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Cyclic groups of semiotic transpositions

1. Group theory of semiotic transpositions

In Toth (2007, pp. 36 ss.), it was shown that the set of prime-signs PS = {.1., .2., .3.} builds together with a binary operation "○" a group, fulfilling the laws of closure, associativity and the existence of both an identity and an inverse element. For the binary operation, Bogarin (1986) proposed the semiotic group-theoretic operation of "symplerosis" σ which turns each element of PS into its inverse element:

σ(.1.) = (.3.)
σ(.2.) = (.2.)
σ(.3.) = (.1.)

Since sign classes and reality thematics consist of sub-signs, and sub-signs consist of prime-signs, also the sign classes and the reality thematics can be investigated by means of group theory, f. ex.:

Sign class:	σ(3.1 2.1 1.3) = (1.3 2.3 3.1)
Reality thematic:	σ(3.1 1.2 1.3) = (1.3 3.2 3.1)

As one can see, the operation of symplerosis does not only build other sign classes from sign classes, but leads to transpositions of these sign classes. The same is true about reality thematics. So, (1.3 2.3 3.1) is a transposition of the regular sign class (3.1 2.3 1.3), namely its full inversion. But the whole immanent system of a sign class and a reality thematic is not complete without the transpositions of partial inversions:

Sign class	Full inversion	Partial Inversions
(3.1 2.3 1.3)	(1.3 2.3 3.1)	(3.1 1.3 2.3)
		(2.3 3.1 1.3)
		(2.3 1.3 3.1)
		(1.3 3.1 2.3)

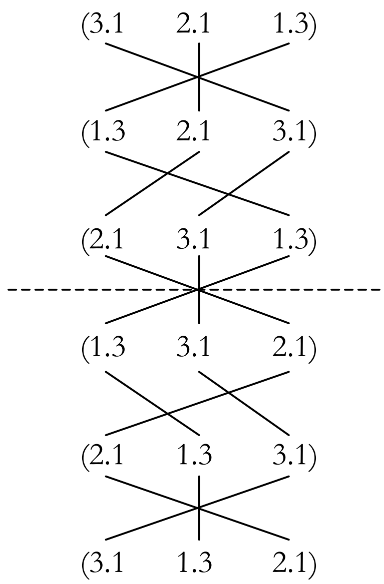
Reality thematic	Full inversion	Partial Inversions
(3.1 3.2 1.3)	(1.3 3.2 3.1)	(3.1 1.3 3.2)
		(3.2 3.1 1.3)
		(3.2 1.3 3.1)
		(1.3 3.1 3.2)

2. Symmetric and asymmetric connections of semiotic transpositions

Since full and partial inversions "scramble" the order of the sub-signs of a sign class and reality thematic, but not the order of the constitutive prime-signs, there are three semiotic connections between each pair of transpositions of sign classes and reality thematics according to the triadic structure of both sign classes and reality thematics. The total number of pair-wise combinations of the 6 transpositions is calculated by

$$K = \frac{n!}{(n - p)! \cdot p!}$$

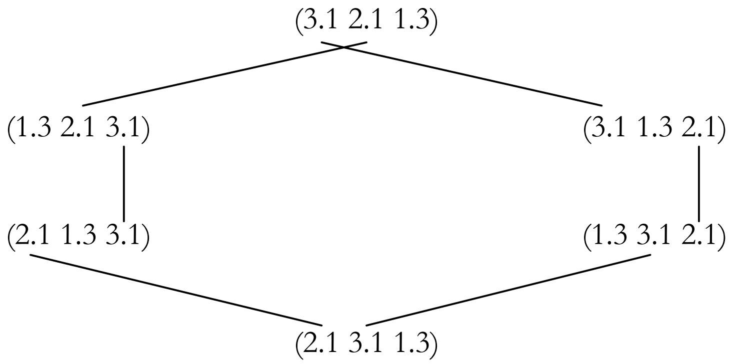
Since n = 6 and p = 2, we get K = 720/(24 · 2) = 15 combinations of sign classes and 15 combinations of reality thematics. If we restrict ourselves to complete combinations of all 6 transpositions of a sign class, we get, e. g., the following three types:

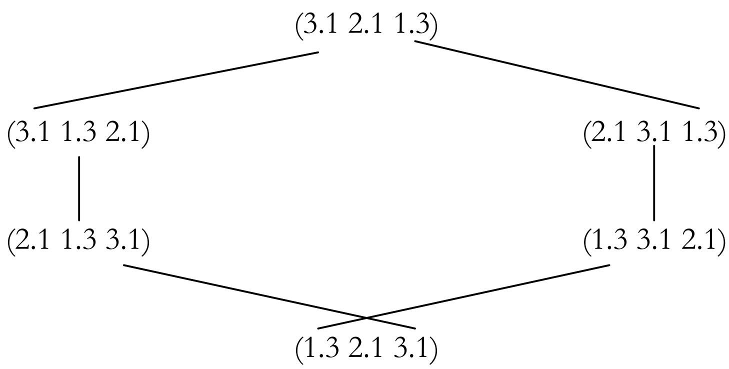


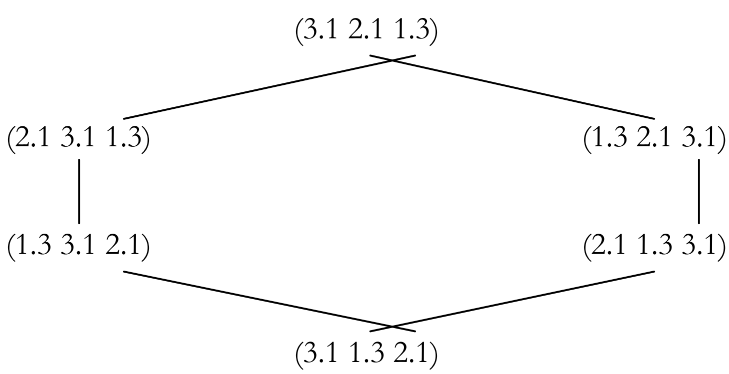
As one can see, only the type of combination to the left is symmetric, while the types in the middle and to the right are not. Without giving a proof or further demonstration, we thus state that in linear combinations of sign connections between transpositions of sign classes and reality thematics, only the one type shown to the left above is symmetric.

3. Symmetric cyclic systems of semiotic transpositions

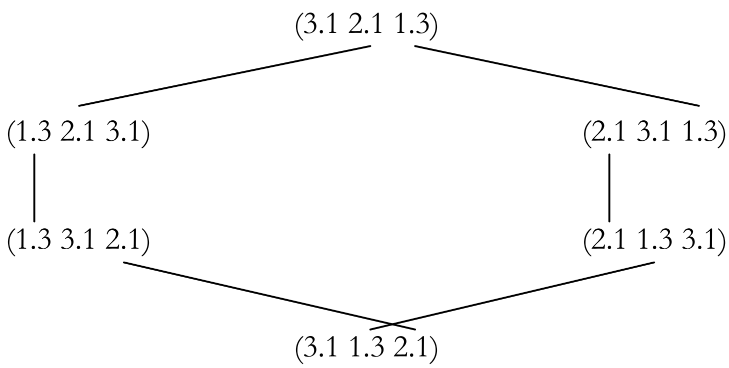
However, if we do not consider linear but cyclic combinations of the 6 transpositions of a sign class or reality thematic, we find that all 6 transpositions can be displayed in symmetric cyclic systems. Here, we understand under a "symmetric cyclic system" each cyclic (non-linear) semiotic system in which 1) all 6 transpositions of a sign class or reality thematic are 2) connected by at least 2 (identical) sub-signs, and 3) the connected sub-signs must show up pair-wise in the same position of the triadic relation of the transpositions. We give three examples of symmetric cyclic systems of semiotic transpositions, all right-handed and starting with the regular sign class (3.1 2.1 1.3), while the first transposition starts in the first graph with (3.1), in the second graph with (2.1) and in the third graph with (1.3):







Now let us have a look at another example for a cyclic symmetric system of transpositions, also starting with (2.2) in the second vertex counterclockwise:



Therefore, we present here, without a proof, two semiotic theorems:

Theorem 1: Each cyclic system of transpositions of a sign class or reality thematic is symmetric.

Theorem 2: All 6 transpositions of a sign class or reality thematic are connected to one another by a exactly 1 sub-sign of identical semiotic value and position.

Thus, theorems 1 and 2 seem to build a basis for the semiotic Law of Determinant-Symmetric Duality Systems (Walther 1982), which states that each sign class and each reality thematic of the system of the 10 sign classes is connected by at least one and at most two sub-signs with the dual-inverse sign class (reality thematic) (3.1 2.2 1.3).

