*)
(▲) = f(□, ⊔*, ◭)
(▲) = f(□, ⊔*, ◨)
(▲) = f(□, ◨, ⊔*)
(▲) = f(□, ◨, ⊏*)
(▲) = f(□, ⊏*, ◭)
(▲) = f(□, ⊏*, ◨)
(▲) = f(□, ○, ⊏)
(▲) = f(◨, ⊏, ○)
(▲) = f(◨, ⊏, ◑)
(▲) = f(◨, ⊔*, □)
(⊏) = f(◭, ◨, ◑)
(⊏) = f(◭, ○, □)
(⊏) = f(◭, ○, ◨)
(⊏) = f(◭, ◑, ◨)
(⊏) = f(▲, □, ○)
(⊏) = f(▲, ◨, ○)
(⊏) = f(▲, ◨, ◑)
(⊏) = f(▲, ■, ○)
(⊏) = f(▲, ■, ◑)
(⊏) = f(▲, ■, ●)
(⊏) = f(▲, ○, □)
(⊏) = f(▲, ○, ◨)
(⊏) = f(▲, ○, ■)
(⊏) = f(▲, ◑, ◨)
(⊏) = f(▲, ◑, ■)
(⊏) = f(▲, ●, ■)
(⊏) = f(□, △, ○)
(⊏) = f(□, ◭, ○)
(⊏) = f(□, ▲, ○)
(⊏) = f(□, ○, △)
(⊏) = f(□, ○, ◭)
(⊏) = f(□, ○, ▲)
(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ▲, ◑)
(⊏) = f(■, ▲, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)

(○) = f(□, ⊏, △)
(○) = f(□, ⊏, ◭)
(○) = f(□, ⊏, ▲)
(○) = f(□, △, ⊓)
(○) = f(□, △, ⊔)
(○) = f(□, △, ⊏)
(○) = f(□, ◭, ⊔)
(○) = f(□, ◭, ⊏)
(○) = f(□, ▲, ⊏)
(○) = f(◨, ⊔, ◭)
(○) = f(◨, ⊏, ◭)
(○) = f(◨, ⊏, ▲)
(○) = f(◨, ◭, ⊔)
(○) = f(◨, ◭, ⊏)
(○) = f(◨, ▲, ⊏)
(○) = f(◨, ▲, ⊏*)
(○) = f(◨, ■, ⊏*)
(○) = f(◨, ⊏*, ▲)
(○) = f(◨, ⊏*, ■)
(○) = f(■, ⊏, ▲)
(○) = f(■, ▲, ⊏)
(○) = f(■, ◨, ⊏*)
(○) = f(■, ⊏*, ◨)
(○) = f(■, ⊏*, ◑)
(○) = f(■, ◑, ⊏*)
(○) = f(⊏*, ◭, ▲)
(○) = f(⊏*, ▲, ◭)
(○) = f(⊏*, ▲, ◨)
(○) = f(⊏*, ▲, ◑)
(○) = f(⊏*, ◨, ▲)
(○) = f(⊏*, ◨, ■)
(○) = f(⊏*, ■, ◨)
(○) = f(⊏*, ■, ◑)
(○) = f(⊏*, ◑, ▲)
(○) = f(⊏*, ◑, ■)
(○) = f(⊏*, ◑, ●)
(○) = f(⊏*, ●, ◑)
(○) = f(◑, ▲, ⊏*)
(○) = f(◑, ■, ⊏*)
(○) = f(◑, ⊏*, ▲)
(○) = f(◑, ⊏*, ■)
(○) = f(◑, ⊏*, ●)
(○) = f(◑, ●, ⊏*)
(○) = f(●, ⊏*, ◑)
(○) = f(●, ◑, ⊏*)

(▲) = f(◨, □, ⊔*)
(▲) = f(◨, □, ⊏*)
(▲) = f(◨, ⊏*, □)
(▲) = f(◨, ⊏*, ○)
(▲) = f(◨, ○, ⊏)
(▲) = f(◨, ○, ⊏*)
(▲) = f(◨, ◑, ⊏)
(▲) = f(■, ⊏, ○)
(▲) = f(■, ⊏, ◑)
(▲) = f(■, ⊏, ●)
(▲) = f(■, ○, ⊏)
(▲) = f(■, ◑, ⊏)
(▲) = f(■, ●, ⊏)
(▲) = f(⊏*, △, ◭)
(▲) = f(⊏*, ◭, △)
(▲) = f(⊏*, ◭, □)
(▲) = f(⊏*, ◭, ○)
(▲) = f(⊏*, □, ◭)
(▲) = f(⊏*, □, ◨)
(▲) = f(⊏*, ◨, □)
(▲) = f(⊏*, ◨, ○)
(▲) = f(⊏*, ○, ◭)
(▲) = f(⊏*, ○, ◨)
(▲) = f(⊏*, ○, ◑)
(▲) = f(⊏*, ◑, ○)
(▲) = f(○, ⊏, □)
(▲) = f(○, ⊏, ◨)
(▲) = f(○, ⊏, ■)
(▲) = f(○, ◭, ⊏*)
(▲) = f(○, □, ⊏)
(▲) = f(○, ◨, ⊏)
(▲) = f(○, ◨, ⊏*)
(▲) = f(○, ■, ⊏)
(▲) = f(○, ⊏*, ◭)
(▲) = f(○, ⊏*, ◨)
(▲) = f(○, ⊏*, ◑)
(▲) = f(○, ◑, ⊏*)
(▲) = f(◑, ⊏, ◨)
(▲) = f(◑, ⊏, ■)
(▲) = f(◑, ◨, ⊏)
(▲) = f(◑, ■, ⊏)
(▲) = f(◑, ⊏*, ○)
(▲) = f(◑, ○, ⊏*)
(▲) = f(●, ⊏, ■)
(▲) = f(●, ■, ⊏)

(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)
(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)
(⊏) = f(◑, ◨, ▲)
(⊏) = f(◑, ■, ▲)
(⊏) = f(●, ▲, ■)
(⊏) = f(●, ■, ▲)

11.

◑	◨	◭	⊔
↓	↓	↓	↓
(◑) = f(⊔, ◭, ◨)	(◨) = f(⊔, ◭, ○)	(◭) = f(⊔, □, ○)	(⊔) = f(△, □, ○)
(◑) = f(⊔, ◨, ◭)	(◨) = f(⊔, ◭, ◑)	(◭) = f(⊔, ◨, ○)	(⊔) = f(△, ○, □)
(◑) = f(⊏, ◭, ◨)	(◨) = f(⊔, ○, ◭)	(◭) = f(⊔, ◨, ◑)	(⊔) = f(◭, □, ○)
(◑) = f(⊏, ▲, ◨)	(◨) = f(⊔, ◑, ◭)	(◭) = f(⊔, ○, □)	(⊔) = f(◭, ◨, ○)
(◑) = f(⊏, ▲, ■)	(◨) = f(⊏, ◭, ○)	(◭) = f(⊔, ○, ◨)	(⊔) = f(◭, ◨, ◑)
(◑) = f(⊏, ◨, ◭)	(◨) = f(⊏, ◭, ◑)	(◭) = f(⊔, ◑, ◨)	(⊔) = f(◭, ○, □)
(◑) = f(⊏, ◨, ▲)	(◨) = f(⊏, ▲, ○)	(◭) = f(⊏, □, ○)	(⊔) = f(◭, ○, ◨)
(◑) = f(⊏, ■, ▲)	(◨) = f(⊏, ▲, ◑)	(◭) = f(⊏, ◨, ○)	(⊔) = f(◭, ◑, ◨)
(◑) = f(◭, ⊔, ◨)	(◨) = f(⊏, ○, ◭)	(◭) = f(⊏, ◨, ◑)	(⊔) = f(□, △, ○)
(◑) = f(◭, ⊏, ◨)	(◨) = f(⊏, ○, ▲)	(◭) = f(⊏, ○, □)	(⊔) = f(□, ◭, ○)
(◑) = f(◭, ◨, ⊔)	(◨) = f(⊏, ◑, ◭)	(◭) = f(⊏, ○, ◨)	(⊔) = f(□, ○, △)
(◑) = f(◭, ◨, ⊏)	(◨) = f(⊏, ◑, ▲)	(◭) = f(⊏, ◑, ◨)	(⊔) = f(□, ○, ◭)
(◑) = f(▲, ⊏, ◨)	(◨) = f(◭, ⊔, ○)	(◭) = f(⊓*, △, ▲)	(⊔) = f(◨, ◭, ○)
(◑) = f(▲, ⊏, ■)	(◨) = f(◭, ⊔, ◑)	(◭) = f(⊓*, ▲, △)	(⊔) = f(◨, ◭, ◑)
(◑) = f(▲, ◨, ⊏)	(◨) = f(◭, ⊏, ○)	(◭) = f(△, ⊓*, ▲)	(⊔) = f(◨, ○, ◭)
(◑) = f(▲, ■, ⊏)	(◨) = f(◭, ⊏, ◑)	(◭) = f(△, ▲, ⊓*)	(⊔) = f(2-2, ◑, ◭)
(◑) = f(▲, ⊏*, ○)	(◨) = f(◭, ○, ⊔)	(◭) = f(△, ▲, ⊔*)	(⊔) = f(○, △, □)
(◑) = f(▲, ○, ⊏*)	(◨) = f(◭, ○, ⊏)	(◭) = f(△, ▲, ⊏*)	(⊔) = f(○, ◭, □)
(◑) = f(◨, ⊔, ◭)	(◨) = f(◭, ◑, ⊔)	(◭) = f(△, ⊔*, ▲)	(⊔) = f(○, ◭, ◨)
(◑) = f(◨, ⊏, ◭)	(◨) = f(◭, ◑, ⊏)	(◭) = f(△, ⊏*, ▲)	(⊔) = f(○, □, △)
(◑) = f(◨, ⊏, ▲)	(◨) = f(▲, ⊏, ○)	(◭) = f(▲, ⊓*, △)	(⊔) = f(○, □, ◭)
(◑) = f(◨, ◭, ⊔)	(◨) = f(▲, ⊏, ◑)	(◭) = f(▲, △, ⊓*)	(⊔) = f(○, ◨, ◭)
(◑) = f(◨, ◭, ⊏)	(◨) = f(▲, ⊔*, □)	(◭) = f(▲, △, ⊔*)	(⊔) = f(◑, ◭, ◨)
(◑) = f(◨, ▲, ⊏)	(◨) = f(▲, □, ⊔*)	(◭) = f(▲, △, ⊏*)	(⊔) = f(◑, ◨, ◭)
(◑) = f(■, ⊏, ▲)	(◨) = f(▲, □, ⊏*)	(◭) = f(▲, ⊔*, △)	
(◑) = f(■, ▲, ⊏)	(◨) = f(▲, ⊏*, □)	(◭) = f(▲, □, ⊔*)	
(◑) = f(■, ⊏*, ○)	(◨) = f(▲, ⊏*, ○)	(◭) = f(▲, ⊏*, △)	
(◑) = f(■, ○, ⊏*)	(◨) = f(▲, ○, ⊏)	(◭) = f(▲, ⊏*, □)	
(◑) = f(⊏*, ▲, ○)	(◨) = f(▲, ○, ⊏*)	(◭) = f(▲, ⊏*, ○)	
(◑) = f(⊏*, ■, ○)	(◨) = f(▲, ◑, ⊏)	(◭) = f(▲, ○, ⊏*)	
(◑) = f(⊏*, ○, ▲)	(◨) = f(⊔*, ▲, □)	(◭) = f(⊔*, ▲, □)	
(◑) = f(⊏*, ○, ■)	(◨) = f(⊔*, □, ▲)	(◭) = f(⊔*, ▲, △)	
(◑) = f(⊏*, ○, ●)	(◨) = f(⊔*, □, ■)	(◭) = f(⊔*, □, ▲)	
(◑) = f(⊏*, ●, ○)	(◨) = f(⊔*, ■, □)	(◭) = f(□, ⊔, ○)	
(◑) = f(○, ▲, ⊏*)	(◨) = f(□, ▲, ⊔*)	(◭) = f(□, ⊏, ○)	
(◑) = f(○, ■, ⊏*)	(◨) = f(□, ▲, ⊏*)	(◭) = f(□, ▲, ⊔*)	
(◑) = f(○, ⊏*, ▲)	(◨) = f(□, ⊔*, ▲)	(◭) = f(□, ▲, ⊏*)	
(◑) = f(○, ⊏*, ■)	(◨) = f(□, ⊔*, ■)	(◭) = f(□, ⊔*, ▲)	
(◑) = f(○, ⊏*, ●)	(◨) = f(□, ■, ⊔*)	(◭) = f(□, ⊏*, ▲)	
(◑) = f(○, ●, ⊏*)	(◨) = f(□, ■, ⊏*)	(◭) = f(□, ○, ⊔)	
(◑) = f(●, ⊏*, ○)	(◨) = f(□, ⊏*, ▲)	(◭) = f(□, ○, ⊏)	

(◑) = f(●, ○, ⊏*)

(◨) = f(□, ⊏*, ■)
(◨) = f(■, ⊔*, □)
(◨) = f(■, □, ⊔*)
(◨) = f(■, □, ⊏*)
(◨) = f(■, ⊏*, □)
(◨) = f(■, ⊏*, ○)
(◨) = f(■, ○, ⊏*)
(◨) = f(⊏*, ▲, □)
(◨) = f(⊏*, ▲, ○)
(◨) = f(⊏*, □, ▲)
(◨) = f(⊏*, □, ■)
(◨) = f(⊏*, ■, □)
(◨) = f(⊏*, ■, ○)
(◨) = f(⊏*, ○, ▲)
(◨) = f(⊏*, ○, ■)
(◨) = f(○, ⊔, ◭)
(◨) = f(○, ⊏, ◭)
(◨) = f(○, ⊏, ▲)
(◨) = f(○, ◭, ⊔)
(◨) = f(○, ◭, ⊏)
(◨) = f(○, ▲, ⊏)
(◨) = f(○, ▲, ⊏*)
(◨) = f(○, ■, ⊏*)
(◨) = f(○, ⊏*, ▲)
(◨) = f(○, ⊏*, ■)
(◨) = f(◑, ⊔, ◭)
(◨) = f(◑, ⊏, ◭)
(◨) = f(◑, ⊏, ▲)
(◨) = f(◑, ◭, ⊔)
(◨) = f(◑, ◭, ⊏)
(◨) = f(◑, ▲, ⊏)

(◭) = f(◨, ⊔, ○)
(◭) = f(◨, ⊔, ◑)
(◭) = f(◨, ⊏, ○)
(◭) = f(◨, ⊏, ◑)
(◭) = f(◨, ○, ⊔)
(◭) = f(◨, ○, ⊏)
(◭) = f(◨, ◑, ⊔)
(◭) = f(◨, ◑, ⊏)
(◭) = f(⊏*, △, ▲)
(◭) = f(⊏*, ▲, △)
(◭) = f(⊏*, ▲, □)
(◭) = f(⊏*, ▲, ○)
(◭) = f(⊏*, □, ▲)
(◭) = f(⊏*, ○, ▲)
(◭) = f(○, ⊔, □)
(◭) = f(○, ⊔, ◨)
(◭) = f(○, ⊏, □)
(◭) = f(○, ⊏, ◨)
(◭) = f(○, ▲, ⊏*)
(◭) = f(○, □, ⊔)
(◭) = f(○, □, ⊏)
(◭) = f(○, ◨, ⊔)
(◭) = f(○, ◨, ⊏)
(◭) = f(○, ⊏*, ▲)
(◭) = f(◑, ⊔, ◨)
(◭) = f(◑, ⊏, ◨)
(◭) = f(◑, ◨, ⊔)
(◭) = f(◑, ◨, ⊏)

12.

◑	◨	◭	⊏
↓	↓	↓	↓
(◑) = f(⊔, ◭, ◨)	(◨) = f(⊔, ◭, ○)	(◭) = f(⊔, □, ○)	(⊏) = f(△, □, ○)
(◑) = f(⊔, ◨, ◭)	(◨) = f(⊔, ◭, ◑)	(◭) = f(⊔, ◨, ○)	(⊏) = f(△, ○, □)
(◑) = f(⊏, ◭, ◨)	(◨) = f(⊔, ○, ◭)	(◭) = f(⊔, ◨, ◑)	(⊏) = f(◭, □, ○)
(◑) = f(⊏, ▲, ◨)	(◨) = f(⊔, ◑, ◭)	(◭) = f(⊔, ○, □)	(⊏) = f(◭, ◨, ○)
(◑) = f(⊏, ▲, ■)	(◨) = f(⊏, ◭, ○)	(◭) = f(⊔, ○, ◨)	(⊏) = f(◭, ◨, ◑)
(◑) = f(⊏, ◨, ◭)	(◨) = f(⊏, ◭, ◑)	(◭) = f(⊔, ◑, ◨)	(⊏) = f(◭, ○, □)
(◑) = f(⊏, ◨, ▲)	(◨) = f(⊏, ▲, ○)	(◭) = f(⊏, □, ○)	(⊏) = f(◭, ○, ◨)
(◑) = f(⊏, ■, ▲)	(◨) = f(⊏, ▲, ◑)	(◭) = f(⊏, ◨, ○)	(⊏) = f(◭, ◑, ◨)
(◑) = f(◭, ⊔, ◨)	(◨) = f(⊏, ○, ◭)	(◭) = f(⊏, ◨, ◑)	(⊏) = f(▲, □, ○)

(◑) = f(◭, ⊏, ◨)
(◑) = f(◭, ◨, ⊔)
(◑) = f(◭, ◨, ⊏)
(◑) = f(▲, ⊏, ◨)
(◑) = f(▲, ⊏, ■)
(◑) = f(▲, ◨, ⊏)
(◑) = f(▲, ■, ⊏)
(◑) = f(▲, ⊏*, ○)
(◑) = f(▲, ○, ⊏*)
(◑) = f(◨, ⊔, ◭)
(◑) = f(◨, ⊏, ◭)
(◑) = f(◨, ⊏, ▲)
(◑) = f(◨, ◭, ⊔)
(◑) = f(◨, ◭, ⊏)
(◑) = f(◨, ▲, ⊏)
(◑) = f(■, ⊏, ▲)
(◑) = f(■, ▲, ⊏)
(◑) = f(■, ⊏*, ○)
(◑) = f(■, ○, ⊏*)
(◑) = f(⊏*, ▲, ○)
(◑) = f(⊏*, ■, ○)
(◑) = f(⊏*, ○, ▲)
(◑) = f(⊏*, ○, ■)
(◑) = f(⊏*, ○, ●)
(◑) = f(⊏*, ●, ○)
(◑) = f(○, ▲, ⊏*)
(◑) = f(○, ■, ⊏*)
(◑) = f(○, ⊏*, ▲)
(◑) = f(○, ⊏*, ■)
(◑) = f(○, ⊏*, ●)
(◑) = f(○, ●, ⊏*)
(◑) = f(●, ⊏*, ○)
(◑) = f(●, ○, ⊏*)

(◨) = f(⊏, ○, ▲)
(◨) = f(⊏, ◑, ◭)
(◨) = f(⊏, ◑, ▲)
(◨) = f(◭, ⊔, ○)
(◨) = f(◭, ⊔, ◑)
(◨) = f(◭, ⊏, ○)
(◨) = f(◭, ⊏, ◑)
(◨) = f(◭, ○, ⊔)
(◨) = f(◭, ○, ⊏)
(◨) = f(◭, ◑, ⊔)
(◨) = f(◭, ◑, ⊏)
(◨) = f(▲, ⊏, ○)
(◨) = f(▲, ⊏, ◑)
(◨) = f(▲, ⊔*, □)
(◨) = f(▲, □, ⊔*)
(◨) = f(▲, □, ⊏*)
(◨) = f(▲, ⊏*, □)
(◨) = f(▲, ⊏*, ○)
(◨) = f(▲, ○, ⊏)
(◨) = f(▲, ○, ⊏*)
(◨) = f(▲, ◑, ⊏)
(◨) = f(⊔*, ▲, □)
(◨) = f(⊔*, □, ▲)
(◨) = f(⊔*, □, ■)
(◨) = f(⊔*, ■, □)
(◨) = f(□, ▲, ⊔*)
(◨) = f(□, ▲, ⊏*)
(◨) = f(□, ⊔*, ▲)
(◨) = f(□, ⊔*, ■)
(◨) = f(□, ■, ⊔*)
(◨) = f(□, ■, ⊏*)
(◨) = f(□, ⊏*, ▲)
(◨) = f(□, ⊏*, ■)
(◨) = f(■, ⊔*, □)
(◨) = f(■, □, ⊔*)
(◨) = f(■, □, ⊏*)
(◨) = f(■, ⊏*, □)
(◨) = f(■, ⊏*, ○)
(◨) = f(■, ○, ⊏*)
(◨) = f(⊏*, ▲, □)
(◨) = f(⊏*, ▲, ○)
(◨) = f(⊏*, □, ▲)
(◨) = f(⊏*, □, ■)
(◨) = f(⊏*, ■, □)
(◨) = f(⊏*, ■, ○)
(◨) = f(⊏*, ○, ▲)

(◭) = f(⊏, ○, □)
(◭) = f(⊏, ○, ◨)
(◭) = f(⊏, ◑, ◨)
(◭) = f(⊓*, △, ▲)
(◭) = f(⊓*, ▲, △)
(◭) = f(△, ⊓*, ▲)
(◭) = f(△, ▲, ⊓*)
(◭) = f(△, ▲, ⊔*)
(◭) = f(△, ▲, ⊏*)
(◭) = f(△, ⊔*, ▲)
(◭) = f(△, ⊏*, ▲)
(◭) = f(▲, ⊓*, △)
(◭) = f(▲, △, ⊓*)
(◭) = f(▲, △, ⊔*)
(◭) = f(▲, △, ⊏*)
(◭) = f(▲, ⊔*, △)
(◭) = f(▲, □, ⊔*)
(◭) = f(▲, ⊏*, △)
(◭) = f(▲, ⊏*, □)
(◭) = f(▲, ⊏*, ○)
(◭) = f(▲, ○, ⊏*)
(◭) = f(⊔*, ▲, □)
(◭) = f(⊔*, ▲, △)
(◭) = f(⊔*, □, ▲)
(◭) = f(□, ⊔, ○)
(◭) = f(□, ⊏, ○)
(◭) = f(□, ▲, ⊔*)
(◭) = f(□, ▲, ⊏*)
(◭) = f(□, ⊔*, ▲)
(◭) = f(□, ⊏*, ▲)
(◭) = f(□, ○, ⊔)
(◭) = f(□, ○, ⊏)
(◭) = f(◨, ⊔, ○)
(◭) = f(◨, ⊔, ◑)
(◭) = f(◨, ⊏, ○)
(◭) = f(◨, ⊏, ◑)
(◭) = f(◨, ○, ⊔)
(◭) = f(◨, ○, ⊏)
(◭) = f(◨, ◑, ⊔)
(◭) = f(◨, ◑, ⊏)
(◭) = f(⊏*, △, ▲)
(◭) = f(⊏*, ▲, △)
(◭) = f(⊏*, ▲, □)
(◭) = f(⊏*, ▲, ○)
(◭) = f(⊏*, □, ▲)
(◭) = f(⊏*, ○, ▲)

(⊏) = f(▲, ◨, ○)
(⊏) = f(▲, ◨, ◑)
(⊏) = f(▲, ■, ○)
(⊏) = f(▲, ■, ◑)
(⊏) = f(▲, ■, ●)
(⊏) = f(▲, ○, □)
(⊏) = f(▲, ○, ◨)
(⊏) = f(▲, ○, ■)
(⊏) = f(▲, ◑, ◨)
(⊏) = f(▲, ◑, ■)
(⊏) = f(▲, ●, ■)
(⊏) = f(□, △, ○)
(⊏) = f(□, ◭, ○)
(⊏) = f(□, ▲, ○)
(⊏) = f(□, ○, △)
(⊏) = f(□, ○, ▲)
(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ▲, ◑)
(⊏) = f(■, ▲, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)
(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)
(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)

(◨) = f(⊏*, ○, ■)	(◭) = f(○, ⊔, □)	(⊏) = f(◑, ◨, ▲)
(◨) = f(○, ⊔, ◭)	(◭) = f(○, ⊔, ◨)	(⊏) = f(◑, ■, ▲)
(◨) = f(○, ⊏, ◭)	(◭) = f(○, ⊏, □)	(⊏) = f(●, ▲, ■)
(◨) = f(○, ⊏, ▲)	(◭) = f(○, ⊏, ◨)	(⊏) = f(●, ■, ▲)
(◨) = f(○, ◭, ⊔)	(◭) = f(○, ▲, ⊏*)	
(◨) = f(○, ◭, ⊏)	(◭) = f(○, □, ⊔)	
(◨) = f(○, ▲, ⊏)	(◭) = f(○, □, ⊏)	
(◨) = f(○, ▲, ⊏*)	(◭) = f(○, ◨, ⊔)	
(◨) = f(○, ■, ⊏*)	(◭) = f(○, ◨, ⊏)	
(◨) = f(○, ⊏*, ▲)	(◭) = f(○, ⊏*, ▲)	
(◨) = f(○, ⊏*, ■)	(◭) = f(◑, ⊔, ◨)	
(◨) = f(◑, ⊔, ◭)	(◭) = f(◑, ⊏, ◨)	
(◨) = f(◑, ⊏, ◭)	(◭) = f(◑, ◨, ⊔)	
(◨) = f(◑, ⊏, ▲)	(◭) = f(◑, ◨, ⊏)	
(◨) = f(◑, ◭, ⊔)		
(◨) = f(◑, ◭, ⊏)		
(◨) = f(◑, ▲, ⊏)		

13.

◑	◨	▲	⊏
(◑) = f(⊔, ◭, ◨)	(◨) = f(⊔, ◭, ○)	(▲) = f(⊏, □, ○)	(⊏) = f(△, □, ○)
(◑) = f(⊔, ◨, ◭)	(◨) = f(⊔, ◭, ◑)	(▲) = f(⊏, □, ○)	(⊏) = f(△, ○, □)
(◑) = f(⊏, ◭, ◨)	(◨) = f(⊔, ○, ◭)	(▲) = f(⊏, ◨, ○)	(⊏) = f(◭, □, ○)
(◑) = f(⊏, ▲, ◨)	(◨) = f(⊔, ◑, ◭)	(▲) = f(⊏, ◨, ◑)	(⊏) = f(◭, ◨, ○)
(◑) = f(⊏, ▲, ■)	(◨) = f(⊏, ◭, ○)	(▲) = f(⊏, ■, ○)	(⊏) = f(◭, ◨, ◑)
(◑) = f(⊏, ◨, ◭)	(◨) = f(⊏, ◭, ◑)	(▲) = f(⊏, ■, ◑)	(⊏) = f(◭, ○, □)
(◑) = f(⊏, ◨, ▲)	(◨) = f(⊏, ▲, ○)	(▲) = f(⊏, ■, ●)	(⊏) = f(◭, ○, ◨)
(◑) = f(⊏, ■, ▲)	(◨) = f(⊏, ▲, ◑)	(▲) = f(⊏, ○, □)	(⊏) = f(◭, ◑, ◨)
(◑) = f(◭, ⊔, ◨)	(◨) = f(⊏, ○, ◭)	(▲) = f(⊏, ○, ◨)	(⊏) = f(▲, □, ○)
(◑) = f(◭, ⊏, ◨)	(◨) = f(⊏, ○, ▲)	(▲) = f(⊏, ○, ■)	(⊏) = f(▲, ◨, ○)
(◑) = f(◭, ◨, ⊔)	(◨) = f(⊏, ◑, ◭)	(▲) = f(⊏, ◑, ◨)	(⊏) = f(▲, ◨, ◑)
(◑) = f(◭, ◨, ⊏)	(◨) = f(⊏, ◑, ▲)	(▲) = f(⊏, ◑, ■)	(⊏) = f(▲, ■, ○)
(◑) = f(▲, ⊏, ◨)	(◨) = f(◭, ⊔, ○)	(▲) = f(⊏, ●, ■)	(⊏) = f(▲, ■, ◑)
(◑) = f(▲, ⊏, ■)	(◨) = f(◭, ⊔, ◑)	(▲) = f(⊓*, △, ◭)	(⊏) = f(▲, ■, ●)
(◑) = f(▲, ◨, ⊏)	(◨) = f(◭, ⊏, ○)	(▲) = f(⊓*, ◭, △)	(⊏) = f(▲, ○, □)
(◑) = f(▲, ■, ⊏)	(◨) = f(◭, ⊏, ◑)	(▲) = f(△, ⊓*, ◭)	(⊏) = f(▲, ○, ◨)
(◑) = f(▲, ⊏*, ○)	(◨) = f(◭, ○, ⊔)	(▲) = f(△, ◭, ⊓*)	(⊏) = f(▲, ○, ■)
(◑) = f(▲, ○, ⊏*)	(◨) = f(◭, ○, ⊏)	(▲) = f(△, ◭, ⊔*)	(⊏) = f(▲, ◑, ◨)
(◑) = f(◨, ⊔, ◭)	(◨) = f(◭, ◑, ⊔)	(▲) = f(△, ◭, ⊏*)	(⊏) = f(▲, ◑, ■)
(◑) = f(◨, ⊏, ◭)	(◨) = f(◭, ◑, ⊏)	(▲) = f(△, ⊏*, ◭)	(⊏) = f(▲, ●, ■)
(◑) = f(◨, ⊏, ▲)	(◨) = f(▲, ⊏, ○)	(▲) = f(◭, ⊓*, △)	(⊏) = f(□, △, ○)
(◑) = f(◨, ◭, ⊔)	(◨) = f(▲, ⊏, ◑)	(▲) = f(◭, △, ⊓*)	(⊏) = f(□, ◭, ○)
(◑) = f(◨, ◭, ⊏)	(◨) = f(▲, ⊔*, □)	(▲) = f(◭, △, ⊔*)	(⊏) = f(□, ▲, ○)

(◑) = f(◨, ▲, ⊏)
(◑) = f(■, ⊏, ▲)
(◑) = f(■, ▲, ⊏)
(◑) = f(■, ⊏*, ○)
(◑) = f(■, ○, ⊏*)
(◑) = f(⊏*, ▲, ○)
(◑) = f(⊏*, ■, ○)
(◑) = f(⊏*, ○, ▲)
(◑) = f(⊏*, ○, ■)
(◑) = f(⊏*, ○, ●)
(◑) = f(⊏*, ●, ○)
(◑) = f(○, ▲, ⊏*)
(◑) = f(○, ■, ⊏*)
(◑) = f(○, ⊏*, ▲)
(◑) = f(○, ⊏*, ■)
(◑) = f(○, ⊏*, ●)
(◑) = f(○, ●, ⊏*)
(◑) = f(●, ⊏*, ○)
(◑) = f(●, ○, ⊏*)

