Prof. Dr. Alfred Toth

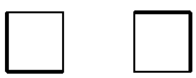
Semiotic shortest-path diagrams

1. The number of shortest paths in n × n grids is calculated by the central binomial coefficients

$$\begin{bmatrix} 2n \\ n \end{bmatrix} \quad \text{or} \quad \frac{(2n)!}{n!^2}$$

i.e., they fall along the center line of Pascal's triangle (for semiotics, cf. Toth 2008a, pp. 186 ss.).

2. In a 1 × 1 grid, there are two shortest paths:



Since a 1 × 1 grid is a network model for the dyadic sign relation $SR_{2,2}$, which has been called "pre-semiotic" sign relation by Ditterich (1990, pp. 29, 81) and which is a sub-matrix of the following semiotic matrix:

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

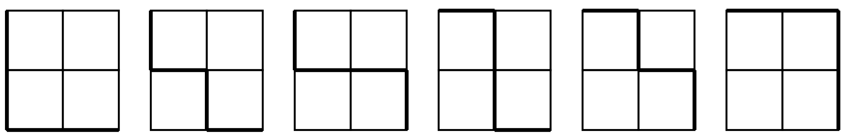
we get for the shortest semiotic paths through the 1 × 1 grid:

1. ((1.1, 2.1), (2.1, 2.2)) ≡ [[α, id1], [id2, α]]
2. ((1.1, 1.2), (1.2, 2.2)) ≡ [[id1, α], [α, id2]]

3. By a 2 × 2 grid, we may either represent $SR_{3,3}$ or the respective sub-matrix of $SR_{4,3}$ (cf. Toth 2008b, c, d):

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

In a 2×2 grid, there are 6 possible shortest paths:



For $SR_{3,3}$, we get:

3. ((1.1, 2.1), (2.1, 3.1), (3.1, 3.2), (3.2, 3.3)) ≡ [[α, id1], [β, id1], [id3, α], [id3, β]]
4. ((1.1, 2.1), (2.1, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡ [[α, id1], [id2, α], [β, id2], [id3, β]]
5. ((1.1, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡ [[α, id1], [id2, α], [id2, β], [β, id3]]
6. ((1.1, 1.2), (1.2, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡ [[id1, α], [α, id2], [β, id2], [id3, β]]
7. ((1.1, 1.2), (1.2, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡ [[id1, α], [α, id2], [id2, β], [β, id3]]
8. ((1.1, 1.2), (1.2, 1.3), (1.3, 2.3), (2.3, 3.3)) ≡ [[id1, α], [id1, β], [α, id3], [β, id3]]