4. Finite and infinite semiotic cycles

In the symmetric semiotic cycles of transpositions given above, we can theoretically drive around clockwise or counterclockwise ad infinitum. However, we are interested in differentiating between finite and infinite semiotic cycles and in determining the length of the cycles. As it is shown below, there are exactly 6 possible cycles for a system of 6 transpositions, which can, however, be summed up into 3 types:

1. Cycle: Total Inversion

(3.1 2.1 1.3) → (1.3 2.1 3.1) → (3.1 2.1 1.3) → ∞
Length of cycle: 3

2. Cycle: Inversion of the last two and the first sub-signs

(3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3) → ∞

This type is identical with the type of inversion of the first and the last two subs-signs:

(3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3) → ∞
Length of cycle: 4

3. Cycle: Inversion of the last one and the first two sub-signs

(3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3) → ∞

This type is identical with the type of inversion of the first two and the last sub-sign:

(3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3) → ∞
Length of cycle: 4

Hence, we get the following complete list of finite and infinite semiotic cycles based on semiotic cyclic groups:

1st cycle

1. (3.1 2.1 1.3) → (1.3 2.1 3.1) → (3.1 2.1 1.3).
2. (3.1 1.3 2.1) → (2.1 1.3 3.1) → (3.1 1.3 2.1) → ∞.
3. (2.1 3.1 1.3) → (1.3 3.1 2.1) → (2.1 3.1 1.3) → ∞.
4. (2.1 1.3 3.1) → (3.1 1.3 2.1) → (2.1 1.3 3.1) → ∞.
5. (1.3 3.1 2.1) → (2.1 3.1 1.3) → (1.3 3.1 2.1) → ∞.
6. (1.3 2.1 3.1) → (3.1 2.1 1.3) → (1.3 2.1 3.1) → ∞.

2nd cycle

1. (3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3).
2. (3.1 1.3 2.1) → (1.3 2.1 3.1) → (2.1 3.1 1.3) → (3.1 1.3 2.1) → ∞.
3. (2.1 3.1 1.3) → (3.1 1.3 2.1) → (1.3 2.1 3.1) → (2.1 3.1 1.3) → ∞.
4. (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3) → (2.1 1.3 3.1).
5. (1.3 3.1 2.1) → (3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1).
6. (1.3 2.1 3.1) → (2.1 3.1 1.3) → (3.1 1.3 2.1) → (1.3 2.1 3.1) → ∞.

3rd Cycle

1. (3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3).
2. (3.1 1.3 2.1) → (2.1 3.1 1.3) → (1.3 2.1 3.1) → (3.1 1.3 2.1) → ∞.
3. (2.1 3.1 1.3) → (1.3 2.1 3.1) → (3.1 1.3 2.1) → (2.1 3.1 1.3) → ∞.
4. (2.1 1.3 3.1) → (3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1).
5. (1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3) → (1.3 3.1 2.1).
6. (1.3 2.1 3.1) → (3.1 1.3 2.1) → (2.1 3.1 1.3) → (1.3 2.1 3.1) → ∞.

Thus, only the following semiotic cycles are finite:

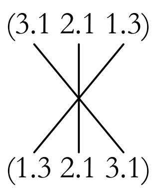
(3.1 2.1 1.3) → (1.3 2.1 3.1) → (3.1 2.1 1.3).
(3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3).
(3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3).

(2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3) → (2.1 1.3 3.1).
(2.1 1.3 3.1) → (3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1).

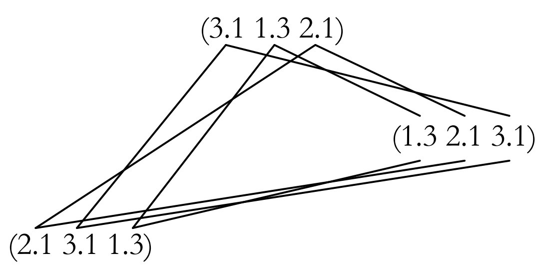
(1.3 3.1 2.1) → (3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1).
(1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3) → (1.3 3.1 2.1).