(◨) = f(▲, □, ⊔*)
(◨) = f(▲, □, ⊏*)
(◨) = f(▲, ⊏*, □)
(◨) = f(▲, ⊏*, ○)
(◨) = f(▲, ○, ⊏)
(◨) = f(▲, ○, ⊏*)
(◨) = f(▲, ◑, ⊏)
(◨) = f(⊔*, ▲, □)
(◨) = f(⊔*, □, ▲)
(◨) = f(⊔*, □, ■)
(◨) = f(⊔*, ■, □)
(◨) = f(□, ▲, ⊔*)
(◨) = f(□, ▲, ⊏*)
(◨) = f(□, ⊔*, ▲)
(◨) = f(□, ⊔*, ■)
(◨) = f(□, ■, ⊔*)
(◨) = f(□, ■, ⊏*)
(◨) = f(□, ⊏*, ▲)
(◨) = f(□, ⊏*, ■)
(◨) = f(■, ⊔*, □)
(◨) = f(■, □, ⊔*)
(◨) = f(■, □, ⊏*)
(◨) = f(■, ⊏*, □)
(◨) = f(■, ⊏*, ○)
(◨) = f(■, ○, ⊏*)
(◨) = f(⊏*, ▲, □)
(◨) = f(⊏*, ▲, ○)
(◨) = f(⊏*, □, ▲)
(◨) = f(⊏*, □, ■)
(◨) = f(⊏*, ■, □)
(◨) = f(⊏*, ■, ○)
(◨) = f(⊏*, ○, ▲)
(◨) = f(⊏*, ○, ■)
(◨) = f(○, ⊔, ◭)
(◨) = f(○, ⊏, ◭)
(◨) = f(○, ⊏, ▲)
(◨) = f(○, ◭, ⊔)
(◨) = f(○, ◭, ⊏)
(◨) = f(○, ▲, ⊏)
(◨) = f(○, ▲, ⊏*)
(◨) = f(○, ■, ⊏*)
(◨) = f(○, ⊏*, ▲)
(◨) = f(○, ⊏*, ■)
(◨) = f(◑, ⊔, ◭)(
(◨) = f(◑, ⊏, ◭)
(◨) = f(◑, ⊏, ▲)

(▲) = f(◭, △, ⊏*)
(▲) = f(◭, ⊔*, △)
(▲) = f(◭, ⊔*, □)
(▲) = f(◭, □, ⊔*)
(▲) = f(◭, □, ⊏*)
(▲) = f(◭, ⊏*, △)
(▲) = f(◭, ⊏*, □)
(▲) = f(◭, ⊏*, ○)
(▲) = f(◭, ○, ⊏*)
(▲) = f(⊔*, △, ◭)
(▲) = f(⊔*, ◭, △)
(▲) = f(⊔*, ◭, □)
(▲) = f(⊔*, □, ◭)
(▲) = f(⊔*, □, ◨)
(▲) = f(⊔*, ◨, □)
(▲) = f(□, ⊏, ○)
(▲) = f(□, ◭, ⊔*)
(▲) = f(□, ◭, ⊏*)
(▲) = f(□, ⊔*, ◭)
(▲) = f(□, ⊔*, ◨)
(▲) = f(□, ◨, ⊔*)
(▲) = f(□, ◨, ⊏*)
(▲) = f(□, ⊏*, ◭)
(▲) = f(□, ⊏*, ◨)
(▲) = f(□, ○, ⊏)
(▲) = f(◨, ⊏, ○)
(▲) = f(◨, ⊏, ◑)
(▲) = f(◨, ⊔*, □)
(▲) = f(◨, □, ⊔*)
(▲) = f(◨, □, ⊏*)
(▲) = f(◨, ⊏*, □)
(▲) = f(◨, ⊏*, ○)
(▲) = f(◨, ○, ⊏)
(▲) = f(◨, ○, ⊏*)
(▲) = f(◨, ◑, ⊏)
(▲) = f(■, ⊏, ○)
(▲) = f(■, ⊏, ◑)
(▲) = f(■, ⊏, ●)
(▲) = f(■, ○, ⊏)
(▲) = f(■, ◑, ⊏)
(▲) = f(■, ●, ⊏)
(▲) = f(⊏*, △, ◭)
(▲) = f(⊏*, ◭, △)
▲) = f(⊏*, ◭, □)
(▲) = f(⊏*, ◭, ○)
(▲) = f(⊏*, □, ◭)

(⊏) = f(□, ○, △)
(⊏) = f(□, ○, ▲)
(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ▲, ◑)
(⊏) = f(■, ▲, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)
(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)
(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)
(⊏) = f(◑, ◨, ▲)
(⊏) = f(◑, ■, ▲)
(⊏) = f(●, ▲, ■)
(⊏) = f(●, ■, ▲)

(◨) = f(◑, ◭, ⊔)
(◨) = f(◑, ◭, ⊏)
(◨) = f(◑, ▲, ⊏)

(▲) = f(⊏*, □, ◨)
(▲) = f(⊏*, ◨, □)
(▲) = f(⊏*, ◨, ○)
(▲) = f(⊏*, ○, ◭)
(▲) = f(⊏*, ○, ◨)
(▲) = f(⊏*, ○, ◑)
(▲) = f(⊏*, ◑, ○)
(▲) = f(○, ⊏, □)
(▲) = f(○, ⊏, ◨)
(▲) = f(○, ⊏, ■)
(▲) = f(○, ◭, ⊏*)
(▲) = f(○, □, ⊏)
(▲) = f(○, ◨, ⊏)
(▲) = f(○, ◨, ⊏*)
(▲) = f(○, ■, ⊏)
(▲) = f(○, ⊏*, ◭)
(▲) = f(○, ⊏*, ◨)
(▲) = f(○, ⊏*, ◑)
(▲) = f(○, ◑, ⊏*)
(▲) = f(◑, ⊏, ◨)
(▲) = f(◑, ⊏, ■)
(▲) = f(◑, ◨, ⊏)
(▲) = f(◑, ■, ⊏)
(▲) = f(◑, ⊏*, ○)
(▲) = f(◑, ○, ⊏*)
(▲) = f(●, ⊏, ■)
(▲) = f(●, ■, ⊏)

14.

◑	■	▲	⊏
↓	↓	↓	↓
(◑) = f(⊔, ◭, ◨)	(■) = f(⊏, ▲, ○)	(▲) = f(⊏, □, ○)	(⊏) = f(△, □, ○)
(◑) = f(⊔, ◨, ◭)	(■) = f(⊏, ▲, ◑)	(▲) = f(⊏, □, ○)	(⊏) = f(△, ○, □)
(◑) = f(⊏, ◭, ◨)	(■) = f(⊏, ▲, ●)	(▲) = f(⊏, ◨, ○)	(⊏) = f(◭, □, ○)
(◑) = f(⊏, ▲, ◨)	(■) = f(⊏, ○, ▲)	(▲) = f(⊏, ◨, ◑)	(⊏) = f(◭, ◨, ○)
(◑) = f(⊏, ▲, ■)	(■) = f(⊏, ◑, ▲)	(▲) = f(⊏, ■, ○)	(⊏) = f(◭, ◨, ◑)
(◑) = f(⊏, ◨, ◭)	(■) = f(⊏, ●, ▲)	(▲) = f(⊏, ■, ◑)	(⊏) = f(◭, ○, □)
(◑) = f(⊏, ◨, ▲)	(■) = f(▲, ⊏, ○)	(▲) = f(⊏, ■, ●)	(⊏) = f(◭, ○, ◨)
(◑) = f(⊏, ■, ▲)	(■) = f(▲, ⊏, ◑)	(▲) = f(⊏, ○, □)	(⊏) = f(◭, ◑, ◨)
(◑) = f(◭, ⊔, ◨)	(■) = f(▲, ⊏, ●)	(▲) = f(⊏, ○, ◨)	(⊏) = f(▲, □, ○)
(◑) = f(◭, ⊏, ◨)	(■) = f(▲, ○, ⊏)	(▲) = f(⊏, ○, ■)	(⊏) = f(▲, ◨, ○)
(◑) = f(◭, ◨, ⊔)	(■) = f(▲, ◑, ⊏)	(▲) = f(⊏, ◑, ◨)	(⊏) = f(▲, ◨, ◑)
(◑) = f(◭, ◨, ⊏)	(■) = f(▲, ●, ⊏)	(▲) = f(⊏, ◑, ■)	(⊏) = f(▲, ■, ○)
(◑) = f(▲, ⊏, ◨)	(■) = f(⊔*, □, ◨)	(▲) = f(⊏, ●, ■)	(⊏) = f(▲, ■, ◑)

(◑) = f(▲, ⊏, ■)
(◑) = f(▲, ◨, ⊏)
(◑) = f(▲, ■, ⊏)
(◑) = f(▲, ⊏*, ○)
(◑) = f(▲, ○, ⊏*)
(◑) = f(◨, ⊔, ◭)
(◑) = f(◨, ⊏, ◭)
(◑) = f(◨, ⊏, ▲)
(◑) = f(◨, ◭, ⊔)
(◑) = f(◨, ◭, ⊏)
(◑) = f(◨, ▲, ⊏)
(◑) = f(■, ⊏, ▲)
(◑) = f(■, ▲, ⊏)
(◑) = f(■, ⊏*, ○)
(◑) = f(■, ○, ⊏*)
(◑) = f(⊏*, ▲, ○)
(◑) = f(⊏*, ■, ○)
(◑) = f(⊏*, ○, ▲)
(◑) = f(⊏*, ○, ■)
(◑) = f(⊏*, ○, ●)
(◑) = f(⊏*, ●, ○)
(◑) = f(○, ▲, ⊏*)
(◑) = f(○, ■, ⊏*)
(◑) = f(○, ⊏*, ▲)
(◑) = f(○, ⊏*, ■)
(◑) = f(○, ⊏*, ●)
(◑) = f(○, ●, ⊏*)
(◑) = f(●, ⊏*, ○)
(◑) = f(●, ○, ⊏*)

(■) = f(⊔*, ◨, □)
(■) = f(□, ⊔*, ◨)
(■) = f(□, ◨, ⊔*)
(■) = f(□, ◨, ⊏*)
(■) = f(□, ⊏*, ◨)
(■) = f(◨, ⊔*, □)
(■) = f(◨, □, ⊔*)
(■) = f(◨, □, ⊏*)
(■) = f(◨, ⊏*, □)
(■) = f(◨, ⊏*, ○)
(■) = f(◨, ○, ⊏*)
(■) = f(⊏*, □, ◨)
(■) = f(⊏*, ◨, □)
(■) = f(⊏*, ◨, ○)
(■) = f(⊏*, ○, ◨)
(■) = f(⊏*, ○, ◑)
(■) = f(⊏*, ◑, ○)
(■) = f(○, ⊏, ▲)
(■) = f(○, ▲, ⊏
(■) = f(○, ◨, ⊏*)
(■) = f(○, ⊏*, ◨)
(■) = f(○, ⊏*, ◑)
(■) = f(○, ◑, ⊏*)
(■) = f(◑, ⊏, ▲)
(■) = f(◑, ▲, ⊏)
(■) = f(◑, ⊏*, ○)
(■) = f(◑, ○, ⊏*)
(■) = f(●, ⊏, ▲)
(■) = f(●, ▲, ⊏)

(▲) = f(⊓*, △, ◭)
(▲) = f(⊓*, ◭, △)
(▲) = f(△, ⊓*, ◭)
(▲) = f(△, ◭, ⊓*)
(▲) = f(△, ◭, ⊔*)
(▲) = f(△, ◭, ⊏*)
(▲) = f(△, ⊏*, ◭)
(▲) = f(◭, ⊓*, △)
(▲) = f(◭, △, ⊓*)
(▲) = f(◭, △, ⊔*)
(▲) = f(◭, △, ⊏*)
(▲) = f(◭, ⊔*, △)
(▲) = f(◭, ⊔*, □)
(▲) = f(◭, □, ⊔*)
(▲) = f(◭, □, ⊏*)
(▲) = f(◭, ⊏*, △)
(▲) = f(◭, ⊏*, □)
(▲) = f(◭, ⊏*, ○)
(▲) = f(◭, ○, ⊏*)
(▲) = f(⊔*, △, ◭)
(▲) = f(⊔*, ◭, △)
(▲) = f(⊔*, ◭, □)
(▲) = f(⊔*, □, ◭)
(▲) = f(⊔*, □, ◨)
(▲) = f(⊔*, ◨, □)
(▲) = f(□, ⊏, ○)
(▲) = f(□, ◭, ⊔*)
(▲) = f(□, ◭, ⊏*)
(▲) = f(□, ⊔*, ◭)
(▲) = f(□, ⊔*, ◨)
(▲) = f(□, ◨, ⊔*)
(▲) = f(□, ◨, ⊏*)
(▲) = f(□, ⊏*, ◭)
(▲) = f(□, ⊏*, ◨)
(▲) = f(□, ○, ⊏)
(▲) = f(◨, ⊏, ○)
(▲) = f(◨, ⊏, ◑)
(▲) = f(◨, ⊔*, □)
(▲) = f(◨, □, ⊔*)
(▲) = f(◨, □, ⊏*)
(▲) = f(◨, ⊏*, □)
(▲) = f(◨, ⊏*, ○)
(▲) = f(◨, ○, ⊏)
(▲) = f(◨, ○, ⊏*)
(▲) = f(◨, ◑, ⊏)
(▲) = f(■, ⊏, ○)

(⊏) = f(▲, ■, ●)
(⊏) = f(▲, ○, □)
(⊏) = f(▲, ○, ◨)
(⊏) = f(▲, ○, ■)
(⊏) = f(▲, ◑, ◨)
(⊏) = f(▲, ◑, ■)
(⊏) = f(▲, ●, ■)
(⊏) = f(□, △, ○)
(⊏) = f(□, ◭, ○)
(⊏) = f(□, ▲, ○)
(⊏) = f(□, ○, △)
(⊏) = f(□, ○, ▲)
(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ▲, ◑)
(⊏) = f(■, ▲, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)
(⊏) = f(○, ◨, ▲)
(⊏) = f(○, ■, ▲)
(⊏) = f(◑, ◭, ◨)
(⊏) = f(◑, ▲, ◨)
(⊏) = f(◑, ▲, ■)
(⊏) = f(◑, ◨, ◭)
⊏) = f(◑, ◨, ▲)
(⊏) = f(◑, ■, ▲)
(⊏) = f(●, ▲, ■)
(⊏) = f(●, ■, ▲)

(▲) = f(■, ⊏, ◑)
(▲) = f(■, ⊏, ●)
(▲) = f(■, ○, ⊏)
(▲) = f(■, ◑, ⊏)
(▲) = f(■, ●, ⊏)
(▲) = f(⊏*, △, ◭)
(▲) = f(⊏*, ◭, △)
(▲) = f(⊏*, ◭, □)
(▲) = f(⊏*, ◭, ○)
(▲) = f(⊏*, □, ◭)
(▲) = f(⊏*, □, ◨)
(▲) = f(⊏*, ◨, □)
(▲) = f(⊏*, ◨, ○)
(▲) = f(⊏*, ○, ◭)
(▲) = f(⊏*, ○, ◨)
(▲) = f(⊏*, ○, ◑)
(▲) = f(⊏*, ◑, ○)
(▲) = f(○, ⊏, □)
(▲) = f(○, ⊏, ◨)
(▲) = f(○, ⊏, ■)
(▲) = f(○, ◭, ⊏*)
(▲) = f(○, □, ⊏)
(▲) = f(○, ◨, ⊏)
(▲) = f(○, ◨, ⊏*)
(▲) = f(○, ■, ⊏)
(▲) = f(○, ⊏*, ◭)
(▲) = f(○, ⊏*, ◨)
(▲) = f(○, ⊏*, ◑)
(▲) = f(○, ◑, ⊏*)
(▲) = f(◑, ⊏, ◨)
(▲) = f(◑, ⊏, ■)
(▲) = f(◑, ◨, ⊏)
(▲) = f(◑, ■, ⊏)
(▲) = f(◑, ⊏*, ○)
(▲) = f(◑, ○, ⊏*)
(▲) = f(●, ⊏, ■)
(▲) = f(●, ■, ⊏)

15.

●	■	▲	⊏
↓	↓	↓	↓
(●) = f(⊏, ▲, ■)	(■) = f(⊏, ▲, ○)	(▲) = f(⊏, □, ○)	(⊏) = f(△, □, ○)
(●) = f(⊏, ■, ▲)	(■) = f(⊏, ▲, ◑)	(▲) = f(⊏, □, ○)	(⊏) = f(△, ○, □)
(●) = f(▲, ⊏, ■)	(■) = f(⊏, ▲, ●)	(▲) = f(⊏, ◨, ○)	(⊏) = f(◭, □, ○)

(●) = f(▲, ■, ⊏)
(●) = f(■, ⊏, ▲)
(●) = f(■, ▲, ⊏)
(●) = f(⊏*, ○, ◑)
(●) = f(⊏*, ◑, ○)
(●) = f(○, ⊏*, ◑)
(●) = f(○, ◑, ⊏*)
(●) = f(◑, ⊏*, ○)
(●) = f(◑, ○, ⊏*)

(■) = f(⊏, ○, ▲)
(■) = f(⊏, ◑, ▲)
(■) = f(⊏, ●, ▲)
(■) = f(▲, ⊏, ○)
(■) = f(▲, ⊏, ◑)
(■) = f(▲, ⊏, ●)
(■) = f(▲, ○, ⊏)
(■) = f(▲, ◑, ⊏)
(■) = f(▲, ●, ⊏)
(■) = f(⊔*, □, ◨)
(■) = f(⊔*, ◨, □)
(■) = f(□, ⊔*, ◨)
(■) = f(□, ◨, ⊔*)
(■) = f(□, ◨, ⊏*)
(■) = f(□, ⊏*, ◨)
(■) = f(◨, ⊔*, □)
(■) = f(◨, □, ⊔*)
(■) = f(◨, □, ⊏*)
(■) = f(◨, ⊏*, □)
(■) = f(◨, ⊏*, ○)
(■) = f(◨, ○, ⊏*)
(■) = f(⊏*, □, ◨)
(■) = f(⊏*, ◨, □)
(■) = f(⊏*, ◨, ○)
(■) = f(⊏*, ○, ◨)
(■) = f(⊏*, ○, ◑)
(■) = f(⊏*, ◑, ○)
(■) = f(○, ⊏, ▲)
(■) = f(○, ▲, ⊏
(■) = f(○, ◨, ⊏*)
(■) = f(○, ⊏*, ◨)
(■) = f(○, ⊏*, ◑)
(■) = f(○, ◑, ⊏*)
(■) = f(◑, ⊏, ▲)
(■) = f(◑, ▲, ⊏)
(■) = f(◑, ⊏*, ○)
(■) = f(◑, ○, ⊏*)
(■) = f(●, ⊏, ▲)
(■) = f(●, ▲, ⊏)

(▲) = f(⊏, ◨, ◑)
(▲) = f(⊏, ■, ○)
(▲) = f(⊏, ■, ◑)
(▲) = f(⊏, ■, ●)
(▲) = f(⊏, ○, □)
(▲) = f(⊏, ○, ◨)
(▲) = f(⊏, ○, ■)
(▲) = f(⊏, ◑, ◨)
(▲) = f(⊏, ◑, ■)
(▲) = f(⊏, ●, ■)
(▲) = f(⊓*, △, ◭)
(▲) = f(⊓*, ◭, △)
(▲) = f(△, ⊓*, ◭)
(▲) = f(△, ◭, ⊓*)
(▲) = f(△, ◭, ⊔*)
(▲) = f(△, ◭, ⊏*)
(▲) = f(△, ⊏*, ◭)
(▲) = f(◭, ⊓*, △)
(▲) = f(◭, △, ⊓*)
(▲) = f(◭, △, ⊔*)
(▲) = f(◭, △, ⊏*)
(▲) = f(◭, ⊔*, △)
(▲) = f(◭, ⊔*, □)
(▲) = f(◭, □, ⊔*)
(▲) = f(◭, □, ⊏*)
(▲) = f(◭, ⊏*, △)
(▲) = f(◭, ⊏*, □)
(▲) = f(◭, ⊏*, ○)
(▲) = f(◭, ○, ⊏*)
(▲) = f(⊔*, △, ◭)
(▲) = f(⊔*, ◭, △)
(▲) = f(⊔*, ◭, □)
(▲) = f(⊔*, □, ◭)
(▲) = f(⊔*, □, ◨)
(▲) = f(⊔*, ◨, □)
(▲) = f(□, ⊏, ○)
(▲) = f(□, ◭, ⊔*)
(▲) = f(□, ◭, ⊏*)
(▲) = f(□, ⊔*, ◭)
(▲) = f(□, ⊔*, ◨)
(▲) = f(□, ◨, ⊔*)
(▲) = f(□, ◨, ⊏*)
(▲) = f(□, ⊏*, ◭)
(▲) = f(□, ⊏*, ◨)
(▲) = f(□, ○, ⊏)
(▲) = f(◨, ⊏, ○)

(⊏) = f(◭, ◨, ○)
(⊏) = f(◭, ◨, ◑)
(⊏) = f(◭, ○, □)
(⊏) = f(◭, ○, ◨)
(⊏) = f(◭, ◑, ◨)
(⊏) = f(▲, □, ○)
(⊏) = f(▲, ◨, ○)
(⊏) = f(▲, ◨, ◑)
(⊏) = f(▲, ■, ○)
(⊏) = f(▲, ■, ◑)
(⊏) = f(▲, ■, ●)
(⊏) = f(▲, ○, □)
(⊏) = f(▲, ○, ◨)
(⊏) = f(▲, ○, ■)
(⊏) = f(▲, ◑, ◨)
(⊏) = f(▲, ◑, ■)
(⊏) = f(▲, ●, ■)
(⊏) = f(□, △, ○)
(⊏) = f(□, ◭, ○)
(⊏) = f(□, ▲, ○)
(⊏) = f(□, ○, △)
(⊏) = f(□, ○, ▲)
(⊏) = f(◨, ◭, ○)
(⊏) = f(◨, ◭, ◑)
(⊏) = f(◨, ▲, ○)
(⊏) = f(◨, ▲, ◑)
(⊏) = f(◨, ○, ◭)
(⊏) = f(◨, ○, ▲)
(⊏) = f(◨, ◑, ◭)
(⊏) = f(◨, ◑, ▲)
(⊏) = f(■, ▲, ○)
(⊏) = f(■, ▲, ◑)
(⊏) = f(■, ▲, ●)
(⊏) = f(■, ○, ▲)
(⊏) = f(■, ◑, ▲)
(⊏) = f(■, ●, ▲)
(⊏) = f(○, △, □)
(⊏) = f(○, ◭, □)
(⊏) = f(○, ◭, ◨)
(⊏) = f(○, ▲, □)
(⊏) = f(○, ▲, ◨)
(⊏) = f(○, ▲, ■)
(⊏) = f(○, □, △)
(⊏) = f(○, □, ◭)
(⊏) = f(○, □, ▲)
(⊏) = f(○, ◨, ◭)

(▲) = f(◨, ⊏, ◑)

(▲) = f(◨, ⊔*, □)

(▲) = f(◨, □, ⊔*)

(▲) = f(◨, □, ⊏*)

(▲) = f(◨, ⊏*, □)

(▲) = f(◨, ⊏*, ○)

(▲) = f(◨, ○, ⊏)

(▲) = f(◨, ○, ⊏*)

(▲) = f(◨, ◑, ⊏)

(▲) = f(■, ⊏, ○)

(▲) = f(■, ⊏, ◑)

(▲) = f(■, ⊏, ●)

(▲) = f(■, ○, ⊏)

(▲) = f(■, ◑, ⊏)

(▲) = f(■, ●, ⊏)

(▲) = f(⊏*, △, ◭)

(▲) = f(⊏*, ◭, △)

(▲) = f(⊏*, ◭, □)

(▲) = f(⊏*, ◭, ○)

(▲) = f(⊏*, □, ◭)

(▲) = f(⊏*, □, ◨)

(▲) = f(⊏*, ◨, □)

(▲) = f(⊏*, ◨, ○)

(▲) = f(⊏*, ○, ◭)

(▲) = f(⊏*, ○, ◨)

(▲) = f(⊏*, ○, ◑)

(▲) = f(⊏*, ◑, ○)

(▲) = f(○, ⊏, □)

(▲) = f(○, ⊏, ◨)

(▲) = f(○, ⊏, ■)

(▲) = f(○, ◭, ⊏*)

(▲) = f(○, □, ⊏)

(▲) = f(○, ◨, ⊏)

(▲) = f(○, ◨, ⊏*)

(▲) = f(○, ■, ⊏)

(▲) = f(○, ⊏*, ◭)

(▲) = f(○, ⊏*, ◨)

(▲) = f(○, ⊏*, ◑)

(▲) = f(○, ◑, ⊏*)

(▲) = f(◑, ⊏, ◨)

(▲) = f(◑, ⊏, ■)

(▲) = f(◑, ◨, ⊏)

(▲) = f(◑, ■, ⊏)

(▲) = f(◑, ⊏*, ○)

(▲) = f(◑, ○, ⊏*)

(▲) = f(●, ⊏, ■)

(⊏) = f(○, ◨, ▲)

(⊏) = f(○, ■, ▲)

(⊏) = f(◑, ◭, ◨)

(⊏) = f(◑, ▲, ◨)

(⊏) = f(◑, ▲, ■)

(⊏) = f(◑, ◨, ◭)

(⊏) = f(◑, ◨, ▲)

(⊏) = f(◑, ■, ▲)

(⊏) = f(●, ▲, ■)

(⊏) = f(●, ■, ▲)

(▲) = f(●, ■, □)

Man kann sich leicht vorstellen, welche astronomische semiotische Komplexität entsteht, wenn nur schon zwei der fünfzehn polykontexturalen Repräsentationssysteme miteinander in Verbindung gesetzt werden. Ein vergleichsweise simples Beispiel findet man im 2. Teil von Toth (2008b, S. 143 ff.). Angesichts der enormen Komplexität dieser kleinen Ausschnitte aus dem "semiotic web", das natürlich durch jede kommunikative, kreative und repräsentative Handlung in einem Teil ihres Netzes aktiviert wird, wird man an Kafkas Diktum erinnert, dass man eigentlich tot zusammenbrechen müsste, würde man nur imstande sein, die ganze auf einen einströmende Information zu apperzipieren, sobald man nur einen Schritt vor seine Haustüre setzt.

Bibliographie

Toth, Alfred, Semiotic Ghost Trains. Klagenfurt 2008 (2008a)
Toth, Alfred, Grundzüge einer Semiotik des Hotelgewerbes. Klagenfurt 2008 (2008b)
Toth, Alfred, Entwurf einer handlungstheoretischen Semiotik. Klagenfurt 2008 (2008c)
Toth, Alfred, Einführung polykontextural-semiotischer Funktionen. In: Electronic Journal for Mathematical Semiotics, 2008d
Toth, Alfred, Die qualitativen polykontextural-semiotischen Funktionen. In: Electronic Journal for Mathematical Semiotics, 2009

Zwei Formen polykontexturaler Referenz

1. Ich beziehe mich hier auf zwei Vorläuferarbeiten über monokontexturale (Toth 2008a) und polykontexturale (Toth 2008b) Referenz. Die erste Möglichkeit polykontexturaler Referenz besteht im Subzeichen selbst, das immer in drei Erscheinungsformen auftreten kann:

(a.b) (Normalform)
(a.b)° (Konverse)
×(a.b) (Duale)

Wie in Toth (2009) gezeigt worden war, fallen Konverse und Duale nur in monokontexturalen und höchstens 3-polykontexturalen Systemen zusammen, so dass dort also

$(a.b)° = \times(a.b)$

gilt. Ab n = 4 haben wir aber die Ungleichung

$(a.b)° \neq \times(a.b)$,

d.h. ab 4 Kontexturen erscheint jedes nicht-selbstduale Subzeichen in drei verschiedenen Formen:

$$\left.\begin{array}{l}(M_{1,4})° = O_{1,4} \\ \times(M_{1,4}) = O_{4,1} \\ O_{1,4} \neq O_{4,1}\end{array}\right\} \qquad M_{1,4} \;/\; O_{1,4} \;/\; O_{4,1}$$

$$\left.\begin{array}{l}(M_{3,4})° = I_{3,4} \\ \times(M_{3,4}) = I_{4,3} \\ I_{3,4} \neq I_{4,3}\end{array}\right\} \qquad M_{3,4} \;/\; I_{3,4} \;/\; I_{4,3}$$

$$\left.\begin{array}{l}(O_{2,4})° = I_{2,4} \\ \times(O_{2,4}) = I_{4,2} \\ I_{2,4} \neq I_{4,2}\end{array}\right\} \qquad O_{2,4} \;/\; I_{2,4} \;/\; I_{4,2}$$

Man kann nun diesen Subzeichen – ähnlich wie dies Kaehr (2009, S. 15) getan hatte, relativ wirklich logisch-erkenntnistheoretische Funktionen (Subjekt, Objekt, subjektives/ objektives Subjekt und Objekt, evtl. weitere wie das Kaehrsche „Abjekt" usw.) zuschreiben und auf diese Weise polykontexturale Referenzsysteme aufbauen.

2. Eine zweite Möglichkeit ergibt sich, wenn man direkt von der 4-polykontexturalen semiotischen Matrix ausgeht

$$\begin{pmatrix} 1.1_{1,3,4} & 1.2_{1,4} & 1.3_{3,4} \\ 2.1_{1,4} & 2.2_{1,2,4} & 2.3_{2,4} \\ 3.1_{3,4} & 3.2_{2,4} & 3.3_{2,3,4} \end{pmatrix}$$

und sie in modalkategoriale Form umschreibt

$$\begin{pmatrix} M_{1,3,4} & M_{1,4} & M_{3,4} \\ O_{1,4} & O_{1,2,4} & O_{2,4} \\ I_{3,4} & I_{2,4} & I_{2,3,4} \end{pmatrix}$$

Dann kann man, ähnlich willkürlich, z.B. die Triaden im Sinne von Hier-, Da-, Dort-Deixis und die Trichotomien im Sinne von Ich, Du, Es interpretieren. Im Gegensatz zur 1. Möglichkeit von man z.B. Möglichkeiten hat, den Numerus (Ich – Wir; Du – Ihr; Er – Sie; Es) einzubringen, ist dies hier bedeutend problematischer (vgl. Toth 2008b).

