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The pre-semiotic system of Tetradic Pentatomies

1. From the sign relation $SR_3 = (3.a\ 2.b\ 1.c)$ we can construct 10 sign classes on the basis of the trichotomic order ($a \leq b \leq c$). From the sign relation $SR_{4,3}$, we can build 15 sign classes on the basis of the same trichotomic order. And from the sign relation $SR_6$, we get 35 sign classes on the basis of the tetratomic order ($a \leq b \leq c \leq d$), cf. Toth (2008b). Furthermore, from $SR_6$, we can construct 3 Trichotomic Triads (Walther 1982), from $SR_{4,3}$, we can build 3 Tetradic Pentatomies, and from $SR_6$, we get 4 Tetratomic Tetrads of dyadic thematization and 4 Tetratomic Tetrads of triadic thematization (Toth 2008a, pp. 182 ss.). Thus, the pre-semiotic sign relation $SR_{4,3}$ which is 4-adic, but 3-ary, leads to a semiotic structure which is 4-adic, but 5-atomic.

2. By aid of the frequency notation introduced in Toth (2008a, pp. 176 s.), we can order the 15 pre-semiotic sign classes of $SR_{4,3}$ according to the structural realities presented in the dual reality thematics. We abbreviate four identical sub-signs by HOM (homogeneous structural reality). The types of thematizations are LEFT ($X \leftarrow Y$), RIGHT ($X \rightarrow Y$), and SWCH (for sandwich). The latter is used for the types of thematizations that cannot be decided if they are leftward or rightward:

\[
\begin{align*}
(1.0\ 1.1\ 1.2\ 1.3) & & 1^4 & & \text{HOM} \\
(2.0\ 1.1\ 1.2\ 1.3) & & 2^1 \leftarrow 1^3 & & \text{LEFT} \\
(2.0\ 2.1\ 1.2\ 1.3) & & 2^2 \leftarrow 1^2 & & \text{SWCH} \\
(2.0\ 2.1\ 2.2\ 1.3) & & 2^3 \rightarrow 1^1 & & \text{RIGHT} \\
(3.0\ 2.1\ 1.2\ 1.3) & & 3^1 \leftarrow 2^1 \leftarrow 1^2 & & \text{LEFT} \\
(2.0\ 2.1\ 2.2\ 2.3) & & 2^4 & & \text{HOM} \\
(3.0\ 1.1\ 1.2\ 1.3) & & 3^1 \leftarrow 1^3 & & \text{LEFT} \\
(3.0\ 3.1\ 1.2\ 1.3) & & 3^2 \leftarrow 1^2 & & \text{SWCH} \\
(3.0\ 3.1\ 3.2\ 1.3) & & 3^3 \rightarrow 1^1 & & \text{RIGHT} \\
(3.0\ 3.1\ 2.2\ 1.3) & & 3^3 \leftarrow 2^1 \leftarrow 1^3 & & \text{RIGHT} \\
(3.0\ 3.1\ 3.2\ 3.3) & & 3^4 & & \text{HOM} \\
(3.0\ 2.1\ 2.2\ 2.3) & & 3^1 \leftarrow 2^3 & & \text{LEFT} \\
(3.0\ 3.1\ 2.2\ 2.3) & & 3^2 \leftarrow 2^2 & & \text{SWCH} \\
(3.0\ 3.1\ 3.2\ 2.3) & & 3^3 \rightarrow 2^1 & & \text{RIGHT} \\
(3.0\ 2.1\ 2.2\ 1.3) & & 3^1 \leftarrow 2^2 \rightarrow 1^1 & & \text{SWCH}
\end{align*}
\]

We recognize that each of the three pentatomies has the following structure ($X, Y, \text{and } Z \in \{1, 2, 3\}$):
3. In SR$_3$ and in SR$_4$, the possibilities of constructing n-atomic n-ads from the sets of the respective sign classes and reality thematics depend on the symmetric structure of thematizations provided by the triadic structural realities presented in the reality thematics of the dual-invariant sign classes (3.1 2.1 1.2 0.3). However, in SR$_{4,3}$, there is no such "eigenreal" sign class (cf. Bense 1992). Moreover, since the category of Zeroness cannot appear in trichotomic values of sign classes and in triadic values of reality thematics, there cannot be any reality thematics that hang together with their dual sign classes by sub-signs which contains (0.). Thus, the only kind of symmetry in the structure of SR$_{4,3}$ lies in the possibility to order its 15 dual representations systems according to the above structure of structural realities.

