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Tetradic sign classes from relational and categorial numbers

1. In Toth (2008b), we had elaborated Bense's introduction of relational and categorial numbers in order to fully characterize sign relations Z^r_k (Bense 1975, pp. 65 s.). Z^r_k includes pre-semiotic media relations (M^o) which connect Z^r_k as a representation scheme of the semiotic space with the ontological space out of which objects are selected to be thetically introduced as meta-objects and thus as signs (Bense 1967, p. 9). This distinction allows to differentiating between the semiotic sign relation

$$SR = (.1., .2., .3.)$$

and the pre-semiotic qualitative-quantitative sign relation

$$PSR = (0., .1., .2., .3.).$$

Since, in Z^r_k , $k \neq 0$, the respective pre-semiotic matrix does not contain the zeroness in trichotomic position. Hence the pre-semiotic matrix is "defective" from the viewpoint of a total-symmetric matrix of Cartesian products over $(.0., .1., .2., .3.)$:

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

From that it follows, too, that sign classes built from the 12 sub-signs in the pre-semiotic matrix will not lead to the system of the 35 tetradic-tetratomic sign classes shown and discussed in Toth (2008a, pp. 179 ss.). If we apply the trichotomic semiotic order in triadic semiotic sign classes:

$$(3.a\ 2.b\ 1.c) \text{ with } a \leq b \leq c$$

to the tetratomic order in tetradic pre-semiotic sign classes:

$$(3.a\ 2.b\ 1.c\ 0.d) \text{ with } a \leq b \leq c \leq d,$$

then we can construct the following 15 pre-semiotic sign classes:

- 1 (3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)
- 2 (3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)
- 3 (3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)
- 4 (3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
- 5 (3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)
- 6 (3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
- 7 (3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
- 8 (3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)
- 9 (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)
- 10 (3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
- 11 (3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
- 12 (3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
- 13 (3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
- 14 (3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
- 15 (3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3),

whose number corresponds to the 15 trito-numbers of the polycontextural contexture T_4 (cf. Kronthaler 1986, p. 34), which underlines the fact that these 15 pre-semiotic sign classes are both quantitative and qualitative sign classes, because the integration of the zeroness into the triadic sign relation bridges the polycontextural border between the ontological space of objects and the semiotic space of signs (cf. Toth 2003, 2008a).

Moreover, we notice that in the system of the 15 pre-semiotic classes, there is, unlike in the system of the 10 semiotic sign classes, no dual-identical sign class corresponding to the triadic “eigenreal” sign class $(3.1\ 2.2\ 1.3) \times (3.1\ 2.2\ 1.3)$, cf. Bense (1992). On the other side, the system of the 15 pre-semiotic sign classes displays, in the system of their dual reality thematics, semiotic structures that do not occur in the system of the 10 semiotic sign classes. In order to “formalize” them, we use the notational system introduced in Toth (2008a, pp. 176 ss.). The abbreviation HOM stands for homogeneous thematizations, LEFT and RIGHT refer to the direction of thematizations (indicated by arrows), and SWCH for “sandwich thematization” points to the fact that in the respective structural realities two realities are thematizing and two are thematized. Then we get the following types of tetradic thematizations of the 15 pre-semiotic sign classes:

1. Homogeneous thematizations:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$	1^4	HOM
11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$	2^4	HOM
15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$	3^4	HOM

2. Dyadic thematizations

2.1. Dyadic-leftward thematizations

2	(3.1 2.1 1.1 0.2) × (2.0 <u>1.1 1.2 1.3</u>)	$2^1 \leftarrow 1^3$	LEFT
3	(3.1 2.1 1.1 0.3) × (3.0 <u>1.1 1.2 1.3</u>)	$3^1 \leftarrow 1^3$	LEFT
12	(3.2 2.2 1.2 0.3) × (3.0 <u>2.1 2.2 2.3</u>)	$3^1 \leftarrow 2^3$	LEFT

2.2. Dyadic-rightward thematizations

7	(3.1 2.2 1.2 0.2) × (<u>2.0 2.1 2.2</u> 1.3)	$2^3 \rightarrow 1^1$	RIGHT
10	(3.1 2.3 1.3 0.3) × (<u>3.0 3.1 3.2</u> 1.3)	$3^3 \rightarrow 1^1$	RIGHT
14	(3.2 2.3 1.3 0.3) × (<u>3.0 3.1 3.2</u> 2.3)	$3^3 \rightarrow 2^1$	RIGHT

2.3. Sandwich-Thematizations (only centripetal)

4	(3.1 2.1 1.2 0.2) × (<u>2.0 2.1 1.2 1.3</u>)	$2^2 \leftrightarrow 1^2$	SWCH
6	(3.1 2.1 1.3 0.3) × (<u>3.0 3.1 1.2 1.3</u>)	$3^2 \leftrightarrow 1^2$	SWCH
13	(3.2 2.2 1.3 0.3) × (<u>3.0 3.1 2.2 2.3</u>)	$3^2 \leftrightarrow 2^2$	SWCH