Bibliographie

Kaehr, Rudolf, Xanadus textemes. In: www.mathematical-semiotics.com/.../Monok.%20u.%20%20polyk%20Umg%20Sit..pdf (2009)

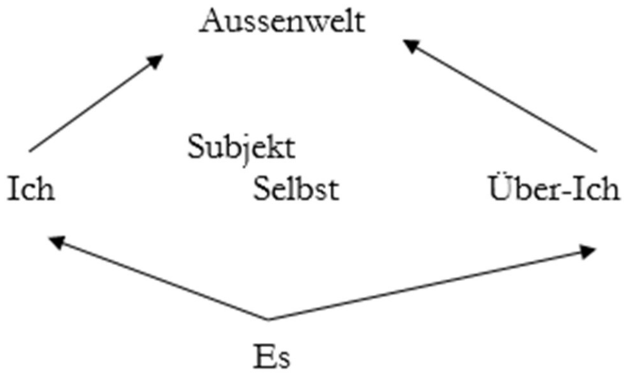
Toth, Alfred, Reference in theoretical semiotics. In: In: Electronic Journal for Mathematical Semiotics, 2008a

Toth, Alfred, Reference in poly-contextural semiotics. In: In: Electronic Journal for Mathematical Semiotics, 2008b

Toth, Alfred, Subjekt und Objekt und ihr jeweiliges Anderes. In: Electronic Journal of Mathematical Semiotics, 2009

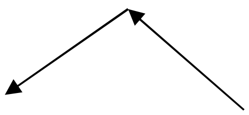
Kann das Diamantenmodell Lacans in die komplexe semiotische Ebene eingebettet werden?

1. Perner schematisierte in seinem Rezensionsaufsatz zu einer deutschen Edition von Lacans „Spiegelstadium“ dessen Diamantenmodell:



Anderseits findet man das folgende Dreiecksmodell für die Lacansche Unterscheidung von Symbolik, Realem und Imaginärem:

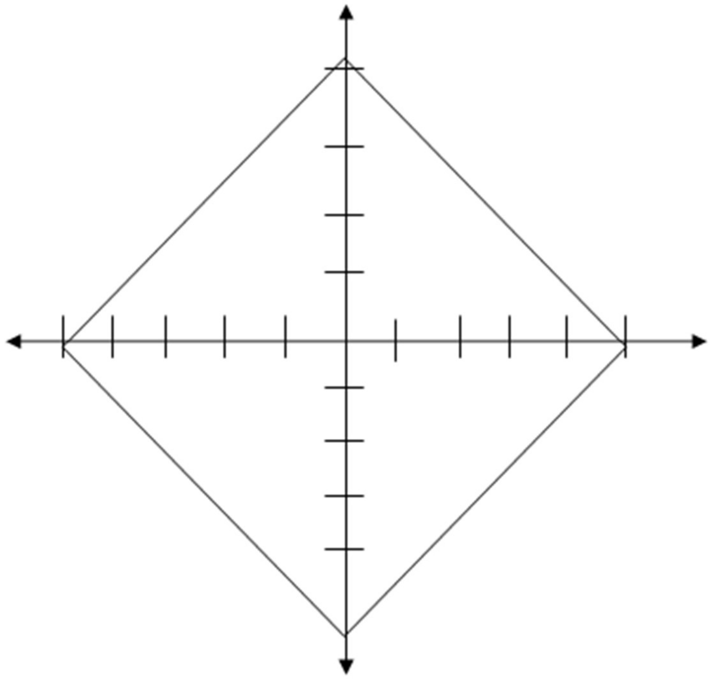
Imaginäres



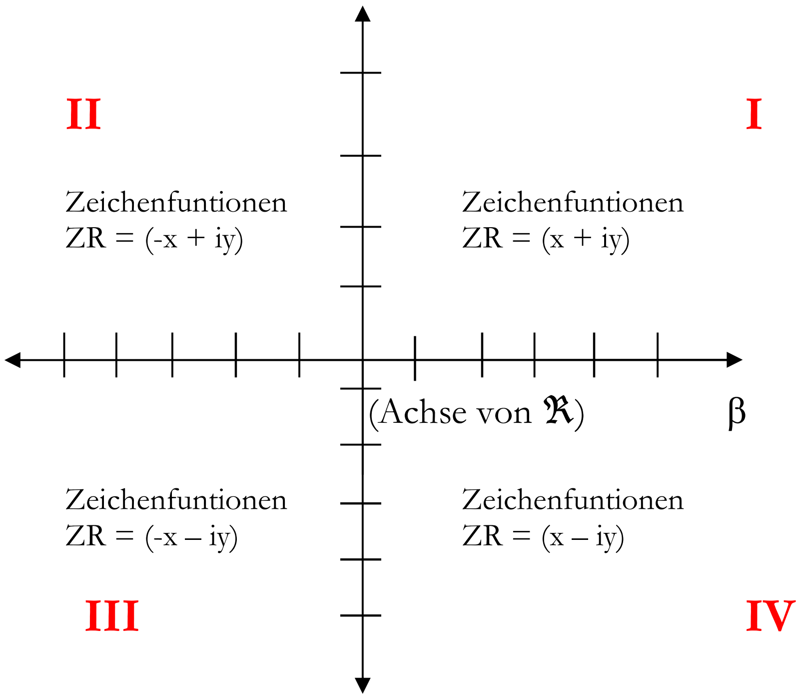
Symbolisches.......... Reales,

wobei zwischen dem Symbolischen und dem Realen das mysteriöse „Object A“ vermittelt.

2. Wenn man vom Lacanschen Dreieck ausgeht, kann man es zu einem Rhombus, also einem Diamanten, vervollständigen und danach in die in Toth (2009) eingeführte komplexe semiotische Ebene einbetten:



Die zugrunde liegende semiotische (Gaußsche) Zahlenebene sieht dann wie folgt aus:



Die im I. Quadranten zu liegen kommende Ecke des Rhombus wäre dann das „Reale", die im II. Q. zu liegen kommende Ecke das Symbolische 1, die im III. Q. zu liegen kommende Ecke das Imaginäre, und die im IV. Quadranten zu liegende Ecke des Rhombus wäre eine 2. Symbolische Sphäre. 1. und 2. symbolische Sphären entsprächen damit der semiotischen Unterscheidung zwischen idealistischem ([-Ω, +β]) und materialistischem ([+Ω, -β]) Zeichenbereich. Das Imaginäre deckte sich ferner mit der güntherschen Meontik, wozu man bereits Bense (1952, S. 78 ff. u. S. 115, Anm. 72) berücksichtige. Speziell zur Lacanschen Problematik der

„Doppelgänger" ist es nötig, zu berücksichtigen, dass diese sowohl im Idealismus als auch im Materialismus wurzeln; vgl. hierzu Günther: „Es wird in Zukunft immer weniger gestattet sein, dasjenige als Geist zu erklären, was in Wahrheit Materie ist. In dieser Verwechslung hat der Glaube an Gespenster seine Wurzel" (Günther 1980, S. 230 f.). – „Gewiss ist es Zeichen mangelnder metaphysischer Begabung, wenn man sich nicht vor Gespenstern und Leichen fürchten kann oder gar keine Veranlagung zum ‚Aberglauben' hat" (Günther 2000, S. 208). Im letzteren Fall handelt es sich eben z.B. um materielose Geister, wie man sie aus Filmen usw. kennt, die durch Wände und Türen gehen, nicht erschossen werden können und für die generell die Gesetze der Physik nicht gelten.

Reales würde dann durch Zeichenklassen und Realitätsthematiken der allgemeinen Form

(3.a 2.b 1.c) × (c.1 b.2 a.3),

Imaginäres durch solche der Form

(-3.-a –2.-b –1.-c) × (-c.-1 –b.-2 –a.-3),

Symbolisch-Idealistisches durch solche der Form

(3.-a 2.-b 1.-c) × (-c.1 –b.2 –a.1),

und Symbolisch-Materialistisches durch solche der Form

(-3.a –2.b –1.c) × (c.-1 b.-2 a.-3)

klassifiziert, wobei man beachte, dass die Realitätsthematiken der symbolisch-materialistischen Zeichenklassen symbolisch-idealistisch sind und umgekehrt! D.h. innerhalb des gesamten, aus Zeichenklasse und Realitätsthematik bestehenden Repräsentationssystems, ist der Unterschied zwischen Materialismus und Idealismus aufgehoben und vermutlich deshalb intuitiv von Lacan nicht weiter differenziert worden.

Bibliographie

Bense, Max, Die Theorie Kafkas. Köln 1952

Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. Bd. 3. Hamburg 1980

Günther, Gotthard, Die amerikanische Apokalypse. München 2000

Perner, Achim, Einführende Bemerkungen zu Jacques Lacan .... In: http://www.freud-lacan-berlin.de/res/Perner_Einfuehrung_Spiegelstadium.pdf

Toth, Alfred, Komplexe semiotische Analyse. In: In: Electronic Journal for Mathematical Semiotics, 2009

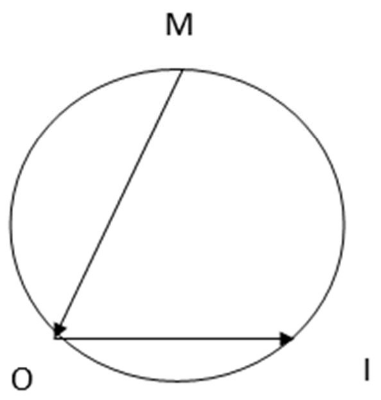
Zeichenrelationen von Bisimulativgeichungen mit leerer Menge und semiotische Diamanten

1. Zu „semiotischen Diamanten“ vgl. Toth (2008, S. 177 ff.) und Kaehr (2008). Sie lassen sich sehr gut mit Hilfe von Kreismodellen darstellen, ähnlich wie dies Günther (1979) mit den Negationszyklen getan hatte. Wie in allen letzten Arbeiten gehen wir von Aczels (1988) Mengentheorie (ohne Plenitudidäts-Axiom) aus und definieren:

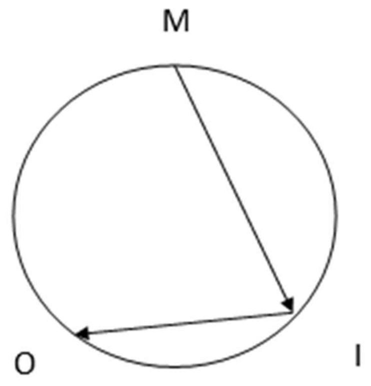
x = {{x}, Ø}, y = {{{x, y}}, Ø}, z = {{{{{x, y, z}}}}, Ø}

Da jede triadische Zeichenrelation 3! = 6 Permutationen hat, bekommen wir die im folgenden präsentierten 6 Darstellungen:

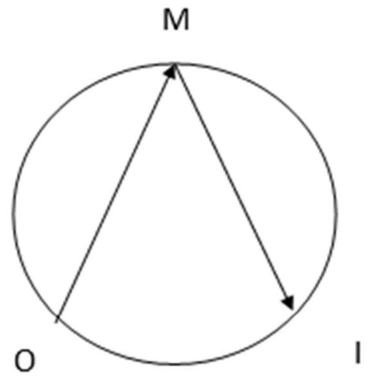
1. (M → O → I) := {{{x}, Ø}, {{{x, y}}, Ø}, {{{{{x, y, z}}}}, Ø}}



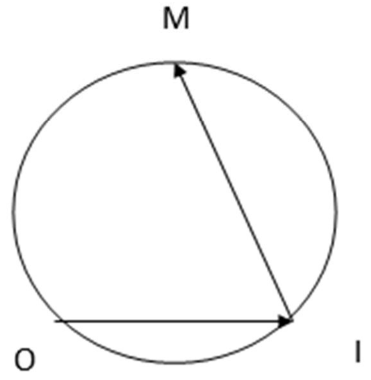
2. (M → I → O) := {{{x}, Ø}, {{{{{x, y, z}}}}, Ø}, {{{x, y}}, Ø}}



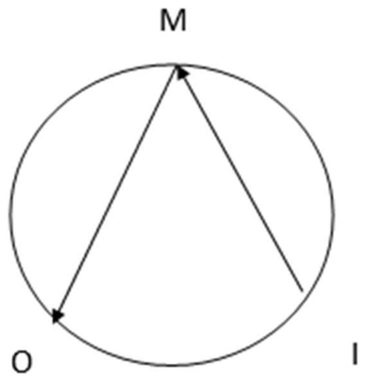
3. (O → M → I) := {{{{x, y}}, Ø}, {{x}, Ø}, {{{{{x, y, z}}}}, Ø}}



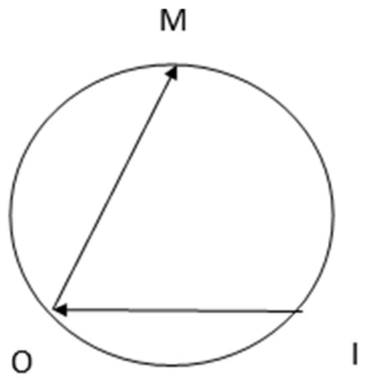
4. (O → I → M) := {{{x, y}}, Ø}, {{{{{x, y, z}}}}, Ø}, {{x}, Ø}}



5. (I → M → O) := {{{{{{x, y, z}}}}, Ø}, {{x}, Ø}, {{{x, y}}, Ø}}



6. (I → O → M) := {{{{{{x, y, z}}}}, Ø}, {{{x, y}}, {{x}, Ø}}



Bibliographie

Aczel, Peter, Non-well-founded sets. Cambridge1988

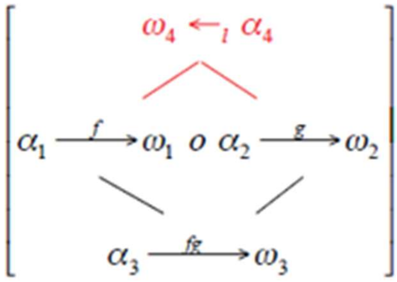
Günther, Gotthard, Beiträge zur Grundlegung einer operationsfähigen Dialektik. 2 Bd. Hamburg 1979

Kaehr, Rudolf, Toth's semiotic diamonds. http://www.thinkartlab.com/pkl/lola/Toth-Diamanten/Toth-Diamanten.pdf (2008)

Toth, Alfred, Semiotische Strukturen und Prozesse. Klagenfurt 2008

Semiotische Diamanten und Realitätsthematiken

1. Der wesentliche Unterschied zwischen einer (algebraischen) Kategorie und einem (polykontexturalen) Diamanten, wie er von Kaehr (2008) eingeführt worden ist, liegt darin, dass letzterer einen sog. Heteromorphismus besitzt, eine Inversionsfunktion der komponierten Abbildungen der in den Diamanten eingebetteten Kategorie (Abb. aus Kaehr 2008, S. 14):



Es ist also

ZR = ($\alpha_1 \rightarrow \omega_1/\omega_2 \rightarrow \alpha_2$), Het. = ($\alpha_2 \rightarrow \alpha_1$),

wobei in Het. die Reihenfolge der Kontexturenzahlen, sofern es 2 oder mehr sind, umgestellt wird:

ZR = ($\alpha_{\alpha.\beta} \rightarrow \omega_{\gamma.\delta} \rightarrow \alpha_{\varepsilon\zeta}$), Het. = ($\alpha_{\zeta,\varepsilon} \rightarrow \alpha_{\beta.\alpha}$).

2. Dagegen ergibt sich ein von der Zeichenklasse verschiedener Diamant, wenn wir die entsprechende Realitätsthematik nehmen:

RR = ($\alpha^\circ_{\zeta,\varepsilon} \rightarrow \omega^\circ_\delta/\omega^\circ_\gamma \rightarrow \alpha^\circ_{\beta.\alpha}$), Het. = ($\alpha^\circ_{\beta.\alpha} \rightarrow \alpha^\circ_{\zeta.\varepsilon}$).

Man lernt hieraus, dass, wenn man die ganzen Dualsysteme betrachtet, die Konversen von Subzeichen nicht mit den Dualia zusammenfallen, den je für die Matrix einer ZR und je für die Matrix einer RR gilt zwar

$(a.b)^{\circ} = \times(a.b) = (b.a)$,

aber wenn man beide „Pole der semiotischen Erkenntnisrelation" (Gfesser), d.h ZR und RR berücksichtigt, gilt

$(a.b)_{\alpha.\beta}{}^{\circ} = (b.a)_{\alpha.\beta} \neq \times(a.b)_{\alpha.\beta} = (b.a)_{\beta.\alpha}$.

3. Die Kaehrsche Entdeckung der Heteromorphismen, die angewandt auf die ZR-Matrize, die Konversen der Subzeichen einer Zeichenklasse, und, angewandt auf die RR-Matrize, deren Dualia liefert, lehrt uns also vor allem, dass es im Falle von semiotischer Repräsentation nicht genügt, die drei Subzeichen einer Zeichenklasse, also z.B.

ZR = (a.b c.d e.f)

zu berücksichtigen, sondern dass zu jedem Subzeichen auch der konverse und der duale „Zwilling" gefunden werden muss:

$$DS = \left\{ \begin{array}{l} (a.b) = \{(a.b), (a.b)^{\circ}, \times(a.b)\} \\ (c.d) = \{(c.d), (c.d)^{\circ}, \times(c.d)\} \\ (e.f) = \{(e.f), (e.f)^{\circ}, \times(e.f)\} \end{array} \right\}$$

Matrixdarstellung für DS = (3.2 2.2 1.3 × 3.1 2.2 2.3):

$$\left(\begin{pmatrix} 1.1 & 1.2 & \underline{e.f} \\ 2.1 & \underline{c.d} & a.b \\ \underline{(e.f)^{\circ}} & \underline{(a.b)^{\circ}} & 3.3 \end{pmatrix} \quad \begin{pmatrix} 1.1 & 1.2 & 1.3 \\ 2.1 & \underline{c.d} & 2.3 \\ \underline{\times(e.f)} & \underline{\times(a.b)} & 3.3 \end{pmatrix} \right)$$

Bibliographie

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2009

27.11.2010

Die Diamantenrelation als Relation über Relationen

1. Während die Peircesche Zeichenrelation, als „Relation über Relationen" geschrieben, wie folgt aussieht:

ZR = (M, ((M, M → O), (M → O → I)))

wird die Kaehrsche Diamantenrelation der Peirceschen Zeichenrelation wie folgt notiert:

Diam(ZR) = ((A | a), (A → B | c), (A → B → C | b_2 ← b_1),

wobei bei a zwischen a^λ und a^ρ zu unterscheiden ist. Der wesentliche Unterschied zwischen ZR und Diam(ZR) liegt also in Berücksichtigung der „heteromorphen" Relationen, d.h. der zeicheninternen Umgebungen, die in monokontexturalen Semiotiken mit den entsprechenden „Homomorphen" zusammenfallen (vgl. z.B. (2.2 → 1.3) = (1.3 → 2.2), d.h. $(2.2 \rightarrow 1.3)^\circ$ = (2.2 ← 1.3), jedoch ($2.2_{\alpha.\beta} \rightarrow 1.3_\gamma$) → ($1.3_\gamma \rightarrow 2.2_{\beta.\alpha}$), d.h. $(1.3_\gamma \rightarrow 2.2_{\alpha.\beta})^\circ \neq (1.3_\gamma \rightarrow 2.2_{\beta.\alpha})$).

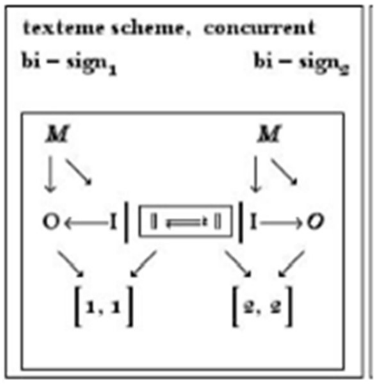
2. Wir können Diam(ZR) wie folgt darstellen:

(A | a):

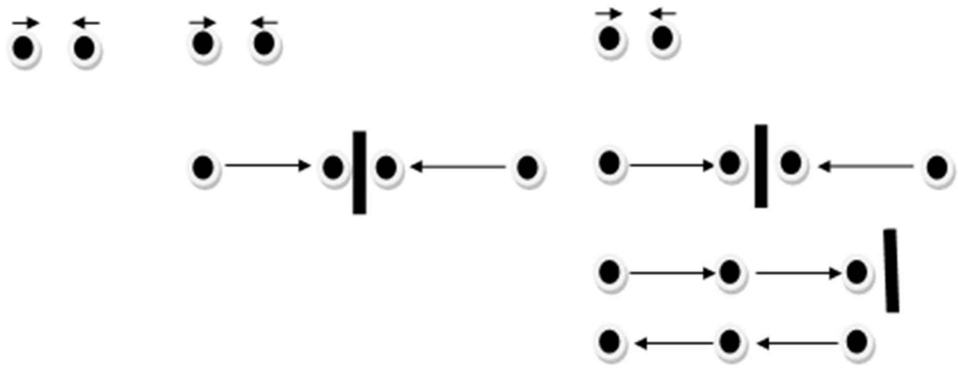
(A → B | c)

$(A \rightarrow B \rightarrow C \mid b_2 \leftarrow b_1)$

Im triadischen Falle gibt es also 3 Möglichkeiten, die drei Kategorien der morphismischen und der heteromorphismischen Relation zu „matchen". Bei monadisch-kontexturellem Zeichenzusammenhang entsteht also das Kaehrsche „Bi-Sign", das für homogenes I-Matching wie folgt aussieht (Bild aus Kaehr 2009):



3. Damit können wir die semiotische Diamantenrelation als Relation über Relationen wie folgt darstellen (vgl. Bense 1979, S. 53):



Bibliographie

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979

Kaehr, Rudolf, Xanadu´s textemes. In: ThinkArtLab 2009

Diamantentheoretische Vermittlung von Ontik und Semiotik

1. Ein wahrgenommenes Objekt wird durch die Wahrnehmung noch zu keinem Zeichen, denn einerseits können Zeichen nur durch willentliche Entscheidung eingeführt werden, und andererseits gibt es nicht-wahrnehmbare Objekte, die trotzdem zu Zeichen erklärt werden können. Das Objekt also, das zum Zeichen erklärt wird, ist somit höchstens in zeitlichem Sinne dem Zeichen vor-gegeben, ansonsten aber keineswegs absolut: vielmehr steht die Wahrnehmung eines Objektes am Anfang eines Prozesses, an dessen Ende die Erklärung dieses Objektes zum Zeichen stehen kann, aber keineswegs stehen muß. Es ist somit falsch, die thetische Einführung direkt bei einem irgendwie absoluten Objekt anzusetzen, und genauso falsch ist es, sie als einen der Wahrnehmung und seinen Phasen (Perzeption, Identifikation, Apperzeption) wesensfremden Prozeß aufzufassen.

2. Die der Semiotik zugehörige Ontik ist somit keine Theorie absoluter, apriorischer, vorgegebener und anderer phantasmagorischer Objekte, sondern eine Theorie der wahrgenommenen Objekte, die nur in dem Fall mit der Semiotik korreliert ist, wenn ein wahrgenommenes Objekt am Ende des ganzen Prozesses tatsächlich zum Zeichen erklärt wird. Es würde ja auch niemand behaupten, daß die Tatsache, daß ich den Stoff-Fetzen in meiner Hosentasche als Nasentuch erkennen und dementsprechend benutzen kann, aus dem Taschentuch bereits ein Zeichen macht. Ein Zeichen wird aus dem Taschentuch erst dann, wenn ich es (in möglichst ungebrauchtem Zustand) verknote und es dergestalt in einem Bedeutungs- und Sinnzusammenhang einbette – z.B. als Erinnerungszeichen, daß ich morgen meine Tochter früher von der Schule abhole. Gerade weil die Ontik eine Theorie wahrgenommener Objekte ist, muß man sich jedoch bewußt machen, daß mit dem Absolutheitsanspruch auch die Unikalitätstheorie von Objekten fällt: Wir können ein Objekt erstens nur deshalb wahrnehmen, weil es sich von einem (wie auch immer gearteten) Hintergrund abhebt, d.h. von einer Umgebung, in der sie gerade *nicht* sind. Zweitens benötigen wird zur Identifikation eines Objektes als eines bestimmten Etwas eine Funktion, welche das betreffende Objekt einer oder mehreren Klassen von ähnlichen Objekten zuweist. (Selbst das unikale Objekt des Morgen- bzw. Abendsterns gehört zur Klasse der Planeten, das Einhorn zur Klasse der Tiere, die Meerjungfrau gehört

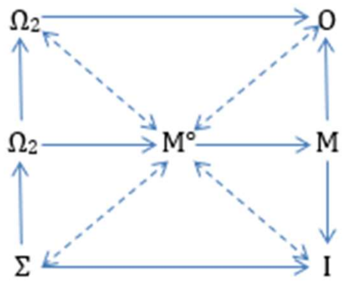
gleichzeitig zur Klasse der Menschen und der Tiere [Fische], denn auch unsere sog. imaginären Objekte sind in Wahrheit stets Patchworks aus Versatzstücken realer Objekte, d.h. also, daß Objekte stets nicht-leeren Klassen von Objektklassen, sog. Objektfamilien, angehören.) Drittens muß nach der Wahrnehmung und anschließenden Identifikation eines Objektes dessen Erkenntnis treten. Z.B. nehme ich erstens ein Etwas wahr, zweitens identifiziere ich dieses Etwas durch Zuordnung zur Klasse der Bäume als ein Stück Holz, drittens aber erkenne ich in diesem Stück Holz vielleicht seine mögliche Verwendung als Brennmaterial, d.h. als sog. Scheit.[7] Zur Erkenntnisstufe von Objekten gehören offenbar Benses "Werkzeugrelation", die als präsemiotisch ausgewiesen ist (Bense 1981, S. 33), sowie Wiesenfarths Gestalttheorie (Wiesenfarth 1979).

3. Geht man von einer Ontik als Theorie wahrgenommener Objekte aus, die erstens als solche, d.h. als wahrgenommene Objekte, zweitens als in Objektfamilien identifizierte Objekte und drittens als von Subjekten im Erkenntnisprozeß apperzipierte Objekte erscheinen, kann man nach dem Vorschlag von Toth (2011) das folgende verdoppelte System konstruieren, in dem das Seiende als der Inbegriff wahrgenommener Objekte im Verhältnis zu seinem Sein in der Form von Dualitätsbeziehungen erscheint:

$[A \rightarrow I]$	$[I \rightarrow A]$
$[[A \rightarrow I] \rightarrow A]$	$[A \rightarrow [I \rightarrow A]]$
$[[A \rightarrow I] \rightarrow A] \rightarrow I]]$	$[I \rightarrow [A \rightarrow [I \rightarrow A]]$
Seiendes	Sein

Dieses ontische System läßt sich jedoch nicht direkt auf das zugehörige semiotische System abbilden, weil nach Bense (1975, S. 45 ff.) ein System von disponiblen Mittel zwischen Ontik und Semiotik vermittelt. In Toth (2012a) hatten wir daher die Zeichengenese als der Theorie systemischer Übergänge zwischen Ontik und Semiotik wie folgt skizziert:

[7] Es wäre eine interessante Aufgabe, den Wortschatz verschiedener Sprachen (bzw. verschiedener Kulturstufen) darauf hin durchzuforsten, welche Teilklassen von Wörtern primär perzipierte (z.B. Berg), identifizierte (z.B. Stein) oder apperzipierte (z.B. Kiesel) Objekte bezeichnen. Die ausschließliche Konzentration auf Zeichen unter Vernachläßigung ihrer bezeichneten Objekte hat auch solche Studien bisher verunmöglicht. Eine Ausnahme, bei der allerdings statt von der Semiotik von der Linguistik ausgegangen wird, ist Leisi (1953).



