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Semiotic Hankel and Toeplitz matrices

1. In his contribution for a festschrift for Max Bense, Gotthard Günther introduced orthogonal matrices for ontological constants in polycontextural systems (1984/85). Günther was convinced that he had found "the arithmetic of mediation" and "the philosophical place of the number" (Günther 1991, pp. xxix, xxx). Without mentioning it, Günther used Hankel matrices (1991, pp. 421 s.). He noticed that the diagonals of the matrices "always divide the square, which they separate, in two areas of higher and lower reflection" (1991, p. 423). In Günther's 12×12 Hankel matrix, "the diagonal 12 undoubtedly belongs to the upper structural area of the inverse images which appear only once; the apex of the lower region, of the multiplying images, reaches as highest number only 11, and this just once. Thus, there exists, from top to bottom, a decrease of reflexivity which has been implied since ever by classical metaphysics, as far as it had dealt with speculations about the Beyond, like for example in Dionysios Areopagita. Generally, we can say that ontological systems, as far as they depend on different values, always possess borders which are dictated by the laws of orthogonality" (1991, p. 423).

2. We now introduce Hankel and Toeplitz matrices to semiotics. If we compare the Hankel Matrix for for SR_3:

1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3
1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	1.1
1.3	2.1	2.2	2.3	3.1	3.2	3.3	1.1	1.2
2.1	2.2	2.3	3.1	3.2	3.3	1.1	1.2	1.3
2.2	2.3	3.1	3.2	3.3	1.1	1.2	1.3	2.1
2.3	3.1	3.2	3.3	1.1	1.2	1.3	2.1	2.2
3.1	3.2	3.3	1.1	1.2	1.3	2.1	2.2	2.3
3.2	3.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1
3.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2

to the Hankel Matrix for $SR_{4,3}$:

0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3
0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1
0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2
1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3
1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1
1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2
2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3
2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1
2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2
3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3
3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1
3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2,

we recognize that the minor diagonals that correspond to the sub-signs of the dual-invariant, eigenreal sign class (3.1 2.2 1.3) and which build the side-diagonal in the semiotic 3×3 matrix of the prime-signs:

1.1	1.2	3.1
2.1	2.2	2.3
3.1	3.2	3.3,

(cf. Bense 1992), appear once above the side diagonal of the matrices consisting of (3.3), separated only by the minor diagonal of (3.2) from it, and once beneath it. Now, while in SR_3 the lower minor diagonals of eigenreality are only separated by one diagonal from the side diagonal, they are separated in $SR_{4,3}$ by four minor diagonals and thus lie much deeper in the area of subjectivity according to Günther.

3. If we now have a look at the corresponding Toeplitz matrices for SR_3:

3.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2
3.2	3.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1
3.1	3.2	3.3	1.1	1.2	1.3	2.1	2.2	2.3
2.3	3.1	3.2	3.3	1.1	1.2	1.3	2.1	2.2
2.2	2.3	3.1	3.2	3.3	1.1	1.2	1.3	2.1
2.1	2.2	2.3	3.1	3.2	3.3	1.1	1.2	1.3
1.3	2.1	2.2	2.3	3.1	3.2	3.3	1.1	1.2
1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	1.1
1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3

and $SR_{4,3}$:

3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2
3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1
3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3
2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2
2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3	2.1
2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2	1.3
1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1	1.2
1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3	1.1
1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2	0.3
0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1	0.2
0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	0.1
0.1	0.2	0.3	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3,

we recognize the same structural positions of the diagonals of eigenreality as in the Hankel matrices, although in the Toeplitz matrices, (3.3) builds the main diagonal, and the areas of objectivity and subjectivity or of the order of the images and reverse images of the diagonals of eigenreality in the sense of Günther are reversed.

From this little study, we can conclude that eigenreality is a semiotic phenomenon that is most closely related to polycontextural ontology and logic via their common phenomenon of orthogonality. In both types of matrices, eigenreality appears both in the objective and in the subjective areas, i.e. both as images and reverse images of their constitutive sub-signs. The distance between subjective and objective eigenreality increases in the transition of the matrices from SR_3 to $SR_{4,3}$. Since $SR_{4,3}$ is the pre-semiotic sign model for quantitative-qualitative semiotic representation systems (cf. Toth 2008a-e), the increasing distance of eigenreality in the Hankel and Toeplitz matrices implies that the transition from SR_3 to $SR_{4,3}$ is accompanied by deeper embedding of eigenreality in subjectivity.

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