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Signs and qualitative numbers

1. As Rudolf Kaehr (2009) has shown in an impressive article, it is not enough to introduce the Peicean fundamental categories Firstness, Secondness and Thirdness in order to scoop out the full mathematical potential that lies in sign relations. It is necessary, too, to introduce their inner environments:

Firstness:	Peirce:	A
	Kaehr:	$A \mid a$
Secondness:	Peirce:	$A \rightarrow B$
	Kaehr	$A \rightarrow B \mid c$
Thirdness:	Peirce:	$A \rightarrow C$
	Kaehr:	$A \rightarrow C \mid b_1 \leftarrow b_2$

As one can see best under Thirdness, this means that with a morphism, also its corresponding hetero-morphism must be introduced. In his book "Toward Diamonds" (Kaehr 2007), Kaehr had illustrated the interplay between morphisms and hetero-morphisms with an auto-trip: I can only approach Stuttgart, when I am leaving Heilbronn at the same time. I.e., with each step foward, I also make a step backward. The steps backwards are the environment of the steps foward.

If we start with the semiotic 3×3 matrix and assume that signs work in 3 contextures, we obtain the following 3-contextural 3×3 matrix

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix}$$

and on its basis the 10 Peircean sign classes and their dual reality thematics, contextuated in 3 contextures:

$$(3.1_3\ 2.1_1\ 1.1_{1,3}) \quad \times \quad (1.1_{3,1}\ 1.2_1\ 1.3_3)$$
$$(3.1_3\ 2.1_1\ 1.2_1) \quad \times \quad (2.1_1\ 1.2_1\ 1.3_3)$$
$$(3.1_3\ 2.1_1\ 1.3_3) \quad \times \quad (3.1_3\ 1.2_1\ 1.3_3)$$
$$(3.1_3\ 2.2_{1,2}\ 1.2_1) \quad \times \quad (2.1_1\ 2.2_{2,1}\ 1.3_3)$$
$$(3.1_3\ 2.2_{1,2}\ 1.3_3) \quad \times \quad (3.1_3\ 2.2_{2,1}\ 1.3_3)$$
$$(3.1_3\ 2.3_2\ 1.3_3) \quad \times \quad (3.1_3\ 3.2_2\ 1.3_3)$$
$$(3.2_2\ 2.2_{1,2}\ 1.2_1) \quad \times \quad (2.1_1\ 2.2_{2,1}\ 2.3_2)$$
$$(3.2_2\ 2.2_{1,2}\ 1.3_3) \quad \times \quad (3.1_3\ 2.2_{2,1}\ 2.3_2)$$
$$(3.2_2\ 2.3_2\ 1.3_3) \quad \times \quad (3.1_3\ 3.2_2\ 2.3_2)$$
$$(3.3_{2,3}\ 2.3_2\ 1.3_3) \quad \times \quad (3.1_3\ 3.2_2\ 3.3_{3,2})$$

2. Now we will have a look at the qualitative numbers of the first 3 contextures. As it is known, qualitative numbers exist in three number areas (the possible term "number field" does not hold for qualitative numbers, which are not based on identity logic). They are called proto-, deutero- and trito-numbers. Therefore, the interface between the three number areas and thee 3 contextures are:

Proto	Deutero	Trito	
0	0	0	C1
00 01	00 01	00 01	C2
000 001 012	000 001 012	000 001 010 011 012	C3

While in C = 1 and C = 2 there is no difference between proto-, deutero- and trito-numbers, in C = 3, proto- and deutero-numbers are split up into 5 trito-numbers. This means: A sign class which lies in 1 contexture, e.g.

$(3.a\ 2.b\ 1.c)$

is trivially the same in all three qualitative number areas. A sign class which lies in 2 contextures, e.g. the complex sign classes introduced in Toth (2007, pp. 52 ss.), e.g.