Dieses System beruht somit erstens auf der ontischen Dualrelation

$[[I \rightarrow A], [[[A \rightarrow [I \rightarrow A]], [[[A \rightarrow I] \rightarrow A] \rightarrow I]]]]$
$\times$
$[[A \rightarrow I], [[[A \rightarrow I] \rightarrow A], [[[A \rightarrow I] \rightarrow A] \rightarrow I]]]]$

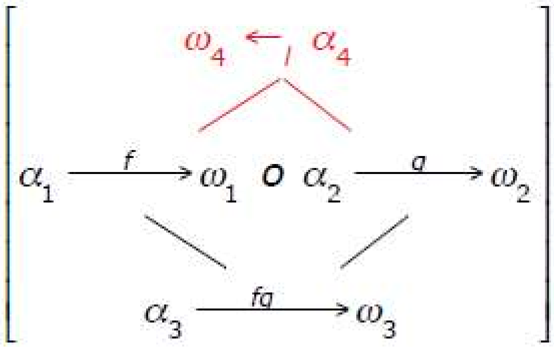
und zweitens auf der semiotischen Dualrelation

ZTh = ((3.a), (2.b), (1.c))
$\times$
RTh = ((c.1), (b.2), (a.3))

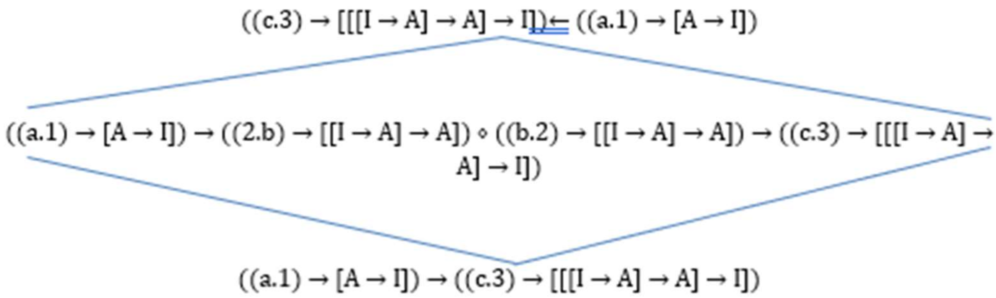
die nach Toth (2012b) in der Form von zwei chiastischen Relationen

$\chi[((3.a), (2.b), (1.c)), [[A \rightarrow I], [[[A \rightarrow I] \rightarrow A], [[[A \rightarrow I] \rightarrow A] \rightarrow I]]]]$
$\chi[((c.1), (b.2), (a.3)), [[I \rightarrow A], [[[A \rightarrow [I \rightarrow A]], [[[A \rightarrow I] \rightarrow A] \rightarrow I]]]]$

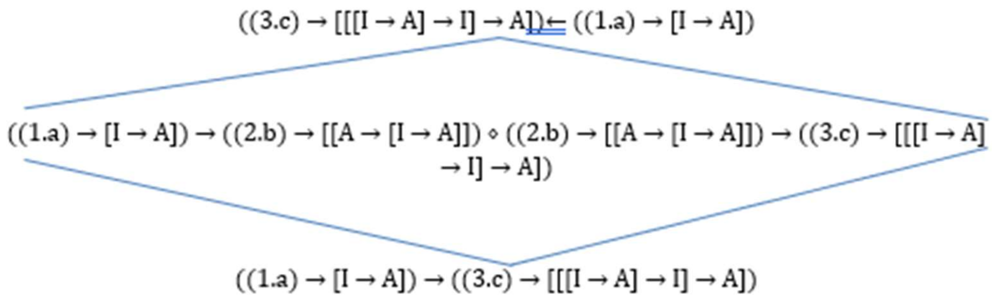
dargestellt werden kann. Inhaltlich bedeutet dies also, daß über die Kontexturgrenzen zwischen Objekt und Zeichen (bzw. Ontik und Semiotik) hinaus ein "sympathetisches" Verhältnis besteht erstens zwischen dem Sein und der Realitätsthematik und zweitens zwischen dem Seienden und der Zeichenthematik. Wegen dieser Überkreuz-Beziehungen, welche die klassische Logik hinter sich lassen und die von G. Günther eingeführte Proemialrelation zu ihrer logischen Fundierung benötigen, kann man nun das von R. Kaehr (2007, S. 58) vorgeschlagene Diamantenmodell, in dem sowohl kategoriale als auch von Kaehr so genannte "saltatorische" Morphismen vereinigt sind, zur Darstellung der verdoppelten chiastischen Beziehungen zwischen Ontik und Semiotik in der Form eines ontisch-semiotischen Vermittlungssystems verwenden:



Dann bekommen wir als ersten den realitätsthematisch-ontischen (Seiendes) Diamanten:



und als zweiten den zeichentheoretisch-ontischen (Sein) Diamanten:



Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007

Leisi, Ernst, Der Wortinhalt. Leipzig 1953

Toth, Alfred, Dreiteilung der semiotischen Systemtheorie. In: Electronic Journal for Mathematical Semiotics, 2011

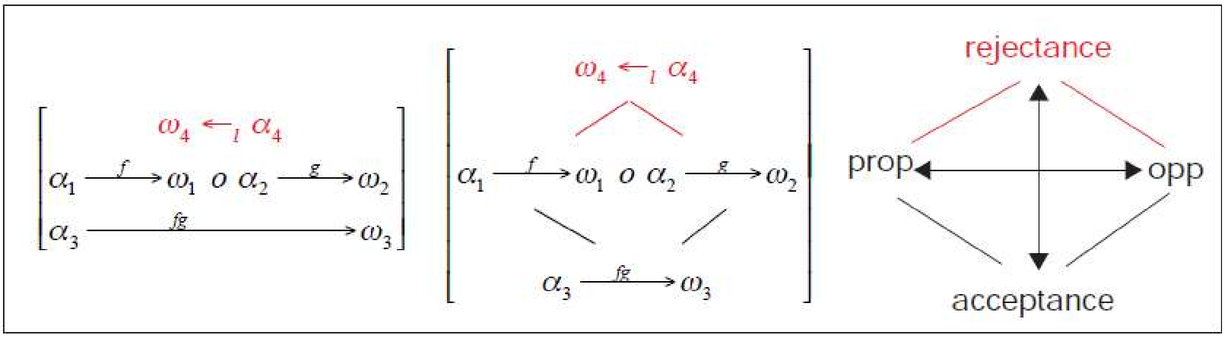
Toth, Alfred, Disponibilität als zeichengenetische Vermittlung. In: Electronic Journal for Mathematical Semiotics, 2012a

Toth, Alfred, Präsemiotische Vermittlung von Ontik und Semiotik. In: Electronic Journal for Mathematical Semiotics, 2012b

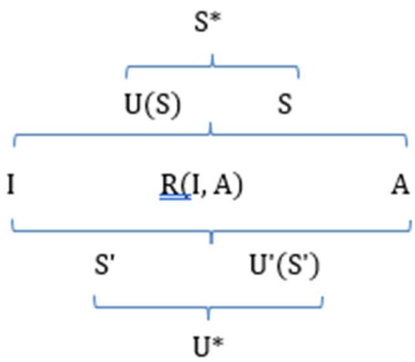
Wiesenfarth, Gerhard, Untersuchungen zur Kennzeichnung von Gestalt mit informationstheoretischen Methoden. Diss. Stuttgart 1979

Systeme mit Rändern als 3-stufige Diamanten

1. In Toth (2012) war gezeigt worden, daß man die von Rudolf Kaehr entwickelten "saltarialen" Diamanten, die im Sinne der Komplementarität von (kategorialen) Morphismen und (saltarialen) "Heteromorphismen" zu den Kategorien der Kategorientheorie komplementär sind



(Kaehr 2007, S. 11), systemtheoretisch wie folgt darstellen kann



worin also die durch Apostroph markierten Terme zu den entsprechenden Termen der Systemdefinition S*

$S^* = [A, \mathcal{R}[A, I], I]$

für den Fall $\mathcal{R}[S, U] = \emptyset$ komplementär sind.

2. Im Falle von Systemen mit "Rändern"

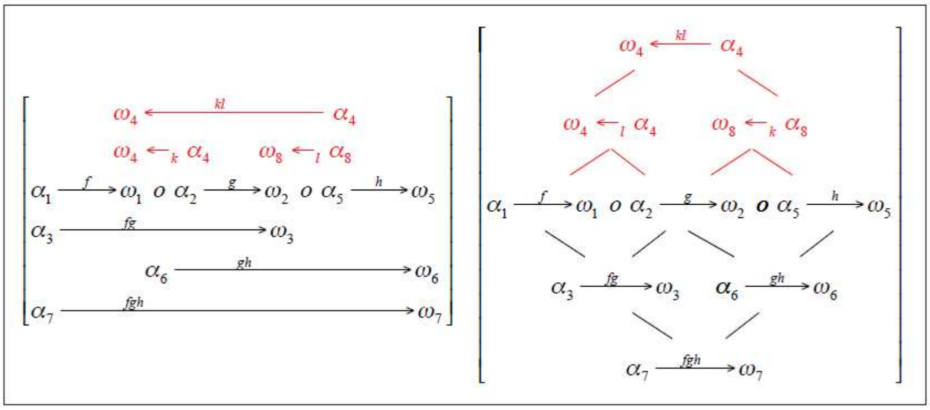
$S^{**} = [S, \mathcal{R}[S, U], U]$

mit $\mathcal{R}[S, U] \neq \emptyset$

bzw. in Verallgemeinerung für jedes gerichtete Paar von Teilsystemen

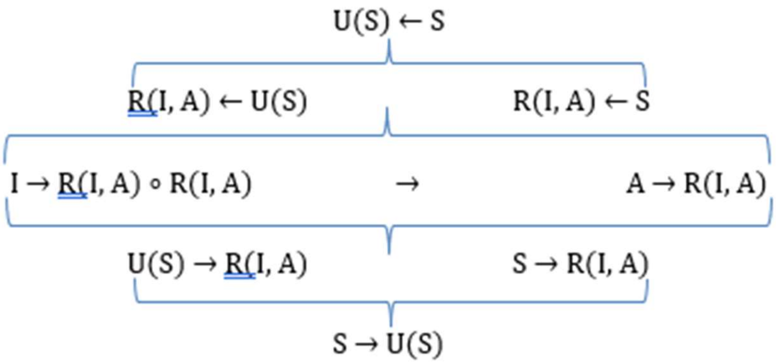
S* = [S_1, $\mathcal{R}$[S_1, S_2], S_2]

kann man somit das 3-stufige Diamanten-Modell verwenden, das Kaehr wie folgt skizziert hatte



(Kaehr 2007, S. 12)

Eine mögliche systemtheoretische Belegung der als Systemvariablen interpretierten kategorialen und saltarialen Domänen, Codomänen und Abbildungen sieht wie folgt aus



Literatur

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007

Toth, Alfred, Systemische Perspektive und kategoriale Diamanten. In: Electronic Journal for Mathematical Semiotics, 2012

Vermittelte und unvermittelte perspektivische Relationen

1. Wir gehen aus von der Objektrelation (vgl. Toth 2012a)

$O = [[\Omega_i, \Omega_i], [\Sigma_k, \Sigma_l]]$

sowie von der von Bense (1979, S. 53) definierten Zeichenrelation

$Z = (M \rightarrow ((M \rightarrow O) \rightarrow (M \rightarrow O \rightarrow I)))$.

Wegen der in Toth (2012b) dargelegten ontisch-semiotischen Isomorphie (vgl. dazu auch Menne [1992, S. 39 ff.] sowie Klaus [1973, S. 59 ff.]) haben wir damit die beiden Transformationen

t_1: $O \rightarrow S^*/\times S^* = [[\Omega_i, \Omega_i], [\Sigma_k, \Sigma_l]] \rightarrow$

$S^* = [x_0^1, [x^2_1, [x^3_2, [x^4_3, [x^5_4, [x^6_5, \ldots, [x^{n+1}_n]_n]$

$\times S^* = [[x^{n+1}_n], \ldots, [x^6_5, [x^5_4, [x^4_3, [x^3_2, [x^2_1, [x^1_0]_n]$

t_2: $Z \rightarrow S^*/\times S^* = (M \rightarrow ((M \rightarrow O) \rightarrow (M \rightarrow O \rightarrow I))) \rightarrow$

$S^* = [x_0^1, [x^2_1, [x^3_2, [x^4_3, [x^5_4, [x^6_5, \ldots, [x^{n+1}_n]_n]$

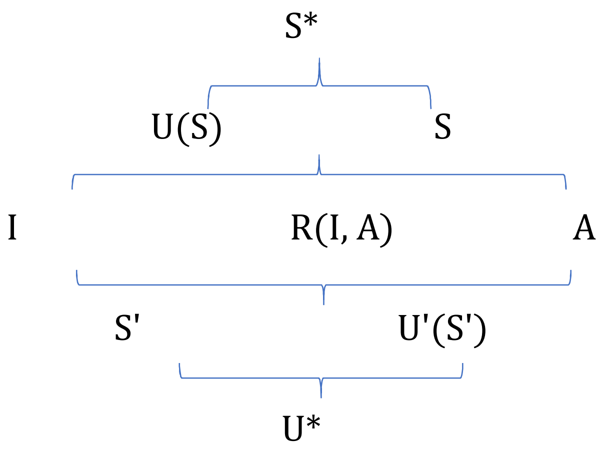
$\times S^* = [[x^{n+1}_n], \ldots, [x^6_5, [x^5_4, [x^4_3, [x^3_2, [x^2_1, [x^1_0]_n]$.

Da man ein System mit oder ohne Rand durch

$S^* = [S, \mathcal{R}[S, U], U]$ (mit $\mathcal{R}[S, U] = \emptyset$ oder $\mathcal{R}[S, U] \neq \emptyset$)

definieren kann (vgl. Toth 2012b), kann man ferner, wie bereits in Toth (2012c) gezeigt, nicht nur Zeichen-, sondern auch Objektrelationen im Sinne der von Kaehr (2007) vorgeschlagenen "saltarialen" Diamanten-Modelle formal darstellen. Damit ergibt sich für beiden möglichen Fälle leerer und nicht-leerer Ränder

2.1. für $\mathcal{R}[S, U] = \emptyset$ den 2-stufigen Diamanten



2.2. für den Fall $\mathcal{R}$[S, U] ≠ Ø den 3-stufigen Diamanten

$$U(S) \leftarrow S$$

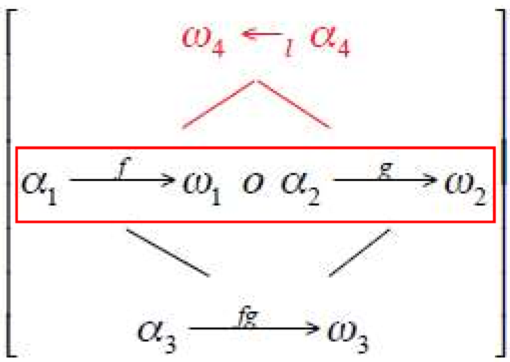
$$R(I, A) \leftarrow U(S) \qquad\qquad R(I, A) \leftarrow S$$

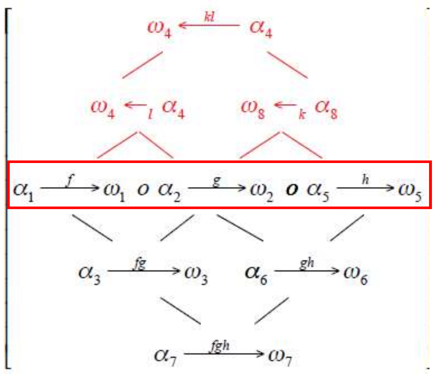
$$I \rightarrow R(I, A) \circ R(I, A) \qquad \rightarrow \qquad A \rightarrow R(I, A)$$

$$U(S) \rightarrow R(I, A) \qquad\qquad S \rightarrow R(I, A)$$

$$S \rightarrow U(S).$$

Wie man erkennt, liegt die wesentliche Unterscheidung zwischen diamantentheoretischer 2- und 3-Stufigkeit in der Unvermitteltheit bzw. Vermitteltheit





der in den Modellen zentralen Abbildungen, weshalb wir von 2- und 3-stufigen Diamanten sprechen. Zur Illustration seien im folgenden charakteristische perspektivische Relationen beigebracht.

2.1. Unvermittelte perspektivische Relationen

Grob gesagt, sprechen wir von unvermittelten perspektivischen Relationen, wenn in einer systemischen Dichotomie beide Referenzpunkte konstant sind, wenn also z.B. ein Raum einmal von A nach B und einmal von B nach A betrachtet wird.

2.1.1. [Vorn → Hinten] vs. [Hinten → Vorn]

Restelbergstr. 290, 8044 Zürich

2.1.2. Unten vs. Oben

Aemtlerstr. 86, 8003 Zürich

2.1.3. Außen vs. Innen

Döltschiweg 35, 8055 Zürich

2.2. Vermittelte perspektivische Relationen

Bei vermittelten perspektivischen Relationen gibt es zwei Fälle: 1. In einer systemischen Dichotomie [A, B] ist nur entweder A oder B konstant. 2. Zwei Dichotomien [A, B] und [C, D] enthalten je ein Teilsystem, das zur gleichen Objektfamilie gehört. ein Beispiel für Fall 1 ist [Vorne Links, Hinten Rechts]. Ein Beispiel für Fall 2 ist [Hauseingang, Wohnungseingang], wo also die beiden Eingänge verschiedenen Einbettungsstufen des Systems angehören.

2.2.1. Beispiele für Fall 1

Carl Spittelerstr. 6, 8053 Zürich

Mühlebachstr. 12, 8008 Zürich

2.2.2. Beispiele für Fall 2

2.2.2.1. [Eingang → Treppenhaus] vs. [Wohnungseingang → Wohnung]

Weststr. 194, 8003 Zürich

2.2.2.2. [Treppenhaus → Hauseingang] vs. [Wohnungseingang → Wohnung]

Hier liegt also gegenüber 2.2.2.1. in einem der dichotomischen Glieder die konverse perspektivische Relation vor, d.h. es handelt sich um eine Kombination von Fall 1 und Fall 2.

Mühlebachstr. 12, 8008 Zürich

2.2.2.3. Aufgangstreppe vs. Hauseingangsstufen vs. Maisonette-Treppe

Die erste Treppe verbindet also die Umgebung mit dem System, die zweite zwei Elemente des Adsystems des Systems, nämlich den Zugang und den (selbst adessiven) Hauseingang, und die dritte Treppe verbindet zwei eingebettete Teilsysteme der Wohnung, die selbst ein Teilsystem des Systems ist.

Sonneggstr. 88, 8006 Zürich

Mühlebachstr. 12, 8008 Zürich

Literatur

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007

Klaus, Georg, Semiotik und Erkenntnistheorie. 3. Aufl. München 1973

Menne, Albert, Einführung in die Methodologie. 3. Aufl. Darmstadt 1992

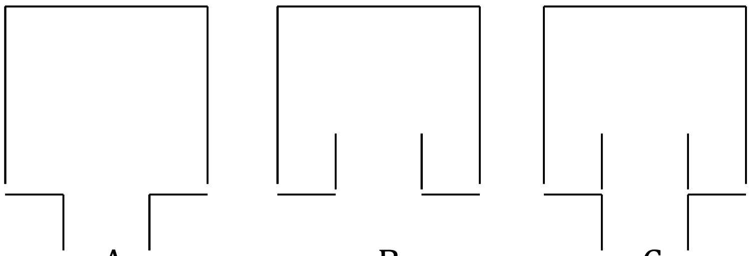
Toth, Alfred, Perspektive vs. Kontexturgrenze. In: Electronic Journal for Mathematical Semiotics, 2012a

Toth, Alfred, Ontisch-semiotische Isomorphie. In: Electronic Journal for Mathematical Semiotics, 2012b

Toth, Alfred, Systeme mit Rändern als 3-stufige Diamanten. In: Electronic Journal for Mathematical Semiotics, 2012c

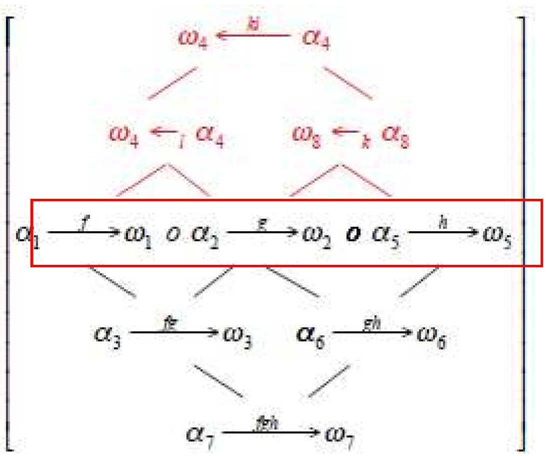
Türräume als vermittelte perspektivische Relationen

1. Die von mir bereits verschiedentlich behandelten Türräume oder Windfänge (vgl. Toth 2012a) können in den folgenden drei Haupttypen auftreten



A B C,

d.h. als adessiver, exessiver und konvers exessiver (zentraler) Typ. Im folgenden wird im Anschluß an Toth (2012b) gezeigt, daß man das von Kaehr (2007) konstruierte 3-stufige Diamantenmodell "saltarialer" Abbildungen



dazu benutzen kann, Türräume im Sinne von vermittelten perspektivischen Relationen darzustellen, d.h. von Relationen, welche durch die beiden Transformationen

t_1: $O \rightarrow S^*/{\times}S^* = [[\Omega_i, \Omega_i], [\Sigma_k, \Sigma_l]] \rightarrow$

$$S^* = \quad [x_0{}^1, [x^2{}_1, [x^3{}_2, [x^4{}_3, [x^5{}_4, [x^6{}_5, ..., [x^{n+1}{}_n]\,_n]$$

$$\times S^* = \quad [[x^{n+1}{}_n], ..., [x^6{}_5, [x^5{}_4, [x^4{}_3, [x^3{}_2, [x^2{}_1, [x^1{}_0]\,_n]$$

t_2: $Z \to S^*/\times S^* = \quad (M \to ((M \to O) \to (M \to O \to I)))\to$

$$S^* = \quad [x_0^1, [x^2_1, [x^3_2, [x^4_3, [x^5_4, [x^6_5, ..., [x^{n+1}_n]_n]$$

$$\times S^* = \quad [[x^{n+1}_n], ..., [x^6_5, [x^5_4, [x^4_3, [x^3_2, [x^2_1, [x^1_0]_n].$$

darstellbar sind (vgl. Toth 2012c).

2.1. Exessive Türräume

Minervastr. 93, 8032 Zürich

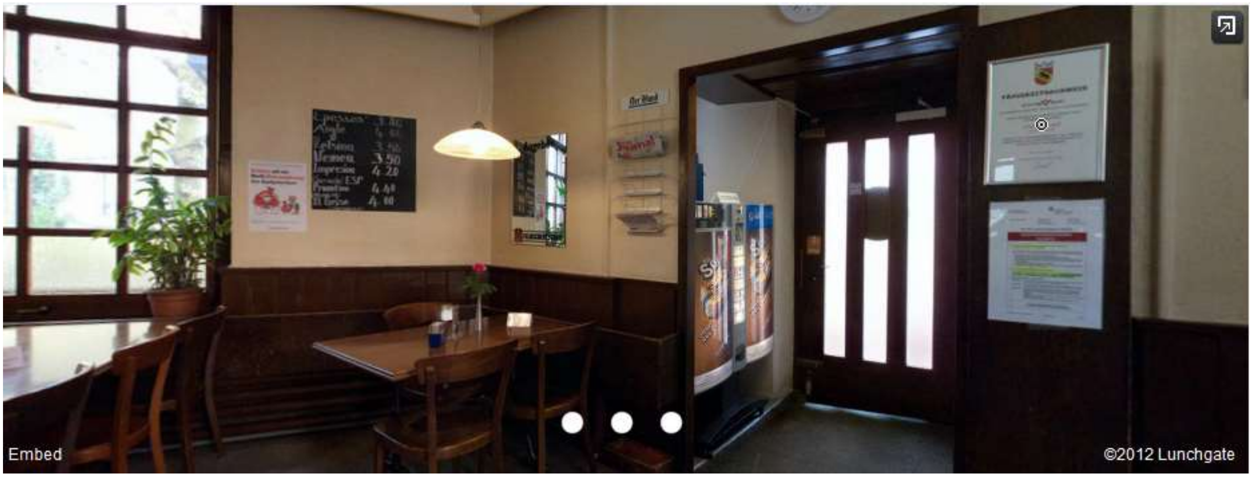
Ehem. Rest. Gessnerallee, Schützengasse 32, 8001Zürich

Rest. Rheinfelder Bierhalle, Niederdorfstr. 76, 8001 Zürich

Rest. Metzgerhalle, Schaffhauser-str. 354, 8050 Zürich

Das folgende Paar von Bildern zeigt eine exessive perspektivische Relation sowie ihre Konverse.

Claragraben 154a, 4057 Basel



Rest. Brunnhof, Lilienweg 20, 3007 Bern

2.2. Konvers-exessive Türräume

Café Bar Central, Central 1, 8001 Zürich

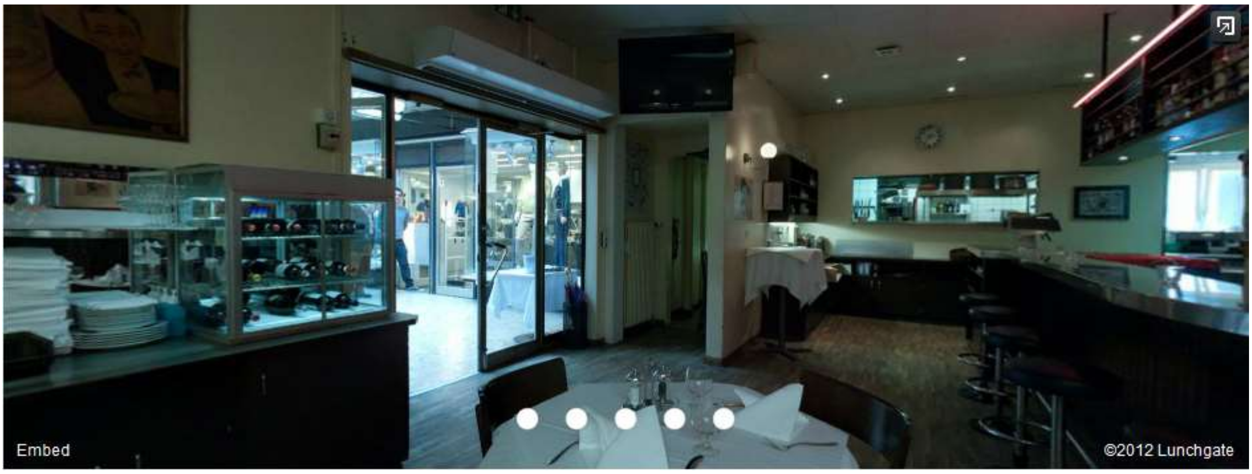
Café Boy, Kochstr. 2, 8004 Zürich

2.3. Adessive Türräume

Wehntalerstr. 431, 8046 Zürich

Rest. Grünwald, Regensdorferstr. 237, 8049 Zürich

2.4. Daneben gibt es natürlich die Null-Türräume. Sie stellen nach dem zuvor Gesagten unvermittelte perspektivische Relationen dar, d.h. es genügt zu ihrer formalen Darstellung das ebenfalls von Kaehr (2007) gegebene 2-stufige Diamantenmodell.



Rest. Aarbergerhof, Aarbergergasse 40, 3011 Bern

Literatur

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007

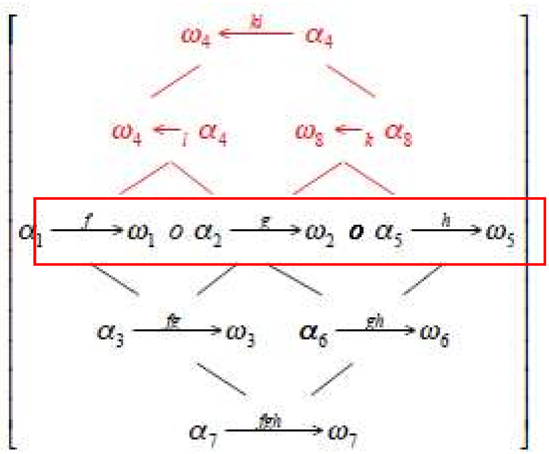
Toth, Alfred, Türräume I-II. In: Electronic Journal for Mathematical Semiotics, 2012a

Toth, Alfred, Vermittelte und unvermittelte perspektivische Relationen. In: Electronic Journal for Mathematical Semiotics, 2012b

Toth, Alfred, Systeme mit Rändern als 3-stufige Diamanten. In: Electronic Journal for Mathematical Semiotics, 2012c

Schienen, Brücken und Schleusen

1. Mit Hilfe des von R. Kaehr (2007) entwickelten und im Rahmen der systemtheoretischen Objekttheorie u.a. in Toth (2012a, b) bereits angewandten Modells 3-stufiger Diamanten,



welche nicht nur die morphismischen Abbildungen der Kategorietheorie, sondern auch die ihnen komplementären "heteromorphismischen Abbildungen der zur Theorie der Kategorien komplementären "Saltatorien"-Theorie in sich vereinigen, kann man, wie zuletzt in Toth (2012c) gezeigt, Vermittlungsstrukturen in den beiden systemtheoretischen Basis-Transformationen, der ontischen

t_1: $O \rightarrow S^*/\times S^* = [[\Omega_i, \Omega_j], [\Sigma_k, \Sigma_l]] \rightarrow$

$S^* = \quad [x_0{}^1, [x^2{}_1, [x^3{}_2, [x^4{}_3, [x^5{}_4, [x^6{}_5, ..., [x^{n+1}{}_n]{}_n]$

$\times S^* = \quad [[x^{n+1}{}_n], ..., [x^6{}_5, [x^5{}_4, [x^4{}_3, [x^3{}_2, [x^2{}_1, [x^1{}_0]{}_n]$

und der semiotischen

t_2: $Z \rightarrow S^*/\times S^* = \quad (M \rightarrow ((M \rightarrow O) \rightarrow (M \rightarrow O \rightarrow I))) \rightarrow$

$S^* = \quad [x_0{}^1, [x^2{}_1, [x^3{}_2, [x^4{}_3, [x^5{}_4, [x^6{}_5, ..., [x^{n+1}{}_n]{}_n]$

$\times S^* = \quad [[x^{n+1}{}_n], ..., [x^6{}_5, [x^5{}_4, [x^4{}_3, [x^3{}_2, [x^2{}_1, [x^1{}_0]{}_n]$.

formalisieren. Zur Illustration dieser gleichzeitig vermittelten und vermittelnden objektalen Transformationen seien im folgenden einige architektonische Elemente aufgezeigt, die wir, in sinnfälliger Abweichung von ihrer üblichen Verwendung, mit Schienen, Brücken und Schleusen bezeichnen.

2.1. Schienen

Hölderlinstr. 12, 8032 Zürich

Unterwerkstr. 11, 8052 Zürich

2.2. Brücken

Dolderstr. 40, 8032 Zürich

Walchestr. 25, 8001 Zürich

2.3. Schleusen

Seefeldstr. 75, 8008 Zürich

Fröhlichstr. 49, 8008 Zürich   Mühlebachstr. o.N., 8008 Zürich

Literatur

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007

Toth, Alfred, Türräume I-II. In: Electronic Journal for Mathematical Semiotics, 2012a

Toth, Alfred, Vermittelte und unvermittelte perspektivische Relationen. In: Electronic Journal for Mathematical Semiotics, 2012b

Toth, Alfred, Bi-Objekte und perspektivische Relationen. In: Electronic Journal for Mathematical Semiotics, 2012c

Kontexturierte semiotische Morphismen

1. Wie in Toth (2014a-c) dargelegt, ist die peircesche Zeichenrelation

$Z = R(M, O, I)$

trotz ihrer zehnfach ausdifferenzierbaren Realitätsthematiken und den von ihnen präsentierten strukturellen Realitäten logisch 2-wertig, denn der die Subjektposition in Z repräsentierende Interpretantenbezug kann nur das Ich-Subjekt der aristotelischen Logik abbilden. Am deutlichsten wird dies bei Benses Definition des semiotischen Kommunikationsschemas (vgl. Bense 1971, S. 39 ff.)

$K = (O \rightarrow M \rightarrow I)$,

in dem als Sender der Objektbezug auftritt, der, genau wie in der 2-wertigen Logik (vgl. Günther 1991, S. 176), mit dem Es-Objekt gleichzeitig das Du-Subjekt repräsentiert.

2. Statt die peircesche Zeichenrelation zu erweitern, d.h. logische Mehrwertigkeit mit höherer relationaler n-adizität zu koppeln, wurde daher vorgeschlagen, die von Bense (1975, S. 101) eingeführte semiotische Matrix für jede der 3 × 3 = 9 als Einträge fungierenden Subrelationen zu kontexturieren

$(1.1)_i \quad (1.2)_i \quad (1.3)_i$

$(2.1)_i \quad (2.2)_i \quad (2.3)_i$

$(3.1)_i \quad (3.2)_i \quad (3.3)_i$

mit $i \in \{ich, du, er\}$. Dadurch wird also die Subjektdeixis vom Interpretanten-bezug auf die von ihm qua

$ZR = (M \subset ((M \subset O) \subset (M \subset O \subset I)))$

(vgl. Bense 1979, S. 53) semiotisch inkludierten Mittel- und Objektbezüge ausgedehnt, d.h. das gesamte triadisch-trichotomische System, welches die

Matrix repräsentiert, ist nun ich-, du- oder er-deiktisch oder durch Kombinationen dieser Deixen darstellbar.