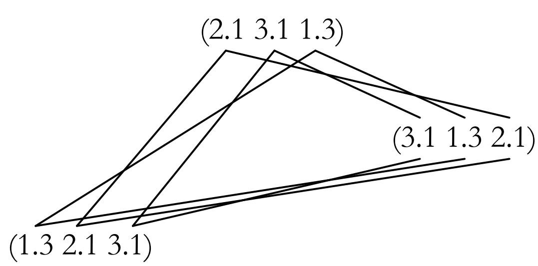
If we write all (finite and infinite) full cycles as graphs, we get the following representative systems of all 3 semiotic cycles:

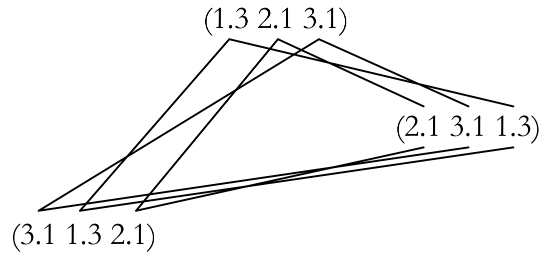
1st cycle:



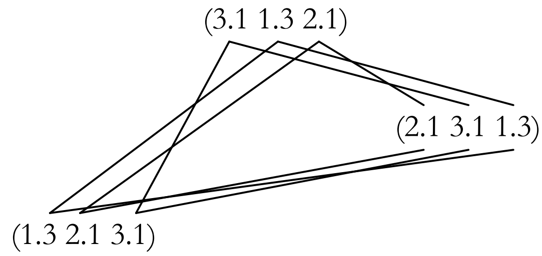
2nd cycle:

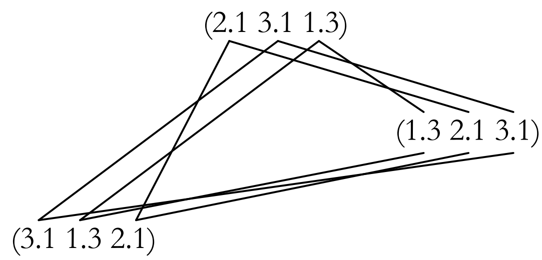


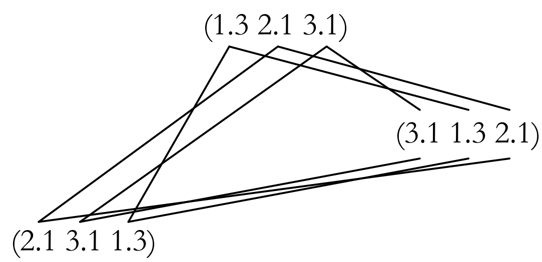




3rd cycle:







5. Sign classes, reality thematics and their transpositions as semiotic signals

If one wants to transmit a signal in a line, all inductive and capacity-bound environmental influences impede the transfer already after few meters. One solution of this problem is not to transmit a signal alone, but together with an identical signal of reverse polarity or a zero-signal: A signal have at its source the time function f(t), the reference signal have the negative time function –f(t); on the way between source and target, an impeding time function s(t) be added; the original signal be transmitted on line A and the reference signal on line B. Then, at the target, line A bears the time function g(t) = f(t) + s(t), and line B the time function h(t) = -f(t) + s(t). We thus get:

$g(t) – h(t) = [f(t) + s(t)] – [-f(t) + s(t)] = 2 f(t)$, and it shows that the noise s(t) has disappeared. Instead, we get a signal-amplitude double as high at the target.

If the signal h(t) is transmitted as zero-signal, we get the following equation:

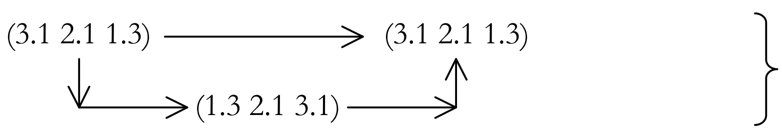
$g(t) – h(t) = [f(t) + s(t)] – [-f(t) + s(t)] = f(t)$, with $h(t) = s(t)$.

In this case, the receiver gets the single signal-amplitude with eliminated noise (Parr 1995).