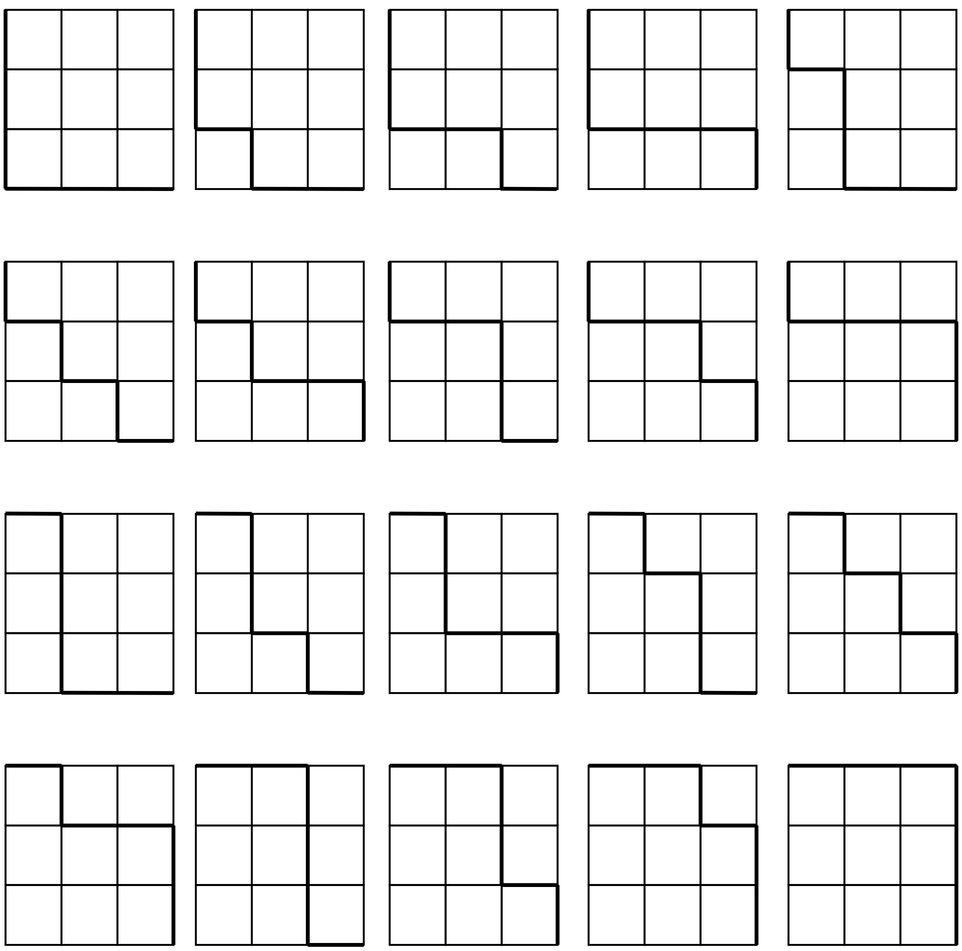
If we take the upper part of the above semiotic matrix for $SR_{4,3}$, we get:

9. ((0.1, 1.1), (1.1, 2.1), (2.1, 2.2), (2.2, 2.3)) ≡ [[γ, id1], [α, id1], [id2, α], [id2, β]]
10. ((0.1, 1.1), (1.1, 1.2), (1.2, 2.2), (2.2, 2.3)) ≡ [[γ, id1], [id1, α], [α, id2], [id2, β]]
11. ((0.1, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 2.3)) ≡ [[γ, id1], [id1, α], [id1, β], [α, id3]]
12. ((0.1, 0.2), (0.2, 1.2), (1.2, 2.2), (2.2, 2.3)) ≡ [[id0, α], [γ, id2], [α, id2], [id2, β]]
13. ((0.1, 0.2), (0.2, 1.2), (1.2, 1.3), (1.3, 2.3)) ≡ [[id0, α], [γ, id2], [id1, β], [α, id3]]
14. ((0.1, 0.2), (0.2, 0.3), (0.3, 1.3), (1.3, 2.3)) ≡ [[id0, α], [id0, β], [γ, id3], [α, id3]]

4. By a 3 × 3 grid, we represent $SR_{4,4}$ with the following semiotic matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3

In a 3 × 3 grid, there are 20 possible shortest paths (Dickau 2008):



Then we get for $SR_{4,4}$:

15. ((0.0, 1.0), (1.0, 2.0), (2.0, 3.0), (3.0, 3.1), (3.1, 3.2), (3.2, 3.3)) ≡
[[γ, id0], [α, id0], [β, id0], [id3, γ], [id3, α], [id3, β]]