3. Treten kombinierte Deixen auf, z.B. im folgenden Fall

$DS = [(3.1)_{ich.du}, (2.2)_{ich,du}, (1.3)_{ich,du}] \times [(3.1)_{du,ich}, (2.2)_{du,ich}, (1.3)_{du,ich}]$,

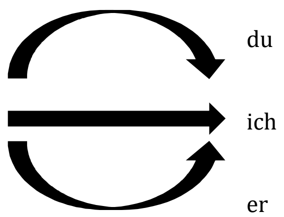
so hat dies, wie man anhand dieses Beispiels sieht, empfindliche Konsequenzen für die bisherige, auf der Universalität der Eigenrealität (vgl. Bense 1992) gegründete Semiotik, denn hier gilt

$\times[(3.1)_{ich.du}, (2.2)_{ich,du}, (1.3)_{ich,du}] \neq [(3.1)_{du,ich}, (2.2)_{du,ich}, (1.3)_{du,ich}]$.

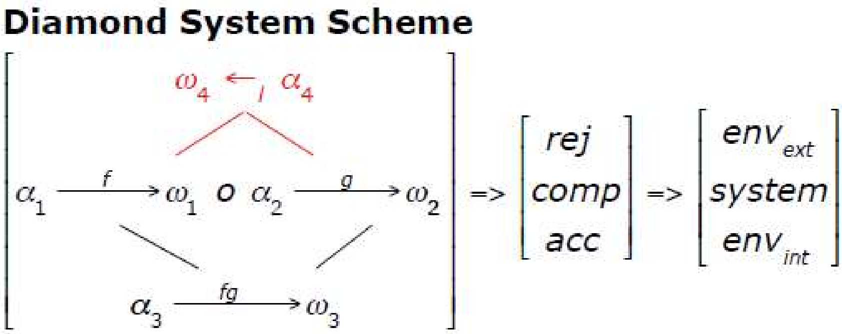
Das bedeutet also, daß nicht nur die bisher Domänen bzw. Codomänen semiosischer Abbildungen repräsentierenden Subrelationen, sondern auch die Abbildungen selbst, die semiosischen Morphismen, deiktisch kontexturiert sind, d.h. wir bekommen

$(id_1)_i$	$(\alpha)_i$	$(\beta\alpha)_i$
$(\alpha°)_i$	$(id_2)_i$	$(\beta)_i$
$(\alpha°\beta°)_i$	$(\beta°)_i$	$(id_3)_i$

Das bedeutet also, daß für jedes Paar von Subrelationen der Form $S_1 = <a.b>$ und $S_2 = <c.d>$ jeweils drei mögliche kontexturierte Morphismen

<a.b>  <c.d>

existieren, die natürlich aus dem Rahmen der ebenfalls logisch 2-wertigen Kategorietheorie fallen (vgl. dazu Toth 1997, S. 21 ff.). Sie fallen allerdings ebenfalls aus dem Rahmen der von Kaehr (2007, S. 2 ff.) eingeführten Differenzierung zwischen Morphismen und Heteromorphismen, vgl. das folgende Schema aus Kaehr (2007, S. 2)



darin die schwarz markierten Pfeile die Morphismen und der recht markierte Pfeil den zugehörigen Heteromorphismus bedeuten. Rein theoretisch kann man zwar Entsprechendes auch für die kontexturierte Semiotik konstruieren, denn z.B. gibt es nicht nur die Konversionen

$\langle a.b \rangle_{ich,du} \rightarrow \langle c.d \rangle_{du,er}$

$\langle a.b \rangle_{ich,du} \leftarrow \langle c.d \rangle_{du,er}$,

sondern auch die weiteren Konversionen

$\langle b.a \rangle_{du,ich} \rightarrow \langle d.c \rangle_{er,du}$

$\langle b.a \rangle_{ich,du} \leftarrow \langle d.c \rangle_{du,er}$,

aber da Semiosen im Gegensatz zu kategorie- und diamantentheoretischen Abbildungen aus prinzipiellen Gründen keine umkehrbaren Abbildungen darstellen (vgl. Bense 1981, S. 124 ff.), verbietet sich eine Interpretation der Abbildung

$\langle b.a \rangle_{ich,du} \leftarrow \langle d.c \rangle_{du,er}$

im Sinne eines "semiotischen Heteromorphismus" von selbst. Kontexturierte semiotische Morphismen stellen somit neben den 2-wertigen kategorialen und den mehr-wertigen diamantentheoretischen Morphismen eine dritte, sich weder mit den einen noch mit den anderen deckende Klasse qualitativer Abbildungen dar.

Literatur

Bense, Max, Zeichen und Design. Baden-Baden 1971

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979

Bense, Max, Axiomatik und Semiotik. Baden-Baden 1981

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007

Toth, Alfred, Entwurf einer Semiotisch-Relationalen Grammatik. Tübingen 1997

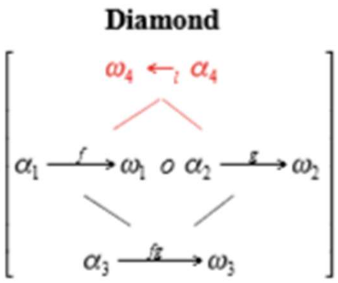
Toth, Alfred, Nicht-minimale Semiotiken. In: Electronic Journal for Mathematical Semiotics, 2014a

Toth, Alfred, Semiotische Deixis und Kontexturen. In: Electronic Journal for Mathematical Semiotics, 2014b

Toth, Alfred, Ontische Objekt- und Subjektkonjunktionen. In: Electronic Journal for Mathematical Semiotics, 2014c

System, Umgebung und ihre Vermittlung im kaehrschen diamond-Modell

1. Bekanntlich wurde der diamond von Rudolf Kaehr (2007) als Modell für eine qualitative Kategorientheorie eingeführt.



Ein diamond besteht in seiner unteren Hälfte auf einer Komposition (Konkatenation) regulärer morphismischer Abbildungen der Form

$(a \to b) \circ (b \to c) = (a \to c)$,

die allerdings kontexturiert sind,

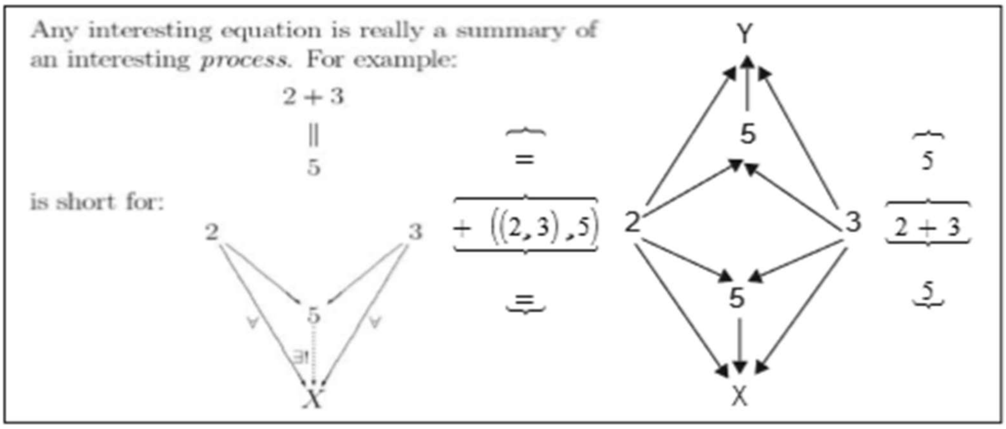
und in seiner oberen Hälfte aus der von Kaehr als Heteromorphismus bezeichneten Abbildung

$(a \leftarrow c)$,

die ebenfalls kontexturiert ist, so daß

$(a \to c)^{-1} \neq (a \leftarrow c)$.

Am besten hat diese polykontextural bedingte Nicht-Umkehrbarkeit Kaehr selbst am Beispiel der „diamondization of arithmetic“ dargestellt und kommentiert (vgl. Kaehr (2008, S. 72).

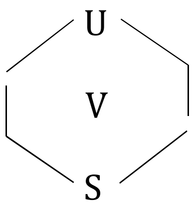


How is the diamond operation, 2+2=5, to read? The first diagram gives an explanation of the processes involved into the addition. That is, for all numbers 2 of X and all numbers 3 of X there is exactly one number 5 of X representing the addition 2+3. This is the classic operational or categorial approach to addition (Baez).

The second diagram shows the diamond representation of the addition 2+3. The wordings are the same, one for X, and one for Y. The equation is *stable* in respect of the acceptional addition and the rejectional addition iff X=Y. That is, iff the numbers and the operations belong to isomorphic arithmetical systems, then they are equivalent. If X would be a totally different arithmetical system to Y then some disturbance of the harmony between both would happen. Nevertheless, because of their rejectional direction, numbers of Y might "run" in reverse order to X and coincide at the point of X=Y.

The meaning of a sign is defined by its use. Thus, the numeral "5" belonging to the system X, has not exactly the same meaning as the numeral "5" belonging to the system Y. They may be isomorphic, hetero-morphic, equivalent, but they are not equal. Equality is given intra-contextually for terms of X only, or for terms of Y only. But not for terms between X and Y. In other words, the equation is realized as an equivalence only if it has a model in the rejectional, i.e., in the environmental or context system. Otherwise, that is, without the environmental system, the arithmetical system of the acceptance system, here X, has to be accepted as unique, fundamental and pre-given.

2. Somit kann man den diamond systemtheoretisch wie folgt darstellen



d.h. für die Konkatenation V der Morphismen

$V = (a \rightarrow b) \circ (b \rightarrow c) = (a \rightarrow c)$

gilt somit

$V = S \cap U$

mit

$U = S^{-1}$.

Damit bekommen wir die bereits in Toth (2015) eingeführte ontische Systemrelation

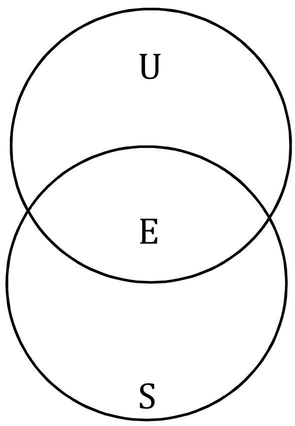
$S^* = (S, U, E)$,

wobei

$E = V(S, U)$,

d.h. der kaehrsche diamond ist triadisch und nicht dyadisch, auch wenn Kaehr lediglich „system“ und „environment“ unterscheidet.

Somit kann man einen diamond mengentheoretisch wie folgt darstellen



S* muß daher ordnungstheoretisch durch

$S^* = (S, E, U)$

und die peircesche Zeichenrelation vermöge ontisch-semiotischer Isomorphie durch

$Z = (O, M, I)$

redefiniert werden. Damit ergeben sich nun die interessanten neuen Teilisomorphien

$S \cong O$

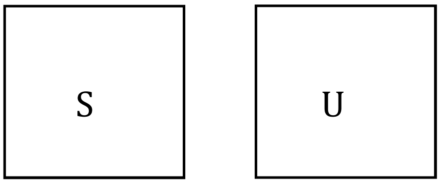
$E = M$

U = I,

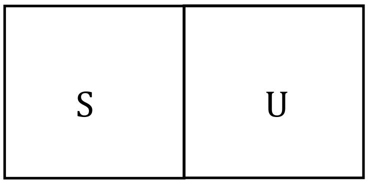
womit also die neue Systemrelation außerdem der von Bense (1971) eingeführten Kommunikationsrelation K = (O, M, I) isomorph ist. Vor allem aber fungieren Abschlüsse E nun nicht mehr drittheitlich, sondern erstheitlich, und zwar konform dem Mittelbezug als „Medium“ (Peirce), das zwischen Objekt und Interpretant innerhalb von Z vermittelt. Die Umgebung ist somit nicht mehr zweit-, sondern drittheitlich, d.h. als semiotisches Objekt fungiert das System, und als semiotischer Interpretant die Umgebung, die also durch Abschlüsse vermittelt werden.

3. E wird damit zum ontischen Rand, von dem die folgenden Typen unterschieden wurden (vgl. Toth 2019)

3.1. E(S) $\cap$ E(U) = Ø

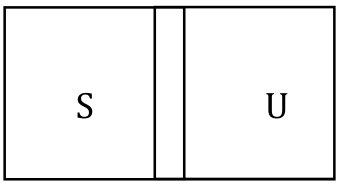


3.2. E(S) $\cap$ E(U) ≠ Ø

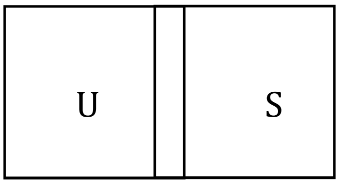


3.3. E(S) $\subset$ U oder E(U) $\subset$ S

3.3.1. E(U) $\subset$ S



3.3.2. E(S) ⊂ U



Literatur

Bense, Max, Zeichen und Design. Baden-Baden 1971

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007. Digitalisat: www.vordenker.de/rk/rk_Diamond-Theory_collection-of-papers-and-fragments_2007.pdf

Toth, Alfred, Zu einer triadischen System-Definition. In: Electronic Journal for Mathematical Semiotics, 2015

Toth, Alfred, Drei Definitionen von Rändern. In: Electronic Journal for Mathematical Semiotics, 2019

Diamondisierung 4-kontexturaler Tritozeichen

1. Bekanntlich benötigt eine minimale vollständige polykontexturale und damit qualitative Semiotik 4 Kontexturen (vgl. Toth 2019a). Ferner muß statt der von Bense (1975, S. 35 ff.) eingeführten triadisch-trichotomischen Zeichenrelation

$ZR^{3.3} = (3.x, 2.y, 1.z)$

mit

x, y, z ∈ (1, 2, 3)

sowie also der quadratischen 3×3-Matrix

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

von einer triadisch-pentatomischen Zeichenrelation

$ZR^{3.5} = (3.x, 2.y, 1.z)$

mit

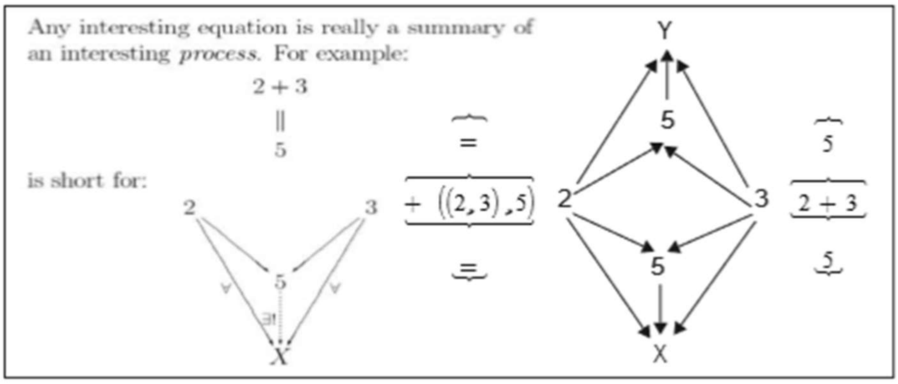
x, y, z ∈ (1, 2, 3, 4, 5)

sowie der somit nicht-quadratischen 3×5-Matrix

	.1	.2	.3	.4	.5
1.	1.1	1.2	1.3	1.4	1.5
2.	2.1	2.2	2.3	2.4	2.6
3.	3.1	3.2	3.3	3.4	3.5

ausgegangen werden.

2. Will man das System der Tritozeichen bzw. Semio-Morphogramme für K = 4 „diamondisieren“, so sollte man dem folgenden Modell mit Erläuterungen aus Kaehr (2007, S. 72) folgen.

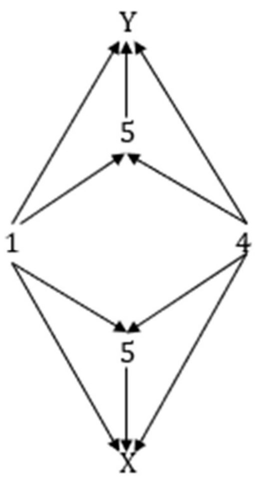


How is the diamond operation, 2+2=5, to read? The first diagram gives an explanation of the processes involved into the addition. That is, for all numbers 2 of X and all numbers 3 of X there is exactly one number 5 of X representing the addition 2+3. This is the classic operational or categorial approach to addition (Baez).

The second diagram shows the diamond representation of the addition 2+3. The wordings are the same, one for X, and one for Y. The equation is *stable* in respect of the acceptional addition and the rejectional addition iff X=Y. That is, iff the numbers and the operations belong to isomorphic arithmetical systems, then they are equivalent. If X would be a totally different arithmetical system to Y then some disturbance of the harmony between both would happen. Nevertheless, because of their rejectional direction, numbers of Y might "run" in reverse order to X and coincide at the point of X=Y.

The meaning of a sign is defined by its use. Thus, the numeral "5" belonging to the system X, has not exactly the same meaning as the numeral "5" belonging to the system Y. They may be isomorphic, hetero-morphic, equivalent, but they are not equal. Equality is given intra-contextually for terms of X only, or for terms of Y only. But not for terms between X and Y. In other words, the equation is realized as an equivalence only if it has a model in the rejectional, i.e., in the environmental or context system. Otherwise, that is, without the environmental system, the arithmetical system of the acceptance system, here X, has to be accepted as unique, fundamental and pre-given.

Für $ZR^{3,5}$ ist also vom folgenden arithmetischen diamond-Modell auszugehen.



Dabei ist also das System X

$X = (1 + 4) = 5$

und die Umgebung Y

$Y = 5 = (4 + 1)$,

allerdings mit einem beträchtlichen Unterschied, welcher die von Kaehr hervorgehobene Möglichkeit $X \neq Y$ betrifft. Bildet man nämlich Subzeichen auf Morphogramme ab (vgl. Toth 2019b), so ist die Nicht-Peano-Ordnung der trichotomischen Stellenwerte

1, 2, 4, 5, 3,

da

$V(2,3) = (4, 5)$

ist.

Somit ist

$X = (1, 2, 4, 5, 3)$,

es ist jedoch

$Y = (3, 5, 4, 2, 1)$.

Man vgl. nun die Peanoabbildung einer vorwärts laufenden auf eine rückwärts laufende Zahlenfolge

1	2	3	4	5
↓	↓	↓	↓	↓
5	4	3	2	1,

mit der Identitätsabbildung $(3 \rightarrow 3)$ mit der Nicht-Peanoabbildung, die X und Y zugrunde liegt

1 2 4 5 3

↓ ↓ ↓ ↓ ↓

3 5 4 2 1,

darin die Identitätsabbildung (4 → 4) gibt.

Wegen

(3 → 3) ≠ (4 → 4)

folgt nun

X ≠ Y.

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

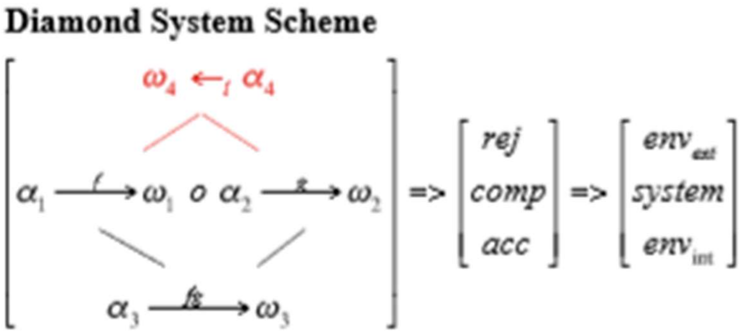
Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007. Digitalisat: www.vordenker.de/rk/rk_Diamond-Theory_collection-of-papers-and-fragments_2007.pdf

Toth, Alfred, Eine minimale vollständige polykontexturale Semiotik für K = 4. In: Electronic Journal for Mathematical Semiotics, 2019a

Toth, Alfred, Abbildungen von Subzeichen auf Morphogramme. In: Electronic Journal for Mathematical Semiotics, 2019b

Zwei Probleme in der kaehrschen Diamond Theory

1. Bekanntlich erweitert das von Rudolf Kaehr (2007) eingeführte diamond-Modell die Komposition von Morphismen um eine rückwärts gerichtete Abbildung, die Kaehr „Heteromorphismus" nennt. Während die Komposition zweier Morphismen als System eingeführt wird, stellen sowohl die konkatenierte Abbildung (Akzeption) als auch die Abbildung der Domäne der 2. Abbildung auf die Codomäne der 1. Abbildung innerhalb der Kompositon (Rejektion) Umgebungen dar: Die Konkatenation wird in interne und der Heteromorphismus als externe Umgebung definiert. Jedes System – egal, ob es logisch, mathematisch oder semiotisch mit Hilfe eines diamonds dargestellt wird –, besitzt somit nicht nur eine, sondern zwei Umgebungen (vgl. Kaehr 2007, S. 68).



Während also der mittlere und der untere Teil des diamonds mit Hilfe der quantitativen Kategorientheorie definierbar sind, ist es der obere Teil nicht. Diese auch „jumpoid" oder „saltatory" genannte Abbildung ist qualitativ, weil sie sich von der Komposition durch die Kontexturen unterscheidet. Ein damond erfordert somit mindestens 4 Kontexturen

$$2.2_4 \leftarrow_f 2.2_4$$

$$1.3_1 \rightarrow_f 2.2_1 \circ 2.2_2 \rightarrow_g 3.1_2$$

$$1.3_3 \rightarrow_{gf} 3.1_3$$

Bei 1-kontexturalen Systemen wie etwa der Peirce-Bense-Semiotik, wird somit der diamond trivial, vgl. etwa

$$2.2_1 \leftarrow_f \quad 2.2_1$$

$$1.3_1 \rightarrow_f \quad 2.2_1 \circ \quad 2.2_1 \rightarrow_g \quad 3.1_1$$

$$1.3_1 \rightarrow_{gf} \quad 3.1_1$$

denn hier gilt

$(2.2_1 \rightarrow 2.2_1)^{-1} = (2.2_1 \leftarrow 2.2_1)$.

2. Wie man sieht, funktioniert innerhalb der Semiotik das diamond-Modell ohne Probleme, solange man die quantitativen Subrelationen der Zeichenzahlen kontexturiert. Was aber geschieht, wenn man direkt von semiotischen Morphogrammen ausgeht (vgl. Toth 2019)?

2.1. Problem 1

Bestimmte Morphogramme tauchen nur in einer der drei qualitativen Zahlen pro Kontextur auf. Ein Beispiel ist (011), das nur Tritozahl sein kann. Wenn nun diese Zahl Codomäne einer ersten Abbildung und Domäne einer zweiten Abbildung einer morphismischen Komposition ist,

$$011 \leftarrow_f \quad 011$$

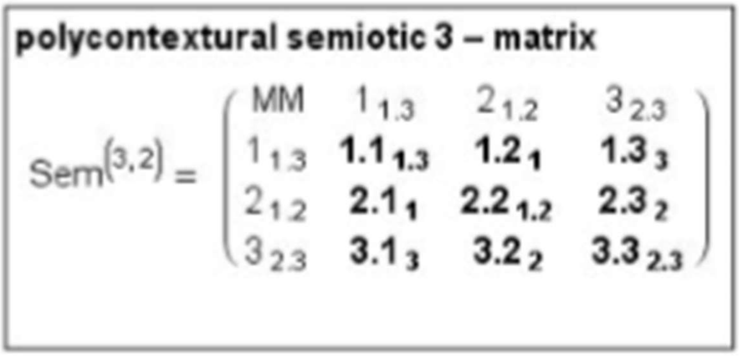
$$001 \rightarrow_f \quad 011 \circ \quad 011 \rightarrow_g \quad 012$$

$$001 \rightarrow_{gf} \quad 012$$

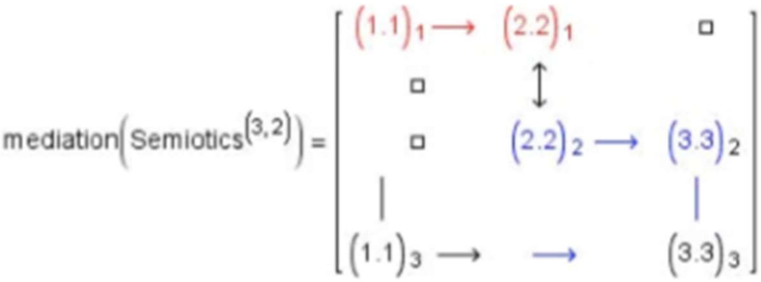
dann wird der diamond, wie schon oben im monokontexturalen semiotischen Beispiel, trivial, da erstens natürlich alle Zahlen des diamonds aus K = 3 sind, aber (011) im Gegensatz zu (001) und (012) nicht Proto- oder Tritozahl sein kann.

2.2. Problem 2

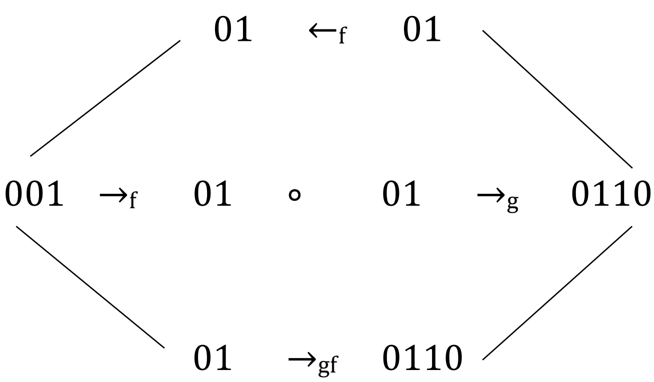
Während die kontexturierten Subzeichen im nachstehenden diamond-Modell (Kaehr 2009, S. 71)



im triadischen Falle nur in einer oder zwei Kontexturen liegen können – und zwar aufgrund des folgenden Mediationsschemas der semiotischen Matrix aus 3 Teilmatrizen (vgl. Kaehr 2009, S. 192),



stellt sich bei Morphogrammen, da ja die Länge eines Morphogramms die Kontextur (und umgekehrt) eindeutig bestimmt, die interessante Frage, wie man denn Morphogramme aus verschiedenen Kontexturen wie etwa im folgenden Modell aufeinander abbildet



Hier gilt also: 1. Jedes Morphogramm liegt in einer und nur einer Kontextur. 2. Der diamond enthält Morphogramme aus 3 Kontexturen

f: $001 \rightarrow 01$

g: $01 \rightarrow 0110$,

d.h. K(01) = 2, K(001) = 3 und K(0110) = 4.

Literatur

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Theory_collection-of-papers-and-fragments_2007.pdf

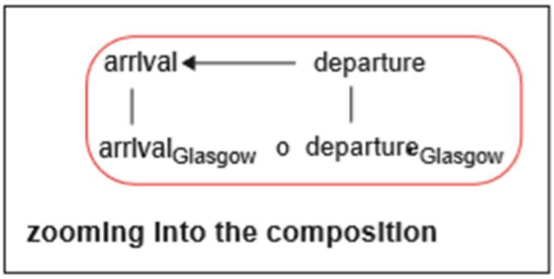
Kaehr, Rudolf, Diamond-Semiotic Short Studies. Glasgow 2009. Digitalisat: www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Eine minimale vollständige polykontexturale Semiotik für K = 4. In: Electronic Journal for Mathematical Semiotics, 2019

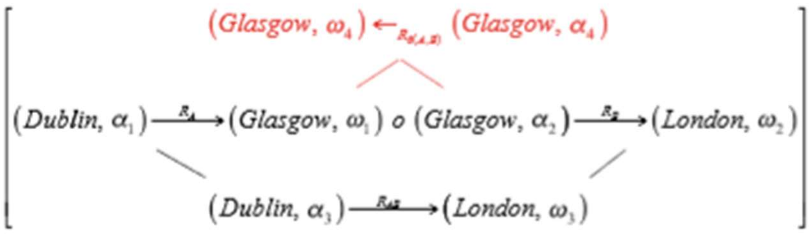
Der semiotische diamond

1. Eine der bedeutendsten theoretischen Neuerungen in der polykontexturalen Logik und der auf ihr basierenden qualitativen Mathematik stellt Rudolf Kaehrs Entdeckung des „diamond" dar (vgl. Kaehr 2007). Der diamond wird zunächst informell eingeführt: Wenn ich mit dem Zug von Dublin nach London fahre und in Glasgow umsteige, dann fungiert Glasgow sofort als Ankunftsort des ersten Teils der Reise als auch als Abfahrtsort des zweiten Teils. Kategorientheoretisch kann man also das Umsteigen als „Komposition" zweier Morphismen – des ersten und zweiten Teils der Reise – auffassen. Nach Kaehr (2007, S. 18 ff.) ist es nun so, daß die Komposition ein Gegenstück hat, und zwar eine Abbildung, welche in der Umkehrung der Abbildung zwischen Glasgow als Ankunfts- und Abfahrtsort besteht. Diese in der quantitativen Kategorientheorie nicht definierte Abbildung nennt Kaehr „Heteromorphismus":

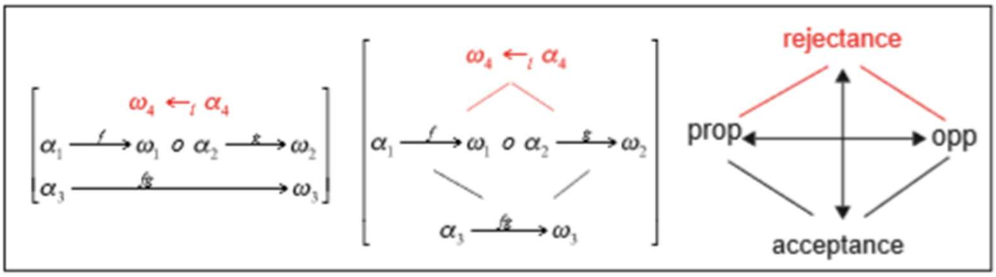
Bemerkenswert ist also, daß der Komposition des Hinwegs eine Abbildung des Herwegs korrespondiert, wobei diese Abbildung allerdings nicht eine Konversion der gesamten, komponierten, Abbildung, sondern nur der Komposition darstellt:



Nach Kaehr läßt sich die gesamte Hin- und Rückreise in der Form eines diamond formal wie folgt darstellen:



Die heteromorphe Abbildung korrespondiert also der homomorphen, konkatanierten, Abbildung:



2. Auffällig ist also, daß der kategorientheoretische diamond sich nicht mit den vier Möglichkeiten des Tetralemmas deckt, das bekanntlich wie folgt definiert ist:

X (affirmation)

$\neg X$ (negation)

$X \wedge \neg X$ (both)

$\neg(X \vee \neg X) \iff X \wedge \neg X \iff \emptyset$ (neither)

Das kategorientheoretische Modell des Tetralemmas sieht dann allerdings wie folgt aus:

$Dublin_1$ ⟵――――――――――――――― $London_3$

$Dublin_1$ → $Glasgow_1$ ∘ $Glasgow_2$ → $London_3$

$Dublin_1$ ―――――――――――――――⟶ $London_3$

Hier gilt zwar ebenfalls ($Glasgow_1$) ≠ ($Glasgow_2$), aber der Umkehrung der komponierten Abbildung ($Dublin_1$) → ($London_3$) korrespondiert wieder eine komponierte Abbildung, und zwar diejenige der ganzen Reise und nicht nur der Umkehrung von Ankunft und Abfahrt am Umsteigebahnhof ($Dublin_1$) ← ($London_3$). Um als auch die letztere zu formalisieren und den Diamanten zu vervollständigen, müßte dieser wie folgt aussehen:

Dublin$_1$ ⟵ London$_3$

Glasgow$_1$ ← Glasgow$_2$

Dublin$_1$ → Glasgow$_1$ ∘ Glasgow$_2$ → London$_3$

Glasgow$_1$ → Glasgow$_2$

Dublin$_1$ ⟶ London$_3$

Mittels Zahlen ausgedrückt haben wir also

1_1 ⟵ 3_3

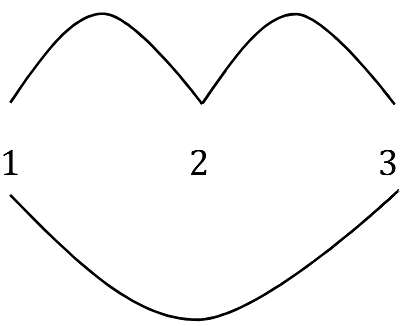
2_1 ← 2_2

1_1 → 2_1 ∘ 2_2 → 3_3

2_1 → 2_2

1_1 ⟶ 3_3

Vgl. dazu die Darstellung mittels Abbildungszahlen (Toth 2019a, b) für n = 3.