3. Triadic thematizations

3.1. Triadic-leftward thematization

5	(3.1 2.1 1.2 0.3) × (3.0 2.1 <u>1.2 1.3</u>)	$3^1 \leftrightarrow 2^1 \leftarrow 1^2$	LEFT
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3.2. Triadic-rightward thematization

9	(3.1 2.2 1.3 0.3) × (<u>3.0 3.1 2.2</u> 1.3)	$3^2 \rightarrow 2^1 \leftrightarrow 1^3$	RIGHT
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3.3. Sandwich-thematization (centrifugal)

8	(3.1 2.2 1.2 0.3) × (3.0 <u>2.1 2.2</u> 1.3)	$3^1 \leftarrow 2^2 \rightarrow 1^1$	SWCH
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It is easy to recognize that the 15 reality thematics of the system of the tetradic pre-semiotic sign classes can not be organized into a system of tetratomic tetrads analogous to the system of trichotomic triads (cf. Walther 1982). The latter is symmetric by aid of the determinant of the eigenreal sign class (3.1 2.2 1.3), and since there is no eigenreality in the system of the 15 pre-semiotic sign classes, they can not be constructed as n-adic m-ary semiotic systems in which $n = m$ like in the case of the tetratomic tetrads constructed out of the 35 tetradic-tetratomic sign classes in Toth (2008a, pp. 180 ss.).

However, it is possible to construct a system of **triadic pentatomies** out of the system of the 15 pre-semiotic sign classes:

1	(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)	1 ⁴	HOM
2	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)	2 ¹ ← 1 ³	LEFT
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)	2 ² ↔ 1 ²	SWCH
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)	2 ³ → 1 ¹	RIGHT
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)	3 ¹ ↔ 2 ¹ ← 1 ²	LEFT
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)	2 ⁴	HOM
3	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)	3 ¹ ← 1 ³	LEFT
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)	3 ² ↔ 1 ²	SWCH
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)	3 ³ → 1 ¹	RIGHT
9	(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)	3 ² → 2 ¹ ↔ 1 ³	RIGHT
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)	3 ⁴	HOM
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)	3 ¹ ← 2 ³	LEFT
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)	3 ² ↔ 2 ²	SWCH
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)	3 ³ → 2 ¹	RIGHT
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)	3 ¹ ← 2 ² → 1 ¹	SWCH

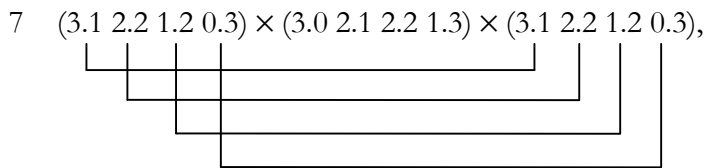
We recognize that each of these pentatomies has the following structure. X, Y, and Z ∈ {1, 2, 3}:

X⁴ HOM
X¹ ← Y³ LEFT
X² ↔ Y² SWCH
X³ → Y¹ RIGHT
X¹ ← X² → Z¹ SWCH

Thus, although the structural realities presented in the tetratomic reality thematics are tetradic, zeroness appears as triadic sign value and thus in the sign classes, but not as tetradic value and thus not in the reality thematics. In other words: In order to describe the realities presented by the tetradic pre-semiotic sign classes, **three** semiotic categories (X, Y, Z) are sufficient. Therefore, according to Bense (1975, pp. 64 ss. and Toth 2008b), the X, Y, Z refer to the **categorical numbers**, and the “exponents” in the above frequency notation of structural realities refer to the **relational numbers**. Using this frequency notation, we are able, on the basis of the above pentatomic structure of tetradic realities, to construct the system of the triadic pentatomies from the system of the 15 pre-semiotic sign classes based on the pre-semiotic sign relation PSR = (3.a 2.b 1.c 0.d), the tetratomic pre-semiotic order (a ≤ b ≤ c ≤ d) and the restriction that zeroness must not appear in trichotomic position.

This n-adic m-ary semiotic system for n = 3 and m = 5 thus connects by its n-adic value the pre-semiotic system of the 15 sign classes with the triadic system of the 10 sign classes which therefore appear as a morphogrammatic fragment of the system of the 15 pre-semiotic sign classes, on the one side, and with a pentadic-m-ary system of ≤ 126 sign classes (cf. Toth 2008a, pp. 186 ss.) whose fragment the system of the 15 pre-semiotic sign classes is, on the other side (cf. Toth 2003, pp. 54 ss.).

Finally, one should notice that the absence of a dual-identical sign class in order to express eigenreality in the system of the 15 pre-semiotic sign classes leads to the fact that these pre-semiotic sign classes cannot be dualized, but must be triadized (cf. Kronthaler 1992, p. 293). Triadization is thus the minimal condition to transform one of the 15 pre-semiotic sign classes by reversing both the order of its dyadic sub-relations and of its monadic prime-signs back to its original sign class structure:



The following study is the first contribution to **Pre-semiotics** in the sense of the theory of the pre-semiotic sign classes, their **trial** reality thematics and their associated system of triadic pentatomies. The main aim of Pre-semiotics is to formally analyze and describe the “never-land” between the Ontological and the Semiotic Space in the sense of Bense (1975, p. 65) and to disclose the pre-semiotic relations in the network of the abyss between sign and object.

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