$(\pm 3.\pm a\ \ \pm 2.\pm b\ \ \pm 1.\pm c)$

is also the same in all three qualitative number areas, since this is the field of Aristotelian logic and complex number theory based on it.

However, a sign class which lies in 3 contextures, e.g.

$(3.1_3\ 2.2_{1,2}\ 1.2_1) \equiv (3.1_{[[000],[001],[010],[011],[012]}\ 2.2_{<[0],[00],[01]>}\ 1.2_{[o]})$

is "eineindeutig-mehrmöglich" (one-to-one pluri-valent) and corresponds exactly with the multi-ordinality of A. Korzybski" (Kronthaler 1986, p. 60). This means, from the correspondence between sign, resp. sub-sign and qualitative number, we have for $(3.1_3\ 2.2_{1,2}\ 1.2_1)$:

$(3.1_3) = 3.1_{[[000]}; 3.1_{[001]}; 3.1_{[010]}; 3.1_{[011]}; 3.1_{[012)}$
$(2.2_{1,2}) = 2.2_{[00),}\ 2.2_{01]}$
$(1.2_1) = 1.2_{[0]}$

However, if we would write

[[000],[001],[010],[011],[012], <[0],[00],[01]>, [o])

instead of

$(3.1_3\ 2.2_{1,2}\ 1.2_1)$,

the notation of the sign-class with trito-numbers could mean the following sign relations in 3 contextures:

(3.1 2.2 1.2), (3.1 2.2 2.1), (1.3 2.2 1.2), (1.3 2.2 2.1),

since for converse sub-sign-relations we have

$C((a.b)) = C\ ((a.b)°)$.

Also note that for all sub-signs with 2 or more indices, the respective qualitative numbers are members of ordered sets:

[[000],[001],[010],[011],[012], **<[0],[00],[01]>**, [o]),

while the sets of qualitative numbers per contexture are non-ordered:

[[000],[001],[010],[011],[012]] =
[[012], [011], [010], [001], [000]] =
[[011], [010], [012], [000], [001]] = ...

Therefore, the abolishment of eigenreality in polycontextural semiotics which can be numerically shown by the disequation

$(3.1_3\ 2.2_{1,2}\ 1.3_3) \neq \times(3.1_3\ 2.2_{1,2}\ 1.3_3) = (3.1_3\ 2.2_{2,1}\ 1.3_3)$ with $(2.2_{1,2}) \neq (2.2_{2,1})$

shows that the dissolution of identity which abolishes eigenreality, is nothing else than the total-reflection of the ordered set of n-contextural indices for any n; cf. for n = 4:

$(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) \neq \times(3.1_{3,4}\ 2.2_{1,2,4}\ 1.3_{3,4}) = (3.1_{3,4}\ 2.2_{4,2,1}\ 1.3_{3,4})$ with $(2.2_{1,2,4}) \neq (2.2_{4,2,1})$.

However, we also see that with increasing n:

n = 2: $(2.2_{1,2}) \neq (2.2_{2,1})$.
n = 3: with $(2.2_{1,2,4}) \neq (2.2_{4,2,1}) \neq (2.2_{4,1,2}) \neq (2.2_{2,1,4}) \neq (2.2_{2,4,1}) \neq (2.2_{4,1,2})$,

the qualitative-mathematical distance between identity-based monocontextural sign relations and identity-abolished polycontextural sign relations grows, and it grows exactly with $n! = 1 \cdot 2 \cdot 3 \cdot ... \cdot n$, so that we can measure the abyss or contextural border between an identity-based relation and a non-identity-based relation as

$CB = (n! - 2)$.

Thus, CB gives us the number of permutations between a morphism and its heteromorphism. In the case of n =4, i.e. in a 5-contextural semiotic systems, we have $(4! - 2) = 22$, i.e.

$$\underbrace{(a.b)_{i,j,k,l} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a.b)_{l,k,j,i}}_{}$$

(n! –2) = 22 permutations = contextural border (CB)

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