In the following, we will display a semiotic graph of the three Tetradic Pentatomies of SR$_{4,3}$ which is determined by the semiotic connections between the 15 reality thematics:

In SR$_{4,3}$, there is no eigenreal sign class and thus, the 15 sign classes cannot be transformed into a “determinant-symmetric duality system” (Walther 1982, p. 18). Moreover, there are sign classes and reality thematics that are not connected with any other sign classes and reality thematics. Since zeroness does not appear in trichotomic position in sign classes and in triadic position in reality thematic, there are no tetradic sign connections.

Example for triadic sign connection:

(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)

Example for tetradic sign connection:

(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
Example for dyadic sign connection:

\[(3.1 \ 2.1 \ 1.1 \ 0.3) \times (3.0 \ 1.1 \ 1.2 \ 1.3)\]  
\[\| \]  
\[(3.1 \ 2.1 \ 1.2 \ 0.2) \times (2.0 \ 2.1 \ 1.2 \ 1.3)\]

Example for monadic sign connection:

\[(3.3 \ 2.3 \ 1.3 \ 0.3) \times (3.0 \ 3.1 \ 3.2 \ 3.3)\]  
\[\| \]  
\[(3.2 \ 2.2 \ 1.2 \ 0.3) \times (3.0 \ 2.1 \ 2.2 \ 2.3)\]

Example for \(\emptyset\)-adic sign connection:

\[(3.1 \ 2.1 \ 1.1 \ 0.1) \times (1.0 \ 1.1 \ 1.2 \ 1.3)\]  
\[\| \]  
\[(3.2 \ 2.2 \ 1.2 \ 0.2) \times (2.0 \ 2.1 \ 2.2 \ 2.3)\]

4. If we compare now the system of the three Tetradic Pentatomies of SR\(_{4,3}\) with the system of the three Triadic Trichotomies of SR\(_{3}\) plus the eigenreal sign class \((3.1 \ 2.2 \ 1.3)\):

![Diagram showing the connections between sign classes and reality thematics.](image)

we recognize that here, each sign class and its reality thematic is connected by at least one and maximal two sub-signs with the dual-identical sign class \((3.1 \ 2.2 \ 1.3) \times (3.1 \ 2.2 \ 1.3)\). Since in SR\(_{3}\) for each pair of prime-signs \((a,b)\), \((b,a)\) also appears in the semiotic matrix and thus in the sign classes and reality thematics, the semiotic matrix of SR\(_{3}\) is symmetric, while the semiotic matrix of SR\(_{4,3}\) is non-symmetric, but not asymmetric, since zeroness at least appears in the triadic positions, i.e. in the columns of the matrix. Therefore, the system of the Trichotomic Triads of SR\(_{3}\) forms a lattice (cf. Beckmann 1976), while the system of the
Tetradic Pentatomic does not. Nevertheless, triadic sign connection appears only between the dual-identical sign class and its reality thematic \((3.1 \ 2.2 \ 1.3) \times (3.1 \ 2.2 \ 1.3)\).

Example for dyadic sign connection:

\[(3.1 \ 2.1 \ 1.2) \times (2.1 \ 1.2 \ 1.3)\]

\[\text{Example for monadic sign connection:}\]

\[(3.1 \ 2.1 \ 1.1) \times (1.1 \ 1.2 \ 1.3)\]

\[(3.3 \ 2.3 \ 1.3) \times (3.1 \ 3.2 \ 3.3)\]

Example for \(\emptyset\)-adic sign connection:

\[(3.1 \ 2.1 \ 1.1) \times (1.1 \ 1.2 \ 1.3)\]

\[(3.3 \ 2.3 \ 1.3) \times (3.1 \ 3.2 \ 3.3)\]

5. We can sum up:

- A sign relation \(\text{SR}_{a,b}\) with \(a \neq b\) leads to semiotic systems of structural realities of \(n\)-atomic \(m\)-ads with \(n \neq m\) (whereby in most cases \(a \neq n\) and \(b \neq m\)).
- If \(n = m\) and \(n, m > 3\), there is more than one system of structural realities of \(n-(m-)\) atomic \(n-(m-)\)ads; cf. Toth (2008a, pp. 197 ss., 186 ss.).
- If \(a \neq b\), then for every sub-sign \((a.b)\), not every sub-sign \((b.a)\) appears in the respective semiotic matrix, and the respective system of sign classes and reality thematics does not form a semiotic lattice.
- The main reason for \(a \neq b\) and \(n \neq m\), respectively, is the lack of a dual-identical sign class.
- In a semiotic system of structural realities, which does not have a dual-identical sign class, there is no distinct sign class or reality thematic that hangs together with each other sign class or reality thematic by at least one sub-sign.

**Bibliography**

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