By aid of semiotic transpositions, we are now able to define fully inverse sign classes and reality thematics as inverse semiotic signals, but also to enlarge the pure physical possibilities of signal transmission by the semiotic framework that enables to use also partially inverted sign classes and transpositions. Moreover, depending on the length of the finite semiotic cycles, we can present here the beginnings of a semiotic theory of signals that goes way beyond the physical signal theory, not only because here, signals are understood as signs consisting of form, content and meaning, but also in technical respect, as the following schemes may show:

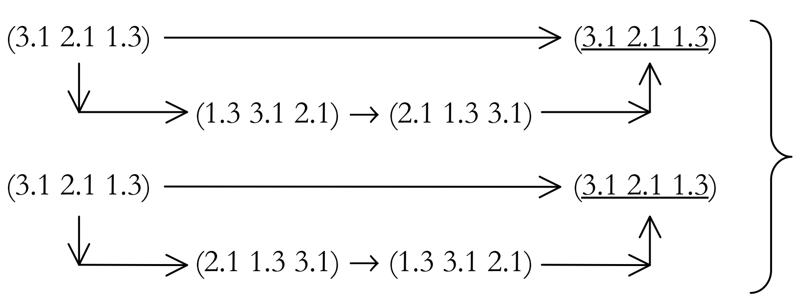
1st type of semiotic signal transfer

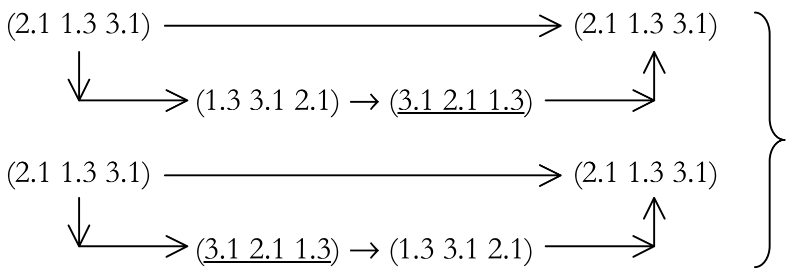
The first type of semiotic signal transfer has a cyclic length of 3 (as we have done above, we count all n vertices of the respective graphs):

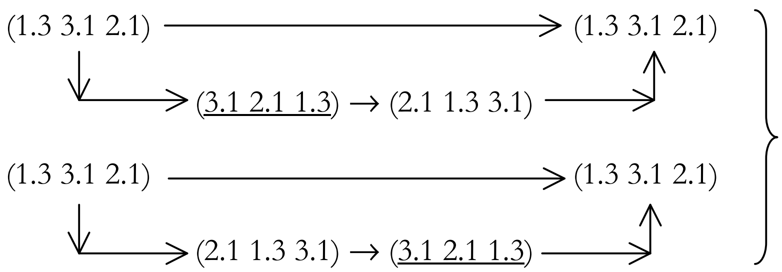


2nd type of semiotic signal transfer

The second type of semiotic signal transfer has a cyclic length of 4. It shows up in 3 sub-types:



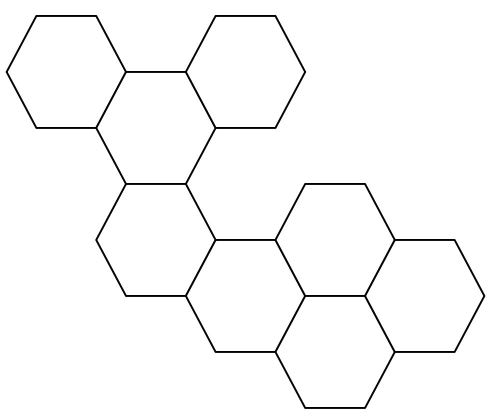




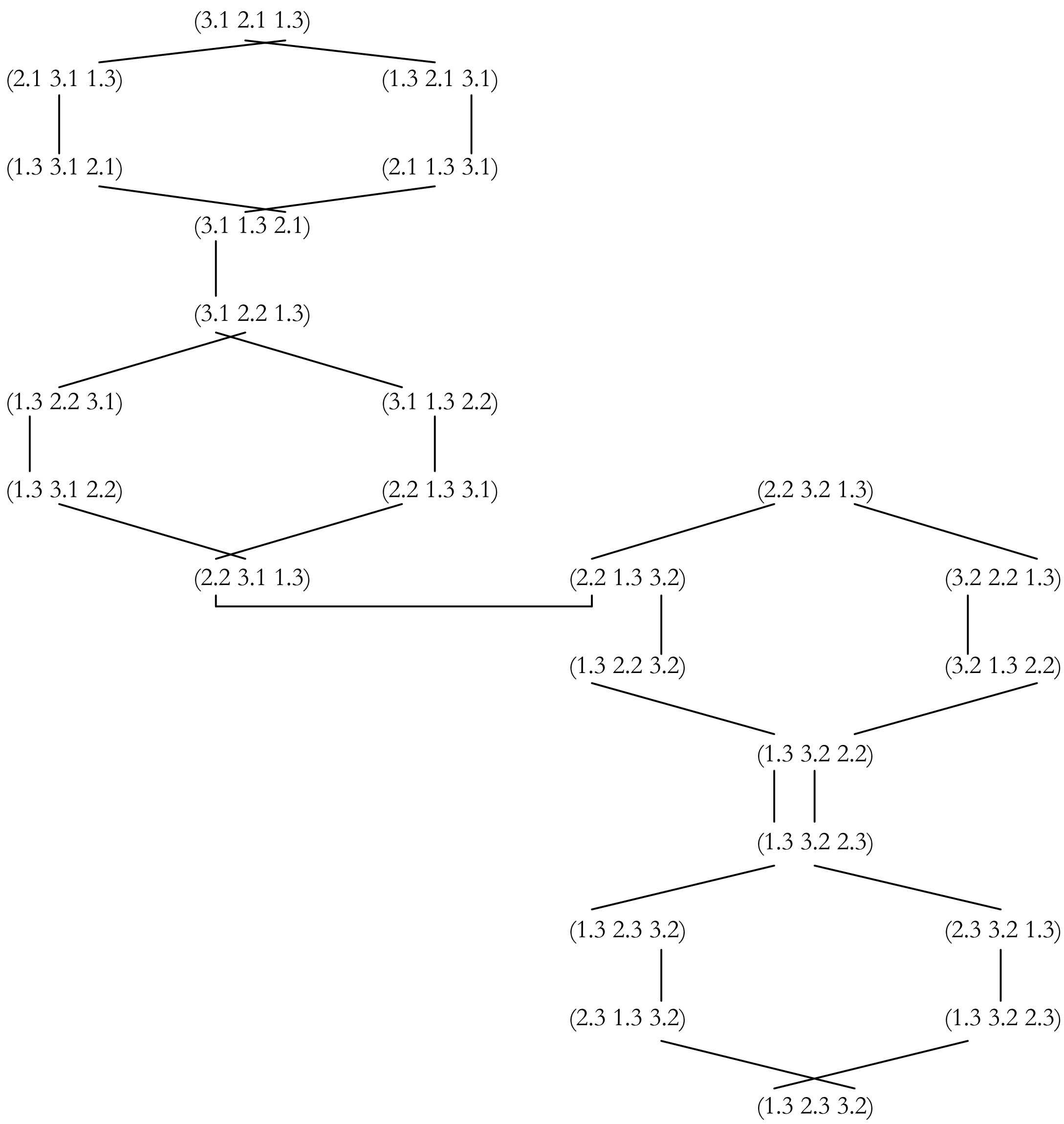
Since these schemes have the general structure of communication schemes with feedback, transpositions may also play a crucial role in cybernetic systems, whereby the transpositions may fulfill semiotic feedback relations.

6. Hexagonal lattices of cyclic semiotic systems

Hexagonal lattices (cf. Hermes 1967, pp. 25 ss.; for semiotic lattices cf. Walther 1979, pp. 137 s.) are the natural way of combining the 6 transpositions of each sign class and reality thematic amongst themselves as well as amongst each 6 transpositions of all 10 sign classes and reality thematics. The connection of each edge and each vertex in a hexagonal semiotic system is guaranteed by the above stated two semiotic theorems as well as by the semiotic Law of Determinant-Symmetric Duality Systems. From the purely abstract standpoint, the following drawing shows a fragment of a hexagonal lattice-system based on shared vertices:



A very small fragment of cyclic semiotic systems of the transpositions of the sign classes (3.1 2.1 1.3), (3.1 2.2 1.3) and (3.2 2.2 1.3) is presented above displaying both connections by vertices and by edges of the triadic semiotic relations:



Again without giving a proof, we thus state another semiotic theorem:

Theorem 3: Hexagonal semiotic lattices can be built from each sign class and reality thematic and each of their transpositions according to Theorems 1 and 2.

Since the Law of Determinant-Symmetric Duality systems showed to be a consequence of theorems 1 and 2, and since theorem 3 is based on theorems 1 and 2, we may finish with the conclusion that by applying group theory to semiotics, semiotic cycles lead necessarily to hexagonal semiotic lattices, if not only full inversions of the sign classes and reality thematics, but also partial inversions, and hence the full combinatorial power of cyclic semiotic systems is taken into consideration.

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