(1 → 2 → 3)

(1 → 3)

Die Unterschiede zwischen der Darstellung mittels eines Diamanten und derjenigen mittels Abbildungszahlen sind also:

1. Die Abbildungszahlen werden zwar ohne Kompositionen dargestellt, aber sie lassen sich problemlos in Form von Kompositionen notieren:

$1 \rightarrow 2 \circ 2 \rightarrow 3$

$1 \rightarrow 3$

2. Abbildungszahlen sind zwar a priori nicht different für die Richtung der Abbildungen, aber sie lassen sich wiederum problemlos dazu ergänzen:

$1 \leftarrow 2 \circ 2 \leftarrow 3$

$1 \leftarrow 3$,

d.h. wir haben dann

$1 \leftarrow 3$,

$1 \leftarrow 2 \circ 2 \leftarrow 3$

$1 \rightarrow 2 \circ 2 \rightarrow 3$

$1 \rightarrow 3$.

3. Wie man anhand der letzten Darstellung sieht, sind allerdings die Morphismen bei den Abbildungszahlen immer vollständig, was die Domänen und die Codomänen betrifft, d.h. es gibt keine Abbildungen

$2_1 \rightarrow 2_2$

$2_1 \leftarrow 2_2$

mit

$2_1 \neq 2_2$.

Der Grund liegt darin, die beiden obigen Abbildungen für die rein quantitative Kategorientheorie nicht definiert sind, d.h. von deren Standpunkt aus liegen, um wieder das informelle Beispiel heranzuziehen, Glasgow als Ankunftsbahnhof und Glasgow als Abfahrtsbahnhof immer in derselben Kontextur – und damit auch sowohl auf der Hinfahrt als auch auf der Rückfahrt.

Tatsächlich ist nun gerade die nicht-quantitative Ungleichung ($2_1 \neq 2_2$) dafür verantwortlich, daß der Kaehrsche diamond im Gegensatz zum Tetralemma und im Gegensatz zu den Abbildungszahlen als qualitativ gedeutet wird. In der Polykontexturalitätstheorie wird ja jedem Subjekt eine 2-wertige Logik abgebildet, und diese n 2-wertigen Logiken werden durch Transperatoren zu einem „disseminierten" Verband. Ein solcher Transoperator liegt nun gerade in der konversen Abbildung ($2_1 \leftarrow 2_2$) vor, für die also gilt

$(2_1 \rightarrow 2_2) \neq (2_1 \leftarrow 2_2)$,

da die Zahl 2 sich in zwei verschiedenen Kontexturen K = 1 und K = 2 befindet, also einmal auf der Hinreise von Dublin nach London und einmal auf der Rückreise von London nach Dublin. (En potamoîs toîs autoîs embainomen kai ouk embainomen.)

Die Frage ist nur, wie hier eine Kontextur definiert wird. Wird sie für ein Objekt definiert, würde das bedeuten, daß der Bahnhof von Glasgow auf dem Hinweg ein anderer ist als auf dem Rückweg. Wird sie für ein Subjekt definiert, dann würde das implizieren, daß der Bahnhof von Glasgow für ein Subjekt X ein anderer ist als für ein Subjekt Y. Daraus würde allerdings folgen, daß auch die Komposition heteromorphismisch sein müßte, für den Fall, daß sowohl X als auch Y von Dublin via Glasgow nach London reisen.

Ein weiteres Problem stellt sich bei Namenabbildungen auf ein und dasselbe Objekt. Vgl. zur Illustration den folgenden Kartenausschnitt:

Hier wird die Grenze zwischen den Züricher Stadtquartieren Wipkingen (PLZ: 8037) und Höngg (PLZ: 8049) durch zwei Namenabbildungen der gleichen Straße markiert. Hier findet also ebenfalls eine morphismische Komposition der Form

$(1_1 \rightarrow 2_1 \circ 2_2 \rightarrow 3_3)$

statt, wobei je nach der Richtung der Abbildung

2_1 = Nordstraße / Ottenbergstraße

2_2 = Ottenbergstraße / Nordstraße

ist, denn die Nordstraße gehört politisch zu 8037 Zürich-Wipkingen, die Ottenbergstraße aber zu 8049 Zürich-Höngg. Dieser Fall liegt somit anders als derjenige des Bahnhofs von Glasgow, denn auf diesen ist ja ein und derselbe Name abgebildet.

Damit erhebt sich aufs Neue die Frage, auf welcher Basis Kontexturen eigentlich definiert werden. Im letzten Beispiel gehört die Grenze (G) zwischen den beiden Straßen nämlich zur Vereinigung der beiden Kontexturen, also

$G \subset (K = 1 \cup K = 2)$,

während im ersten Beispiel der Bahnhof von Glasgow einer und derselben Kontextur angehört, obwohl er von Kaehr als bikontextural eingeführt wird. Offenbar sind bei Kaehr also die Kontexturen, wie es ja die polykontexturale Basistheorie verlangt, ausschließlich nach den Subjekten definiert. Daraus folgt dann aber, daß bei der „Kontexturengrenze" im letzten Beispiel gar keine Grenze vorliegt, denn sie gilt ja unabhängig vom Subjekt und ist rein nach dem Ort und damit vermöge der Ortsfunktionalität ontischer Objekte $\Omega = f(\omega)$ nach dem Objekt definiert. Dieses ist in der polykontexturalen Logik allerdings im Gegensatz zum Objekt, das weiterhin im Sinne Hegels als „plattes, totes" Objekt aufgefaßt wird, nicht-iterierbar. Wir stehen also vor einem Dilemma.

Literatur

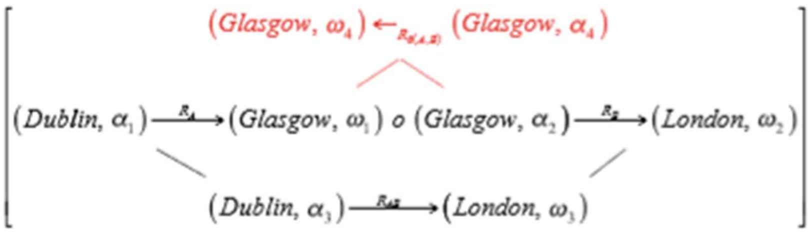
Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007. Digitalisat: www.vordenker.de/rk/rk_Diamond-Theory_collection-of-papers-and-fragments_2007.pdf

Toth, Alfred, Abbildungszahlen. In: Electronic Journal for Mathematical Semiotics, 2019a

Toth, Alfred, Nicht-eingebettete und eingebettete Abbildungszahlen. In: Electronic Journal for Mathematical Semiotics, 2019b

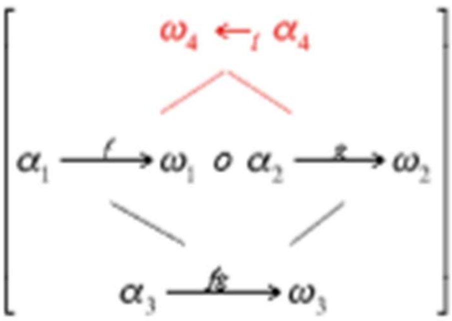
Der vollständige semiotische diamond

1. Der kategorientheoretische diamond kann nach Kaehr (2007) wie folgt informell eingeführt werden



.

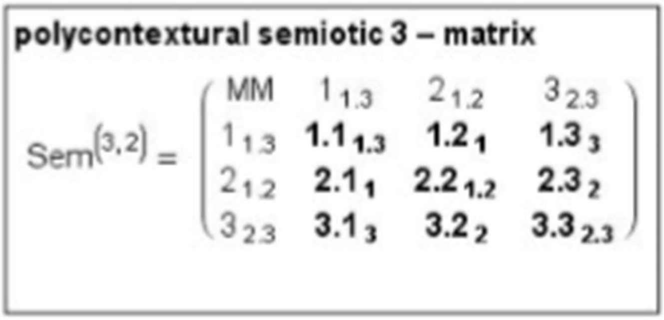
Wie man sieht, unterscheidet sich ein diamond von einer morphismischen Komposition dadurch, daß er sich nach dem Vorbild des Tetralemmas richtet. Glasgow fungiert sowohl als Ankunftsort als auch als Abfahrtsort auf der Reise von Dublin nach London und liegt daher in zwei verschiedenen Kontexturen. Der Komposition der Codomäne der ersten Abbildung und der Domäne der zweiten Abbildung korrespondiert allerdings eine konverse Abbildung, die wiederum in einer anderen Kontextur liegt. Dieser „qualitative Sprung" entspricht den güntherschen Transoperatoren und erzeugt die Übergänge zwischen verschiedenen Kontexturen innerhalb eines logischen Verbundsystems.



2. Wie wir allerdings bereits in Toth (2019) gezeigt hatten, enthält der diamond sowohl in seinem quantitativen als auch in seinem qualitativen Teil keine weiteren Teilabbildungen. So ist beispielsweise der „Heteromorphismus" keineswegs die Konversion des komponierten Morphismus, denn dieser umfaßt ja die ganze Abbildung, jener aber nur diejenige der Kompositionsstelle.

1_1	$\longleftarrow$					3_2
1_1	$\leftarrow$	2_3		2_3	$\leftarrow$	3_2
		2_3	$\leftarrow$	2_3		
1_1	$\rightarrow$	2_1	$\circ$	2_2	$\rightarrow$	3_2
		2_1	$\rightarrow$	2_2		
1_1	$\rightarrow$	2_1		2_2	$\rightarrow$	3_2
1_1					$\longrightarrow$	3_2

Wenn wir von einer kontexturierten Zeichenklasse ausgehen, die wir anhand der ebenfalls von Kaehr eingeführten kontexturierten semiotischen Matrix (vgl. Kaehr 2009) konstruieren,


polycontextural semiotic 3 – matrix

$$Sem^{(3,2)} = \begin{pmatrix} MM & 1_{1.3} & 2_{1.2} & 3_{2.3} \\ 1_{1.3} & \mathbf{1.1}_{1.3} & \mathbf{1.2}_{1} & \mathbf{1.3}_{3} \\ 2_{1.2} & \mathbf{2.1}_{1} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3_{2.3} & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

läßt sich die Polykontexturalität der qualitativen Zeichenklassen im Gegensatz zur Monokontexturalität der quantitativen am besten anhand der nach Bense (1992) „eigenrealen“ Zeichenklasse

$\times(3.1, 2.2, 1.3) = (3.1, 2.2, 1.3)$

aufzeigen, die sich nun als

$\times(3.1_3, 2.2_{1.2}, 1.3_3) \neq (3.1_3, 2.2_{2.1}, 1.3_3)$,

präsentiert, in deren Dualsystem also in Ungleichung durch

$\times 2.2_{1.2} \neq 2.2_{2.1}$

entsteht.

1.3_3	$\longleftarrow$					3.1_3
1.3_3	$\leftarrow$	$2.2_{2.1}$		$2.2_{2.1}$	$\leftarrow$	3.1_3
		$2.2_{2.1}$	$\leftarrow$	$2.2_{2.1}$		
1.3_3	$\rightarrow$	$2.2_{1.2}$	$\circ$	$2.2_{1.2}$	$\rightarrow$	3.1_3
		$2.2_{1.2}$	$\rightarrow$	$2.2_{1.2}$		
1.3_3	$\rightarrow$	$2.2_{1.2}$		$2.2_{1.2}$	$\rightarrow$	3.1_3
1.3_3	$\longrightarrow$					3.1_3

Die hier präsentierten vollständigen diamonds enthalten also nicht nur die quantitativen, sondern auch die qualitativen Teilabbildungen eines triadischen semiotischen Dualsystems der Form

$DS = (3.x, 2.y, 1.z) \times (1.z, 2.y, 3.x)$

3.x	→	2.y	2.y	→	3.x
2.y	→	1.z	1.z	→	2.y

3.x → 1.z 1.z → 3.x,

wogegen sich das gesamte Dualsystem mit Kaehrs diamond nicht darstellen läßt.

Literatur

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, The Book of Diamonds. Glasgow 2007. Digitalisat: www.vordenker.de/rk/rk_Diamond-Theory_collection-of-papers-and-fragments_2007.pdf

Kaehr, Rudolf, Diamond-Semiotic Short Studies. Glasgow 2009. Digitalisat: www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Der semiotische diamond. In: Electronic Journal for Mathematical Semiotics, 2019

Die identitätslogische Basis der theoretischen Semiotik

1. Eine noch immer nicht zweifelsfrei beantwortete Frage lautet: Ist die theoretische Semiotik mono- oder polykontextural? Einerseits hatte Bense (1975, S. 168 ff.) nachgewiesen, daß dem durch die vollständige Induktion darstellbaren Zählen der Zahl das Generieren der Zeichen entspricht, andererseits hatte Bayer festgestellt; „Eine Analogie zu Günthers Reflexionstheorie fällt ins Auge: er unterscheidet zwischen der zweiwertigen Reflexion, in der das Seiende als Bewußtseinsfremdes erlebt wird, und der Reflexion des Bewußtseins auf sich selbst als Gegensatz zu diesem Sein. Setzen wir nun statt 'Reflexion' 'Repräsentation', so gewinnen wir die Unterscheidung zwischen der Repräsentation eines anderen und der Repräsentation der Repräsentation selbst in der semiotischen Reflexion, also der Reflexion auf das Zeichen selbst“ (1994, S. 24).

2. Falls die Semiotik monokontextural ist, ist sie identitätslogisch, da für Monokontexturen (L = 2) gilt

$1 \equiv 2$,

d.h.

$L = (0, 1) = L^{-1} = (1, 0)$.

Ist die Semiotik dagegen polykontextural, so gilt im minimalen Fall (L = 3)

$1 \equiv 2$

$2 \equiv 3$

$1 \equiv 3$,

d.h.

$L_1 = (0, 1), L_2 = (1, 2), L_3 = (0, 2)$

mit $L_1 \neq L_2 \neq L_3$

aber $L_1 = L_1^{-1}, L_2 = L_2^{-1}, L_3 = L_3^{-1}$,

da innerhalb jeder Kontextur nach Günther die monokontexturale Logik gilt.

3. Tatsächlich weist die Semiotik, wie man aus der semiotischen Matrix (Bense 1975, S. 37) ablesen kann, nicht nur eine, sondern drei Identitäten auf, nämlich die sog. genuinen Subzeichen

$(1.1) = id_1$

$(2.2) = id_2$

$(3.3) = id_3$.

Bildet man genau jene Zeichenklassen, welche diese identitiven Morphismen aufweisen, auf die letzteren ab, so erhält man ein Teilsystem von 6/10 Zeichenklassen.

(3.1, 2.1, 1.1) → $(1.1) = id_1$

(3.1, 2.2, 1.2)

(3.1, 2.2, 1.3)

→ $(2.2) = id_2$

(3.2, 2.2, 1.2)

(3.2, 2.2, 1.3)

(3.3, 2.3, 1.3) → $(3.3) = id_3$.

Die restlichen 4/10 Zeichenklassen sind rein formal identitätslogisch unbestimmt, d.h. wir finden folgende Abbildungen

(3.1, 2.1, 1.2)

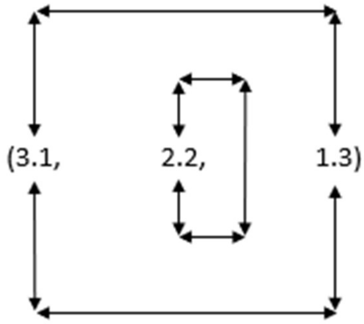
(3.1, 2.1, 1.3)

→ $((1.1), (2.2), (3.3)) = (id_1, id_2, id_3)$

(3.1, 2.3, 1.3)

(3.2, 2.3, 1.3).

Nach Bense (1992, S. 76) werden die 10 Zeichenklassen durch die eigenreale Zeichenklasse (3.1, 2.2, 1.3) innerhalb eines sog. determinantensymmetrischen Dualitätssystems reguliert:



Als reales Modell bestimmte Bense (1992, S. 56) das Möbiusband, denn trotz der 3-fachen Identitätslogik gilt Identität zwischen Subjekt- und Objektpol der Repräsentation vermöge Dualinvarianz

$\times(3.1, 2.2, 1.3) = (3.1, 2.2, 1.3)$,

d.h. wir haben $L = (0, 1) = L^{-1} = (1, 0)$, und damit ist die Semiotik dennoch ein ($L = 2$)-System, sie ist also trotz der Fähigkeit der eigenreale Zeichenklasse, auf sich selbst repräsentierend zu reflektieren, monokontextural!

4. Nach Kaehr (2009) kann man die theoretische Semiotik auf verblüffend einfache Weise in ein polykontexturales System transformieren. Im einfachsten Falle, bei ($L = 3$), werden die Subzeichen der Matrix wie folgt kontexturiert.

categorical 3 – contextural semiotic matrix

$$Sem^{(3,2)}{}_{cat} = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & \mathbf{id_{1,3}} & \alpha_1 & \alpha_3 \\ 2 & \alpha^{\circ}{}_1 & \mathbf{id_{1,2}} & \alpha_2 \\ 3 & \alpha^{\circ}{}_3 & \alpha^{\circ}{}_2 & \mathbf{id_{2,3}} \end{pmatrix}$$

Dadurch kann man nun zwar Abbildungen kontexturierter Zeichenklasse auf die identitätslogischen Kategorien, aber keine solche auf identitive Morphismen konstruieren:

$(3.1_3, 2.1_1, 1.1_{1.3}) \quad \rightarrow \quad (1.1) \neq id_1$

$(3.1_3, 2.2_{1.2}, 1.2_1)$

$(3.1_3, 2.2_{1.2}, 1.3_3)$

$\rightarrow \quad (2.2_{1.2}) \neq id_2$

$(3.2_2, 2.2_{1.2}, 1.2_1)$

$(3.2_2, 2.2_{1.2}, 1.3_3)$

$(3.3_{2.3}, 2.3_2, 1.3_3) \quad \rightarrow \quad (3.3_{2.3}) \neq id_3$

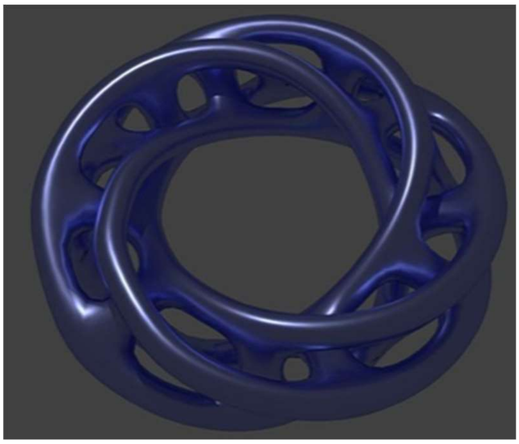
$(3.1_3, 2.1_1, 1.2_1)$

$(3.1_3, 2.1_1, 1.3_3) \quad \rightarrow \quad ((1.1_{1.3}), (2.2_{1.2}), (3.3_{2.3})) \neq (id_1, id_2, id_3)$

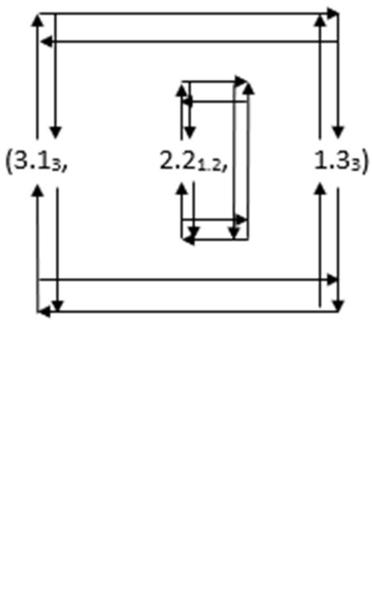
$(3.1_3, 2.3_2, 1.3_3)$

$(3.2_2, 2.3_2, 1.3_3)$.

Das zugehörige Modell ist also kein einfaches Möbiusband wie im monokontexturalen Fall, sondern doppeltes Möbiusband wie dasjenige im nachstehenden Bild



mit dem zugehörigen Regelungsschema



denn es gilt ja

$\times(3.1_3, 2.2_{1.2}, 1.3_3) \neq (3.1_3, 2.2_{2.1}, 1.3_3)$,

d.h.

$\times(3.1_3) = \times(1.3_3)$

aber

$\times(2.2_{1.2}) \neq (2.2_{2.1})$

und somit

$\times(3.1_3, 2.2_{1.2}, 1.3_3) = ((3.1_3, 2.2_1, 1.3_3), (3.1_3, 2.2_2, 1.3_3))$

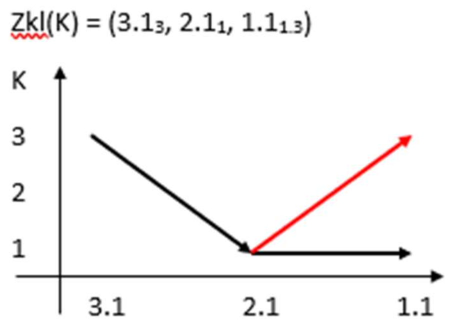
mit

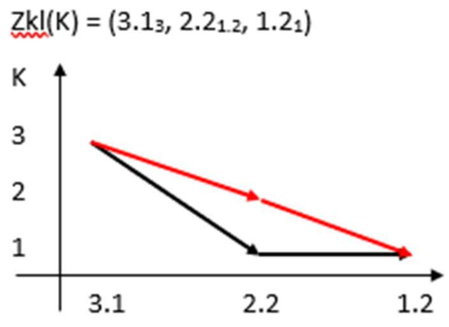
$\times(3.1_3, 2.2_1, 1.3_3) = (3.1_3, 2.2_1, 1.3_3)$

$\times(3.1_3, 2.2_2, 1.3_3) = (3.1_3, 2.2_2, 1.3_3)$,

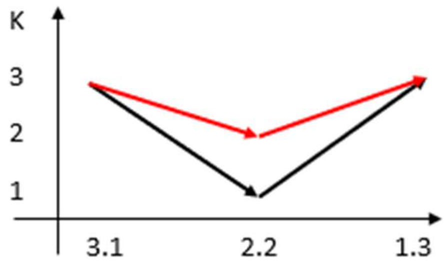
diese Zeichenklasse liegt also in 2 Kontexturen.

In 2 Kontexturen liegen also die gleichen 6/10 Zeichenklassen, welche genuine Subzeichen aufweisen. Sei Zkl = f(K), dann könen wir sie wie folgt graphentheoretisch darstellen.

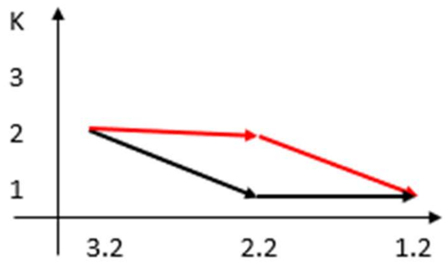




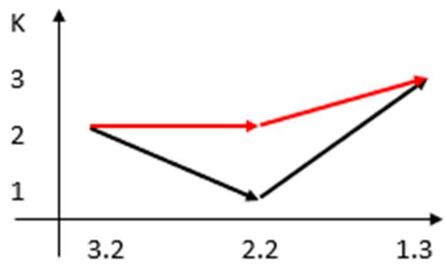
$Zkl(K) = (3.1_3, 2.2_{1.2}, 1.3_3)$



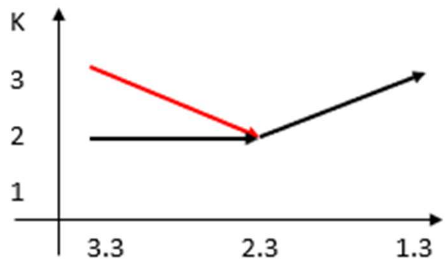
´$Zkl(K) = (3.2_2, 2.2_{1.2}, 1.2_1)$



$Zkl(K) = (3.2_2, 2.2_{1.2}, 1.3_3)$



$Zkl(K) = (3.3_{2.3}, 2.3_2, 1.3_3)$



Bei den Zkl(K)-Graphen entstehen also mit Ausnahme von $(3.1_3, 2.1_1, 1.1_{1.3})$ und $(3.3_{2.3}, 2.3_2, 1.3_3)$ abgeschlossene kontexturelle REPRÄSENTATIONSRÄUME, bisher unbekannte topologische Zeichengebilde, welche an die von Steffen (1981, S. 48 ff.) entdeckten „generativen Einflußfelder" erinnern.

Literatur

Bayer, Udo, Semiotik und Ontologie. In: Semiosis 74-76, 1994, S. 3-34

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Steffen, Werner, Zum semiotischen Aufbau ästhetischer Zustände von Bildwerken. Diss. Stuttgart 1981

Kontexturierte semiotische Relationen

1. In Toth (2019) hatten wir kontexturierte triadisch-trichotomische Zeichenklassen der abstrakten Form

$ZKl^{3,3} = f(K)$ mit $K \in (1, 2, 3)$

über der folgenden kontexturierten Matrix aus Kaehr (2009)

categorical 3 – contextural semiotic matrix

$$Sem^{(3,2)}{}_{cat} = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & \mathbf{id_{1,3}} & \alpha_1 & \alpha_3 \\ 2 & \alpha^\circ{}_1 & \mathbf{id_{1,2}} & \alpha_2 \\ 3 & \alpha^\circ{}_3 & \alpha^\circ{}_2 & \mathbf{id_{2,3}} \end{pmatrix}$$

betrachtet. Wir wollen im folgenden unsere dort gewonnenen Erkenntnisse auf das Gesamtsystem der $3^3 = 27$ über ZKl = (3.x, 2.y, 1.z) mit x, y, z $\in$ (1, 2, 3) erzeugbaren semiotischen Relationen übertragen, d.h. wir heben im folgenden die triadische Inklusionsrestriktion ($x \leq y \leq z$) auf.

2. Wenn man die kaehrsche Semiotik Matrix betrachtet, fallen zwei Sachverhalte auf, die wir als Theoreme notieren können.

THEOREM 1: Duale Subzeichen liegen in den gleichen Kontexturen.

THEOREM 2: Homogene Subzeichen von $ZR^{3,3}$ liegen in zwei Kontexturen.

Aus Theorem 2 folgt als Lemma, daß die Anzahl homogener Subzeichen der Anzahl der Kontexturen entspricht.

Zunächst seien die 27 semiotischen Relationen als kontexturierte Zeichenklassen notiert.

$(3.1_3, 2.1_1, \underline{1.1}_{1.3})$	$(3.1_3, \underline{2.2}_{1.2}, \underline{1.1}_{1.3})$	$(3.1_3, 2.3_2, \underline{1.1}_{1.3})$
$(3.1_3, 2.1_1, 1.2_1)$	$(3.1_3, \underline{2.2}_{1.2}, 1.2_1)$	$(3.1_3, 2.3_2, 1.2_1)$
$(3.1_3, 2.1_1, 1.3_3)$	$(3.1_3, \underline{2.2}_{1.2}, 1.3_3)$	$(3.1_3, 2.3_2, 1.3_3)$

$(3.2_2, 2.1_1, \underline{1.1}_{1.3})$	$(3.2_2, \underline{2.2}_{1.2}, \underline{1.1}_{1.3})$	$(3.2_2, 2.3_2, \underline{1.1}_{1.3})$
$(3.2_2, 2.1_1, 1.2_1)$	$(3.2_2, \underline{2.2}_{1.2}, 1.2_1)$	$(3.2_2, 2.3_2, 1.2_1)$
$(3.2_2, 2.1_1, 1.3_3)$	$(3.2_2, \underline{2.2}_{1.2}, 1.3_3)$	$(3.2_2, 2.3_2, 1.3_3)$

$(\underline{3.3}_{2.3}, 2.1_1, \underline{1.1}_{1.3})$	$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, \underline{1.1}_{1.3})$	$(\underline{3.3}_{2.3}, 2.3_2, \underline{1.1}_{1.3})$
$(\underline{3.3}_{2.3}, 2.1_1, 1.2)$	$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, 1.2)$	$(\underline{3.3}_{2.3}, 2.3_2, 1.2_1)$
$(\underline{3.}3_{2.3}, 2.1_1, 1.3_3)$	$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, 1.3_3)$	$(\underline{3.3}_{2.3}, 2.3_2, 1.3_3)$

Wie im folgenden gezeigt wird, können diese 27 Zkl(K) nach der Anzahl der Abbildungen in 8 nicht-diskrete Gruppen eingeteilt werden.