16. ((0.0, 1.0), (1.0, 2.0), (2.0, 2.1), (2.1, 3.1), (3.1, 3.2), (3.2, 3.3)) ≡
    [[γ, id0], [α, id0], [id2, γ], [β, id1], [id3, α], [id3, β]]
17. ((0.0, 1.0), (1.0, 2.0), (2.0, 2.1), (2.1, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡
    [[γ, id0], [α, id0], [id2, γ], [id2, α], [β, id2], [id3, β]]
18. ((0.0, 1.0), (1.0, 2.0), (2.0, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡
    [[γ, id0], [α, id0], [id2, γ], [id2, α], [id2, β], [β, id3]]
19. ((0.0, 1.0), (1.0, 1.1), (1.1, 2.1), (2.1, 3.1), (3.1, 3.2), (3.2, 3.3)) ≡
    [[γ, id0], [id1, γ], [α, id1], [β, id1], [id3, α], [id3, β]]
20. ((0.0, 1.0), (1.0, 1.1), (1.1, 2.1), (2.1, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡
    [[γ, id0], [id1, γ], [α, id1], [id2, α], [β, id2], [id3, β]]
21. ((0.0, 1.0), (1.0, 1.1), (1.1, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡
    [[γ, id0], [id1, γ], [α, id1], [id2, α], [id2, β], [β, id3]]
22. ((0.0, 1.0), (1.0, 1.1), (1.1, 1.2), (1.2, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡
    [[γ, id0], [id1, γ], [id1, α], [α, id2], [β, id2], [id3, β]]
23. ((0.0, 1.0), (1.0, 1.1), (1.1, 1.2), (1.2, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡
    [[γ, id0], [id1, γ], [id1, α], [α, id2], [id2, β], [β, id3]]
24. ((0.0, 1.0), (1.0, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 2.3), (2.3, 3.3)) ≡
    [[γ, id0], [id1, γ], [id1, α], [id1, β], [α, id3], [β, id3]]
25. ((0.0, 0.1), (0.1, 1.1), (1.1, 2.1), (2.1, 3.1), (3.1, 3.2), (3.2, 3.3)) ≡
    [[id0, γ], [γ, id1], [α, id1], [β, id1], [id3, α], [id3, β]]
26. ((0.0, 0.1), (0.1, 1.1), (1.1, 2.1), (2.1, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡
    [[id0, γ], [γ, id1], [α, id1], [id2, α], [β, id2], [id3, β]]
27. ((0.0, 0.1), (0.1, 1.1), (1.1, 2.1), (2.1, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡
    [[id0, γ], [γ, id1], [α, id1], [id2, α], [id2, β], [β, id3]]
28. ((0.0, 0.1), (0.1, 1.1), (1.1, 1.2), (1.2, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡
    [[id0, γ], [γ, id1], [id1, α], [α, id2], [β, id2], [id3, β]]
29. ((0.0, 0.1), (0.1, 1.1), (1.1, 1.2), (1.2, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡
    [[id0, γ], [γ, id1], [id1, α], [α, id2], [id2, β], [β, id3]]
30. ((0.0, 0.1), (0.1, 1.1), (1.1, 1.2), (1.2, 1.3), (1.3, 2.3), (2.3, 3.3)) ≡
    [[id0, γ], [γ, id1], [id1, α], [id1, β], [α, id3], [β, id3]]
31. ((0.0, 0.1), (0.1, 0.2), (0.2, 1.2), (1.2, 2.2), (2.2, 3.2), (3.2, 3.3)) ≡
    [[id0, γ], [id0, α], [γ, id2], [α, id2], [β, id2], [id3, β]]
32. ((0.0, 0.1), (0.1, 0.2), (0.2, 1.2), (1.2, 2.2), (2.2, 2.3), (2.3, 3.3)) ≡
    [[id0, γ], [id0, α], [γ, id2], [α, id2], [id2, β], [β, id3]]
33. ((0.0, 0.1), (0.1, 0.2), (0.2, 1.2), (1.2, 1.3), (1.3, 2.3), (2.3, 3.3)) ≡
    [[id0, γ], [id0, α], [γ, id2], [id1, β], [α, id3], [β, id3]]
34. ((0.0, 0.1), (0.1, 0.2), (0.2, 0.3), (0.3, 1.3), (1.3, 2.3), (2.3, 3.3)) ≡
    [[id0, γ], [id0, α], [id0, β], [γ, id3], [α, id3], [β, id3]]

As for our previous studies, the present one is a contribution, too, for more complex semiotic networks such as the SRG-model presented in Toth (1997).

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