2.1. Abbildungen auf (1.1)

$(3.1_3, 2.1_1, \underline{1.1}_{1.3}) \rightarrow ((3.1_3, 2.1_1, \underline{1.1}_1), (3.1_3, 2.1_1, \underline{1.1}_3))$

$(3.1_3, 2.3, \underline{1.1}_{1.3}) \rightarrow ((3.1_3, 2.3, \underline{1.1}_1), (3.1_3, 2.3, \underline{1.1}_3))$

$(3.2_2, 2.1_1, \underline{1.1}_{1.3}) \rightarrow ((3.2_2, 2.1_1, \underline{1.1}_1), (3.2_2, 2.1_1, \underline{1.1}_3))$

$(3.2_2, 2.3_2, \underline{1.1}_{1.3}) \rightarrow ((3.2_2, 2.1_1, \underline{1.1}_1), (3.2_2, 2.1_1, \underline{1.1}_3))$

2.2. Abbildungen auf (2.2)

$(3.1_3, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((3.1_3, \underline{2.2}_1, 1.2_1), (3.1_3, \underline{2.2}_2, 1.2_1))$

$(3.1_3, \underline{2.2}_{1.2}, 1.3_3) \rightarrow ((3.1_3, \underline{2.2}_1, 1.2_1), (3.1_3, \underline{2.2}_2, 1.2_1))$

$(3.2_2, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((3.2_2, \underline{2.2}_1, 1.2_1), (3.2_2, \underline{2.2}_2, 1.2_1))$

$(3.2_2, \underline{2.2}_{1.2}, 1.3_3) \rightarrow ((3.2_2, \underline{2.2}_1, 1.3_3), (3.2_2, \underline{2.2}_2, 1.3_3))$

2.3. Abbildungen auf (3.3)

$(\underline{3.3}_{2.3}, 2.1_1, 1.2_1) \rightarrow ((\underline{3.3}_2, 2.1_1, 1.2_1), (\underline{3.3}_3, 2.1_1, 1.2_1))$

$(\underline{3.3}_{2.3}, 2.1_1, 1.3_3) \rightarrow ((\underline{3.3}_2, 2.1_1, 1.3_3), (\underline{3.3}_3, 2.1_1, 1.3_3))$

$(\underline{3.3}_{2.3}, 2.3_2, 1.2_1) \rightarrow ((\underline{3.3}_2, 2.3_2, 1.2_1), (\underline{3.3}_3, 2.3_2, 1.2_1))$

$(\underline{3.3}_{2.3}, 2.3_2, 1.3_3) \rightarrow ((\underline{3.3}_2, 2.3_2, 1.3_3), (\underline{3.3}_3, 2.3_2, 1.3_3))$

2.4. Abbildungen auf ((1.1), (2.2))

$(3.1_3, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow ((3.1_3, \underline{2.2}_1, \underline{1.1}_1), (3.1_3, \underline{2.2}_1, \underline{1.1}_3),$

$(3.1_3, \underline{2.2}_2, \underline{1.1}_1), (3.1_3, \underline{2.2}_2, \underline{1.1}_3))$

$(3.2_2, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow ((3.2_2, \underline{2.2}_1, \underline{1.1}_1), (3.2_2, \underline{2.2}_1, \underline{1.1}_3),$

$(3.2_2, \underline{2.2}_2, \underline{1.1}_1), (3.2_2, \underline{2.2}_2, \underline{1.1}_3))$

2.5. Abbildungen auf ((2.2), (3.3))

$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((\underline{3.3}_2, \underline{2.2}_1, 1.2_1), (\underline{3.3}_2, \underline{2.2}_2, 1.2_1)$

$(\underline{3.3}_3, \underline{2.2}_1, 1.2_1), (\underline{3.3}_3, \underline{2.2}_2, 1.2_1))$

$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((\underline{3.3}_2, \underline{2.2}_1, 1.2_1), (\underline{3.3}_2, \underline{2.2}_2, 1.2_1)$

$(\underline{3.3}_3, \underline{2.2}_1, 1.2_1), (\underline{3.3}_3, \underline{2.2}_2, 1.2_1))$

2.6. Abbildungen auf ((1.1), (3.3))

$(\underline{3.3}_{2.3}, 2.1_1, \underline{1.1}_{1.3}) \rightarrow ((\underline{3.3}_2, 2.1_1, \underline{1.1}_1), (\underline{3.3}_2, 2.1_1, \underline{1.1}_3)$

$(\underline{3.3}_3, 2.1_1, \underline{1.1}_1), (\underline{3.3}_3, 2.1_1, \underline{1.1}_3))$

$(\underline{3.3}_{2.3}, 2.3_2, \underline{1.1}_{1.3}) \rightarrow ((\underline{3.3}_2, 2.3_2, \underline{1.1}_1), (\underline{3.3}_2, 2.3_2, \underline{1.1}_3)$

$(\underline{3.3}_3, 2.3_2, \underline{1.1}_1), (\underline{3.3}_3, 2.3_2, \underline{1.1}_3))$

2.7. Abbildungen auf ((1.1), (2.2), (3.3))

$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow ((\underline{3.3}_2, \underline{2.2}_1, \underline{1.1}_1), (\underline{3.3}_2, \underline{2.2}_1, \underline{1.1}_3), (\underline{3.3}_2, \underline{2.2}_2, \underline{1.1}_1), (\underline{3.3}_2, \underline{2.2}_2, \underline{1.1}_3), (\underline{3.3}_3, \underline{2.2}_1, \underline{1.1}_1), (\underline{3.3}_3, \underline{2.2}_1, \underline{1.1}_3), (\underline{3.3}_3, \underline{2.2}_2, \underline{1.1}_1), (\underline{3.3}_3, \underline{2.2}_2, \underline{1.1}_3))$

2.8. Keine identitive Abbildungen

$(3.1_3, 2.1_1, 1.2_1)$

$(3.1_3, 2.3_2, 1.2_1)$

$(3.1_3, 2.1_1, 1.3_3)$

$(3.1_3, 2.3_2, 1.3_3)$

$(3.2_2, 2.1_1, 1.2_1)$

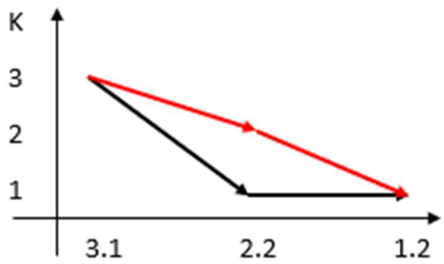
$(3.2_2, 2.3_2, 1.2_1)$

$(3.2_2, 2.1_1, 1.3_3)$

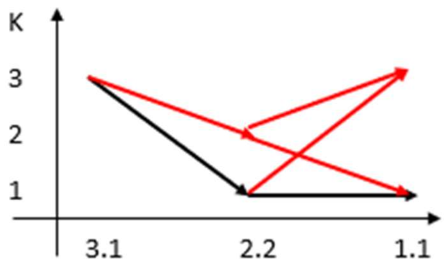
$(3.2_2, 2.3_2, 1.3_3)$

3. Wie bereits in Toth (2019) gezeigt, weisen Zkl(K) mit K > 1 sog. abgeschlossene kontexturelle Repräsentationsräume auf, die für die 10/27 regulären Zkl(K) aufgezeichnet wurden. Die 17 irregulären Zkl(K) weisen erstens andere Typen und zweitens wegen der Möglichkeit, daß eine ZKl(K) mehr als ein homogenes Subzeichen erhält auch komplexere Repräsentationsräume auf. Wir geben je ein Beispiel für Zkl(K) mit K = 2, 4, 8-

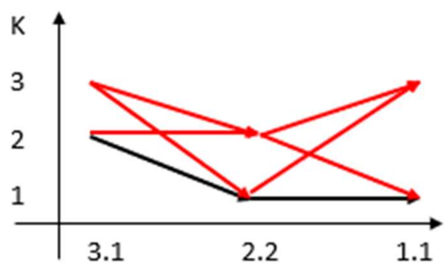
3.1. Zkl(K = 2) = $(3.1_3, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((3.1_3, \underline{2.2}_1, 1.2_1), (3.1_3, \underline{2.2}_2, 1.2_1))$



3.2. Zkl(K = 4) = $(3.1_3, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow ((3.1_3, \underline{2.2}_1, \underline{1.1}_1), (3.1_3, \underline{2.2}_1, \underline{1.1}_3),$
$(3.1_3, \underline{2.2}_2, \underline{1.1}_1), (3.1_3, \underline{2.2}_2, \underline{1.1}_3))$



3.3. Zkl(K = 8) = $(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow ((\underline{3.3}_2, \underline{2.2}_1, \underline{1.1}_1), (\underline{3.3}_2, \underline{2.2}_1, \underline{1.1}_3), (\underline{3.3}_2, \underline{2.2}_2, \underline{1.1}_1), (\underline{3.3}_2, \underline{2.2}_2, \underline{1.1}_3), (\underline{3.3}_3, \underline{2.2}_1, \underline{1.1}_1), (\underline{3.3}_3, \underline{2.2}_1, \underline{1.1}_3), (\underline{3.3}_3, \underline{2.2}_2, \underline{1.1}_1), (\underline{3.3}_3, \underline{2.2}_2, \underline{1.1}_3))$



Literatur

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Die identitätslogische Basis der theoretischen Semiotik. In: Electronic Journal for Mathematical Semiotics, 2019

Beweis der Existenz ontischer Kontexturen

1. Bekanntlich wurde die Kontextur durch Günther (1976-80) als der Geltungsbereich einer 2-wertigen (aristotelischen) Logik definiert. Eine monokontexturale Logik weist nur ein L = (0, 1) auf, darin 0 die Objekt- und 1 die Subjektposition bezeichnet. Eine polykontexturale Logik ist also ein System von n Kontexturen der Form L, die logisch durch Rejektionswertfunktoren und mathematisch durch Transoperatoren (Kronthaler 1986) miteinander verbunden sind. Da für jedes L gilt $L = (0, 1) = L^{-1} = (1, 0)$ (vgl. Günther 2000, S. 230 f.), kann man eine Logik L auch durch $0 \equiv 1$ definieren. Das bedeutet also, daß die Negation nichts enthalten kann, was die Position, die sie spiegelt, bereits enthält – et vice versa. Logisch Neues resultiert also erst in einem minimal 3-kontexturalen System ($0 \equiv 1$, $1 \equiv 2$, $0 \equiv 2$). Nun sind Ausdrücke der Form $x \equiv y$ logisch 1-stellige Operationen, denn dieser Ausdruck bedeutet ja, um Wittgenstein zu zitieren, nichts anderes, also daß man die linke und die rechte Seite des ≡-Operators vertauschen kann. Es dürfte nun einleuchten, daß der ≡-Operator nur auf semiotischer Ebene sinnvoll ist, denn auf ontischer Ebene gibt es keine zwei geschiedenen Objekte, die identisch sind. Identität tritt in der Objektwelt nur als Selbstidentität auf. Von hier aus folgt also bereits automatisch die Existenz von ontischen Kontexturen. Jedes Objekt bildet mit einem Subjekt eine separate Kontextur. Zwei Autos, die gleich sind, sind dennoch nicht verschieden, was schon daraus folgt, daß sie nicht den gleichen ontischen Ort einnehmen können: $\mathbf{\Omega}_m(\omega_i) \neq \mathbf{\Omega}_n(\omega_j)$ mit $i \neq j$, d.h. aus $\mathbf{\Omega}(\omega_i) = \mathbf{\Omega}(\omega_j)$ mit $i \neq j$ folgt $\mathbf{\Omega}_m = \mathbf{\Omega}_n$. Dennoch wollen wir uns im folgenden, ausgehend von realen ontischen Modellen, um eine formalere Beweisführung der Existenz ontischer Konteturen bemühen.

2. Während aus der monokontexturalen Darstellung der semiotischen Eigenrealität durch Bense (1992)

$\times(3.1, 2.2, 1.3) \equiv (3.1, 2.2, 1.3)$

folgt, was nichts anderes als O ≡ 1 von L ist, da die Zeichenklasse die Subjektposition und ihre duale koordinierte Realitätsthematik die Objektposition der verdoppelten semiotischen Repräsentation des Zeichens kodiert, erhalten wir durch die polykontexturalen Darstellung der gleichen Zeichenklasse nach einem Vorschlag von Kaehr (2009)

$\times(3.1_3, 2.2_{1.2}, 1.3_3) \neq (3.1_3, 2.2_{2.1}, 1.3_3)$,

d.h.

$\times(3.1_3) = \times(1.3_3)$

aber

$\times(2.2_{1.2}) \neq (2.2_{2.1})$

und somit

$\times(3.1_3, 2.2_{1.2}, 1.3_3) = ((3.1_3, 2.2_1, 1.3_3), (3.1_3, 2.2_2, 1.3_3))$

mit

$\times(3.1_3, 2.2_1, 1.3_3) = (3.1_3, 2.2_1, 1.3_3)$

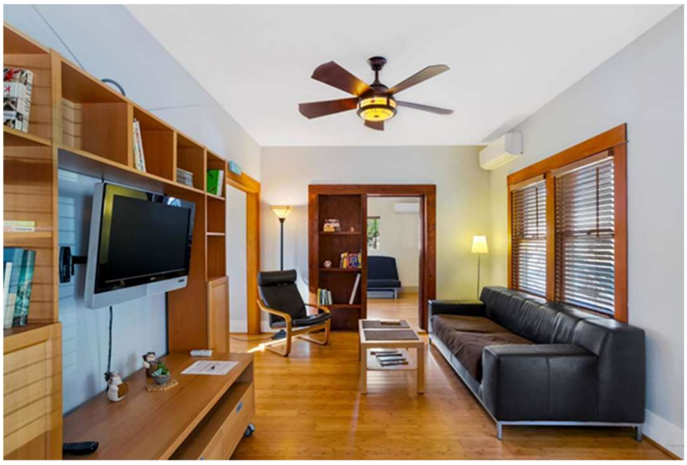
$\times(3.1_3, 2.2_2, 1.3_3) = (3.1_3, 2.2_2, 1.3_3)$,

diese Zeichenklasse liegt also in 2 Kontexturen. Es gibt somit weder Dualidentität noch Eigenrealität.

Betrachten wir nun das folgende Teilsystem des Wohnzimmers eines Hauses in Tucson, AZ, indem wir die ontische Relation R = (Vorn, Hinten) zugrunde legen. (Die dualisierten Photos wurden jeweils mit einem Bildprogramm durch Vf. hergestellt.)

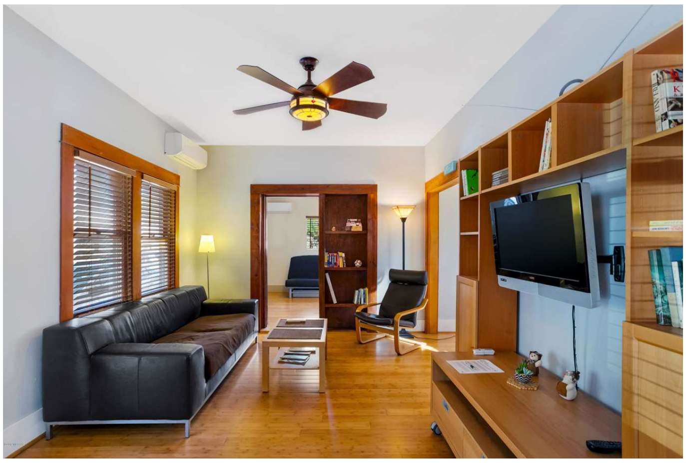
Vorn → Hinten

×(Hinten ⟶ Vorn)



(Vorn ⟶ Hinten) ≠ ×(Hinten ⟶ Vorn)

Hinten ⟶ Vorn



×(Vorn ⟶ Hinten)

Wir bekommen also sofort

(Hinten → Vorn) ≠ ×(Vorn → Hinten).

Allgemein gilt also in der Ontik für ein Paar dichotomischer Relationen x und y

(x → y) ≠ ×(y → x)

(y → x) ≠ ×(x → y),

also auch

(x → y → z) ≠ ×(z → y → x)

(z → y → x) ≠ ×(x → y → z).

Vermöge ontisch-semiotischer Isomorphie folgt also

(3.1 → 2.2 → 1.3) ≠ ×(1.3 → 2.2 → 3.1)

(1.3 → 2.2 → 3.1) ≠ ×(3.1 → 2.2 → 1.3),

das gilt aber nach Kaehr 2009 (vgl. Toth 2019a, b) gdw.

K(3.1) = K(1.3) ≠ K(2.2)

(K(3.1) = K(1.3) vermöge des 1. Theorems in Toth 2019b). □

Literatur

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Günther, Gotthard, Beiträge zur Formalisierung einer operationsfähigen Dialektik. 3 Bde. Hamburg 1976-1980.

Günther, Gotthard, Die amerikanische Apokalypse. München 2000

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Kronthaler, Engelbert, Grundlegung einer Mathematik der Qualitäten. Frankfurt am Main 1986

Toth, Alfred, Die identitätslogische Basis der theoretischen Semiotik. In: Electronic Journal for Mathematical Semiotics, 2019a

Toth, Alfred, Kontexturierte semiotische Relationen. In: Electronic Journal for Mathematical Semiotics, 2019b

Kontexturelle semiotische Matrizen

1. Wie zuletzt in Toth (2019a, b) gezeigt, kann man, einem Vorschlag Kaehrs folgend, die Semiotik dadurch in ein polykontexturales System transformieren, daß man auf die Subzeichen der von Bense (1975, S. 37) eingeführten semiotischen Matrix Kontexturenzahlen abbildet. Die kontexturelle Matrix aus Kaehr (2009) sieht dann für die triadisch-trichotomische Semiotik ($S^{3,3}$) wie folgt aus.

categorical 3 – contextural semiotic matrix

$Sem^{(3,2)}_{cat}$ =

MM	1	2	3
1	$id_{1.3}$	α_1	α_3
2	α°_1	$id_{1.2}$	α_2
3	α°_3	α°_2	$id_{2.3}$

Dabei gelten offenbar folgende Theoreme (vgl. Toth 2019b):

THEOREM 1: Duale Subzeichen liegen in den gleichen Kontexturen.

Lemma 1: Die Kontexturen triadischer und trichotomischer Peircezahlen sind gleich.

THEOREM 2: Homogene Subzeichen von $ZR^{3,3}$ liegen in zwei Kontexturen.

Lemma 2: Die Anzahl homogener Subzeichen ist gleich der der Anzahl der Kontexturen.

Lemma 3: Die Anzahl der Kontexturen ist gleich der Stelligkeit der Relation.

Vermöge der Lemmata 2 und 3 sieht dann eine kontexturelle Matrix für $S^{4,4}$ wie folgt aus (Kaehr 2009).

4 – contextural semiotic matrix

$Sem^{(4,2)}$ =

MM	1	2	3	4
1	$\mathbf{1.1}_{1.3.4}$	$1.2_{1.3}$	$1.3_{1.4}$	$1.4_{3.4}$
2	$2.1_{1.3}$	$\mathbf{2.2}_{1.2.3}$	$2.3_{1.2}$	$2.4_{2.3}$
3	$3.1_{1.4}$	$3.2_{1.2}$	$\mathbf{3.3}_{1.2.4}$	$3.4_{2.4}$
4	$4.1_{3.4}$	$4.2_{3.2}$	$4.3_{2.4}$	$\mathbf{4.4}_{2.3.4}$

2. Bekanntlich gibt es nun in der monokontexturalen Semiotik von Peirce und Bense genau eine Zeichenklasse, die mit ihrer dual koordinierten Realitätsthematik identisch ist, d.h. bei der

Zkl ≡ Rth

gilt. Es handelt sich um die von Bense (1992) als eigenreale bezeichnete Zeichenklasse des Zeichens selbst, bei der also Subjekt- und Objektpol der verdoppelten Repräsentation koinzidieren

$\times(3.1, 2.2, 1.3) = (3.1, 2.2, 1.3)$.

Sobald wir allerdings von der kaehrschen kontexturellen Matrix für $S^{3,3}$ ausgehen, wird die Dualidentität und mit ihr die Eigenrealität aufgehoben:

$\times(3.1_3, 2.2_{1.2}, 1.3_3) \neq (3.1_3, 2.2_{2.1}, 1.3_3)$,

d.h.

$\times(3.1_3) = \times(1.3_3)$,

aber

$\times(2.2_{1.2}) \neq (2.2_{2.1})$

und somit

$\times(3.1_3, 2.2_{1.2}, 1.3_3) = ((3.1_3, 2.2_1, 1.3_3), (3.1_3, 2.2_2, 1.3_3))$

mit

$\times(3.1_3, 2.2_1, 1.3_3) = (3.1_3, 2.2_1, 1.3_3)$

$\times(3.1_3, 2.2_2, 1.3_3) = (3.1_3, 2.2_2, 1.3_3)$,

diese Zeichenklasse liegt also in 2 Kontexturen.

Die Frage, die sich uns nun stellt, ist: Ist es möglich, die Eigenrealität auch innerhalb der polykontexturalen Semiotik beizubehalten? Wie man leicht sieht, hängt die Beantwortung dieser Frage damit zusammen, ob es gelingt, Subzeichen zu kontexturieren, ohne gegen Lemma 2 zu verstoßen.

Offenbar muß dazu das Theorem 1 aufgehoben werden. Wir weisen also duale Subzeichen der Form $\times(x.y) = (y.x)$ je verschiedenen Kontexturen zu:

$\times(x.y)_i \neq (y.x)_j$

mit i ≠ j. Die Kontexturen sollen entsprechend der Progression der Peircezahlen (1 → 2 → 3) gezählt werden.

$1.1_{1.2}$	$2.2_{5.6}$	$3.3_{3.4}$
1.2_1	2.1_2	
1.3_3	3.1_4	
2.3_5	3.2_6	

$ZR^{3,3}$ liegt nun also in 6 statt in 3 Kontexturen. Dann erhalten wir folgende neue kontexturelle Matrix

	1	2	3
1	$1.1_{1.2}$	1.2_1	1.3_3
2	2.1_2	$2.2_{5.6}$	2.3_5
3	3.1_4	3.2_6	$3.3_{3.4}$.

Die monokontextural eigenreale Zeichenklasse ist somit auch in dieser polykontexturalen Semiotik eigenreal

$\times(3.1_4, 2.2_{5.6}, 1.3_3) = (3.1_4, 2.2_{5.6}, 1.3_3)$.

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Die identitätslogische Basis der theoretischen Semiotik. In: Electronic Journal for Mathematical Semiotics, 2019a

Toth, Alfred, Kontexturierte semiotische Relationen. In: Electronic Journal for Mathematical Semiotics, 2019b

Die Abbildung von Zkl auf K(Zkl)

1. Eine minimale, triadisch-trichotomische, Zeichenklasse hat die allgemeine Form

Zkl = (3.x, 2.y, 1.z)

mit x, y, z ∈ (1, 2, 3),

d.h. die die Zeichenklassen erzeugende Matrix ist eine quadratische Matrix mit m = n = 3.

Da die drei Peicezahlen P = (1, 2, 3) qualitativ sind, insofern 1 mit der Kategorie der Möglichkeit, 2 mit der Kategorie der Wirklichkeit und 3 mit der Kategorie der Notwendigkeit assoziiert wird, liegt der Gedanke einer kontexturalen 3-teilung der 3-adisch-3-chotomischen Semiotik nahe. Diese besitzt ja im Gegensatz zur einen Identität der 2-wertigen Logik auch drei Identitäten, und diese werden durch die drei homogenen Subzeichen (x.x) mit x = (1, 2, 3), also (1.1), (2.2) und (3.3), repräsentiert.

Allerdings dauerte es bis zum Erscheinen der Arbeit Kaehr (2009), bis eine kontexturierte Matrix vorgelegt wurde.

categorical 3 – contextural semiotic matrix

$$Sem^{(3,2)}_{cat} = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & \mathbf{id_{1,3}} & \alpha_1 & \alpha_3 \\ 2 & \alpha^\circ_1 & \mathbf{id_{1,2}} & \alpha_2 \\ 3 & \alpha^\circ_3 & \alpha^\circ_2 & \mathbf{id_{2,3}} \end{pmatrix}$$

Wie man leicht sieht (vgl. auch Toth 2019a-c) gelten folgende Theoreme und Lemmata.

THEOREM 1: Duale Subzeichen liegen in den gleichen Kontexturen.

Lemma 1: Die Kontexturen triadischer und trichotomischer Peircezahlen sind gleich.

THEOREM 2: Homogene Subzeichen von $ZR^{3,3}$ liegen in zwei Kontexturen.

Lemma 2: Die Anzahl homogener Subzeichen ist gleich der der Anzahl der Kontexturen.

Lemma 3: Die Anzahl der Kontexturen ist gleich der Stelligkeit der Relation.

2. Rein theoretisch lassen sich über Zkl = (3.x, 2.y, 1.z) mit x, y, z $\in$ (1, 2, 3) $3^3 = 27$ Zeichenklassen erzeugen. Allerdings wurde aus diesen von Peirce und Bense durch die trichotomische Inklusionsrestriktion $x \leq y \leq z$ eine Teilmenge von 10 Zeichenklassen herausgefiltert, welche die Berechnung der Zkl(K), d.h. der kontexturierten Zeichenklassen, schwierig macht. Vermöge der obigen Sätze liegen nur diejenigen Zeichenklassen in 1 Kontextur, bei denen keine Abbildung möglich ist, also diejenigen, die keine Subzeichen der Form S = (x.x) enthalten.

$(3.1_3, 2.1_1, \underline{1.1}_{1.3}) \quad \rightarrow \quad ((3.1_3, 2.1_1, \underline{1.1}_1), (3.1_3, 2.1_1, \underline{1.1}_3))$

$(3.1_3, 2.1_1, 1.2_1)$

$(3.1_3, 2.1_1, 1.3_3)$

$(3.1_3, \underline{2.2}_{1.2}, 1.2_1) \quad \rightarrow \quad ((3.1_3, \underline{2.2}_1, 1.2_1), (3.1_3, \underline{2.2}_2, 1.2_1))$

$(3.1_3, \underline{2.2}_{1.2}, 1.3_3) \quad \rightarrow \quad ((3.1_3, \underline{2.2}_1, 1.2_1), (3.1_3, \underline{2.2}_2, 1.2_1))$

$(3.1_3, 2.3_2, 1.3_3)$

$(3.2_2, \underline{2.2}_{1.2}, 1.2_1) \quad \rightarrow \quad ((3.2_2, \underline{2.2}_1, 1.2_1), (3.2_2, \underline{2.2}_2, 1.2_1))$

$(3.2_2, \underline{2.2}_{1.2}, 1.3_3) \quad \rightarrow \quad ((3.2_2, \underline{2.2}_1, 1.3_3), (3.2_2, \underline{2.2}_2, 1.3_3))$

$(3.2_2, 2.3_2, 1.3_3)$

$(\underline{3.3}_{2.3}, 2.3_2, 1.3_3) \quad \rightarrow \quad ((\underline{3.3}_2, 2.3_2, 1.3_3), (\underline{3.3}_3, 2.3_2, 1.3_3))$

Total: 16 K-Zeichenklassen.

Wir haben damit

Zkl	Zkl(K)
10	16.

3. Noch drastischer sieht das Verhältnis der Anzahl der nicht-kontexturierten und der kontexturierten Zeichenklassen im Gesamtsystem der 27 $ZR^{3,3}$ aus.

Wie im folgenden gezeigt wird, können diese 27 Zkl(K) nach der Anzahl der Abbildungen in 8 nicht-diskrete Gruppen eingeteilt werden.

$(3.1_3, 2.1_1, \underline{1.1}_{1.3}) \quad \rightarrow \quad ((3.1_3, 2.1_1, \underline{1.1}_1), (3.1_3, 2.1_1, \underline{1.1}_3))$

$(3.1_3, 2.3, \underline{1.1}_{1.3}) \rightarrow ((3.1_3, 2.3, \underline{1.1}_1), (3.1_3, 2.3, \underline{1.1}_3))$

$(3.2_2, 2.1_1, \underline{1.1}_{1.3}) \rightarrow ((3.2_2, 2.1_1, \underline{1.1}_1), (3.2_2, 2.1_1, \underline{1.1}_3))$

$(3.2_2, 2.3_2, \underline{1.1}_{1.3}) \rightarrow ((3.2_2, 2.1_1, \underline{1.1}_1), (3.2_2, 2.1_1, \underline{1.1}_3))$

$(3.1_3, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((3.1_3, \underline{2.2}_1, 1.2_1), (3.1_3, \underline{2.2}_2, 1.2_1))$

$(3.1_3, \underline{2.2}_{1.2}, 1.3_3) \rightarrow ((3.1_3, \underline{2.2}_1, 1.2_1), (3.1_3, \underline{2.2}_2, 1.2_1))$

$(3.2_2, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((3.2_2, \underline{2.2}_1, 1.2_1), (3.2_2, \underline{2.2}_2, 1.2_1))$

$(3.2_2, \underline{2.2}_{1.2}, 1.3_3) \rightarrow ((3.2_2, \underline{2.2}_1, 1.3_3), (3.2_2, \underline{2.2}_2, 1.3_3))$

$(\underline{3.3}_{2.3}, 2.1_1, 1.2_1) \rightarrow ((\underline{3.3}_2, 2.1_1, 1.2_1), (\underline{3.3}_3, 2.1_1, 1.2_1))$

$(\underline{3.3}_{2.3}, 2.1_1, 1.3_3) \rightarrow ((\underline{3.3}_2, 2.1_1, 1.3_3), (\underline{3.3}_3, 2.1_1, 1.3_3))$

$(\underline{3.3}_{2.3}, 2.3_2, 1.2_1) \rightarrow ((\underline{3.3}_2, 2.3_2, 1.2_1), (\underline{3.3}_3, 2.3_2, 1.2_1))$

$(\underline{3.3}_{2.3}, 2.3_2, 1.3_3) \rightarrow ((\underline{3.3}_2, 2.3_2, 1.3_3), (\underline{3.3}_3, 2.3_2, 1.3_3))$

$(3.1_3, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow ((3.1_3, \underline{2.2}_1, \underline{1.1}_1), (3.1_3, \underline{2.2}_1, \underline{1.1}_3),$

$(3.1_3, \underline{2.2}_2, \underline{1.1}_1), (3.1_3, \underline{2.2}_2, \underline{1.1}_3))$

$(3.2_2, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow ((3.2_2, \underline{2.2}_1, \underline{1.1}_1), (3.2_2, \underline{2.2}_1, \underline{1.1}_3),$

$(3.2_2, \underline{2.2}_2, \underline{1.1}_1), (3.2_2, \underline{2.2}_2, \underline{1.1}_3))$

$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((\underline{3.3}_2, \underline{2.2}_1, 1.2_1), (\underline{3.3}_2, \underline{2.2}_2, 1.2_1)$

$(\underline{3.3}_3, \underline{2.2}_1, 1.2_1), (\underline{3.3}_3, \underline{2.2}_2, 1.2_1))$

$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, 1.2_1) \rightarrow ((\underline{3.3}_2, \underline{2.2}_1, 1.2_1), (\underline{3.3}_2, \underline{2.2}_2, 1.2_1)$

$(\underline{3.3}_3, \underline{2.2}_1, 1.2_1), (\underline{3.3}_3, \underline{2.2}_2, 1.2_1))$

$(\underline{3.3}_{2.3}, 2.1_1, \underline{1.1}_{1.3}) \rightarrow ((\underline{3.3}_2, 2.1_1, \underline{1.1}_1), (\underline{3.3}_2, 2.1_1, \underline{1.1}_3)$

$(\underline{3.3}_3, 2.1_1, \underline{1.1}_1), (\underline{3.3}_3, 2.1_1, \underline{1.1}_3))$

$(\underline{3.3}_{2.3}, 2.3_2, \underline{1.1}_{1.3}) \rightarrow ((\underline{3.3}_2, 2.3_2, \underline{1.1}_1), (\underline{3.3}_2, 2.3_2, \underline{1.1}_3)$

$(\underline{3.3}_3, 2.3_2, \underline{1.1}_1), (\underline{3.3}_3, 2.3_2, \underline{1.1}_3))$

$(\underline{3.3}_{2.3}, \underline{2.2}_{1.2}, \underline{1.1}_{1.3}) \rightarrow$ $((\underline{3.3}_2, \underline{2.2}_1, \underline{1.1}_1), (\underline{3.3}_2, \underline{2.2}_1, \underline{1.1}_3), (\underline{3.3}_2, \underline{2.2}_2, \underline{1.1}_1), (\underline{3.3}_2, \underline{2.2}_2, \underline{1.1}_3), (\underline{3.3}_3, \underline{2.2}_1, \underline{1.1}_1), (\underline{3.3}_3, \underline{2.2}_1, \underline{1.1}_3), (\underline{3.3}_3, \underline{2.2}_2, \underline{1.1}_1), (\underline{3.3}_3, \underline{2.2}_2, \underline{1.1}_3))$

$(3.1_3, 2.1_1, 1.2_1)$

$(3.1_3, 2.3_2, 1.2_1)$

$(3.1_3, 2.1_1, 1.3_3)$

$(3.1_3, 2.3_2, 1.3_3)$

$(3.2_2, 2.1_1, 1.2_1)$

$(3.2_2, 2.3_2, 1.2_1)$

$(3.2_2, 2.1_1, 1.3_3)$

$(3.2_2, 2.3_2, 1.3_3)$

Total: 64 K-Zeichenklassen

Wir haben also

Zkl	Zkl(K)
10	16
27	64.

4. Gehen wir von $ZR^{3,3}$ zu $ZR^{4,4}$ über, so ergeben sich natürlich $4^4 = 256$ Zkl. Wird die erweiterte (tetradisch-tetratomische) Inklusionsordnung auf sie angewandt, werden nur 35 Zkln herausgefiltert (vgl. Toth 2007, S. 179 ff.)

1	3.0	2.0	1.0	0.0	×	0.0	0.1	0.2	0.3	0^4
2	3.0	2.0	1.0	0.1	×	1.0	0.1	0.2	0.3	$1^1 0^3$
3	3.0	2.0	1.0	0.2	×	2.0	0.1	0.2	0.3	$2^1 0^3$
4	3.0	2.0	1.0	0.3	×	3.0	0.1	0.2	0.3	$3^1 0^3$
5	3.0	2.0	1.1	0.1	×	1.0	1.1	0.2	0.3	$1^2 0^2$
6	3.0	2.0	1.1	0.2	×	2.0	1.1	0.2	0.3	$2^1 1^1 0^2$
7	3.0	2.0	1.1	0.3	×	3.0	1.1	0.2	0.3	$3^1 1^1 0^2$
8	3.0	2.0	1.2	0.2	×	2.0	2.1	0.2	0.3	$2^2 0^2$
9	3.0	2.0	1.2	0.3	×	3.0	2.1	0.2	0.3	$3^1 2^1 0^2$
10	3.0	2.0	1.3	0.3	×	3.0	3.1	0.2	0.3	$3^2 0^2$
11	3.0	2.1	1.1	0.1	×	1.0	1.1	1.2	0.3	$1^3 0^1$
12	3.0	2.1	1.1	0.2	×	2.0	1.1	1.2	0.3	$2^1 1^2 0^1$
13	3.0	2.1	1.1	0.3	×	3.0	1.1	1.2	0.3	$3^1 1^2 0^1$
14	3.0	2.1	1.2	0.2	×	2.0	2.1	1.2	0.3	$2^2 1^1 0^1$
15	**3.0**	**2.1**	**1.2**	**0.3**	**×**	**3.0**	**2.1**	**1.2**	**0.3**	$\mathbf{3^1 2^1 1^1 0^1}$
16	3.0	2.1	1.3	0.3	×	3.0	3.1	1.2	0.3	$3^2 1^1 0^1$
17	3.0	2.2	1.2	0.2	×	2.0	2.1	2.2	0.3	$2^3 0^1$
18	3.0	2.2	1.2	0.3	×	3.0	2.1	2.2	0.3	$3^1 2^2 0^1$
19	3.0	2.2	1.3	0.3	×	3.0	3.1	2.2	0.3	$3^2 2^1 0^1$
20	3.0	2.3	1.3	0.3	×	3.0	3.1	3.2	0.3	$3^3 0^1$
21	3.1	2.1	1.1	0.1	×	1.0	1.1	1.2	1.3	1^4
22	3.1	2.1	1.1	0.2	×	2.0	1.1	1.2	1.3	$2^1 1^3$
23	3.1	2.1	1.1	0.3	×	3.0	1.1	1.2	1.3	$3^1 1^3$
24	3.1	2.1	1.2	0.2	×	2.0	2.1	1.2	1.3	$2^2 1^2$
25	3.1	2.1	1.2	0.3	×	3.0	2.1	1.2	1.3	$3^1 2^1 1^2$
26	3.1	2.1	1.3	0.3	×	3.0	3.1	1.2	1.3	$3^2 1^2$
27	3.1	2.2	1.2	0.2	×	2.0	2.1	2.2	1.3	$2^3 1^1$
28	3.1	2.2	1.2	0.3	×	3.0	2.1	2.2	1.3	$3^1 2^2 1^1$
29	3.1	2.2	1.3	0.3	×	3.0	3.1	2.2	1.3	$3^2 2^1 1^1$
30	3.1	2.3	1.3	0.3	×	3.0	3.1	3.2	1.3	$3^3 1^1$
31	3.2	2.2	1.2	0.2	×	2.0	2.1	2.2	2.3	2^4
32	3.2	2.2	1.2	0.3	×	3.0	2.1	2.2	2.3	$3^1 2^3$
33	3.2	2.2	1.3	0.3	×	3.0	3.1	2.2	2.3	$3^2 2^2$
34	3.2	2.3	1.3	0.3	×	3.0	3.1	3.2	2.3	$3^3 2^1$
35	3.3	2.3	1.3	0.3	×	3.0	3.1	3.2	3.3	3^4

Aufgrund von Lemma 2 werden den Zkl im folgenden die Anzahlen der homogenen Subzeichen zugeordnet und bestimmt, in wie vielen Kontexturen die Zkl liegen.

1:	2	8:	1	15:	1	22:	2	29:	2
2:	1	9:	1	16:	1	23:	2	30:	1
3:	1	10:	1	17:	2	24:	1	31:	2
4:	1	11:	2	18:	2	25:	1	32:	2

5:	2	12:	2	19:	2	26:	1	33:	2
6:	2	13:	2	20:	1	27:	2	34:	1
7:	2	14:	1	21:	2	28:	2	35:	2

Total ergeben sich also nur 55 Zkl(K). Diese geringe Anzahl verdankt sich der Tatsache, daß vermöge der tetradisch-tetratomischen Inklusionsordnung, wie schon bei $ZR^{3,3}$, keine Zkl auftritt, die mehr als 1 homogenes Subzeichen hat. Wir haben also bis jetzt

ZR	Zkl	Zkl(K)
$Z^{3,3}$	10	16
$Z^{3,3}$	27	64
$Z^{4,4}$	35	55
$Z^{4,4}$	256	?

Während die Anzahl der Zkl den figurierten Zahlen folgt (vgl. Toth 2007, S. 186 ff.), bilden die Zkl(K) allerdings eine bisher unbekannte Zahlenfolge, die mit 16, 64, 55 beginnt. Damit existiert auch (bisher) keine Möglichkeit, Zkl($Z^{4,4}$) algorithmisch zu berechnen, und dasselbe gilt natürlich für die $Z^{n,n}$ mit $n > 4$.

Literatur

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Zwischen den Kontexturen. Klagenfurt 2007

Toth, Alfred, Die identitätslogische Basis der theoretischen Semiotik. In: Electronic Journal for Mathematical Semiotics, 2019a

Toth, Alfred, Kontexturierte semiotische Relationen. In: Electronic Journal for Mathematical Semiotics, 2019b

Toth, Alfred, Kontexturelle semiotische Matrizen. In: Electronic Journal for Mathematical Semiotics, 2019c

Das semiotische Differential

1. Wie zuletzt in Toth (2019a, b) gezeigt, kann man, einem Vorschlag Kaehrs folgend, die Semiotik dadurch in ein polykontexturales System transformieren, daß man auf die Subzeichen der von Bense (1975, S. 37) eingeführten semiotischen Matrix Kontexturenzahlen abbildet. Die kontexturelle Matrix aus Kaehr (2009) sieht dann für die triadisch-trichotomische Semiotik ($S^{3,3}$) wie folgt aus.

categorical 3 – contextural semiotic matrix

$$Sem^{(3,2)}_{cat} = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & \mathbf{id_{1.3}} & \alpha_1 & \alpha_3 \\ 2 & \alpha^\circ{}_1 & \mathbf{id_{1.2}} & \alpha_2 \\ 3 & \alpha^\circ{}_3 & \alpha^\circ{}_2 & \mathbf{id_{2.3}} \end{pmatrix}$$

2. Beim Übergang von nicht-kontextruellen Zeichenklassen (Zkl) zu kontexturellen Zeichenklassen (Zkl(K)) ändert sich bei der Dualisation, welche Subjekt- und Objektrepräsentation austauscht, für alle Zeichenklassen, die keine homogenen Subzeichen (identitive Morphismen) aufweisen nichts, vgl. etwa

$\times(3.1_3, 2.1_1, 1.2_1) = (2.1_1, 1.2_1, 1.3_3)$

$\times(3.1_3, 2.1_1, 1.2_1) = (2.1_1, 1.2_1, 1.3_3)$.

Dagegen wird die Ordnung der verdoppelten Kontexturenzahlen bei den Zeichenklassen mit homogenen Subzeichen vertauscht. Bei der monokontextural gesehen dualidentischen Zeichenklasse des Zeichens selbst wird damit die von Bense (1992) festgestellte Eigenrealität aufgehoben

$\times(3.1_3, 2.2_{1.2}, 1.3_3) = (3.1_3, 2.2_{2.1}, 1.3_3)$,

d.h.

$(3.1_3, 2.2_{1.2}, 1.3_3) \neq (3.1_3, 2.2_{2.1}, 1.3_3)$.

Die Frage, die sich uns nun stellt, ist: Ist es möglich, die Eigenrealität auch innerhalb der polykontexturalen Semiotik beizubehalten? Offenbar muß dazu das Theorem 1 aus Toth (2019b) aufgehoben werden:

THEOREM 1: Duale Subzeichen liegen in den gleichen Kontexturen.

Wir weisen also duale Subzeichen der Form $\times$(x.y) = (y.x) je verschiedenen Kontexturen zu (vgl. Toth 2019c):

$\times(x.y)_i \neq (y.x)_j$

mit i ≠ j. Die Kontexturen sollen entsprechend der Progression der Peircezahlen (1 → 2 → 3) gezählt werden.

$1.1_{1.2}$ $2.2_{5.6}$ $3.3_{3.4}$

1.2_1 2.1_2

1.3_3 3.1_4

2.3_5 3.2_6

$ZR^{3,3}$ liegt nun also in 6 statt in 3 Kontexturen. Dann erhalten wir folgende neue kontexturelle Matrix

	1	2	3
1	$1.1_{1.2}$	1.2_1	1.3_3
2	2.1_2	$2.2_{5.6}$	2.3_5
3	3.1_4	3.2_6	$3.3_{3.4}$.

Die monokontexturalen Zeichenklassen mit homogenen Subzeichen sind somit auch in dieser polykontexturalen Semiotik eigenreal, vgl. die neue formale Struktur der Eigenrealität

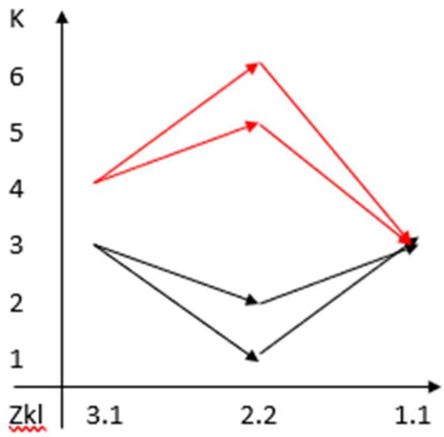
$\times(3.1_4, 2.2_{5.6}, 1.3_3) = (3.1_4, 2.2_{5.6}, 1.3_3)$.

3. Wenn wir uns nun der Zkl(K)-Graphen bedienen (vgl. Toth 2019b), so können wir die beiden Kontexturierungsweisen, also im Falle der eigenrealen Zkl

$(3.1_3, 2.2_{1.2}, 1.3_3) \neq \times(3.1_3, 2.2_{2.1}, 1.3_3)$

$(3.1_4, 2.2_{5.6}, 1.3_3) = \times(3.1_4, 2.2_{5.6}, 1.3_3)$,

in einen modifizierten, d.h. 6-kontexturalen Graphen eintragen.



Die differentielle, je nach Paar von Zkl und Zkl(K) offene oder abgeschlossene topologische Fläche zwischen den beiden Graphen bestimmen wir als das semiotische Differential

$\Delta_{sem} = \Delta(Zkl_i(Kl_j), Zkll_k(Kl_l))$.

Δ_{sem} ist damit ein neues semiotisches Maß zur Bestimmung der Differenz zwischen kontexturierten Zeichenklassen mit und ohne Gültigkeit von Theorem 1.

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Bense, Max, Die Eigenrealität der Zeichen. Baden-Baden 1992

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Die identitätslogische Basis der theoretischen Semiotik. In: Electronic Journal for Mathematical Semiotics, 2019a

Toth, Alfred, Kontexturierte semiotische Relationen. In: Electronic Journal for Mathematical Semiotics, 2019b

Toth, Alfred, Kontexturelle semiotische Matrizen. In: Electronic Journal for Mathematical Semiotics, 2019c

Das erkenntnistheoretische Differential

1. Bekanntlich besteht ein semiotisches Dualsystem der Form

D = ((3.x, 2.y, 1.z) × (z.1, y.2, x.3))

aus der den erkenntnistheoretischen Subjektpol repräsentierenden Zeichenklasse Zkl = (3.x, 2.y, 1.z) und der den erkenntnistheoretischen Objektpol repräsentierenden Realitätsthematik Rth = (z.1, y.2, x.3). Voraussetzung für den hier neu einzuführenden Begriff des erkenntnistheoretischen Differentials ist, daß die dergestalt semiotisch verdoppelte Repräsentation der Erkenntnis „keine vollständige Separation zwischen (materialer) Welt und (intelligiblem) Bewußtsein zuläßt“ (Bense 1979, S. 18 f.). Es besteht demnach stets eine Differenz zwischen dem Repräsentamen der Zkl und dem Präsentamen der Rth.

2. Diese Differenz wollen wir im Anschluß an Toth (2019a) als erkenntnistheoretisches Differential einführen

$\Delta_{erk} = \Delta(Zkl_i(K_j), Rth_i(K_j))$.

Wir gehen also wieder (vgl. Toth 2019b) aus von $ZKl^{3,3} = f(K)$ mit K ∈ (1, 2, 3) über der folgenden kontexturierten Matrix aus Kaehr (2009).

categorical 3 – contextural semiotic matrix

$$Sem^{(3,2)}_{cat} = \begin{pmatrix} MM & 1 & 2 & 3 \\ 1 & \mathbf{id_{1,3}} & \alpha_1 & \alpha_3 \\ 2 & \alpha^\circ_1 & \mathbf{id_{1,2}} & \alpha_2 \\ 3 & \alpha^\circ_3 & \alpha^\circ_2 & \mathbf{id_{2,3}} \end{pmatrix}$$

Es gelten folgende Sätze (vgl. Toth 2019c).

THEOREM 1: Duale Subzeichen liegen in den gleichen Kontexturen.

Lemma 1: Die Kontexturen triadischer und trichotomischer Peircezahlen sind gleich.

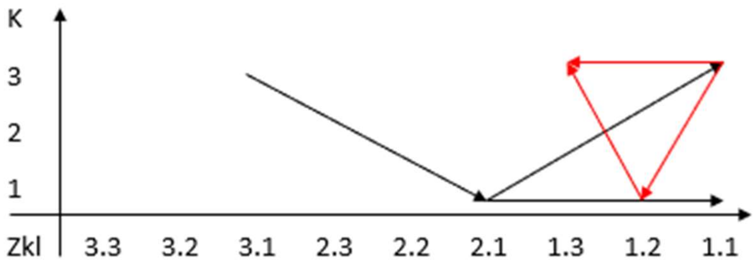
THEOREM 2: Homogene Subzeichen von $ZR^{3,3}$ liegen in zwei Kontexturen.

Lemma 2: Die Anzahl homogener Subzeichen ist gleich der der Anzahl der Kontexturen.

Lemma 3: Die Anzahl der Kontexturen ist gleich der Stelligkeit der Relation.

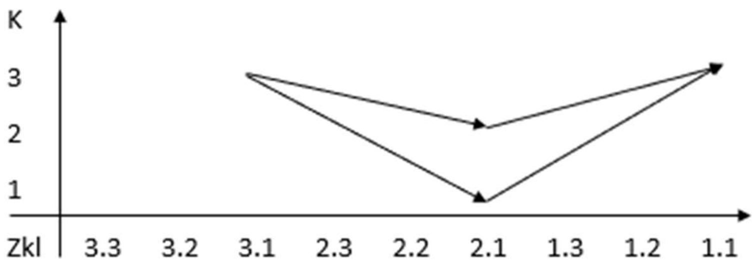
Aufgrund von Lemma 2 wollen wir im folgenden drei Dualsysteme exemplarisch betrachten: D = ((3.1, 2.1, 1.1) × (1.1, 1.2, 1.3), das je 1 identitiven Morphismus besitzt, die mit ihrer dual koordinierten Realitätsthematik identische eigenreale Zeichenklasse (3.1, 2.2, 1.3) und D = ((3.1, 2.1, 1.2) × (2.1, 1.2, 1.3)), das keinen identitiven Morphismus enthält. (In den 10/27 regulären Dualsystemen treten keine verdoppelten homogenen Subzeichen auf. Drei homogene Subzeichen treten nur in der Klasse der Kategorienrealität auf.)

2.1. $\Delta_{erk} = \Delta(Zkl_i(K_j), Rth_i(K_j)) = \Delta((3.1_3, 2.1_1, 1.1_{1.3}), (1.1_{1.3}, 1.2_1, 1.3_3))$



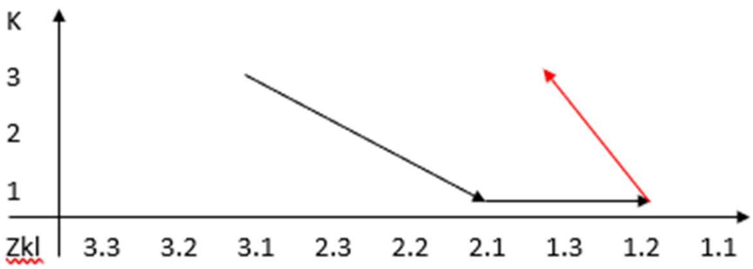
$\Delta_{erk}((3.1_3, 2.1_1, 1.1_{1.3}), (1.1_{1.3}, 1.2_1, 1.3_3)) > 0$

2.2. $\Delta_{erk} = \Delta(Zkl_i(K_j), Rth_i(K_j)) = \Delta((3.1_3, 2.2_{1.2}, 1.3_3), (3.1_3, 2.2_{1.2}, 1.3_3))$



$\Delta_{erk}((3.1_3, 2.2_{1.2}, 1.3_3), (3.1_3, 2.2_{1.2}, 1.3_3)) = 0$

2.3. $\Delta_{erk} = \Delta(Zkl_i(K_j), Rth_i(K_j)) = \Delta((3.1_3, 2.1_1, 1.2_1), (2.1_1, 1.2_1, 1.3_3))$



$\Delta_{erk}((3.1_3, 2.1_1, 1.2_1), (2.1_1, 1.2_1, 1.3_3)) < 0$

Positive Differenz weisen also alle Dualsysteme mit homogenen Subzeichen auf. Negative Differenz, d.h. einen erkenntnistheoretischen Überschuß, zeigen Dualsysteme ohne homogene Subzeichen. Exakt gleich 0 ist erwartungsgemäß die erkenntnistheoretische Differenz zwischen Zkl und Rth des eigenrealen Dualsystems.

Literatur

Bense, Max, Die Unwahrscheinlichkeit des Ästhetischen. Baden-Baden 1979

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Das semiotische Differential. In: Electronic Journal for Mathematical Semiotics, 2019a

Toth, Alfred, Die identitätslogische Basis der theoretischen Semiotik. In: Electronic Journal for Mathematical Semiotics, 2019b

Toth, Alfred, Die Abbildung von Zkl auf Zkl(K). In: Electronic Journal for Mathematical Semiotics, 2019c

Die Abbildung von Kontexturenzahlen auf Subzeichen und Kategorien

1. In Toth (2020) hatten wir gezeigt, daß man mit Hilfe der Kontexturenzahlen die Zeichentrichotomien neu ordnen kann. Im folgenden werden die kontexturierten 10 Zeichenklassen auf die Menge ihrer Kontexturenzahlen abgebildet.

$(3.1_3, 2.1_1, \underline{1.1}_{1.3}) \quad \rightarrow \quad (3, 1, (1.3))$

$(3.1_3, 2.1_1, 1.2_1) \quad \rightarrow \quad (3, 1, 1)$

$(3.1_3, 2.1_1, 1.3_3) \quad \rightarrow \quad (3, 1, 3)$

$(3.1_3, \underline{2.2}_{1.2}, 1.2_1) \quad \rightarrow \quad (3, (1.2), 1)$

$(3.1_3, \underline{2.2}_{1.2}, 1.3_3) \quad \rightarrow \quad (3, (1.2), 3)$

$(3.2_2, \underline{2.2}_{1.2}, 1.2_1) \quad \rightarrow \quad (2, (1.2), 1)$

$(3.2_2, \underline{2.2}_{1.2}, 1.3_3) \quad \rightarrow \quad (2, (1.2), 3)$

$(3.1_3, 2.3_2, 1.3_3) \quad \rightarrow \quad (3, 2, 3)$

$(3.2_2, 2.3_2, 1.3_3) \quad \rightarrow \quad (2, 2, 3)$

$(\underline{3.3}_{2.3}, 2.3_2, 1.3_3) \quad \rightarrow \quad ((2.3), 2, 3)$

2. Da man ferner die Subzeichen der kleinen Matrix auf die semiotischen Morphismen abbilden kann (vgl. Toth 1997, S. 21 ff.), erhält man

1.1	1.2	1.3		id1	α	$\beta\alpha$
2.1	2.2	2.3	$\rightarrow$	$\alpha°$	id2	β
3.1	3.2	3.3		$\alpha°\beta°$	$\beta°$	id3,

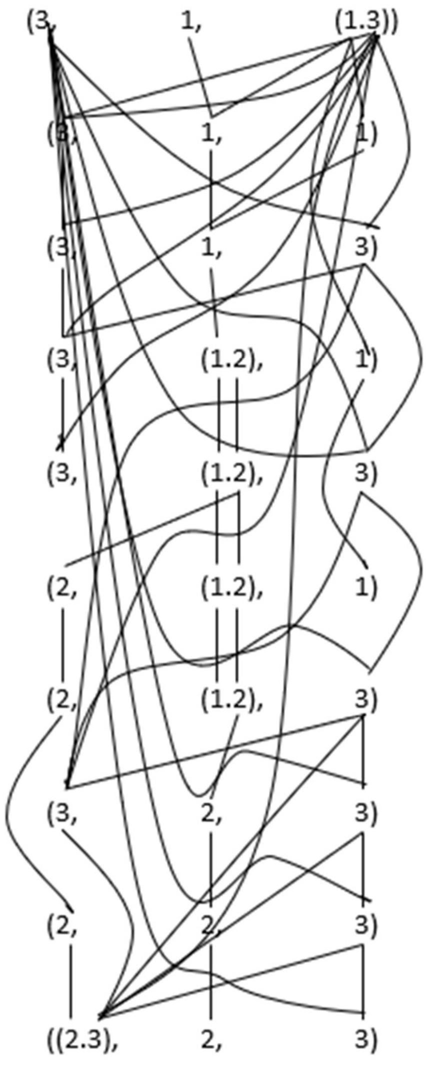
also

id1 = (1.3), id2 = (1.2), id3 = (2.3)

$\alpha = \alpha° = 1$, $\beta = \beta° = 2$, $\beta\alpha = \alpha°\beta° = 3$.

Während somit die Abbildung der kontexturierten Zeichenklassen auf die Kontexturenzahlen bijektiv ist, ist die Abbildung der semiotischen Morphismen auf die Kontexturenzahlen nicht-bijektiv, da duale Subzeichen gleiche Kontexturenzahlen abgebildet bekommen (vgl. Toth 2019)

3. Wegen der Bijektivität der ersteren Abbildung ist es allerdings möglich, das System der 10 Mengen von Kontexturenzahlen als semiotisches Verbundsystem darzustellen. Der folgende Graph enthält aus Gründen der Übersichtlichkeit nur die wichtigsten transitorischen Übergänge.



Wenn man sich bewußt macht, daß Zeichenklassen nicht notwendig 3-stellig sein müssen und da auch die Zahl der Kontexturen beliebg erhöht werden kann (vgl. Kaehr 2009), stellt also der obige Graph einer m-kontexturalen n-wertigen Semiotik ein morphogrammatisches Fragment eines semiotischen disseminierten Verbundsystems dar.

Literatur

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Entwurf einer Semiotisch-Relationalen Grammatik. Tübingen 1997

Toth, Alfred, Die Abbildung von Zkl auf Zkl(K). In: Electronic Journal for Mathematical Semiotics, 2019

Toth, Alfred, Die Ordnung der Zeichentrichotomien durch die Kontexturenzahlen. In: Electronic Journal for Mathematical Semiotics, 2020

Semiotische Kontexturübergänge

1. In Toth (2020) hatten wir gezeigt, daß die Subzeichen der von Bense (1975, S. 37) eingeführten semiotischen Matrix nicht-bijektiv auf ihre Kontexturenzahlen (Kaehr 2009) abgebildet werden können:

1.1	1.2	1.3		(1.3)	1	3
2.1	2.2	2.3	→	1	(1.2)	2
3.1	3.2	3.3		3	2	(2.3).

Die Semiosen zwischen Paaren von Subzeichen korrespondieren somit den Abbildungen zwischen Paaren von Kontexturenzahlen und ermöglichen damit eine formale Darstellung semiotischer Transgressionen.

2. Da der folgende Satz gilt (vgl. Toth 2019),

THEOREM 1: Duale Subzeichen liegen in den gleichen Kontexturen.

sind kontexturelle Abbildungen nur dann bijektiv, wenn ein homogenes Subzeichen auf ein anderes abgebildet wird.

(1.3) → (1.3) = (1.1) → (1.1)

(1.3) → 1 = (1.1) → (1.2)/(1.1) → (2.1)

(1.3) → 3 = (1.1) → (1.3) / (1.1) → (3.1)

(1.3) → (1.2)= (1.1) → (2.2)

(1.3) → 2 = (1.1) → (2.3) / (1.1) → (3.2)

(1.3) → (2.3)= (1.1) → (3.3)

1 → 1 = (1.2) → (1.2) / (1.2) → (2.1) / (2.1) → (1.2) / (2.1) → (2.1)

1 → 3 = (1.2) → (1.3) / (1.2) → (3.1) / (2.1) → (1.3) / (2.1) → (3.1)

1 → (1.2) = (1.2) → (2.2) / (2.1) → (2.2)

$1 \rightarrow 2 \quad = (1.2) \rightarrow (2.3)\ /\ (1.2) \rightarrow (3.2)\ /\ (2.1) \rightarrow (2.3)\ /\ (2.1) \rightarrow (3.2)$

$1 \rightarrow (2.3) \quad = (1.2) \rightarrow (3.3)$

$3 \rightarrow 3 \quad = (1.3) \rightarrow (1.3)\ /\ (1.3) \rightarrow (3.1)\ /\ (3.1) \rightarrow (1.3)\ /\ (3.1) \rightarrow (3.1)$

$3 \rightarrow (1.2) \quad = (1.3) \rightarrow (2.2)\ /\ (3.1) \rightarrow (2.2)$

$3 \rightarrow 2 \quad = (1.3) \rightarrow (2.3)\ /\ (1.3) \rightarrow (3.2)\ /\ (3.1) \rightarrow (2.3)\ /\ (3.1) \rightarrow (3.2)$

$3 \rightarrow (2.3) \quad = (1.3) \rightarrow (3.3)\ /\ (3.1) \rightarrow (3.3)$

$(1.2) \rightarrow (1.2) = (2.2) \rightarrow (2.2)$

$(1.2) \rightarrow 2 \quad = (2.2) \rightarrow (2.3)\ /\ (2.2) \rightarrow (3.2)$

$(1.2) \rightarrow (2.3) = (2.2) \rightarrow (3.3)$

$2 \rightarrow 2 \quad = (2.3) \rightarrow (2.3)\ /\ (2.3) \rightarrow (3.2)\ /\ (3.2) \rightarrow (2.3)\ /\ (3.2) \rightarrow (3.2)$

$2 \rightarrow (2.3) \quad = (2.3) \rightarrow (3.3)\ /\ (3.2) \rightarrow (3.3)$

Die Rechtsmehrdeutigkeit semiotischer Kontexturübergänge besteht somit aus 2 oder 4 Codomänen, je nachdem, ob eines oder kein homogenes Subzeichen an der Abbildung beteiligt ist.

Literatur

Bense, Max, Semiotische Prozesse und Systeme. Baden-Baden 1975

Kaehr, Rudolf, Diamond Semiotic Short Studies. Glasgow, UK 2009. Digitalisat: http://www.vordenker.de/rk/rk_Diamond-Semiotic_Short-Studies_2009.pdf

Toth, Alfred, Kontexturelle semiotische Matrizen. In: Electronic Journal for Mathematical Semiotics, 2019

Toth, Alfred, Morphismen als semiotische Transoperatoren. In: Electronic Journal for Mathematical Semiotics, 2020